Supersymmetric Ward-Takahashi identity in one-loop lattice perturbation theory: General procedure

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The one-loop corrections to the lattice supersymmetric Ward-Takahashi identity (WTi) are investigated in the off-shell regime. In the Wilson formulation of the $N = 1$ supersymmetric Yang-Mills theory, supersymmetry is broken by the lattice, by the Wilson term, and is softly broken by the presence of the gluino mass. However, the renormalization of the supercurrent can be realized in a scheme that restores the continuum supersymmetric WTi (once the on-shell condition is imposed). The general procedure used to calculate the renormalization constants and mixing coefficients for the local supercurrent is presented. The supercurrent not only mixes with the gauge invariant operator T_μ. An extra mixing with other operators coming from the WTi appears. This extra mixing survives in the continuum limit in the off-shell regime and cancels out when the on-shell condition is imposed and the renormalized gluino mass is set to zero. Comparison with numerical results is also presented.

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I. INTRODUCTION

Supersymmetry (SUSY) or fermion-boson symmetry is one of the most exciting topics in field theory. From a theoretical point of view, SUSY plays a fundamental role in string theory. There are many strong phenomenological motivations for believing that SUSY is realized in nature in a spontaneously broken form. The SUSY breaking mechanisms are requested in order to produce a low energy four-dimensional effective action with a residual $N = 1$ SUSY. On the other hand, nonperturbative studies of supersymmetric gauge theories turn out to have remarkably rich properties which are of great physical interest, as has been pointed out in [1]. For this reason, much effort has been dedicated to formulating a lattice version of supersymmetric theories (for a recent review in SUSY on the lattice with a complete list of relevant references, see [2]). More recently, related interesting results in SUSY can be found in [3–12]. Some of these formulations try to realize chiral gauge theories on the lattice with an exact chiral gauge symmetry [13–16]. The lattice formalism is a powerful tool to extract nonperturbative dynamics of field theories and may be able to provide additional information and confirm or improve theoretical expectations.

To formulate SUSY on the lattice, we follow the ideas of Curci and Veneziano [17]. They propose to give up manifest SUSY on the lattice, and instead to restore it in the continuum limit. In [17], the Wilson formulation for the $N = 1$ supersymmetric Yang-Mills (SYM) theory, which is the simplest SUSY gauge theory and corresponds to the SUSY gluodynamics, is adopted. For SU(N_c) it has ($N_c^2 - 1$) gluons and the same number of massless Majorana fermions (gluinos), in the adjoint representation of the color group.

SUSY is broken explicitly by the Wilson term and the finite lattice spacing. In addition, a soft breaking due to the introduction of the gluino mass is present. In [17], it is proposed that SUSY can be recovered in the continuum limit by tuning the bare gauge coupling, g_0, and the gluino mass, $m_{\tilde{g}_0}$, to the SUSY point, $m_{\tilde{g}_0} = 0$, which also coincides with the chiral point. In [18–24], the DESY-Münster-Roma collaboration has investigated these issues for the SU(2) gauge group [some first results have been obtained for SU(3) [25]], simulating the theory with a dynamical gluino using the multibosonic algorithm [26] with a two-step variant called the TSMB algorithm [27] (while quenched results are in [28]).

Another independent way to study the SUSY (chiral) limit in the Wilson formulation of Curci and Veneziano is through the study of the SUSY WTi. On the lattice, it contains explicit SUSY breaking terms and the SUSY limit is defined to be the point in the parameter space where these breaking terms vanish and the SUSY WTi recovers its continuum form. These issues have been investigated numerically in [23] in the on-shell regime.

In this paper, the general procedure used to determine the renormalization constants and mixing coefficients for the local definition of the supercurrent in the off-shell regime is explained. It is shown that, when the operator insertion involves elementary fields, the supercurrent not only mixes with the gauge invariant operator T_μ, as has been claimed in [17]. The supercurrent contains also non-Lorentz covariant terms which survive in the continuum, in the off-shell regime. These non-Lorentz breaking terms cancel out when the on-shell condition on the gluino is imposed and the continuum SUSY WTi is recovered. Preliminary studies have been presented in [29,30].

The paper is organized as follows. In Sec. II the Curci and Veneziano lattice formulation of the $N = 1$ SYM theory is presented, together with the lattice action and the vertices used for the calculation. In Sec. III the SUSY WTi on the lattice are written and the renormalization procedure explained. The calculation of the renormalization constant for the supercurrent is presented in Sec. IV. Discussions and outlook are summarized in Sec. V. In Appendices A, B, and C, some details of the calculation are shown.

II. LATTICE FORMULATION

In the Wilson formulation of the $N = 1$ SYM theory [17], the gluonic part of the action is the standard plaquette one:

$$S_g = \frac{\beta}{2}\sum_x \sum_{\mu\nu}\left[1 - \frac{1}{N_c}\mathrm{Re}\,\mathrm{Tr}\,P_{\mu\nu}(x)\right], \qquad (2.1)$$

where the plaquette operator is defined as [31]

$$P_{\mu\nu}(x) = U_\nu^\dagger(x)U_\mu^\dagger(x+\hat{\nu})U_\nu(x+\hat{\mu})U_\mu(x), \qquad (2.2)$$

and the bare coupling is given by $\beta \equiv 2N_c/g_0^2$. For Wilson fermions, the fermionic part of the action reads

$$S_f = \sum_x a^4\,\mathrm{Tr}\left[\frac{1}{2a}[\bar{\lambda}(x)(\gamma_\mu - r)U_\mu^\dagger(x)\lambda(x+a\hat{\mu})U_\mu(x) - \bar{\lambda}(x+a\hat{\mu})(\gamma_\mu + r)U_\mu(x)\lambda(x)U_\mu^\dagger(x)] + \left(m_0 + \frac{4r}{a}\right)\bar{\lambda}(x)\lambda(x)\right], \qquad (2.3)$$

where m_0 is the gluino bare mass and a is the lattice spacing. The fermionic field (gluino), $\lambda(x) = \lambda^a(x)T^a$, is a Majorana spinor in the adjoint representation of the gauge group. The symbol Tr implies the trace over the color indices. The normalization is given by $\mathrm{Tr}(T^aT^b) = \frac{1}{2}\delta_{ab}$. In this paper, only the case $N_c = 2$ is considered, for which the adjoint gluino field is expressed in terms of Pauli matrices σ_k as

$$\lambda = \sum_{k=1}^{3}\frac{1}{2}\sigma_k\lambda^k. \qquad (2.4)$$

The gluino field $\lambda(x)$ satisfies the Majorana condition

$$\lambda(x) = C\bar{\lambda}^T(x), \qquad (2.5)$$

where $C = \gamma_2\gamma_0$ is the charge conjugation operator. Our matrix convention for the Euclidean γ matrices is as follow:

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (2.6)$$

and

$$\gamma_k = \begin{pmatrix} 0 & -i\sigma^k \\ i\sigma^i & 0 \end{pmatrix}. \qquad (2.7)$$

The matrix γ_5 is defined to be

$$\gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad (2.8)$$

and the matrix $\sigma_{\mu\nu}$ is

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]. \qquad (2.9)$$

The anticommutator property is

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}. \qquad (2.10)$$

Finally, the Wilson parameter is fixed to be $r = 1$.

