Supersymmetry breaking as the origin of flavor

Javier Ferrandis*
Department of Physics and Astronomy, University of Hawaii at Manoa, 2505 Correa Road, Honolulu, Hawaii 96822, USA

Naoyuki Haba†
Institute of Theoretical Physics, University of Tokushima, Tokushima 770-8502, Japan
(Received 17 April 2004; published 16 September 2004)

We present an effective flavor model for the radiative generation of fermion masses and mixings based on a $SU(5)_V \times U(2)_H$ symmetry. We assume that the original source of flavor breaking resides in the supersymmetry-breaking sector. Flavor violation is transmitted radiatively to the fermion Yukawa couplings at low energy through finite supersymmetric threshold corrections. This model can fit the fermion mass ratios and Cabibbo-Kobayashi-Maskawa matrix elements, explain the nonobservation of proton decay, and overcome the present constraints on flavor changing processes through an approximate radiative alignment between the Yukawa and the soft trilinear sector. The model predicts relations between dimensionless fermion mass ratios in the three fermion sectors, and the quark mixing angles, $|V_{us}| \approx [m_d/m_s]^{1/2} \approx [m_u/m_c]^{1/4} \approx 3[m_e/m_\mu]^{1/2}$ and $\frac{1}{2}|V_{cb}/V_{us}| \approx [m_s^3/m_b^2 m_d]^{1/2} \approx [m_c^3/m_t^2 m_u]^{1/2} \approx \frac{1}{9}[m_\mu^3/m_\tau^2 m_e]^{1/2}$, which are confirmed by the experimental measurements.

DOI: 10.1103/PhysRevD.70.055003 PACS numbers: 12.60.Jv, 11.30.Hv

I. INTRODUCTION

It is commonly assumed that the flavor mixing in the supersymmetric limit of the minimal supersymmetric standard model (MSSM) is the same as in the standard model. Accordingly most of the supersymmetric (SUSY) models of flavor proposed to date have tried to explain the fermion mass hierarchies by breaking flavor symmetries in the superpotential. This explanation, however, comes easily into conflict with the present experimental constraints on flavor changing processes. To solve this problem, it has been proposed that there is flavor conservation in the supersymmetry-breaking sector (universality) or alternatively that the flavor violation in the supersymmetry-breaking sector is aligned with the flavor violation in the Yukawa sector to a high degree (alignment). Although some flavor models have been proposed that predict universality or alignment, such conditions are usually satisfied at a high energy scale and are spoiled through renormalization group effects. Flavor, when it originates in the superpotential, is transmitted to the soft supersymmetry-breaking sector through the renormalization group running from the unification scale down to the electroweak scale, easily overcoming the present constraints on flavor changing processes. There is also a third possibility, the so-called decoupling scenario, that assumes that the masses of the first and second generation sfermions are heavy enough to suppress all flavor-violating processes below the present limits.

The presence of flavor violation in the superpotential, in the particular case of supersymmetric grand unified models, causes another problem: the existence of dimension-five operators that accelerate proton decay, thereby ruling out minimal SUSY SU(5) models.

There is an alternative possibility, which has not received much attention until recently [1]. The lighter fermion masses may be a higher order radiative effect as suggested by the observed fermion mass hierarchies. This is not a new idea; following the suggestion by Weinberg [2,3] of a mechanism to generate radiatively the electron mass from a tree-level muon mass several proposals were published. The program, however, was considered more difficult to implement in the context of supersymmetric models since, as pointed out by Ibañez, if supersymmetry is spontaneously broken only tiny fermion masses could be generated radiatively [4]. On the other hand, the presence of soft supersymmetry-breaking terms allows for the radiative generation of quark and charged lepton masses through sfermion-gaugino loops. The gaugino mass provides the violation of fermionic chirality required by a fermion mass, while the soft breaking terms provide the violation of chiral flavor symmetry. This idea was suggested in 1983 by Buchmuller and Wyler [5] and was later rediscovered in Refs. [6–9]. Additional implications of this possibility were subsequently studied in Refs. [10–13], but no complete flavor model implementing a radiative generation of fermion masses has been proposed to date.

In this paper we present a supersymmetric model, based on a U(2) horizontal flavor symmetry, that generates fermion masses radiatively. In the context of the MSSM, the possibility that the quark mixing and the fermion masses for the first two generations can actually be generated radiatively has been recently pointed out by one of the authors [1]; in this paper we continue that investigation. In the U(2)-flavor model presented here flavor breaking originates in the supersymmetry-

*Electronic address: ferrandis@mac.com
†Electronic address: haba@ias.tokushima-u.ac.jp

breaking sector and is transmitted radiatively to the fermion sector at low energy as mentioned above. It is the main point of this paper to show that supersymmetry-breaking models of flavor exist that not only can fit the fermion mass ratios and the quark mixing angles, but also offer an alternative solution to the SUSY flavor and proton decay problems. The basic conditions that we expect from a unified supersymmetric theory that generates fermion masses radiatively are

(1) A symmetry or symmetries of the superpotential guarantee flavor conservation and precludes tree-level masses for the first and second generations of fermions in the supersymmetric limit.
(2) The supersymmetry-breaking terms receive small corrections, which violate the symmetry of the superpotential and are responsible for the observed flavor physics.

In this case the Yukawa matrices provided as a boundary condition for the MSSM at some high energy scale are of the form,

$$\mathbf{Y}_{D,U,L} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_{b,t,\tau} \end{bmatrix}, \qquad (1)$$

where $\mathbf{Y}_{D,U,L}$ are the 3×3 quark and lepton Yukawa matrices. We will assume that supersymmetry and flavor breaking are linked in such a way that after supersymmetry breaking, nontrivial flavor mixing textures are generated in the supersymmetry-breaking sector. It is known that in an effective field theory format holomorphic trilinear soft supersymmetry-breaking terms can originate, below the supersymmetry-breaking messenger scale M, in operators that couple to the supersymmetry-breaking chiral superfields, $\mathcal{Z}$. These operators are generically of the form,

$$\int d^2\theta \left(\frac{\mathcal{Z}}{M}\right) H_\alpha \phi_L \phi_R + \text{c.c.}, \qquad (2)$$

where $\langle \mathcal{Z} \rangle = \langle \mathcal{Z}_s \rangle + \langle \mathcal{Z}_a \rangle \theta^2$. In general the vacuum expectation value (vev) of the auxiliary component $\langle \mathcal{Z}_a \rangle$ parametrizes the scale of supersymmetry breaking, M_S^2. Flavor violation may arise only in the soft terms, for instance, if supersymmetry-breaking superfields, $\mathcal{Z}$, transform nontrivially under flavor symmetries while the vev of the scalar component of $\mathcal{Z}$ vanishes, $\langle \mathcal{Z}_s \rangle = 0$, or is much smaller than the messenger scale. In this case we expect the 3×3 trilinear soft supersymmetry-breaking matrices to look like

$$\mathbf{A}_{D,U,L} = A\mathcal{O}(\lambda), \qquad (3)$$

where $\mathcal{O}(\lambda)$ represents generically some dimensionless flavor-violating polynomial matrix that can be expanded in powers λ, and a flavor-breaking perturbation parameter, with $\lambda < 1$. Flavor violation is transmitted to the fermion sector, i.e., to the Yukawa couplings, through sfermion-gaugino loops. We require the magnitude of λ, which must be determined *a posteriori* by the ratios between measured fermion masses, to be consistent with present constraints on supersymmetric contributions to flavor changing processes. Finally, we note that only finite corrections can generate off-diagonal entries in the Yukawa matrices, since the structure of the Yukawa matrices given by Eq. (1) is renormalization scale independent; in other words the renormalization group running from the unification scale down to the SUSY spectra decoupling scale cannot generate off-diagonal Yukawa couplings. The opposite, however, is not true, i.e., flavor violation in the Yukawa matrices would transmit to the soft supersymmetry-breaking sector through renormalization group running. This constitutes a common problem of all the theories that locate the origin of flavor breaking in the superpotential.

II. A $U(1)_H$ TOY MODEL

We will start with a two-generation toy model that contains the necessary ingredients to radiatively generate fermion masses and mixings. The model is based on a $U(1)_H$ horizontal flavor symmetry. The particle content of the model is summarized in Table I. It is possible to choose the charge assignments in such a way that only one generation is allowed to have a tree-level Yukawa coupling in the superpotential; see Table I. In this case, the tree-level 2×2 Yukawa matrix is

$$\mathbf{Y} = \begin{bmatrix} 0 & 0 \\ 0 & y \end{bmatrix}, \qquad (4)$$

where all except the (2,2) entry are disallowed by the $U(1)_H$ symmetry. We will assume that there is one supersymmetry-breaking chiral superfield, $\mathcal{Z}$, carrying $U(1)_H$ flavor charge. The field $\mathcal{Z}$ is a spurion whose sole role is to communicate flavor as well as supersymmetry breaking to the matter fields. We will assume a zero vev for the scalar component of $\mathcal{Z}$ but nonzero for its auxiliary component. This condition can be achieved through an O'Raifeartaigh-type model superpotential for the $\mathcal{Z}$ superfield [14]. Trilinear soft supersymmetry-breaking terms are generated by operators generically of the form,

$$\int d^2\theta \left(\frac{\mathcal{Z}}{M_F}\right) f \mathcal{H} \mathcal{L}_a \mathcal{R}_b + \text{c.c.}, \qquad (5)$$

TABLE I. Particle content of a two-generation $U(1)_H$ flavor toy model with radiative fermion mass generation. $\mathcal{G}$ and $\mathcal{F}$ are supersymmetry-breaking superfields, $\mathcal{L}_a$ and $\mathcal{R}_a$ ($a = 1, 2$) are matter superfields, $\mathcal{H}$ is the Higgs superfield, and $\tilde{g}$ is the gaugino superfield.