SUSY is not realized on the lattice because, as the Poincaré algebra, a sector of the superalgebra is lost. SUSY is explicitly broken in the action (2.1) and (2.3) by the lattice itself, by the gluino mass term, and by the Wilson term. Nevertheless, one can still define some transformations that reduce to the continuum supersymmetric ones, in the limit $a \to 0$. One choice is [32,33][1]:

$$\begin{aligned}
\delta U_\mu(x) &= -ag_0U_\mu(x)\bar{\xi}(x)\gamma_\mu\lambda(x) - ag_0\bar{\xi}(x)\gamma_\mu\lambda(x+a\hat{\mu})U_\mu(x),\\
\delta U_\mu^\dagger(x) &= ag_0\bar{\xi}(x)\gamma_\mu\lambda(x)U_\mu^\dagger(x) + ag_0U_\mu^\dagger(x)\bar{\xi}(x)\gamma_\mu\lambda(x+a\hat{\mu}),\\
\delta\lambda(x) &= -\frac{i}{g_0}\sigma_{\rho\tau}\mathcal{G}_{\rho\tau}(x)\xi(x),\\
\delta\bar{\lambda}(x) &= \frac{i}{g_0}\bar{\xi}(x)\sigma_{\rho\tau}\mathcal{G}_{\rho\tau}(x),
\end{aligned} \qquad (2.11)$$

where $\xi(x)$ and $\bar{\xi}(x)$ are infinitesimal Majorana fermionic parameters, while $\mathcal{G}_{\rho\tau}(x)$ is the clover plaquette operator,

$$\begin{aligned}
\mathcal{G}_{\rho\tau}(x) = -\frac{1}{8a^2}[&P_{\rho\tau}(x) - P_{\tau\rho}(x) + P_{-\rho,-\tau}(x)\\
&- P_{-\tau,-\rho}(x) + P_{\tau,-\rho}(x) - P_{-\rho,\tau}(x)\\
&+ P_{-\tau,\rho}(x) - P_{\rho,-\tau}(x)].
\end{aligned} \qquad (2.12)$$

A weak coupling perturbation theory is developed by writing the link variable as

$$U_\mu(x) = e^{-aA_\mu(x)}, \qquad (2.13)$$

and expanding it in terms of g_0. Here the gluon field is defined to be $A_\mu(x) = -ig_0A_\mu^b(x)T^b$.

In order to calculate the one-loop corrections to the SUSY WTi [which correspond to $O(g_0^2)$], we need two kinds of gluon-gluino interaction vertices. The gluon-gluino vertex,

[1]Our definition of the link variable $U_\mu(x)$ differs from that of [32] [see our definition of the plaquette (2.2)]; the two definitions are related by Hermitian conjugation.

$$V_{1\mu}^{ab,c}(p,q) = g_0 f^{abc}\left[\gamma_\mu \cos\left(\frac{p_\mu a}{2} + \frac{q_\mu a}{2}\right) - ir\sin\left(\frac{p_\mu a}{2} + \frac{q_\mu a}{2}\right)\right], \tag{2.14}$$

two-gluons-one-gluino vertex,

$$V_{2\mu\nu}^{ab,cd}(k,p) = \frac{1}{2} a g_0^2 (f^{ace}f^{ebd} + f^{ade}f^{ebc})\left[i\gamma_\mu \sin\left(\frac{p_\mu a}{2} + \frac{q_\mu a}{2}\right) - r\cos\left(\frac{p_\mu a}{2} + \frac{q_\mu a}{2}\right)\right]\delta_{\mu\nu}, \tag{2.15}$$

and the three-gluons vertex,

$$G_{3\mu\nu\lambda}^{abc}(k,k_1,k_2) = ig_0 f^{abc}\left[\delta_{\nu\lambda}\cos\left(\frac{k_\nu a}{2}\right)\sin\left(\frac{k_{2\mu}a}{2} - \frac{k_{1\mu}a}{2}\right) + \delta_{\mu\lambda}\cos\left(\frac{k_{1\lambda}a}{2}\right)\sin\left(\frac{k_\nu a}{2} - \frac{k_{2\nu}a}{2}\right) + \delta_{\mu\nu}\cos\left(\frac{k_{2\mu}a}{2}\right) \times \sin\left(\frac{k_{1\lambda}a}{2} - \frac{k_\lambda a}{2}\right)\right]. \tag{2.16}$$

These vertices are similar to QCD, and the only difference is that the fermion is a Majorana fermion in the adjoint representation of the gauge group instead of the fundamental one.

III. SUSY WTI ON THE LATTICE

The vacuum expectation value of an operator $\mathcal{O}$ is defined to be

$$\langle \mathcal{O} \rangle = \int dU\, d\lambda\, \mathcal{O} e^{-S_{\rm total}}, \tag{3.1}$$

where $S_{\rm total}$ is the total action on the lattice. By applying an infinitesimal local supersymmetric transformation, with a localized transformation parameter $\xi(x)$, the lattice WTi is written as [29],

$$\langle \mathcal{O}\nabla_\mu S_\mu(x)\rangle - 2m_0\langle \mathcal{O}\chi(x)\rangle + \left\langle \frac{\delta \mathcal{O}}{\delta\bar{\xi}(x)}\bigg|_{\xi=0}\right\rangle - \left\langle \mathcal{O}\frac{\delta S_{GF}}{\delta\bar{\xi}(x)}\bigg|_{\xi=0}\right\rangle - \left\langle \mathcal{O}\frac{\delta S_{FP}}{\delta\bar{\xi}(x)}\bigg|_{\xi=0}\right\rangle = \langle \mathcal{O} X_S(x)\rangle, \tag{3.2}$$

where S_{GF} is the gauge fixing term, S_{FP} is the Faddeev-Popov term, and $\{(\delta\mathcal{O})/[\delta\bar{\xi}(x)]\}|_{\xi=0}$ represents the contact terms (see Appendices A and B for definitions). This WTi is also discussed in [34]. $X_S(x)$ is the symmetry breaking term coming from the fact that the action is not fully invariant under (2.11). Usually $X_S(x)$ is a complicated function of the link variables and the fermionic variables [32], and its specific form depends on the choice of the lattice supercurrent.