Fields	$\mathcal{G}$	$\mathcal{Z}$	$\mathcal{L}_1$	$\mathcal{R}_1$	$\mathcal{L}_2$	$\mathcal{R}_2$	$\mathcal{H}$	$\tilde{g}$
$U(1)_H$	0	−1	1	1	0	0	0	0

where $M_{\rm F}$ is the flavor-breaking scale, $\mathcal{H}$ is the Higgs superfield, and f is a dimensionless flavor blind coupling determined by the underlying theory (f is flavor blind because our basic assumption is that the underlying exactly supersymmetric model that generates these nonrenormalizable operators is flavor conserving). We also need a flavor-singlet supersymmetry-breaking superfield, $\mathcal{G}$, to generate a mass for the gaugino, $m_{\tilde{g}}$, from operators of the form,

$$\int d^2\theta\left(\frac{\mathcal{G}}{M}\right)\tilde{g}\,\tilde{g} + \text{c.c.}, \tag{6}$$

where M is the messenger scale. We obtain $m_{\tilde{g}} = \langle\mathcal{G}\rangle/M$. We notice that, if no symmetry inhibits it, $\mathcal{G}$ could generate an additional contribution to the trilinear soft supersymmetry-breaking terms introduced by the operator $\mathcal{G}\mathcal{H}\mathcal{L}_2\mathcal{R}_2$. Assuming the charge assignments in Table I, we obtain from Eq. (5) for the soft trilinear matrix,

$$\mathbf{A} = A\begin{bmatrix} 0 & \lambda \\ \lambda & 1 \end{bmatrix}, \tag{7}$$

where $A = f\langle\mathcal{G}\rangle/M$ and $\lambda = \langle\mathcal{Z}\rangle/(M_{\rm F} m_{\tilde{g}})$. We will assume that $\lambda \lesssim \mathcal{O}(1)$. The parameter λ determines the magnitude of flavor violation. Additionally, soft mass matrices are generated from operators generically of the form,

$$\int d^4\theta\left[\frac{\mathcal{Z}\mathcal{Z}^\dagger}{M_{\rm F}^2} + \frac{\mathcal{G}\mathcal{G}^\dagger}{M^2} + \frac{\rho}{MM_{\rm F}}(\mathcal{G}\mathcal{Z}^\dagger + \mathcal{Z}\mathcal{G}^\dagger)\right]k^2\phi^\dagger\phi, \tag{8}$$

where k^2 is a dimensionless coefficient determined by the underlying theory. For the case study in Table I, we obtain the 2×2 soft mass matrices,

$$\tilde{\mathbf{M}}^2_{\mathcal{L}} = \tilde{\mathbf{M}}^2_{\mathcal{R}} = \tilde{m}^2\begin{bmatrix} 1 & \rho\lambda \\ \rho\lambda & 1 \end{bmatrix}, \tag{9}$$

where $\tilde{m} = k\sqrt{1+\lambda^2}\langle\mathcal{G}\rangle/M$. We note that if we did not allow for mixing between flavor-violating and flavor conserving supersymmetry-breaking fields, i.e., $\rho = 0$, there would be no flavor mixing in the soft mass matrices. In the presence of flavor violation in the soft sector, the left- and right-handed components of the sfermions mix. For instance, in the gauge basis the 4×4 sfermion mass matrix is given by

$$\mathcal{M}^2 = \begin{bmatrix} \tilde{\mathbf{M}}^2_{\mathcal{L}} + v^2\mathbf{Y}^\dagger\mathbf{Y} & \mathbf{A}^\dagger v \\ \mathbf{A}v & \tilde{\mathbf{M}}^2_{\mathcal{R}} + v^2\mathbf{Y}\mathbf{Y}^\dagger \end{bmatrix}, \tag{10}$$

where $v = \langle\mathcal{H}\rangle$, $\tilde{\mathbf{M}}^2_{\mathcal{L}}$, and $\tilde{\mathbf{M}}^2_{\mathcal{R}}$ are the 2×2 right-handed and left-handed soft mass matrices given above (including D terms), $\mathbf{A}$ is the 2×2 soft trilinear matrix, and $\mathbf{Y}$ is the 2×2 tree-level Yukawa matrix. $\mathcal{M}^2$ is diagonalized by a 4×4 unitary matrix, $\mathcal{D}$. In general, the dominant finite 1-loop contribution to the 2×2 Yukawa matrix is given by the gaugino-sfermion loop,

$$(\mathbf{Y})^{\rm rad}_{ab} = \frac{\alpha}{\pi} m_{\tilde{g}}\sum_c \mathcal{D}_{ac}\mathcal{D}^*_{(b+2)c}B_0(m_{\tilde{g}}, m_{\tilde{f}_c}), \tag{11}$$

where $\tilde{f}_c$ ($c = 1,\ldots,4$) are sfermion mass eigenstates, $m_{\tilde{g}}$ is the gaugino mass, and α is the gauge coupling of the theory. B_0 is a known function defined in the Appendix. We observe that the contributions from left- and right-handed sfermion mixings involving the soft masses are much smaller. For simplicity we will assume from now on that there are no CP-violating phases in the soft parameters. Moreover, as a consequence of the approximate sfermion mass degeneracy predicted by the model when $\lambda \ll 1$, the dominant contribution to the radiatively generated Yukawa couplings can be simply expressed as

$$\mathbf{Y}^{\rm rad} = \frac{2\alpha}{\pi} m_{\tilde{g}}\mathbf{A}F(\tilde{m}_f, \tilde{m}_f, m_{\tilde{g}}), \tag{12}$$

where the function F is given in the Appendix. We then obtain a simple expression for the radiatively corrected mass matrix,

$$\mathbf{m} = v(\mathbf{Y} + \mathbf{Y}^{\rm rad}) = m\begin{bmatrix} 0 & \gamma\lambda \\ \gamma\lambda & 1 \end{bmatrix}, \tag{13}$$

where m and γ are given by

$$m = yv(1+\zeta), \qquad \gamma = \frac{\zeta}{1+\zeta}, \tag{14}$$

and

$$\zeta = \frac{2\alpha}{\pi y} m_{\tilde{g}} A F(\tilde{m}, \tilde{m}, m_{\tilde{g}}). \tag{15}$$

The loop factor γ encodes the dependence on the SUSY spectra and parametrizes the breaking of the alignment between the soft trilinear and the Yukawa sectors caused by the presence of the tree-level mass m. Although not diagonal in the gauge basis, the matrix $\mathbf{m}$ can be brought to diagonal form in the mass basis by a unitary diagonalization, $\mathcal{V}^\dagger\mathbf{m}\mathcal{V} = (m_1, m_2)$. Therefore the 1-loop mass matrix of our toy model makes the following prediction for the mass ratio between the radiatively generated mass, m_1, and the tree-level one:

$$\frac{m_1}{m_2} = \gamma^2\lambda^2. \tag{16}$$

The flavor mixing is given by a Cabibbo-Kobayashi-Maskawa (CKM)-like mixing matrix,

$$\mathcal{V} = \begin{bmatrix} 1 - \frac{1}{2}\gamma^2\lambda^2 & \gamma\lambda \\ -\gamma\lambda & 1 - \frac{1}{2}\gamma^2\lambda^2 \end{bmatrix}. \tag{17}$$

We note that the mass ratio and the flavor mixing are determined by two basic parameters of the model, the flavor-breaking parameter λ and the loop suppression factor γ. Moreover, the mass ratio is directly correlated with the flavor mixing angle, $\mathcal{V}_{12}$,

$$\frac{m_1}{m_2} = \gamma^2\lambda^2 = \mathcal{V}_{12}^2. \tag{18}$$

If we could determine experimentally the mixing angle we could predict m_1 or vice versa. This toy model illustrates the mechanism that will be used in a realistic model in the next section.

III. U(2)$_H$ FLAVOR SYMMETRY

In this section we will consider a realistic three generation model based in a horizontal U(2)$_H$ symmetry [15]. We will assume the usual MSSM particle content where third generation matter superfields,

$$\mathcal{Q}_3, \mathcal{D}_3, \mathcal{U}_3, \mathcal{L}_3, \mathcal{E}_3, \tag{19}$$

and up and down electroweak Higgs superfields, $\mathcal{H}_u$ and $\mathcal{H}_d$, are singlets under U(2)$_H$. We will denote them abbreviately by $\phi^{L,R}$. Let us assume that first and second generation left- and right-handed superfields transform as contravariant vectors under U(2)$_H$,

$$\Psi_\mathcal{Q} = \begin{pmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \end{pmatrix}, \qquad \Psi_\mathcal{U} = \begin{pmatrix} \mathcal{U}_1 \\ \mathcal{U}_2 \end{pmatrix}, \qquad \Psi_\mathcal{D} = \begin{pmatrix} \mathcal{D}_1 \\ \mathcal{D}_2 \end{pmatrix}, \tag{20}$$

$$\Psi_\mathcal{L} = \begin{pmatrix} \mathcal{L}_1 \\ \mathcal{L}2 \end{pmatrix}, \qquad \Psi_\mathcal{E} = \begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{pmatrix}. \tag{21}$$

We will denote them abbreviately by $\Psi_a^{L,R}$. We will introduce a set of supersymmetry-breaking chiral superfields,

$$S^{ab}, \mathcal{A}^{ab}, \mathcal{F}^a \qquad (a, b = 1, 2), \tag{22}$$

that transform covariantly as a symmetric tensor, antisymmetric tensor, and vector under U(2)$_H$ [with a U(1) charge opposite to that of the matter doublets]. We will assume that at the minimum only the auxiliary components of the flavor-breaking superfields are nonzero, The most general form for the vevs of the flavor-breaking fields is

$$\langle S\rangle = \begin{pmatrix} v_S & 0 \\ 0 & \mathcal{V}_S \end{pmatrix}\theta^2, \tag{23}$$

$$\langle \mathcal{A}\rangle = \begin{pmatrix} 0 & \mathcal{V}_\mathcal{A} \\ -\mathcal{V}_\mathcal{A} & 0 \end{pmatrix}\theta^2, \tag{24}$$

$$\langle \mathcal{F}\rangle = \begin{pmatrix} v_\mathcal{F} \\ \mathcal{V}_\mathcal{F} \end{pmatrix}\theta^2. \tag{25}$$

We will assume the following particular hierarchy in the flavor-breaking vevs: $v_S \ll \mathcal{V}_S$ and for practical purposes $v_S = 0$. We also assume that $\mathcal{V}_S = \mathcal{V}_\mathcal{F}$ and

$$(v_\mathcal{F}, \mathcal{V}_\mathcal{A}, \mathcal{V}_\mathcal{F}) = (\lambda^2, \lambda^2, \lambda)M_\mathrm{F}\tilde{m}. \tag{26}$$

Here λ is the flavor-breaking perturbation parameter, M_F is the flavor-breaking scale, and $\tilde{m}$ is a new mass scale linked to the flavor-breaking fields that will be important to determine the size of the flavor-breaking effects. We do not have yet a predictive model for the U(2) breaking. This is a relevant point which is under current investigation. In the case a global U(2) symmetry is broken spontaneously we expect the U(2)-gauge fields to get masses of the order of the flavor-breaking scale which can be very heavy in this scenario. Therefore any other phenomenological effects in the low energy model beyond the flavor structure it gives rise to would be very suppressed. These *ad hoc* assumptions will prove *a posteriori* to be very successful in reproducing fermion masses. The only couplings allowed in the superpotential by the U(2)$_H$ horizontal symmetry and the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ vertical symmetry are the third generation ones and the so-called μ term,

$$\lambda_t\mathcal{Q}_3\mathcal{U}_3\mathcal{H}_u + \lambda_b\mathcal{Q}_3\mathcal{D}_3\mathcal{H}_d + \lambda_\tau\mathcal{L}_3\mathcal{E}_3\mathcal{H}_d + \mu\mathcal{H}_u\mathcal{H}_d. \tag{27}$$

We note that, in principle, two other couplings could be allowed in the superpotential: $\mathcal{L}_3\mathcal{H}_u$ and $\mathcal{Q}_3\mathcal{L}_3\mathcal{D}_3$. There are different ways to remove these unwanted couplings. They could be forbidden imposing total fermion number conservation. Alternatively one could impose R-parity conservation defined as $R = (-)^{3B+L+2S}$, where B is the baryonic number, L the leptonic number, and S the spin. A third possibility would be to extend the U(2)$_H$ symmetry to the maximal U(3)$_H$ horizontal symmetry. The breaking of the U(3)$_H$ symmetry in the direction of the third generations would leave us with our U(2)$_H$ symmetry; in such a case this bilinear interaction would not be a U(3)$_H$ singlet. Therefore, at tree level Yukawa matrices are generically of the form,

$$\mathbf{Y} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y \end{bmatrix}. \tag{28}$$

First we need to introduce a flavor-singlet chiral superfield, $\mathcal{G}$, to give masses to gauginos from operators of the form,

$$\int d^2\theta\left(\frac{\mathcal{G}}{M}\right)\tilde{g}\,\tilde{g} + \text{c.c.}, \tag{29}$$

where M is the messenger scale. The gaugino mass generated is given by $m_{\tilde{g}} = \langle\mathcal{G}\rangle/M$. Additionally, trilinear soft supersymmetry-breaking terms are generated by operators generically of the form,

$$\sum_{Z=S,\mathcal{A}}\frac{1}{M_\mathrm{F}}\int d^2\theta Z^{ab}\Psi_a^L\Psi_b^R\mathcal{H}_\alpha + \text{c.c.}, \tag{30}$$

$$\frac{1}{M_\mathrm{F}}\int d^2\theta(\phi^R\mathcal{F}^a\Psi_a^L + \phi^L\mathcal{F}^a\Psi_a^R)\mathcal{H}_\alpha + \text{c.c.}, \tag{31}$$

where M_F is the flavor-breaking scale, $a = 1, 2$ and $\mathcal{H}_\alpha$, $\alpha = u, d$ represents any of the Higgs superfields, $\phi^{L,R}$ stands generically for any of the right- or left-handed flavor-singlet matter superfields, and $\Psi^{L,R}$ stands generically for any of the right- or left-handed flavor-vector matter superfields. The flavor-singlet superfield responsible for generating gaugino masses, $\mathcal{G}$, will couple to matter fields. Additionally, to add more generality to our analysis, we will assume that there could be another flavor singlet, $\mathcal{J}$, which couples to the matter superfields but does not couple to the gaugino superfields. In the most general case there could be two additional operators generating soft trilinears,

$$\frac{1}{M}\int d^2\theta(\kappa\mathcal{G} + \eta\mathcal{J})\phi^L\phi^R\mathcal{H}_\alpha + \text{c.c.}, \tag{32}$$

where κ and η are dimensionless couplings determined by the underlying theory. We define the soft breaking mass generated for the $\mathcal{J}$ field as $m_{\tilde{J}} = \langle \mathcal{J}\rangle/M$. Soft supersymmetry-breaking mass matrices are generated by operators generically of the form,

$$\frac{1}{M_F^2}\int d^4\theta[\mathcal{Z}^\dagger_{ac}\mathcal{Z}^{cb}(\Psi^\dagger)^a\Psi_b + \mathcal{F}^\dagger_a\mathcal{F}^b(\Psi^\dagger)^a\Psi_b] + \frac{1}{M^2}\int d^4\theta(\kappa'^2\mathcal{G}^\dagger\mathcal{G} + \eta'^2\mathcal{J}^\dagger\mathcal{J})[(\Psi^\dagger)^a\Psi_a + \phi^\dagger\phi] + \frac{1}{MM_F} \times\int d^4\theta\rho[(\kappa'\mathcal{G}^\dagger + \eta'\mathcal{J}^\dagger)\mathcal{F}^b\phi^\dagger\Psi_b + \text{H.c.}], \tag{33}$$

where $\mathcal{Z} = \mathcal{S}, \mathcal{A}$. When including the last term in the previous equation we assume that the underlying theory allows the flavor-breaking fields to couple with the flavor singlets $\mathcal{G}$ and $\mathcal{J}$, if this were not possible $\rho = 0$. Regarding the possible appearance of D terms in the scalar potential, they appear when a local symmetry is spontaneously broken by scalar fields. In our case the flavor-breaking fields are F terms and these cannot generate flavor-breaking D terms of the usual kind. After the $\mathrm{U}(2)_H$ flavor breaking the following soft trilinear matrices are generated:

$$\mathbf{A} = A\begin{bmatrix} 0 & \sigma\lambda^2 & \sigma\lambda^2 \\ -\sigma\lambda^2 & \sigma\lambda & \sigma\lambda \\ \sigma\lambda^2 & \sigma\lambda & 1 \end{bmatrix}, \tag{34}$$

where

$$A = (\kappa m_{\tilde{g}} + \eta m_{\tilde{J}}) \tag{35}$$

and the dimensionless parameter σ is defined by

$$\sigma = \frac{\tilde{m}}{A}, \tag{36}$$

where $\tilde{m}$ was defined in Eq. (26). After $\mathrm{U}(2)_H$ flavor breaking the following soft mass matrices are also generated:

$$\tilde{\mathbf{M}}^2_{L,R} = \tilde{m}_f^2\begin{bmatrix} 1 + 2\lambda^4\sigma'^2 & \lambda^3\sigma'^2 & \rho\lambda^2\sigma' \\ \lambda^3\sigma'^2 & 1 + 2\lambda^2\sigma'^2 & \rho\lambda\sigma' \\ \rho\lambda^2\sigma' & \rho\lambda\sigma' & 1 \end{bmatrix}, \tag{37}$$

where

$$\tilde{m}_f^2 = (\kappa'^2 m_{\tilde{g}}^2 + \eta'^2 m_{\tilde{J}}^2) \tag{38}$$

and the dimensionless parameter σ' is defined by

$$\sigma' = \frac{\tilde{m}}{\tilde{m}_f}. \tag{39}$$

We note that in this scenario the amount of flavor violation as well as the nondegeneracy predicted in the soft mass matrices is determined by the ratio $A/\tilde{m}_f$. We note that the presence of mixing between flavor-breaking and flavor-singlet SUSY breaking fields generates flavor-violating soft mass matrices in the entries (13) and (23). We will see later that $\lambda^2 \approx 0.05$ is approximately the Cabbibo angle squared. The sfermion nondegeneracy between first and second generations appears to order $\sigma'^2\lambda^2$.

There are three interesting limits. In the first limit we assume that there is no extra flavor singlet, $\mathcal{J}$, i.e., $\eta = \eta' = 0$, and to simplify we assume $\kappa = \kappa' = 1$. We then obtain

$$A = \tilde{m}_f = m_{\tilde{g}}, \tag{40}$$

$$\sigma = \sqrt{\sigma'} = \tilde{m}/m_{\tilde{g}}. \tag{41}$$

$\tilde{m}$ was defined in Eq. (26). In this case all the supersymmetric spectra are correlated with the gaugino mass. In the second limit the flavor-singlet superfield $\mathcal{G}$ does not couple to the matter fields, i.e., $\kappa = \kappa' = 0$, and to simplify we assume $\eta = \eta' = 1$. We then obtain

$$\tilde{m}_g \neq m_{\tilde{J}}, \tag{42}$$

$$A = \tilde{m}_f = m_{\tilde{J}}, \tag{43}$$

$$\sigma = \sqrt{\sigma'} = \tilde{m}/m_{\tilde{J}}, \tag{44}$$

where $m_{\tilde{J}} = \langle\mathcal{J}\rangle/M$ is in general different from the gaugino mass. The third interesting limit we will consider is especially relevant from a phenomenological point of view. If we assume that

$$A < \tilde{m} \ll m_{\tilde{f}}, \tag{45}$$

then $\sigma' \ll 1$ and $\sigma > 1$. This case would suppress the contributions from soft masses to flavor-violating processes while increasing the soft trilinear contributions to the radiatively generated Yukawa couplings.