Let us define the lattice local supercurrent as

$$S_\mu(x) = -\frac{2i}{g_0}\mathrm{Tr}\{\mathcal{G}_{\rho\tau}(x)\sigma_{\rho\tau}\gamma_\mu\lambda(x)\}, \tag{3.3}$$

while ∇_μ is the symmetric lattice derivative,

$$\nabla_\mu f(x) = \frac{1}{2a}[f(x+a\hat{\mu}) - f(x-a\hat{\mu})], \tag{3.4}$$

and $\chi(x)$ corresponds to the gluino mass term

$$\chi(x) = \frac{i}{g_0}\mathrm{Tr}\{\mathcal{G}_{\rho\tau}(x)\sigma_{\rho\tau}\lambda(x)\}. \tag{3.5}$$

In order to renormalize the lattice WTi, the operator mixing has to be taken into account. The standard way to renormalize the supercurrent is to define a subtracted $\bar{X}_S$, whose expectation value is forced to vanish in the limit $a \to 0$ [35,36]. In the case in which the operator insertion $\mathcal{O}$ in Eq. (3.2) is gauge invariant, X_S mixes with the following operators of equal or lower dimension [28]:

$$X_S(x) = \bar{X}_S(x) - (Z_S - 1)\nabla_\mu S_\mu(x) - 2\tilde{m}\chi(x) - Z_T\nabla_\mu T_\mu(x), \tag{3.6}$$

where the current T_μ reads

$$T_\mu(x) = -\frac{2}{g}\mathrm{Tr}\{\mathcal{G}_{\mu\nu}(x)\gamma_\nu\lambda(x)\}. \tag{3.7}$$

On the other hand, if the operator insertion $\mathcal{O}$ is nongauge invariant (i.e., the one involving elementary fields), the gauge dependence implies that operator mixing with nongauge invariant terms has to be taken into account in the renormalization procedure. In this case Eq. (3.6) is modified as [29,37]

$$X_S(x) = \bar{X}_S(x) - (Z_S - 1)\nabla_\mu S_\mu(x) - 2\tilde{m}\chi(x) - Z_T\nabla_\mu T_\mu(x) - \sum_j Z_{B_j}B_j. \tag{3.8}$$

The B_j's denote the occurrence of mixing, not only with nongauge invariant operators but also mixing with gauge invariant operators which do not vanish in the off-shell regime (but vanish in the on-shell regime). Consider, for example, the gauge invariant operator

$$B_0 = \frac{2}{g}\mathrm{Tr}\{\gamma_\rho[D_\tau\mathcal{G}_{\rho\tau}(x)]\lambda(x)\}, \tag{3.9}$$

which is zero imposing the equations of motion (thus, is not considered in [23]), but in the off-shell regime is nonzero and must be considered [38]. Other nongauge invariant operators, which should be included in B_j are

$$B_1 = \frac{2}{g}\partial_\rho A_\rho \not{\partial}\lambda, \qquad B_2 = \frac{2}{g} A_\rho \partial_\rho \not{\partial}\lambda, \qquad B_3 = \frac{2}{g}\not{A}\partial_\rho\partial_\rho\lambda, \tag{3.10}$$

(also reported in [32]). Finally, non-Lorentz covariant terms coming from $\nabla_\mu S_\mu$, the gauge fixing term and contact terms, which appear in the off-shell regime, should also be taken into consideration. Because the B_j do not appear in the tree-level WTi, Z_{B_j} should be $O(g^2)$ [29].

Substituting (3.8) in (3.2) we obtain the renormalized WTi

$$Z_S\langle \mathcal{O}\nabla_\mu S_\mu(x)\rangle + Z_T\langle \mathcal{O}\nabla_\mu T_\mu(x)\rangle - 2(m_0 - \tilde{m})Z_\chi^{-1}\langle \mathcal{O}\chi^R(x)\rangle + Z_{CT}\left\langle \frac{\delta \mathcal{O}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right\rangle - Z_{GF}\left\langle \mathcal{O}\frac{\delta S_{GF}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right\rangle - Z_{FP}\left\langle \mathcal{O}\frac{\delta S_{FP}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right\rangle + \sum_j Z_{B_j}\langle \mathcal{O}B_j\rangle = 0. \tag{3.11}$$

The contact terms, Faddeev-Popov term, and gauge fixing term should be renormalized; that is why in Eq. (3.11) the renormalization constants Z_{CT}, Z_{GF}, and Z_{FP} are introduced. $\langle \mathcal{O}B_j\rangle$ can in principle do mixing with S_μ and T_μ [29]. This implies that S_μ not only mixes with T_μ as was predicted in [17], but extra mixing with gauge variant operators and/or gauge invariant operators, which do not vanish in the off-shell regime, can appear. These extra mixing vanish by setting the renormalized gluino mass to zero and by imposing the on-shell condition on the gluino.

In the continuum, the existence of a renormalized SUSY WTi,

$$\partial_\mu S_\mu^R = 2m_R Z_\chi \chi, \tag{3.12}$$

is generally assumed, where S_R is the renormalized supercurrent and m_R is the renormalized gluino mass. For $m_R = 0$, we have SUSY while a nonvanishing value of m_R breaks SUSY softly. It is generally assumed that SUSY is not anomalous [Eq. (3.12) holds] and only the mass term is responsible for a soft breaking. However, in [39] the question of whether nonperturbative effects may cause a SUSY anomaly has been raised.

It is tempting to associate the normalized continuum supercurrent as

$$S_\mu^R = Z_S S_\mu + Z_T T_\mu, \tag{3.13}$$

in analogy with the lattice chiral WTi in QCD. This analogy fails, as has been pointed out in [36]. Explicit one-loop calculation may shed some light on this issue. If the correctly normalized supercurrent coincides with (3.13), then it is conserved when $m_R = 0$. This is the restoration of SUSY in the continuum limit [23].

By using general renormalization group arguments (see, for example, [36]), one can show that Z_S, Z_T, and Z_χ, being power subtraction coefficients, do not depend on the renormalization scale μ, defining the renormalization operator in Eq. (3.6). This implies that $Z_S = Z_S(g_0, m_0 a)$, $Z_T = Z_T(g_0, m_0 a)$, and $Z_\chi = Z_\chi(g_0, m_0 a)$.

In this paper, we are interested in calculating the renormalization constant for the local supercurrent (3.11) and compare with Monte Carlo results in [23]. Notice that the relation between the one-loop perturbative calculation and the numerical one is $Z_T Z_S^{-1} \equiv Z_T|_{\text{one-loop}}$. This is because, $Z_S = 1 + O(g_0^2)$, while $Z_T = O(g_0^2)$. So it is enough to calculate the coefficient Z_T in one-loop lattice perturbation theory (LPT). The numerical estimates are [23] $Z_T Z_S^{-1} = -0.039(7)$ for the point-split current and $Z_T Z_S^{-1} = 0.185(7)$ for the local current, both at $\beta = 2.3$. An estimate of $Z_T Z_S^{-1}$ for the point-split current at $\beta = 2.3$ can be obtained from the one-loop perturbative calculation in [32]. At order g_0^2 the value is $Z_T|_{\text{one-loop}} = -0.074$ [32]. In this paper, the calculation of $Z_T|_{\text{one-loop}}$ for the local supercurrent is presented.

In principle, each matrix element in Eq. (3.11) is proportional to each element of the Γ-matrix base

$$\Gamma = \{1, \gamma_5, \gamma_\alpha, \gamma_5\gamma_\alpha, \sigma_{\alpha\rho}\}, \tag{3.14}$$

but in order to determine Z_T it is enough to calculate in Eq. (3.11) the projections over two elements of the base (3.14).