A. The down-type quark sector

In general, 1-loop gluino-squark exchange generates a dominant finite contribution to the 3×3 quark Yukawa mass matrices given by

$$(\mathbf{Y})^{\rm rad}_{ab} = \frac{\alpha_s}{3\pi} m_{\tilde{g}} \sum_c \mathcal{D}_{ac} \mathcal{D}^*_{(b+3)c} B_0(m_{\tilde{g}}, m_{\tilde{d}_c}), \tag{46}$$

where $\tilde{d}_c$ $(c = 1, \ldots, 6)$ are squark mass eigenstates, α_s is the strong coupling constant, $\mathcal{D}$ is a 6×6 down/up-type squark diagonalization matrix, and $m_{\tilde{g}}$ is the gluino mass. The function B_0 is defined in the Appendix. For example, the radiatively generated 3×3 down-type quark mass matrix is generically of the form,

$$\mathbf{m}_D = \langle \mathcal{H}_d \rangle (\mathbf{Y} + \mathbf{Y}^{\rm rad}) = \hat{m}_b \begin{bmatrix} 0 & \gamma_b \lambda^2 & \gamma_b \lambda^2 \\ -\gamma_b \lambda^2 & \gamma_b \lambda & \gamma_b \lambda \\ \gamma_b \lambda^2 & \gamma_b \lambda & 1 \end{bmatrix}, \tag{47}$$

where $\hat{m}_b$ is to first order the running bottom mass,

$$\hat{m}_b = y_b v c_\beta [1 + \zeta_d (1 - y_b \mu t_\beta / A)], \tag{48}$$

$$\zeta_d = \frac{2\alpha_s}{3\pi y_b} m_{\tilde{g}} A F(\tilde{m}_f, \tilde{m}_f, m_{\tilde{g}}). \tag{49}$$

Here $t_\beta = \tan\beta = v_u / v_d$ is the ratio between the vevs of the two MSSM Higgs ($v_u = \langle \mathcal{H}_u \rangle$, $v_d = \langle \mathcal{H}_d \rangle$), μ is the so-called μ term, introduced in Eq. (27), and $v = s_W m_W / \sqrt{2\pi\alpha_e} = 174.5$ GeV (s_W is the weak mixing angle, m_W is the SM W-boson mass, and α_e is the electromagnetic coupling constant). γ_b is a loop factor that has a simple expression in the mass degenerate sfermion case. For example, in the down-type quark sector is given by

$$\gamma_b = \frac{\zeta_d \sigma}{[1 + \zeta_d (1 - y_b \mu t_\beta / A)]}. \tag{50}$$

Here σ is the coefficient introduced in Eq. (34); if there is no extra flavor singlet, $\mathcal{J}$, and $\kappa = 1$, σ would be defined as $\tilde{m}/m_{\tilde{g}}$. γ_b encodes the dependence on the SUSY spectra and parametrizes the breaking of the alignment between soft trilinear and Yukawa sectors caused by the tree-level component to the bottom mass. We observe that there is a special limit of the previous formulas, $y_b \to 0$, where the bottom quark mass could also be generated radiatively. We will not consider that case; we assume that all third generation fermions get a tree-level mass.

The phenomenological implications of a mass matrix of the form given in Eq. (47) have been studied by one of the authors in Ref. [1]; here we reproduce some of the results. Although not diagonal in the gauge basis the matrix $\mathbf{m}_D$ can be brought to diagonal form in the mass basis by a biunitary diagonalization, $(\mathcal{V}^d_L)^\dagger \mathbf{m}_D \mathcal{V}^d_R = (m_d, m_s, m_b)$. The down-type quark mass matrix given by Eq. (47) makes the following predictions for the quark mass ratios:

$$\frac{m_d}{m_s} = \lambda^2 (1 + \gamma_b \lambda - 2\lambda^2) + \mathcal{O}(\lambda^6), \tag{51}$$

$$\frac{m_s}{m_b} = \gamma_b \lambda (1 - \gamma_b \lambda + \lambda^2) + \mathcal{O}(\lambda^4). \tag{52}$$

We can express λ and γ_b as a function of the renormalization scheme and approximately scale independent dimensionless quark mass ratios, to first order,

$$\lambda = \left(\frac{m_d}{m_s}\right)^{1/2}, \qquad \gamma_b = \left(\frac{m_s^3}{m_b^2 m_d}\right)^{1/2}. \tag{53}$$

Using the invariant running quark mass ratios determined from experiment (see Appendix), we can determine λ and γ_b,

$$\lambda = 0.209 \pm 0.019, \tag{54}$$

$$\gamma_b = 0.109 \pm 0.030. \tag{55}$$

The down-type quark diagonalization matrix can be calculated as a function of λ and γ_b; at leading order in λ we obtain

$$|\mathcal{V}^d_L| = \begin{bmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & \gamma_b \lambda^2 \\ \lambda & 1 - \frac{1}{2}\lambda^2 (1 + \gamma_b^2) & \gamma_b \lambda \\ \gamma_b \lambda^4 & \gamma_b \lambda & 1 - \frac{1}{2}\gamma_b^2 \lambda^2 \end{bmatrix}. \tag{56}$$

Using the experimentally determined values for γ_b and λ in Eqs. (54) and (55), we obtain the following central theoretical prediction for the $|\mathcal{V}^d_L|_{\rm theo}$ elements:

$$\begin{bmatrix} 0.976 \pm 0.008 & 0.216 \pm 0.035 & 0.0039 \pm 0.0006 \\ 0.216 \pm 0.035 & 0.974 \pm 0.007 & 0.019 \pm 0.007 \\ 0.000\,15 & 0.019 \pm 0.007 & 0.9993 \pm 0.0001 \end{bmatrix}. \tag{57}$$

If we compare $|\mathcal{V}^d_L|_{\rm theo}$ with the 90% C.L. experimental compilation of CKM matrix elements (see Appendix), we observe that $|\mathcal{V}^d_L|_{\rm theo}$ accounts quite well for the measured SM flavor violation. There is good agreement with the experimental data on CKM matrix elements except in the entry $|V_{cb}|$, where we observe a deficit in the theoretical prediction, which turns out to be approximately one-half of the measured value, i.e., $\gamma_b \lambda = |V_{cb}|/2$. We will see later that to solve this deficit we are forced to generate half of the contribution to $|V_{cb}|$ from flavor violation in the up-type quark sector. There is a simpler alternative solution. We can assume that the flavor mixing in the up-type quark sector does not affect the CKM mixing matrix to leading order in λ while the vev of the $\mathcal{F}$ field is instead given by

$$(v_{\mathcal{F}}, \mathcal{V}_{\mathcal{F}}) = (\lambda^2, 2\lambda)M_{\mathrm{F}}\tilde{m}. \tag{58}$$

In this case we obtain $\mathbf{A}_{12} = \mathbf{A}_{21} = 2\sigma\lambda$, $(\mathbf{m}_D)_{12} = (\mathbf{m}_D)_{21} = 2\gamma_b\lambda$, and $|\mathcal{V}_L^d| = 2\gamma_b\lambda$. We note that the prediction for the entry $|(V_L^d)_{31}|$ is very small even tough compatible with the measured value of $|V_{td}|$, which carries a large uncertainty. Before studying the up-type quark sector we will analyze in the next subsection the predictions of the model for the lepton sector.

B. The lepton sector and the need for SU(5)

The mass matrix in Eq. (47) is very successful in reproducing the down-type quark mass ratios, but it cannot explain correctly the measured mass ratios in the lepton and up-type quark sectors. To account for the mass ratios in the lepton sector we will need to promote the standard model $\mathrm{SU}(3)_c \times \mathrm{U}(2)_L \times \mathrm{U}(1)_Y$ vertical symmetry to the SU(5) symmetry of Georgi and Glashow and assign the U(2) flavor-breaking fields to particular representations under SU(5) as we will explain in detail below.

First, we are going to postulate for the lepton soft trilinear matrix $\mathbf{A}_L$ a simple modification of the texture predicted by the minimal model in Eq. (34). Let us assume that

$$\mathbf{A}_L = A_\tau \begin{bmatrix} 0 & \sigma_l\lambda^2 & \sigma_l\lambda^2 \\ -\sigma_l\lambda^2 & 3\sigma_l\lambda & \sigma_l\lambda \\ \sigma_l\lambda^2 & \sigma_l\lambda & 1 \end{bmatrix}. \tag{59}$$

Here λ was introduced in Eq. (26), and A_τ and σ_l are coefficients analogous to the ones introduced in Eq. (34). If there is no extra flavor singlet $\mathcal{J}$, A is given by $A_\tau = \kappa m_{\tilde{\gamma}}$, where $m_{\tilde{\gamma}}$ is the photino mass. We will show next that this texture can perfectly fit the lepton mass ratios. Later we will explain how one can obtain this texture in a SUSY SU(5) framework. The radiatively generated lepton Yukawa couplings are given in this case by

$$(\mathbf{Y}_L)_{ab}^{\mathrm{rad}} = \frac{\alpha}{2\pi} m_{\tilde{\gamma}} \sum_c \mathcal{D}_{ac}\mathcal{D}^*_{(b+3)c} B_0(m_{\tilde{\gamma}}, m_{\tilde{l}_c}), \tag{60}$$

where $\mathcal{D}$ is the slepton 6×6 diagonalization matrix, $m_{\tilde{l}_c}$ are slepton mass eigenvalues, and α is the running fine structure constant. We now obtain a simple expression for the radiatively corrected lepton mass matrix,

$$\mathbf{m}_L = \hat{m}_\tau \begin{bmatrix} 0 & \gamma_\tau\lambda^2 & \gamma_\tau\lambda^2 \\ -\gamma_\tau\lambda^2 & 3\gamma_\tau\lambda & \gamma_\tau\lambda \\ \gamma_\tau\lambda^2 & \gamma_\tau\lambda & 1 \end{bmatrix}, \tag{61}$$

where $\hat{m}_\tau$ is to first order the running bottom mass,

$$\hat{m}_\tau = y_\tau v c_\beta[1 + \zeta_l(1 - y_\tau\mu t_\beta/A)], \tag{62}$$

$$\zeta_l = \frac{2\alpha}{\pi y_\tau} m_{\tilde{\gamma}} A F(\tilde{m}_f, \tilde{m}_f, m_{\tilde{\gamma}}), \tag{63}$$

and γ_τ is a loop factor that has a simple expression in the mass degenerate sfermion case,

$$\gamma_\tau = \frac{\sigma\zeta_l}{[1 + \zeta_l(1 - y_\tau\mu t_\beta/A)]} \approx \sigma\zeta_l. \tag{64}$$

As in the down-type quark sector, γ_t encodes the dependence on the SUSY spectra and parametrizes the breaking of the alignment between soft trilinear and Yukawa sectors caused by the tree-level component to the tau lepton mass. Although not diagonal in the gauge basis, the matrix $\mathbf{m}_L$ can be brought to diagonal form in the mass basis by a biunitary diagonalization, $(\mathcal{V}_L^l)^\dagger \mathbf{m}_L \mathcal{V}_R^l = (m_e, m_\mu, m_\tau)$. The lepton mass matrix given by Eq. (61) makes the following predictions for the lepton mass ratios:

$$\frac{m_e}{m_\mu} = \frac{1}{9}\lambda^2\left(1 - \frac{2}{9}\lambda^2 + \frac{5}{3}\gamma_\tau\lambda\right) + \mathcal{O}(\lambda^4), \tag{65}$$

$$\frac{m_\mu}{m_\tau} = 3\gamma_\tau\lambda\left(1 + \frac{1}{9}\lambda^2 - \frac{1}{3}\gamma_\tau\lambda\right) + \mathcal{O}(\lambda^3). \tag{66}$$

We can relate λ and γ_τ with dimensionless and approximately renormalization scale independent fermion mass ratios; to first order

$$\lambda = 3\left(\frac{m_e}{m_\mu}\right)^{1/2}, \qquad \gamma_\tau = \frac{1}{9}\left(\frac{m_\mu^3}{m_\tau^2 m_e}\right)^{1/2}. \tag{67}$$

Using the invariant running lepton mass ratios determined from experiment we obtain

$$\lambda = 0.206\,480 \pm 0.000\,002, \tag{68}$$

$$\gamma_\tau = 0.094\,95 \pm 0.0001. \tag{69}$$

Interestingly, these values of λ and γ_τ are consistent with the values of λ and γ_b determined in the down-type quark sector. This surprising coincidence unveils two relations,

$$\left(\frac{m_d}{m_s}\right)^{1/2} \approx 3\left(\frac{m_e}{m_\mu}\right)^{1/2}, \tag{70}$$

$$\left(\frac{m_s^3}{m_b^2 m_d}\right)^{1/2} \approx \frac{1}{9}\left(\frac{m_\mu^3}{m_\tau^2 m_e}\right)^{1/2}. \tag{71}$$

These mass relations may be considered experimental evidence supporting the consistency of this scenario.

To explain the origin of the factor "**3**" in the entry (22) of the lepton mass matrix, let us assume that matter superfields of different families group as usual in the representations $\bar{\mathbf{5}}_a$ and $\mathbf{10}_a$ of SU(5) ($a = 1, 2, 3$), while the Higgs superfields, $\mathcal{H}_u$ and $\mathcal{H}_d$, belong to the representations $\mathbf{5}$ and $\bar{\mathbf{5}}$, respectively. If the flavor symmetric tensor superfield S transforms as the representation $\mathbf{75}$ of

SU(5), then the operator,

$$\frac{1}{M} S^{ab}(\mathbf{75})\mathcal{H}_d \mathbf{10}_a \bar{\mathbf{5}}_b \neq 0 \tag{72}$$

will generate the entries (11) and (22) in the lepton and down-type quark mass matrices. Since the SU(5) tensor product $S\mathcal{H}_d$ includes the representation $\overline{\mathbf{45}}$ of SU(5), it will generate the additional factor "**3**" of Georgi and Jarlskog [15,16] in the lepton mass matrix. Furthermore, the flavor antisymmetric tensor $\mathcal{A}$ must transform as a SU(5) singlet; then the operator

$$\frac{1}{M} \mathcal{A}^{ab}(\mathbf{1})\mathcal{H}_d \mathbf{10}_a \bar{\mathbf{5}}_b \neq 0 \tag{73}$$

will generate correctly the entries (12) and (21) in the down-type quark and lepton mass matrices. Finally, the $U(2)_H$ flavor-vector superfield $\mathcal{F}$ could transform as a singlet or under the representation **24** of SU(5); then the operator

$$\mathcal{F}(\mathbf{1}, \mathbf{24}) \rightarrow \frac{1}{M} \mathcal{F}^a \mathcal{H}_d \mathbf{10}_a \bar{\mathbf{5}}_3 \neq 0 \tag{74}$$

would generate the entries $(a3)$ $(a = 1, 2)$, and analogously the entries $(3a)$ from the operator $\mathcal{F}^a \mathcal{H}_d \mathbf{10}_3 \bar{\mathbf{5}}_a$.

C. The charm-quark mass problem

Assigning the $U(2)_H$-flavor fields S, $\mathcal{A}$, and $\mathcal{F}$ to the representations **75**, **1**, and **1** of SU(5), respectively, implies that two of the associated operators in the up-type quark sector are exactly zero,

$$\frac{1}{M} S^{ab}(\mathbf{75})\mathcal{H}_u \mathbf{10}_a \mathbf{10}_b = 0, \tag{75}$$

$$\frac{1}{M} \mathcal{A}^{ab}(\mathbf{1})\mathcal{H}_u \mathbf{10}_a \mathbf{10}_b = 0, \tag{76}$$

where $a, b = 1, 2$. If this were the case the up-type quark soft trilinear matrix would be of the form,

$$\mathbf{A}_U = A \begin{bmatrix} 0 & 0 & \sigma\lambda^2 \\ 0 & 0 & \sigma\lambda \\ \sigma\lambda^2 & \sigma\lambda & 1 \end{bmatrix}, \tag{77}$$

implying that the up-quark mass is massless and the charm mass is, to first order, $m_c \approx \gamma_t^2 \lambda^2 m_t$. This is inconsistent with the values for γ_b and γ_τ required by phenomenology in the down-type quark and lepton sectors, respectively.

We propose two possible solutions to fix the charm-quark mass problem. One is to extend the $U(2)_H$ flavor-breaking sector. Let us assume that there are two sets of $U(2)_H$ flavor-breaking fields, one set transforming as a SU(5) singlet and the other transforming as a **75** under SU(5),

$$[S(\mathbf{75}), \mathcal{A}(\mathbf{75}), \mathcal{F}(\mathbf{75})], \tag{78}$$

$$[S(\mathbf{1}), \mathcal{A}(\mathbf{1}), \mathcal{F}(\mathbf{1})]. \tag{79}$$

Let us assume that $S(\mathbf{75})$, $\mathcal{A}(\mathbf{1})$, and $\mathcal{F}(\mathbf{1})$ get vacuum expectation values as described in Eqs. (23)–(25) correctly generating the down-type quark and lepton matrices and also the entries $(3a)$ and $(a3)$ in the up-type quark mass matrix. Additionally we assume that $S(\mathbf{1})$ gets a vev

$$\langle S(\mathbf{1})\rangle = \begin{pmatrix} 0 & 0 \\ 0 & \mathcal{V}_{\mathcal{A}} \end{pmatrix} \theta^2, \tag{80}$$

where $\mathcal{V}_{\mathcal{A}} = \lambda^2 M_F \tilde{m}$. This vev will generate an entry (22) in the up-type quark mass matrix of the correct size to explain the charm-quark mass, through the operator

$$\frac{1}{M} S^{ab}(\mathbf{1})\mathcal{H}_u \mathbf{10}_a \mathbf{10}_b \neq 0. \tag{81}$$

A second possibility is to use nonrenormalizable operators with higher order powers of $1/M_F$ [15]. In this case we need to introduce at least one new $U(2)_H$-flavor-singlet scalar field, Σ. If Σ transforms as a representation **24** under SU(5), we could generate the entries (22), (12), and (21) in the up-quark mass matrix from the operators

$$\frac{1}{M^2} \Sigma(\mathbf{24}) S^{ab}(\mathbf{75})\mathcal{H}_u \mathbf{10}_a \mathbf{10}_b \neq 0, \tag{82}$$

$$\frac{1}{M^2} \Sigma(\mathbf{24}) \mathcal{A}^{ab}(\mathbf{1})\mathcal{H}_u \mathbf{10}_a \mathbf{10}_b \neq 0. \tag{83}$$

If $\langle\Sigma\rangle = \lambda M_F$, we would generate an entry (22) of order $\gamma_t \lambda^2$ and entries (12) and (21) of order $\gamma_t \lambda^3$ in the up-type quark mass matrix. Unfortunately, a (12)-(21) entry of order $\gamma_t \lambda^3$ would require a γ_t inconsistent with the value of γ_b calculated in the down-type sector and with γ_t being a loop factor. One way to save the higher order mechanism would be to keep the up-quark massless, removing the $\Sigma(\mathbf{24})\mathcal{A}(\mathbf{1})\mathcal{H}_u \mathbf{10}\,\mathbf{10}$ term by imposing an additional discrete symmetry. For instance, a Z_2 symmetry under which $\mathcal{A}$ and the lepton and down-type quark right-handed fields have parity $(-)$ and the rest of the superfields have parity $(+)$ would do the job.

In these two cases, an up-type quark soft trilinear matrix would be generated of the form,

$$\mathbf{A}_U = A \begin{bmatrix} 0 & 0 & \sigma\lambda^2 \\ 0 & \sigma\lambda^2 & \sigma\lambda \\ \sigma\lambda^2 & \sigma\lambda & 1 \end{bmatrix}. \tag{84}$$

This matrix would account correctly for the charm/top quark mass ratio, consistently with the values of γ_b and γ_τ calculated in previous subsections. Unfortunately it would predict an up-quark mass, $m_u \approx \gamma^2 \lambda^4$, 1 order of magnitude heavier than the measured value. Therefore we are forced to prevent the flavor-breaking field $\mathcal{F}(\mathbf{1})$ from mixing with the up-type Yukawa operators. This could be enforced by imposing an additional discrete or U(1) symmetry. As a consequence we will obtain

$$\mathbf{A}_U = A\begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma\lambda^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{85}$$

Additionally we could assume that the flavor field $\mathcal{F}(\mathbf{75})$ gets a vev of the form,

$$\langle \mathcal{F}(\mathbf{75})\rangle = \begin{pmatrix} 0 \\ -\mathcal{V}_{\mathcal{F}} \end{pmatrix}\theta^2, \tag{86}$$

where $\mathcal{V}_{\mathcal{F}} = \lambda M_{\mathrm{F}}\tilde{m}$. This would generate entries (23) and (32) in $\mathbf{A}_U$ of the form,

$$\mathbf{A}_U = A\begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma\lambda^2 & -\sigma\lambda \\ 0 & -\sigma\lambda & 1 \end{bmatrix}. \tag{87}$$

These two solutions are both phenomenologically viable as we will see in more detail below.

D. The up-quark mass problem

The extension of the $\mathrm{U}(2)_H$ flavor-breaking sector allows us to correctly generate the charm-quark mass. On the other hand, the up quark still remains massless. Although the possibility of a massless up quark has been considered in the past as a solution to the strong *CP* problem, more recent studies of pseudoscalar masses and decay constants, along with other arguments, strongly suggest that the up-quark mass is nonzero [17,18]. We can easily generate the up-quark mass in the scenario with two sets of flavor-breaking superfields through a small perturbation of the vev of the $S(\mathbf{1})$ field. Let us assume that

$$\langle S(\mathbf{1})\rangle = \begin{pmatrix} u & 0 \\ 0 & \mathcal{V}_{\mathcal{A}} \end{pmatrix}\theta^2, \tag{88}$$

where $(u, \mathcal{V}_{\mathcal{A}}) = (\lambda^6, \lambda^2)M\tilde{m}$. Alternatively, we could generate an up-quark mass in the scenario with higher order operators in $1/M$ if we perturb the vev of the $S(\mathbf{75})$ field in the form,

$$\langle S(\mathbf{75})\rangle = \begin{pmatrix} w & 0 \\ 0 & \mathcal{V}_{\mathcal{F}} \end{pmatrix}\theta^2, \tag{89}$$

where $(w, \mathcal{V}_{\mathcal{F}}) = (\lambda^5, \lambda)M\tilde{m}$. We observe that this perturbation of the $S(\mathbf{1})$ or $S(\mathbf{75})$ vevs does not affect the predictions in the down-type quark and lepton sectors.