IV. RENORMALIZATION CONSTANTS

We are now considering each matrix element in Eq. (3.11) with $\mathcal{O}$ (a nongauge invariant operator) given by

$$\mathcal{O} := A_\nu^b(y)\bar{\lambda}^a(z). \tag{4.1}$$

In Fourier transformation (FT), we choose p as the outcoming momentum for the gluon field A_μ and q the incoming momentum for the fermion field λ (see Fig. 1). Each matrix element can be written as

$$\langle A_\nu^b(y)\bar{\lambda}^a(z)C(x)\rangle \overset{FT}{\Rightarrow} D_F(q)[C(p,q)]_{\text{amp}} D_B(p)\delta_{ab}, \tag{4.2}$$

where $[C(p,q)]_{\text{amp}}$ can be, i.e., $\nabla_\mu S_\mu$, $\nabla_\mu T_\mu$, etc., with the external propagators amputated, $D_F(q)$ and $D_B(p)$ are the full fermion and gluon propagators, respectively, while δ_{ab} is the color structure, similar to all diagrams.

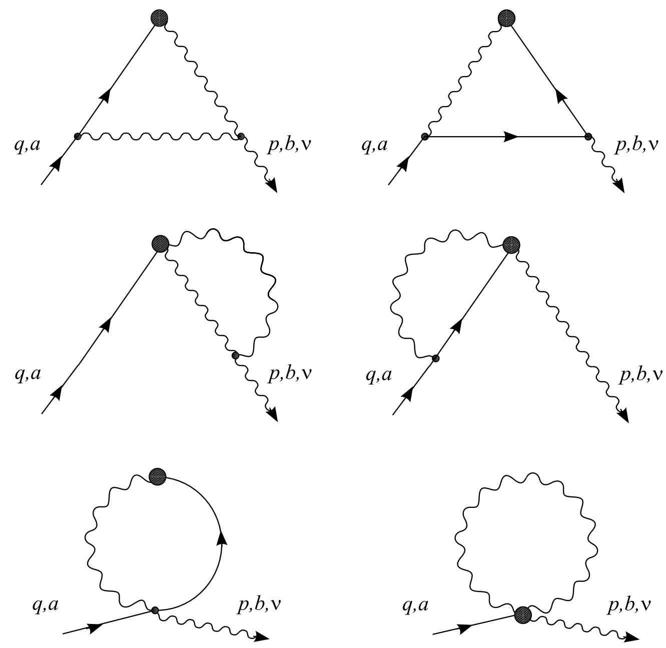

FIG. 1. Diagrams contributing for the supercurrent and the gauge fixing term. The gray blob corresponds to the operator insertion in which flows a momentum $(p - q)$.

The nontrivial part of the calculation is the determination of $[C(p, q)]_{\rm amp}$ for each matrix element in Eq. (3.11). $\langle \mathcal{O}\chi(x)\rangle$ is not considered as we set the renormalized gluino mass to zero.

In order to determine Z_T, one should pick up from each matrix element of Eq. (3.11) those terms which contain the same Lorentz structure as S_μ and T_μ, to tree level. Those operators which do not contain the same tree-level Lorentz structure as S_μ and T_μ do not enter in the determination of Z_T. Below, we present the tree-level values of the different operators of Eq. (3.11). The calculation is straightforward.

For the case of the supercurrent (3.3), the tree-level part reads

$$S_\mu^{(0)}(x) = -\frac{2i}{g}\mathrm{Tr}\{[\partial_\rho A_\tau(x) - \partial_\tau A_\rho(x)]\sigma_{\rho\tau}\gamma_\mu\lambda(x)\}. \tag{4.3}$$

Using $\mathrm{Tr}(T^aT^b) = \frac{1}{2}\delta_{ab}$ for the traces and the antisymmetry of $\sigma_{\rho\tau}$ this expression becomes

$$S_\mu^{(0)}(x) = -2\delta_{ab}\partial_\rho A_\tau^b(x)\sigma_{\rho\tau}\gamma_\mu\lambda^a(x). \tag{4.4}$$

or in FT,

$$\begin{aligned}\tilde{S}_\mu^{(0)}(r) &= \int d^4x\, e^{ir\cdot x}S_\mu^{(0)}(x)\\ &= -2i\delta_{ab}\sigma_{\rho\tau}\gamma_\mu\int d^4x\int\frac{d^4p}{(2\pi)^4}\\ &\quad\times\int\frac{d^4q}{(2\pi)^4}e^{i(r-p+q)x}p_\rho\tilde{A}_\tau^b(p)\tilde{\lambda}^a(q)\\ &= -2i\delta_{ab}\sigma_{\rho\tau}\gamma_\mu\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}\delta(r-p\\ &\quad+q)p_\rho\tilde{A}_\tau^b(p)\tilde{\lambda}^a(q),\end{aligned} \tag{4.5}$$

so we can define the vertex

$$\tilde{S}_{\mu,\tau}^{ab}(p, q) = -2i\delta_{ab}\sigma_{\rho\tau}\gamma_\mu p_\rho. \tag{4.6}$$

Concerning the operator T_μ in Eq. (3.7), the tree-level is

$$T_\mu^{(0)}(x) = i\delta_{ab}[\partial_\mu A_\tau^b(x) - \partial_\tau A_\mu^b(x)]\gamma_\tau\lambda^a(x) \tag{4.7}$$

after the FT; we define the corresponding tree-level vertex,

$$\tilde{T}_{\mu,\tau}^{ab}(p, q) = \delta_{ab}(\not{p}\delta_{\mu\tau} - p_\mu\gamma_\tau). \tag{4.8}$$

The tree-level expression for the amputated matrix element $\langle\mathcal{O}\nabla_\mu S_\mu(x)\rangle$ using the notation in Eq. (4.2) is

$$(\nabla_\mu S_\mu)_{\rm amp}^{(0)} \overset{FT}{\Rightarrow} 2(p-q)_\mu\sigma_{\rho\nu}\gamma_\mu p_\rho, \tag{4.9}$$

while the tree-level expression for the amputated matrix element $\langle\mathcal{O}\nabla_\mu T_\mu(x)\rangle$ is

$$(\nabla_\mu T_\mu)_{\rm amp}^{(0)} \overset{FT}{\Rightarrow} i(\not{p}p_\nu - p^2\gamma_\nu - \not{p}q_\nu + p\cdot q\gamma_\nu). \tag{4.10}$$

In our convention, $\nabla_\mu = i(p-q)_\mu$ is the momentum transfer of the operator insertion.

From Eqs. (4.9) and (4.10), it is easy to see that, for $p = q$, a condition which would greatly simplify the calculation because implies that the operator insertion is at zero momentum, $[\nabla_\mu S_\mu(x)]_{\rm amp}^{(0)} = [\nabla_\mu T_\mu(x)]_{\rm amp}^{(0)} = 0$. So the tree level of $\nabla_\mu S_\mu$ and $\nabla_\mu T_\mu$ cannot be distinguished at zero momentum transfer. In order to determine Z_T, different tree-level values of S_μ and T_μ are needed. To differentiate these tree-level values, general external momenta, p and q, are required.