In these two cases an up-type quark soft trilinear matrix would be generated of the form,

$$\mathbf{A}_U = A\begin{bmatrix} \sigma\lambda^6 & 0 & 0 \\ 0 & \sigma\lambda^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{90}$$

or alternatively if we assume that the field $\mathcal{F}(\mathbf{75})$ gets a vev as in Eq. (86),

$$\mathbf{A}_U = A\begin{bmatrix} \sigma\lambda^6 & 0 & 0 \\ 0 & \sigma\lambda^2 & -\sigma\lambda \\ 0 & -\sigma\lambda & 1 \end{bmatrix}. \tag{91}$$

These textures can both correctly account for the up and charm to top quark mass ratios as we will see in the next subsection.

E. Up-type quark masses and CKM predictions

In the first case considered in Eq. (90) we obtain a simple expression for the radiatively corrected up-type quark mass matrix,

$$\mathbf{m}_U = \hat{m}_t\begin{bmatrix} \gamma_t\lambda^6 & 0 & 0 \\ 0 & \gamma_t\lambda^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{92}$$

where $\hat{m}_t$ is the running top quark mass defined by,

$$\hat{m}_t = y_t v s_\beta (1 + \zeta_u[1 - y_t\mu/(At_\beta)]), \tag{93}$$

$$\zeta_u = \frac{2\alpha_s}{3\pi y_t} m_{\tilde{g}} A F(\tilde{m}_f, \tilde{m}_f, m_{\tilde{g}}), \tag{94}$$

and γ_t is the loop factor given by

$$\gamma_t = \frac{\zeta_u\sigma}{(1 + \zeta_u[1 - y_t\mu/(t_\beta A)])} \approx \zeta_u\sigma \tag{95}$$

analogous to the ones defined in the down-type quark and lepton sectors. In the second case considered in Eq. (91) the radiatively corrected up-type quark mass matrix is given by

$$\mathbf{m}_U = \hat{m}_t\begin{bmatrix} \gamma_t\lambda^6 & 0 & 0 \\ 0 & \gamma_t\lambda^2 & -\gamma_t\lambda \\ 0 & -\gamma_t\lambda & 1 \end{bmatrix}. \tag{96}$$

The phenomenological implications of a mass matrix of a form similar to Eq. (96) have been studied by one of the authors in Ref. [1]. Although not diagonal in the gauge basis, the matrix $\mathbf{m}_U$ can be brought to diagonal form in the mass basis by a unitary diagonalization, $(\mathcal{V}_L^u)^\dagger \mathbf{m}_U \mathcal{V}_R^u = (m_u, m_c, m_t)$. It makes the following predictions for the up-type quark mass ratios:

$$\frac{m_u}{m_c} = \lambda^4[1 + \gamma_t(1 + \gamma_t)] + \mathcal{O}(\gamma^2\lambda^6), \tag{97}$$

$$\frac{m_c}{m_t} = \gamma_t\lambda^2(1 - \gamma_t)(1 - 2\gamma_t^2\lambda^2) + \mathcal{O}(\lambda^6). \tag{98}$$

In both cases we can express λ and γ_t as a function of up-type quark mass ratios, to first order, as

$$\lambda = \left(\frac{m_u}{m_c}\right)^{1/4}, \qquad \gamma_t = \left(\frac{m_c^3}{m_t^2 m_u}\right)^{1/2}. \tag{99}$$

Using the invariant running quark mass ratios determined from experiment (see Appendix) and Eqs. (97)

and (98) we can determine λ and γ_t numerically to be

$$\lambda = 0.225 \pm 0.015, \qquad (100)$$

$$\gamma_t = 0.071 \pm 0.019. \qquad (101)$$

These values for λ and γ_t coincide with the values for λ, γ_b, and γ_τ determined from measured fermion masses in the down-type quark and lepton sectors. This surprising coincidence unveils two more relations between dimensionless fermion mass ratios,

$$\left(\frac{m_d}{m_s}\right)^{1/2} \approx \left(\frac{m_u}{m_c}\right)^{1/4} \approx 3\left(\frac{m_e}{m_\mu}\right)^{1/2}, \qquad (102)$$

$$\left(\frac{m_s^3}{m_b^2 m_d}\right)^{1/2} \approx \left(\frac{m_c^3}{m_t^2 m_u}\right)^{1/2} \approx \frac{1}{9}\left(\frac{m_\mu^3}{m_\tau^2 m_e}\right)^{1/2}. \qquad (103)$$

The up-type quark diagonalization matrix can be calculated as a function of λ and γ_t. The up-quark diagonalization matrix can be used in combination with the down-quark diagonalization matrix to obtain an expression for the CKM mixing matrix, $\mathcal{V}_{\rm CKM} = (\mathcal{V}_L^u)^\dagger \mathcal{V}_L^d$. We are considering two cases: in the first one A_D is given by Eq. (34) and A_U is given by Eq. (91), in the second case $(A_D)_{12}$ gets an additional factor 2 from the $\mathcal{F}(\mathbf{1})$ vev in Eq. (58) while A_U is given by Eq. (90). In both cases using that $\gamma = \gamma_b \approx \gamma_t$ we obtain the same prediction for $|\mathcal{V}_{\rm CKM}|^{\rm theo}$ to leading order in λ,

$$\begin{bmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \gamma\lambda^2 \\ \lambda & 1-\frac{1}{2}\lambda^2(1+2\gamma^2) & 2\gamma\lambda \\ \gamma\lambda^2 & 2\gamma\lambda & 1-2\gamma^2\lambda^2 \end{bmatrix}. \qquad (104)$$

Using the experimentally determined values for λ, γ_b, and γ_t in Eqs. (54), (55), and (101) we obtain the following numerical theoretical prediction for the $|\mathcal{V}_{\rm CKM}|_{\rm theo}$ elements:

$$\begin{bmatrix} 0.976 \pm 0.008 & 0.216 \pm 0.035 & 0.0039 \pm 0.0006 \\ 0.216 \pm 0.035 & 0.974 \pm 0.007 & 0.035 \pm 0.006 \\ 0.0039 \pm 0.0006 & 0.035 \pm 0.006 & 0.9993 \pm 0.0001 \end{bmatrix}. \qquad (105)$$

If we now compare $|\mathcal{V}_{\rm CKM}|_{\rm theo}$ with the 90% C.L. experimental compilation of CKM matrix elements from the Particle Data Group (PDG) compilation (see Appendix), we observe that $|\mathcal{V}_{\rm CKM}|_{\rm theo}$ accounts perfectly for the measured flavor violation in the standard model. There is now in very good agreement with the experimental data on the entry $|V_{cb}|$. We also obtain the same prediction for $|V_{td}|$, $|V_{td}| \approx |V_{ub}|$. The flavor violation in the upper left sector of the up-type quark mass matrix is not constrained by the CKM matrix, since the flavor violation in the up-type quark sector does not affect the entries (12) and (13) to leading order in λ.

F. Higher order Yukawa couplings

Yukawa couplings which are not generated at one loop could be generated at higher orders. For instance, the Yukawa coupling $(\mathbf{Y}_U)_{13}$ could be generated at two loops through a diagram with gluino and Higgs exchange and three soft trilinear vertices: $(\mathbf{A}_D)_{12}$, $(\mathbf{A}_D)_{22}$, and $(\mathbf{A}_U)_{23}$ [19]. We are interested in an overestimation of this 2-loop Yukawa coupling. Assuming that all the sparticles in the loop have masses of the same order, to maximize the loop factor, we obtain

$$(\mathbf{Y}_U)_{13}^{\text{2-loop}} \simeq \left(\frac{2\alpha_s}{3\pi}\right)\left(\frac{1}{4\pi}\right)^2\left(\frac{v}{m_{\tilde{q}}}\right)^2 c_\beta^2 \lambda^4, \qquad (106)$$

where $v = 175$ GeV. The ratio $v/m_{\tilde{q}}$, the c_β factors ($c_\beta = \cos\beta$), and the λ factors come from the three soft trilinear vertices. To facilitate the comparison with the 1-loop generated Yukawa couplings we will express this in powers of λ. Using that $\lambda \simeq 0.2$ and $\gamma \simeq 0.1$, we obtain

$$(\mathbf{Y}_U)_{13}^{\text{2-loop}} \simeq \frac{\gamma\lambda^{10}}{\tan^2\beta}\left(\frac{1\ \text{TeV}}{m_{\tilde{q}}}\right)^2. \qquad (107)$$

We note that this 2-loop generated Yukawa coupling is very suppressed when compared with the 1-loop generated couplings and for all practical purposes it can be considered zero.

IV. SUPPRESSION OF FLAVOR CHANGING PROCESSES BY RADIATIVE ALIGNMENT

It has been pointed out recently [1] that the radiative generation of fermion masses through flavor violation in the soft breaking terms may allow us to overcome the present experimental constraints on some of the supersymmetric contributions to flavor changing processes more easily than other flavor models. This is a necessary requirement for the consistency of any supersymmetric model [20]. Correlations between radiative mass generation and dipole operator phenomenology were first pointed out in Ref. [21]. In this scenario, as a consequence of the approximate radiative alignment between Yukawa and soft trilinear matrices there is an extra suppression of the supersymmetric contributions to flavor changing processes coming from the soft trilinear sector. For calculational purposes it is convenient to rotate the squarks to the so-called SKM (SuperKamiokande) basis, the basis where gaugino vertices are flavor diagonal [22]. In this basis, the entries in the soft trilinear matrices are directly proportional to their respective contributions to flavor-violating processes. For instance, the soft trilinear matrix $\mathbf{A}_D$ in the SKM basis is given by

$$\mathbf{A}_D^{\rm SKM} = (\mathcal{V}_L^d)^\dagger \mathbf{A}_D \mathcal{V}_R^d. \qquad (108)$$

Assuming the soft trilinear matrix $\mathbf{A}_D$ is given by Eq. (34), which corresponds to the proposed solution where one-half of $|V_{cb}|$ is generated in the up-sector, and the diagonalization matrices $\mathcal{V}^d_{L,R}$ are calculated from Eq. (47), we obtain, to leading order in λ and γ_b,

$$\mathbf{A}_D^{\rm SKM} = A_b \begin{bmatrix} -\sigma\lambda^3 & \sigma\theta\lambda^5 & \sigma\lambda^4 \\ 2\sigma\gamma_b\lambda^3 & \sigma\lambda & (\gamma_b - \sigma)\lambda \\ 2\sigma\lambda^2 & (\gamma_b - \sigma)\lambda & 1 + 2\sigma\gamma_b\lambda^2 \end{bmatrix}. \tag{109}$$

Here we will assume that σ, as defined in Eq. (36), is 1. We observe that the entry (21) is suppressed by an additional factor $\gamma_b\lambda$ as a consequence of the radiative alignment between Yukawa and soft trilinear matrices. Moreover in the SKM basis the left-handed down-type squark soft mass matrix is given by

$$(\tilde{\mathbf{M}}_D^2)_{LL}^{\rm SKM} = (\mathcal{V}_L^d)^\dagger (\tilde{\mathbf{M}}_D^2)_{LL} \mathcal{V}_L^d, \tag{110}$$

and analogously for the right-handed soft mass matrix. Assuming the soft trilinear texture from Eq. (37) we obtain for $(\mathbf{M}_D^2)_{LL}^{\rm SKM}$, to leading order in λ and γ_b,

$$m^2_{\tilde{b}_L} \begin{bmatrix} 1 + 2\sigma'^2\lambda^4 & -\sigma'^2\lambda^3 & \sigma'^2\lambda^4 \\ -\sigma'^2\lambda^3 & 1 + 2\sigma'^2\lambda^2 & -\sigma'^2\lambda \\ -\sigma'^2\lambda^4 & -\sigma'^2\lambda & (1 + 2\sigma'^2\gamma_b\lambda^2) \end{bmatrix}. \tag{111}$$

Here σ' was defined in Eq. (39). We note in the limit $\sigma' \ll 1$ the contributions from soft mass matrices to flavor changing processes would be suppressed. The following results would not change much assuming instead the second solution we proposed in the previous section. Using the values given by Eqs. (54) and (55) for λ and γ_b the amount of soft flavor violation required to fit quark masses and mixing angles is determined.

The entry most constrained experimentally in the soft mass matrices is the entry (12), which in our scenario gives the dominant contribution to the K_L-K_S mass difference. If we assume that $\sigma' = 1$, i.e., $A_b = m_{\tilde{q}}$, then we obtain that the squark spectra must be $m_{\tilde{q}} > 2$ TeV to avoid the saturation of the experimental measurement, $\Delta m_K = (3.490 \pm 0.006) \times 10^{-12}$ MeV [17]. This constraint is considerably milder if the gluino-squark mass ratio is much larger or smaller than 1. On the other hand if $A_b = m_{\tilde{q}}/2$ the flavor-violating soft mass matrices receive an additional $1/4$ factor through the σ' coefficient defined in Eq. (39). In this case we would saturate the experimental measurement for $m_{\tilde{q}} > 400$ GeV and we would predict $\Delta m_K < 7 \times 10^{-13}$ for $m_{\tilde{q}} > 1$ TeV. Furthermore, in the limit $\sigma' \ll 1$ this contribution goes to zero and the soft trilinear contribution dominates. The soft trilinear contribution because of the extra suppression factor $\gamma_b\lambda$ is very suppressed. Assuming a large value of $\tan\beta$, $m_{\tilde{q}} > 700$ GeV and any gluino-squark mass ratio it generates a contribution to Δm_K below the experimental uncertainty.

The entry (13) in the soft mass matrix $(\tilde{\mathbf{M}}_D^2)_{LL}^{\rm SKM}$ gives also the dominant contribution to Δm_B. We note that the entry (13) appears as a consequence of the possible mixing between flavor-breaking and flavor-singlet SUSY breaking fields through operators of the form,

$$\frac{1}{MM_{\rm F}} \int d^4\theta \rho [(\kappa' \mathcal{G}^\dagger + \eta' \mathcal{J}^\dagger) \mathcal{F}^b \phi^\dagger \Psi_b + {\rm H.c.}]. \tag{112}$$

The texture under consideration predicts a contribution to Δm_B for $m_{\tilde{q}} > 600$ GeV and any gluino mass ratio which is below uncertainty of the experimental measurement $\Delta m_B = (3.22 \pm 0.05) \times 10^{-10}$ MeV. We note that this constraint can be avoided if operators of the form in Eq. (112) are not allowed by the underlying supersymmetric theory, i.e., if we assume that $\rho = 0$.

Finally, from the measured $b \to s\gamma$ decay rate, one can obtain limits on the entry (23) [23]. In general the flavor-violating soft trilinear gives the dominant contribution to this process. Assuming a large value of $\tan\beta$, $\tan\beta > 30$, $A_b \simeq m_{\tilde{q}}$, and $m_{\tilde{q}} > 500$ GeV we obtain a contribution to $B(b \to s\gamma) < 3.4 \times 10^{-5}$, which is still below the uncertainty of the experimental measurement, $B(b \to s\gamma) = (3.3 \pm 0.4) \times 10^{-4}$. Using known expressions [22,23] we can also calculate the soft mass, i.e., LL, contribution to $B(b \to s\gamma)$. This is in general suppressed when compared with the LR, i.e., soft trilinear contribution, by a factor,

$$\frac{1}{6}\left(\frac{m_b}{m_{\tilde{g}}}\right) \frac{(\delta_{12}^d)_{LL}}{(\delta_{12}^d)_{LR}} \approx 3 \times 10^{-3} t_\beta \left(\frac{m_{\tilde{b}}}{m_{\tilde{g}}}\right), \tag{113}$$

where we used that

$$(\delta_{12}^d)_{LR} = (\gamma_b - 1)\lambda \left(\frac{A_b}{m_{\tilde{b}}}\right)\left(\frac{v}{m_{\tilde{b}}}\right)\frac{1}{t_\beta}, \tag{114}$$

$$(\delta_{12}^d)_{LL} = \lambda, \tag{115}$$

and v and γ_τ are given by $v = 175$ GeV and $\gamma_b \approx 0.1$. Even considering very large $\tan\beta$ values the LL contribution in this model is 1 order of magnitude smaller than the LR contribution to this process. Therefore the approximate constraints on the supersymmetric spectra calculated above from the LR contribution to $B(b \to s\gamma)$, while ignoring the LL contributions, are still valid.

We can perform a similar analysis of flavor changing processes in the lepton sector. As in the squark sector it is convenient for calculational purposes, to rotate the sleptons to a basis where gaugino vertices are flavor diagonal. The soft trilinear matrix $\mathbf{A}_L$ in the SKM basis is given by

$$\mathbf{A}_L^{\rm SKM} = (\mathcal{V}_L^l)^\dagger \mathbf{A}_L \mathcal{V}_R^l. \tag{116}$$

Assuming the soft trilinear texture from Eq. (59) with $\sigma = 1$ we obtain, to leading order in λ and γ_τ,

$$\mathbf{A}_L^{\rm SKM} = A_\tau \begin{bmatrix} \frac{1}{3}\lambda^3 & \frac{2}{3}\gamma_\tau\lambda^3 & \frac{2}{3}\lambda^2 \\ \frac{4}{3}\gamma_\tau\lambda^3 & 3\lambda & (\gamma_l - 1)\lambda \\ \frac{4}{3}\lambda^2 & (\gamma_l - 1)\lambda & (1 + 2\gamma_\tau\lambda^2) \end{bmatrix}. \quad (117)$$

Using the values given by Eqs. (68) and (69) for λ and γ_τ, the amount of soft lepton flavor violation is determined. The entry (12) contributes to $B(\mu \to e\gamma)$, which is the most experimentally constrained lepton flavor-violating process. For the texture under consideration, assuming a large value of tanβ, tan$\beta \approx 50$, $A_b \simeq m_{\tilde{l}}$, a photino lighter than the sleptons and $m_{\tilde{l}} > 1$ TeV we obtain a branching fraction $\Gamma_{\mu\to e\gamma} < 8 \times 10^{-12}$, which is still below the current experimental limit, $\Gamma^{\rm exp}_{\mu\to e\gamma} < 1.2 \times 10^{-11}$. The predictions for $\Gamma_{\tau\to e\gamma}$ and $\Gamma_{\tau\to\mu\gamma}$ are proportional to the entries (13) and (23), respectively, in the soft trilinear matrix. For the texture under consideration, using the same parameter space limits, we obtain $\Gamma_{\tau\to e\gamma} < 10^{-10}$ and $\Gamma_{\tau\to\mu\gamma} < 2 \times 10^{-9}$; these two predictions are far below the experimental limits, $\Gamma^{\rm exp}_{\tau\to e\gamma} < 2.7 \times 10^{-6}$ and $\Gamma^{\rm exp}_{\tau\to\mu\gamma} < 1.1 \times 10^{-6}$.

There are also contributions to $B(\mu \to e\gamma)$ coming from flavor-violating soft masses. In the SKM basis the left-handed charged slepton soft mass matrix is given by

$$(\tilde{\mathbf{M}}_L^2)^{\rm SKM}_{LL} = (\mathcal{V}_L^l)^\dagger (\tilde{\mathbf{M}}_L^2)_{LL} \mathcal{V}_L^l. \quad (118)$$

Assuming the soft trilinear texture from Eq. (59) and $\sigma' = 1$ we obtain, to leading order in λ and γ_τ,

$$(\mathbf{M}_L^2)^{\rm SKM}_{LL} = m^2_{\tilde{\tau}_L} \begin{bmatrix} 1 & \frac{1}{3}\lambda^3 & \frac{2}{3}\lambda^2 \\ -\frac{1}{3}\lambda^3 & 1 + 2\lambda^2 & -\lambda \\ \frac{2}{3}\lambda^2 & -\lambda & (1 + 2\gamma_\tau\lambda^2) \end{bmatrix}. \quad (119)$$

The contribution of the flavor-violating soft masses to $B(\mu \to e\gamma)$ is suppressed compared with the contribution from the soft trilinear terms by a factor

$$\frac{1}{6}\left(\frac{m_\mu}{m_{\tilde{\gamma}}}\right)\frac{(\delta^l_{12})_{LL}}{(\delta^l_{12})_{LR}} \approx 5 \times 10^{-4} t_\beta \left(\frac{m_{\tilde{l}}}{m_{\tilde{\gamma}}}\right), \quad (120)$$

where we used that

$$(\delta^l_{12})_{LR} = \frac{2}{3}\gamma_\tau\lambda^3\left(\frac{A_\tau}{m_{\tilde{l}}}\right)\left(\frac{v}{m_{\tilde{l}}}\right)\frac{1}{t_\beta}, \quad (121)$$

$$(\delta^l_{12})_{LL} = \tfrac{1}{3}\lambda^3, \quad (122)$$

and v and γ_τ are given by $v = 175$ GeV and $\gamma_\tau = 0.95$. We note that even for a large value of tanβ the LL contribution is much smaller than the LR contribution in this model.