The value of the projections over γ_α and $\gamma_\alpha\gamma_5$ for the different matrix elements in Eq. (3.11) has been performed. Denoting $\frac{1}{4}\mathrm{tr}[\gamma_\alpha(\nabla_\mu S_\mu)_{\rm amp}]$ the projection over γ_α and $\frac{1}{4}\mathrm{tr}[\gamma_\alpha\gamma_5(\nabla_\mu S_\mu)_{\rm amp}]$ the projection over $\gamma_\alpha\gamma_5$ (tr is the trace over the gamma matrices which should not be confused with Tr, the trace over the color indices), it is easy to demonstrate that

$$\begin{aligned}\tfrac{1}{4}\mathrm{tr}[\gamma_\alpha(\nabla_\mu S_\mu)_{\rm amp}^{(0)}] \overset{FT}{\Rightarrow} 2i(&p_\alpha p_\nu - p_\alpha q_\nu - p^2\delta_{\alpha\nu}\\ &+ p\cdot q\delta_{\alpha\nu}),\end{aligned} \tag{4.11}$$

where $\frac{1}{4}\mathrm{tr}(\gamma_\mu\gamma_\rho) = \delta_{\mu\rho}$ and $\frac{1}{4}\mathrm{tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = \varepsilon_{\mu\nu\rho\sigma}$, while

$$\tfrac{1}{4}\mathrm{tr}[\gamma_\alpha\gamma_5(\nabla_\mu S_\mu)^{(0)}_{\mathrm{amp}}] \overset{FT}{\Rightarrow} 2ip_\rho q_\sigma \varepsilon_{\nu\alpha\rho\sigma}. \tag{4.12}$$

Also,

$$\tfrac{1}{4}\mathrm{tr}[\gamma_\alpha(\nabla_\mu T_\mu)^{(0)}_{\mathrm{amp}}] \overset{FT}{\Rightarrow} i(p_\alpha p_\nu - p_\alpha q_\nu - p^2\delta_{\alpha\nu} + p\cdot q\delta_{\alpha\nu}) \tag{4.13}$$

and

$$\tfrac{1}{4}\mathrm{tr}[\gamma_\alpha\gamma_5(\nabla_\mu T_\mu)^{(0)}_{\mathrm{amp}}] \overset{FT}{\Rightarrow} 0. \tag{4.14}$$

Concerning the gauge fixing term, the tree-level value can be read from Eq. (B9)

$$-\left(\frac{\delta S_{GF}}{\delta\xi(x)}\bigg|_{\xi=0}\right)^{(0)}_{\mathrm{amp}} \overset{FT}{\Rightarrow} -2i\not{p}p_\nu, \tag{4.15}$$

and the projections are

$$-\frac{1}{4}\mathrm{tr}\left[\gamma_\alpha\left(\frac{\delta S_{GF}}{\delta\xi(x)}\bigg|_{\xi=0}\right)^{(0)}_{\mathrm{amp}}\right] \overset{FT}{\Rightarrow} F - 2ip_\alpha p_\nu, \tag{4.16}$$

and

$$\frac{1}{4} - \mathrm{tr}\left[\gamma_\alpha\gamma_5\left(\frac{\delta S_{GF}}{\delta\xi(x)}\bigg|_{\xi=0}\right)^{(0)}_{\mathrm{amp}}\right] \overset{FT}{\Rightarrow} 0. \tag{4.17}$$

For the contact terms, the tree level can be seen directly from Eq. (B2) (with $a \to 0$)

$$\left\langle\frac{\delta\mathcal{O}}{\delta\xi(x)}\bigg|_{\xi=0}\right\rangle^{(0)} = 2i\delta(x-y)\gamma_\nu\langle\lambda^a(y)\bar{\lambda}^b(z)\rangle + \delta(x-y)\langle A^a_\nu(y)\sigma_{\rho\tau}\mathcal{G}^b_{\rho\tau}(z)\rangle \tag{4.18}$$

or in FT (in the limit $m_0 \to 0$)

$$2i\gamma_\nu\left(\frac{1}{i\not{q}}\right)\delta_{ab} - 2ip_\rho\frac{1}{p^2}\sigma_{\rho\nu}\delta_{ab}. \tag{4.19}$$

The projections are

$$\frac{1}{4}\mathrm{tr}\left[\gamma_\alpha\left(\frac{\delta\mathcal{O}}{\delta\xi(x)}\bigg|_{\xi=0}\right)^{(0)}_{\mathrm{amp}}\right] \overset{FT}{\Rightarrow} 2i(p_\alpha q_\nu - p\cdot q\delta_{\nu\alpha} + p^2\delta_{\alpha\nu}) \tag{4.20}$$

and

$$\frac{1}{4}\mathrm{tr}\left[\gamma_\alpha\gamma_5\left(\frac{\delta\mathcal{O}}{\delta\xi(x)}\bigg|_{\xi=0}\right)^{(0)}_{\mathrm{amp}}\right] \overset{FT}{\Rightarrow} -2ip_\rho q_\sigma\varepsilon_{\nu\alpha\rho\sigma}. \tag{4.21}$$

Finally, the tree-level vertex for the operator in Eq. (3.9) is

$$(\tilde{B}_0)^{ab}_\tau(p,q) = i\delta_{ab}(p_\rho p_\tau - p^2\delta_{\rho\tau})\gamma_\rho \tag{4.22}$$

while the projection is

$$\tfrac{1}{4}\mathrm{tr}[\gamma_\alpha(B_0)^{(0)}_{\mathrm{amp}}] \overset{FT}{\Rightarrow} i(p_\alpha p_\nu - p^2\delta_{\alpha\nu}). \tag{4.23}$$

For the operators in Eq. (3.10) we have

$$\tfrac{1}{4}\mathrm{tr}[\gamma_\alpha(B_1)_{\mathrm{amp}}] \overset{FT}{\Rightarrow} ip_\nu q_\alpha, \qquad \tfrac{1}{4}\mathrm{tr}[\gamma_\alpha(B_2)_{\mathrm{amp}}] \overset{FT}{\Rightarrow} iq_\nu q_\alpha, \qquad \tfrac{1}{4}\mathrm{tr}[\gamma_\alpha(B_3)_{\mathrm{amp}}] \overset{FT}{\Rightarrow} iq^2\delta_{\alpha\nu}, \tag{4.24}$$

and

$$\tfrac{1}{4}\mathrm{tr}[\gamma_\alpha\gamma_5(B_{0,1,2,3})^{(0)}_{\mathrm{amp}}] \overset{FT}{\Rightarrow} 0. \tag{4.25}$$

The renormalization constants can be written as a power of g_0

$$Z_{\mathrm{operator}} = Z^{(0)}_{\mathrm{operator}} + g_0^2 Z^{(2)}_{\mathrm{operator}} + \cdots, \tag{4.26}$$

and also for the operators a similar expansion can be done:

$$\langle\mathrm{Operator}\rangle = \langle\mathrm{Operator}\rangle^{(0)} + g_0^2\langle\mathrm{Operator}\rangle^{(2)} + \cdots, \tag{4.27}$$

where $\langle\mathrm{Operator}\rangle^{(2)}$ is the one-loop correction while $\langle\mathrm{Operator}\rangle^{(0)}$ is the tree-level value.