To summarize, the flavor violation present in the soft supersymmetry-breaking sector, which is necessary in this scenario to generate fermion masses and quark mixings radiatively, is not excluded by the present experimental constraints. These constraints are not especially stronger than in other supersymmetric flavor models. The approximate radiative alignment between radiatively generated Yukawa matrices and soft trilinear terms helps to suppress some of the supersymmetric contributions to these processes, especially the contribution to $B(\mu \to e\gamma)$.

V. PROTON DECAY SUPPRESSION

It is generally believed that strong experimental limits on proton decay place stringent constraints on supersymmetric grand unified models. Nevertheless, we will see this assertion is very dependent on the mechanism that generates the Yukawa couplings. For instance, in the case of a generic minimal supersymmetric SU(5) model [24], the superpotential, omitting SU(5) and flavor indices, is given by

$$\mathcal{W}_{\rm SU(5)} = \frac{1}{4}\mathbf{Y}_U \mathbf{10}\,\mathbf{10}\mathcal{H}_u + \sqrt{2}\mathbf{Y}_D \mathbf{10}\,\bar{\mathbf{5}}\mathcal{H}_d + \cdots, \quad (123)$$

where $\mathbf{10}$ and $\bar{\mathbf{5}}$ are matter chiral superfields belonging to representations $\mathbf{10}$ and $\bar{\mathbf{5}}$ of SU(5), respectively. As in the supersymmetric generalization of the SM, to generate fermion masses we need two sets of Higgs superfields, $\mathcal{H}_u$ and $\mathcal{H}_d$, belonging to representations $\mathbf{5}$ and $\bar{\mathbf{5}}$ of SU(5). After integrating out the colored Higgs triplet, the presence of Yukawa couplings in the superpotential leads to effective dimension-five interactions which, omitting flavor indices, are of the form,

$$\mathcal{W}_{\rm dim5} \propto \frac{1}{\mathrm{M}_{\mathcal{H}_c}}\left[\frac{1}{2}\mathbf{Y}_U\mathbf{Y}_D(QQ)(QL) + \mathbf{Y}_U\mathbf{Y}_D(UE)(UD)\right], \quad (124)$$

where $\mathrm{M}_{\mathcal{H}_c}$ is the colored Higgs mass, and operators $(QQ)(QL)$ and $(UE)(UD)$ are totally antisymmetric in color indices. Therefore, flavor conservation in the superpotential would imply their cancellation in the exactly supersymmetric theory,

$$(QQ)(QL) \equiv 0, \quad (125)$$

$$(UE)(UD) \equiv 0. \quad (126)$$

In our scenario, we started by assuming that there is a $\mathrm{U}(2)_H$ horizontal symmetry that guarantees the flavor conservation in superpotential of the supersymmetric unified theory. Flavor-violating couplings are generated only at low energy after supersymmetry breaking. The operators that generate flavor violation are of the form,

$$\frac{1}{M}(S, \mathcal{A})\mathcal{H}_d 10\,\bar{5}, \qquad \frac{1}{M}S\mathcal{H}_u 10\,10. \quad (127)$$

Integrating out the colored Higgs we could generate in principle baryon number violating operators of the form,

$$\propto S\mathcal{A}(10\,\bar{5})(10\,10), \tag{128}$$

but one of our basic assumptions is that at the U(2) minimum the U(2) flavor-breaking fields, S and $\mathcal{A}$, are F terms, therefore these operators exactly cancel and one cannot generate directly dimension-five operators. Dimension-five operators could be generated at higher orders. Since tree-level interactions with colored Higgsinos are possible only for the third family, the generation of a dimension-five proton decay operator would require two flavor mixing couplings between the first and the third generation. On the other hand, the Yukawa coupling of the form $(\mathbf{Y}_U)_{13}$ is first generated at two loops and very suppressed, as pointed out in Eq. (107). As a consequence radiatively generated dimension-five operators leading to proton decay are very suppressed in this scenario, when compared with ordinary SUSY grand unified theory (GUT) predictions, which generate flavor in the superpotential. Regarding the next dominant decay mode arising from dimension-six operators via GUT gauge bosons, it has been shown that using the SuperKamiokande limit, $\tau(p \to \pi^0 e^+) > 5.3 \times 10^{33}$ years, a lower bound on the heavy gauge boson mass, M_V, can be extracted, $M_V > 6.8 \times 10^{15}$ GeV. Furthermore, the proton decay rate for $M_V = M_{\rm GUT}$ is far below the detection limit that can be reached within the next years [25].

VI. SUMMARY

Many recipes have been attempted to cook the observed fermion mass hierarchies. We have shown in this paper that a tastier dish may require the right mix of horizontal symmetries, grand unified symmetries, and radiative mass generation. We have proposed an effective flavor-breaking model based on a U(2) horizontal symmetry which is implemented by supersymmetry-breaking fields. As a consequence, flavor breaking originates in the soft supersymmetry-breaking terms and is transmitted to the Yukawa sector at low energy. The approximate radiative alignment between soft trilinear matrices and the radiatively generated Yukawa matrices at low energy helps to suppress the supersymmetric contributions to flavor changing processes. The model allows us to successfully fit the six fermion mass ratios and the quark mixing angles with just two parameters. It also predicts new quantitative relations between dimensionless fermion mass ratios in the three fermion sectors, and the quark mixing angles,

$$|V_{us}| \approx \left[\frac{m_d}{m_s}\right]^{1/2} \approx \left[\frac{m_u}{m_c}\right]^{1/4} \approx 3\left[\frac{m_e}{m_\mu}\right]^{1/2}, \tag{129}$$

$$\frac{1}{2}\left|\frac{V_{cb}}{V_{us}}\right| \approx \left[\frac{m_s^3}{m_b^2 m_d}\right]^{1/2} \approx \left[\frac{m_c^3}{m_t^2 m_u}\right]^{1/2} \approx \frac{1}{9}\left[\frac{m_\mu^3}{m_\tau^2 m_e}\right]^{1/2}, \tag{130}$$

which are confirmed by the experimental measurements. Moreover, the requirement of flavor conservation in the superpotential of the grand unified theory implies the suppression of the problematic dimension-five operators which otherwise would accelerate proton decay.

ACKNOWLEDGMENTS

N. H. would like to thank the University of Hawaii for its hospitality. We thank Professor Xerxes Tata for his comments. We thank R. Dermisek for his suggestions regarding the higher order generation of Yukawa couplings and proton decay operators. We also thank H. Guler for many suggestions. This work was supported by the D.O.E. Grant No. DE-FG03-94ER40833 and by the Grants No. 4039207, No. 14046208, and No. 14740164 of the Ministry of Education and Science of Japan.

APPENDIX

For the calculation of the dimensionless fermion mass ratios used in the main text, running fermion masses were used. These were calculated through scaling factors including known QCD and QED renormalization effects, which can be determined using known solutions to the SM renormalization group equations. For the charged leptons our starting point is the well-known physical masses. For the top quark the starting point is the pole mass from the PDG collaboration [17],

$$m_t = 174.3 \pm 5.1 \text{ GeV}. \tag{A1}$$

For the bottom and charm quarks the running masses, $m_b(m_b)_{\overline{MS}}$ and $m_c(m_c)_{\overline{MS}}$, from Refs. [26,27] are used,

$$m_b(m_b)_{\overline{MS}} = 4.25 \pm 0.25 \text{ GeV}, \tag{A2}$$

$$m_c(m_c)_{\overline{MS}} = 1.26 \pm 0.05 \text{ GeV}; \tag{A3}$$

for the light quarks u, d, and s, the starting point is the normalized $\overline{MS}$ values at $\mu = 2$ GeV. Original extractions [28,29] quoted in the literature have been rescaled as in [17]

$$m_s(2 \text{ GeV})_{\overline{MS}} = 117 \pm 17 \text{ MeV}, \tag{A4}$$

$$m_d(2 \text{ GeV})_{\overline{MS}} = 5.2 \pm 0.9 \text{ MeV}, \tag{A5}$$

$$m_u(2 \text{ GeV})_{\overline{MS}} = 2.9 \pm 0.6 \text{ MeV}. \tag{A6}$$

For completeness we include here some functions used in the main text. The B_0 and $F(x, y, z)$ functions, which are used in the calculation of the 1-loop finite corrections, are given by

$$B_0(m_1, m_2) = 1 + \ln\left(\frac{Q^2}{m_2^2}\right) + \frac{m_1^2}{m_2^2 - m_1^2}\ln\left(\frac{m_2^2}{m_1^2}\right), \tag{A7}$$

where Q is the renormalization scale,

$$F(x,y,z) = \frac{(x^2y^2\ln\frac{y^2}{x^2} + y^2z^2\ln\frac{z^2}{y^2} + z^2x^2\ln\frac{x^2}{z^2})}{(x^2-y^2)(y^2-z^2)(z^2-x^2)} > 0. \tag{A8}$$

For completeness we also include the 90% C.L. experimental compilation of CKM matrix elements from the PDG compilation [17],

$$|\mathcal{V}_{\rm CKM}^{\rm exp}| = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{bmatrix} 0.974\,85 \pm 0.000\,75 & 0.2225 \pm 0.0035 & 0.003\,65 \pm 0.001\,15 \\ 0.2225 \pm 0.0035 & 0.9740 \pm 0.0008 & 0.041 \pm 0.003 \\ 0.0009 \pm 0.005 & 0.0405 \pm 0.0035 & 0.999\,15 \pm 0.000\,15 \end{bmatrix}. \tag{A9}$$

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