Substituting Eqs. (4.26) and (4.27) into Eq. (3.11), to order g_0^2, we obtain

$$(1 + g_0^2 Z_S^{(2)})[\langle\mathcal{O}\nabla_\mu S_\mu(x)\rangle^{(0)} + g_0^2\langle\mathcal{O}\nabla_\mu S_\mu(x)\rangle^{(2)}] + g_0^2 Z_T^{(2)}\langle\mathcal{O}\nabla_\mu T_\mu(x)\rangle^{(0)} + (1 + g_0^2 Z_{CT}^{(2)})\left(\left\langle\frac{\delta\mathcal{O}}{\delta\bar{\xi}(x)}\bigg|_{\xi=0}\right\rangle^{(0)} + g_0^2\left\langle\frac{\delta\mathcal{O}}{\delta\bar{\xi}(x)}\bigg|_{\xi=0}\right\rangle^{(2)}\right) - (1 + g_0^2 Z_{GF}^{(2)})\left(\left\langle\mathcal{O}\frac{\delta S_{GF}}{\delta\bar{\xi}(x)}\bigg|_{\xi=0}\right\rangle^{(0)} + g_0^2\left\langle\mathcal{O}\frac{\delta S_{GF}}{\delta\bar{\xi}(x)}\bigg|_{\xi=0}\right\rangle^{(2)}\right) - g_0^2 Z_{FP}^{(2)}\left\langle\mathcal{O}\frac{\delta S_{FP}}{\delta\bar{\xi}(x)}\bigg|_{\xi=0}\right\rangle^{(0)} + g_0^2\sum_j Z_{B_j}^{(2)}\langle\mathcal{O}B_j(x)\rangle^{(0)} = 0. \tag{4.28}$$

At tree level we have $Z_S^{(0)} = 1$, $Z_T^{(0)} = 0$, $Z_{CT}^{(0)} = 1$, $Z_{GF}^{(0)} = 1$, $Z_{FP}^{(0)} = 0$, $Z_{B_i}^{(0)} = 0$, so the lattice WTi is

$$\langle \mathcal{O}\nabla_\mu S_\mu(x)\rangle^{(0)} + \left\langle \frac{\delta\mathcal{O}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right\rangle^{(0)} - \left\langle \mathcal{O}\frac{\delta S_{GF}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right\rangle^{(0)} = 0, \tag{4.29}$$

which holds in our lattice calculation. Equation (4.29) was previously determined in the continuum [40]. To order g_0^2 the lattice WTi is

$$\langle \mathcal{O}\nabla_\mu S_\mu(x)\rangle^{(2)} + Z_S^{(2)}\langle \mathcal{O}\nabla_\mu S_\mu(x)\rangle^{(0)} + Z_T^{(2)}\langle \mathcal{O}\nabla_\mu T_\mu(x)\rangle^{(0)} + \left\langle \frac{\delta\mathcal{O}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right\rangle^{(2)} + Z_{CT}^{(2)}\left\langle \frac{\delta\mathcal{O}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right\rangle^{(0)} - Z_{GF}^{(2)}\left\langle \mathcal{O}\frac{\delta S_{GF}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right\rangle^{(0)} - \left\langle \mathcal{O}\frac{\delta S_{GF}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right\rangle^{(2)} - \sum_j Z_{B_j}^{(2)}\langle \mathcal{O}B_j(x)\rangle^{(0)} = 0. \tag{4.30}$$

Notice that the Faddeev-Popov term $\langle \mathcal{O}[\delta S_{FP}/\delta\bar{\xi}(x)]|_{\xi=0}\rangle^{(0)}$ in Eq. (4.28) is already $O(g_0^2)$ (see Appendix B) and does not contribute to one-loop order. In Fig. 1, the Feynman diagrams for $\langle \mathcal{O}\nabla_\mu S_\mu(x)\rangle^{(2)}$ and $\langle \mathcal{O}[\delta S_{GF}/\delta\bar{\xi}(x)]|_{\xi=0}\rangle^{(2)}$ are shown, while in Fig. 2 the nonzero contribution to contact terms are presented.

Let us substitute the tree-level values of the operators in Eq. (4.30) using the projections over γ_α,

$$\frac{1}{4}\mathrm{tr}[\gamma_\alpha(\nabla_\mu S_\mu)^{(2)}_{\mathrm{amp}}] + Z_S^{(2)}2i(p_\alpha p_\nu - p_\alpha q_\nu - p^2\delta_{\alpha\nu} + p\cdot q\delta_{\alpha\nu}) + Z_T^{(2)}i(p_\alpha p_\nu - p_\alpha q_\nu - p^2\delta_{\alpha\nu} + p\cdot q\delta_{\alpha\nu}) + \frac{1}{4}\mathrm{tr}\left[\gamma_\alpha\left(\frac{\delta\mathcal{O}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right)^{(2)}_{\mathrm{amp}}\right] + Z_{CT}^{(2)}2i(p_\alpha q_\nu - p\cdot q\delta_{\alpha\nu} + p^2\delta_{\alpha\nu}) - Z_{GF}^{(2)}2ip_\alpha p_\nu - \frac{1}{4}\mathrm{tr}\left[\gamma_\alpha\left(\frac{\delta S_{GF}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right)^{(2)}_{\mathrm{amp}}\right] + \frac{1}{4}Z_{B_j}^{(2)}\mathrm{tr}\langle\gamma_\alpha\mathcal{O}B_j\rangle^{(0)} = 0, \tag{4.31}$$

and the projections over $\gamma_\alpha\gamma_5$,

$$\frac{1}{4}\mathrm{tr}[\gamma_\alpha\gamma_5(\nabla_\mu S_\mu)^{(2)}_{\mathrm{amp}}] + Z_S^{(2)}2ip_\rho q_\sigma\varepsilon_{\nu\alpha\rho\sigma} - Z_{CT}^{(2)}2ip_\rho q_\sigma\varepsilon_{\nu\alpha\rho\sigma} + \frac{1}{4}\mathrm{tr}\left[\gamma_\alpha\gamma_5\left(\frac{\delta\mathcal{O}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right)^{(2)}_{\mathrm{amp}}\right] - \frac{1}{4}\mathrm{tr}\left[\gamma_\alpha\gamma_5\left(\frac{\delta S_{GF}}{\delta\bar{\xi}(x)}\Big|_{\xi=0}\right)^{(2)}_{\mathrm{amp}}\right] + \frac{1}{4}Z_{B_j}^{(2)}\mathrm{tr}\langle\gamma_\alpha\gamma_5\mathcal{O}B_j\rangle^{(0)} = 0. \tag{4.32}$$

Our claim is that, in order to calculate $Z_T^{(2)}$ we can substitute $\frac{1}{4}Z_{B_i}^{(2)}\mathrm{tr}\langle\gamma_\alpha\mathcal{O}B_i\rangle^{(0)} \to Z_{B_0}^{(2)}i(p_\alpha p_\nu - p^2\delta_{\alpha\nu}) + Z_{B_1}^{(2)}ip_\nu q_\alpha + Z_{B_2}^{(2)}iq_\nu q_\alpha + Z_{B_3}^{(2)}iq^2\delta_{\alpha\nu}$, and $\frac{1}{4}Z_{B_i}^{(2)}\mathrm{Tr}\langle\gamma_\alpha\gamma_5\mathcal{O}B_j\rangle^{(0)} \to 0$, where Z_{B_j} correspond to the renormalization constant in Eqs. (4.23), (4.24), and (4.25). No other Z_{B_j} are needed, because there are no other B_j's that would contribute with the same Lorentz structures appearing in the tree-level of Eqs. (4.31) and (4.32).

Each matrix element in Eqs. (4.31) and (4.32) has been calculated for general p and q (off-shell regime). To deal with the IR divergencies and renormalize to one-loop order, the Kawai procedure is used [41], with the help of tabulated results in [42,43]. Once p and q have been extracted from the propagators through the Kawai procedure, the rest of the integral depends on the loop momenta which is numerically integrated. A similar renormalization procedure has been used to calculate the three-loop beta function in QCD with Wilson fermions [44] and the three-loop free energy in QCD with Wilson fermions [45,46] (for a complete study of the off-shell WTi in QCD see [47]). Typically, each matrix element contains ≈ 1000 terms (in particular dilogarithm functions depending on both external momenta which come from the diagrams with three propagators in Fig. 1). After the numerical integration, one can simplify the results in order to read the value of Z_T by setting

$$p^2 = q^2 \quad \text{and} \quad p\cdot q = 0, \tag{4.33}$$

(see Appendix C). This is still an off-shell condition (because even if $p^2 = q^2$, there are no other conditions on this expression, i.e., $q^2 = 0$), but drastically reduces the number and difficulty of the expressions (for example, the dilogarithm terms simplify).

Let us introduce, for simplicity, the notation $\Delta \equiv \mathcal{O}\nabla_\mu S_\mu(x) + \{(\delta\mathcal{O})/[\delta\bar{\xi}(x)]\}|_{\xi=0} - \mathcal{O}\{(\delta S_{GF})/[\delta\bar{\xi}(x)]\}|_{\xi=0}$. Using Eq. (4.33), we get the following dependence on p and q for $\mathrm{tr}\langle\gamma_\alpha\Delta\rangle^{(2)}$ and $\mathrm{tr}\langle\gamma_\alpha\gamma_5\Delta\rangle^{(2)}$,

$$\mathrm{tr}\langle\gamma_\alpha\Delta\rangle^{(2)} \overset{FT}{\Rightarrow} A_1 q^2 \hat{p}_\alpha \hat{p}_\nu + A_2 q^2 \hat{p}_\alpha \hat{q}_\nu + (A_3 + M_3) q^2 \delta_{\alpha\nu} + M_1 q^2 \hat{p}_\nu \hat{q}_\alpha + M_2 q^2 \hat{q}_\alpha \hat{q}_\nu + P_1 q^2 \hat{p}_\nu^2 \delta_{\nu\alpha} + P_2 q^2 \hat{q}_\nu^2 \delta_{\nu\alpha} + \cdots \tag{4.34}$$

and

$$\mathrm{tr}\langle\gamma_\alpha\gamma_5\Delta\rangle^{(2)} \overset{FT}{\Rightarrow} A_4 q^2 \hat{p}_\rho \hat{q}_\sigma \varepsilon_{\nu\alpha\rho\sigma}, \tag{4.35}$$

where the dots in Eq. (4.34) indicate that, because the simplification in Eq. (4.33) is used, some momenta dependence are missing or mixed with others, i.e., $p \cdot q \delta_{\nu\alpha}$ does not appear, while $p^2\delta_{\nu\alpha}$ is mixed with $q^2\delta_{\nu\alpha}$, (see Appendix C for notation).

It is also interesting to see the Lorentz structure of the supercurrent,

$$\begin{aligned}\mathrm{tr}\langle\gamma_\alpha S_\mu\rangle^{(2)} \overset{FT}{\Rightarrow} & N_1 q \hat{p}_\mu \hat{p}_\nu \hat{p}_\alpha + N_2 q \hat{q}_\mu \hat{p}_\nu \hat{p}_\alpha + N_3 q \hat{p}_\mu \hat{q}_\nu \hat{p}_\alpha + N_4 q \hat{q}_\mu \hat{q}_\nu \hat{p}_\alpha + N_5 q \hat{p}_\mu \hat{p}_\nu \hat{q}_\alpha + N_6 q \hat{q}_\mu \hat{p}_\nu \hat{q}_\alpha + N_7 q \hat{p}_\mu \hat{q}_\nu \hat{q}_\alpha \\ & + N_8 q \hat{q}_\mu \hat{q}_\nu \hat{q}_\alpha + Q_1 q \hat{p}_\alpha \delta_{\mu\nu} + Q_2 q \hat{q}_\alpha \delta_{\mu\nu} + Q_3 q \hat{p}_\nu \delta_{\mu\alpha} + Q_4 q \hat{q}_\nu \delta_{\mu\alpha} + Q_5 q \hat{p}_\mu \delta_{\nu\alpha} + Q_6 q \hat{q}_\mu \delta_{\nu\alpha} \\ & + R_1 q \hat{p}_\mu \delta_{\mu\nu\alpha} + R_2 q \hat{q}_\mu \delta_{\mu\nu\alpha} + \cdots,\end{aligned} \tag{4.36}$$

where the coefficients A_i, M_j, Q_k, are typically of the form

$$[C_n + C_m \mathrm{Ln}(a^2 q^2)], \tag{4.37}$$

while P_i, N_j, R_k, do not contain $\mathrm{Ln}(a^2q^2)$ terms. Here, C_n are lattice constants or numbers coming from the numerical integration and C_m are rational numbers coming from the Kawai procedure. Notice that the Lorentz structures multiplying P_i, R_k in Eqs. (4.34) and (4.36) are non-Lorentz covariant, even in the continuum limit ($a \to 0$).

From Eqs. (4.31), (4.32), (4.34), and (4.35), the following conditions can be derived:

$$\begin{aligned} A_1 &= -2iZ_S^{(2)} - iZ_T^{(2)} + 2iZ_{GF}^{(2)} - iZ_{B_0}^{(2)}, \\ A_3 + M_3 &= 2iZ_S^{(2)} + iZ_T^{(2)} - 2iZ_{CT}^{(2)} + iZ_{B_0}^{(2)} - iZ_{B_3}^{(2)}, \\ M_1 &= -iZ_{B_1}^{(2)}, \qquad M_2 = -iZ_{B_2}^{(2)} \end{aligned} \tag{4.38}$$

and

$$\begin{aligned} A_2 &= 2iZ_S^{(2)} + iZ_T^{(2)} - 2iZ_{CT}^{(2)}, \\ A_4 &= -2iZ_S^{(2)} + 2iZ_{CT}^{(2)}. \end{aligned} \tag{4.39}$$

The last two conditions can be explicitly solved for $Z_T^{(2)}$:

$$Z_T^{(2)} = -iA_2 - iA_4. \tag{4.40}$$

Equation (4.40) is the only possible solution of the system (4.31) and (4.32) for $Z_T^{(2)}$. Our result is $Z_T^{(2)}|_{\text{one-loop}} = 0.664$. A VEGAS Monte Carlo routine to perform the one-loop integration with 200×10^6 points, using the GNU Scientific Library[2] is used. To estimate the error, we take the value given by the program which is $\approx 10^{-5}$ for each integral. The calculation, once p and q has been extracted from the propagators, involves around 1300 different one-loop integrals. For each diagram, typically we have 100 different integrals. That means that the error is around 10^{-3}.

Let us compare our perturbative result with the numerical one [23], $Z_T^{NUM} \equiv Z_T/Z_S = 0.185(7)$. One has to observe that the definition used here for S_μ is not the same as in [23]. It is easy to demonstrate that $Z_T^{NUM} = \frac{1}{2} Z_T^{PT}$ [48]. To compare with the numerical results, one has to divide the perturbative value by two which gives $Z_T^{PT} = \frac{1}{2} Z_T^{(2)}|_{\text{one-loop}} = 0.332$. We are currently increasing the precision of the numerical integration to 400×10^6 points. A detailed presentation of the results in Eqs. (4.34) and (4.35) together with the result of each diagram is under way [49].

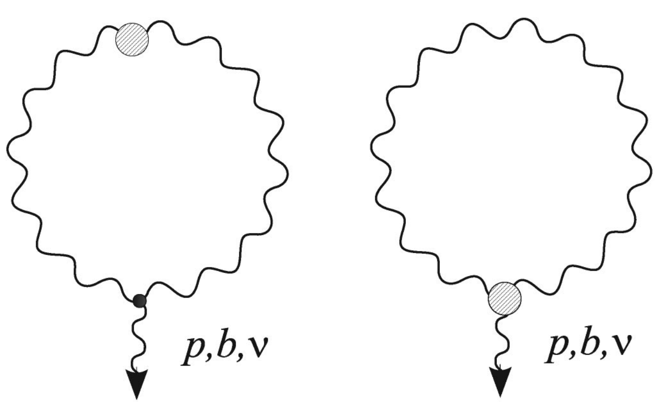

FIG. 2. Nonzero diagrams contributing to the contact terms.

V. DISCUSSION AND CONCLUSIONS

In this paper, the SUSY WTi in one-loop LPT has been investigated. A general procedure in order to get the renormalization constant for the supercurrent has been presented. In LPT it is possible to determine the value of the renormalization constant for the supercurrent from the off-shell regime of the SUSY WTi. The computation of each matrix elements of the WTi has been carried out using the symbolic language MATHEMATICA. The programs were completely written by the author together with the numerical code used for the integration. All the

[2] http//:www.gnu.org/software/gsl/

contributions have been calculated in the off-shell regime and, in order to get the value of the renormalization constant, a simplification in the external momenta (which still keeps the off-shell regime) has been applied. We are currently increasing the precision of the numerical integration, and a detailed presentation of the results is the subject of a forthcoming paper [49]. A reasonably good agreement of our perturbative result for the renormalization constant, $Z_T^{PT} = 0.332$, in comparison with the numerical one $Z_T^{NUM} = 0.185(7)$, has been achieved, taking into consideration the fact that in the numerical simulation $g_0^2 = 4/2.3$, which still corresponds to the nonperturbative region. We observe that, at least at one-loop order in perturbation theory, Z_T is finite. This result may have some theoretical implications which we are currently investigating. Also, the determination of Z_S, using another kind of gamma projection, is under investigation. It would be interesting to calculate Z_S in order to check the trace anomaly and the exact renormalization expression for Eq. (3.13). An important point to stress here is that, even in the continuum limit, we observe in Eq. (4.34) Lorentz breaking terms, which comes from the fact that we substituted X_S by Eq. (3.8). It would be interesting to see whether Eq. (4.34) is the continuum off-shell WTi. The nice point is that, once the Z_T has been determined, we can impose the on-shell condition on the gluino mass. The Lorentz breaking terms cancel out from Eq. (4.34) and the continuum WTi is recovered. At least to one-loop order, we do not observe a SUSY anomaly in $N = 1$ SYM, although a more careful study is required.

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APPENDIX A: PERTURBATIVE CALCULATION

In this appendix, we follow the lines of [29,40]. The lattice SUSY transformations of the gauge field $A_\mu(x)$ are not equal to the continuum ones. On the lattice the transformation of the gauge link $U_\mu(x)$ determines the transformation properties of $A_\mu(x)$. Writing the link variable as

$$U_\mu(x) = e^{-aA_\mu[x+(a/2)\hat{\mu}]}, \tag{A1}$$

for the SUSY transformations of the gauge link we use the symmetric choice [33]

$$\delta U_\mu(x) = -ag_0 U_\mu(x)\bar{\xi}(x)\gamma_\mu\lambda(x) - ag_0\bar{\xi}(x + a\hat{\mu})\gamma_\mu\lambda(x + a\hat{\mu})U_\mu(x).$$

These two equations determine the transformation behavior of the field $A_\mu(x)$ [29]. The FT for the gauge field is defined in the usual way:

$$A_\mu^b(x) = \int d^4k\,\tilde{A}_\mu^b(k)e^{ik\cdot[x+(a/2)\hat{\mu}]}. \tag{A2}$$

Collecting all terms until order g_0^2, we can write down the variation of the gauge field $A_\mu^b(x)$ as [29]

$$\begin{aligned}\delta A_\mu^b(x) = {}& i[\bar{\xi}(x)\gamma_\mu\lambda^b(x) + \bar{\xi}(x + a\hat{\mu})\gamma_\mu\lambda^b(x + a\hat{\mu})] \\ &+ \frac{i}{2}ag_0 f_{abc}[\bar{\xi}(x)\gamma_\mu\lambda^c(x) - \bar{\xi}(x + a\hat{\mu})\gamma_\mu\lambda^c(x \\ &+ a\hat{\mu})]A_\mu^a - \frac{i}{24}a^2g_0^2(2\delta_{ab}\delta_{cd} - \delta_{ac}\delta_{bd} \\ &- \delta_{ad}\delta_{bc})A_\mu^c A_\mu^d[\bar{\xi}(x)\gamma_\mu\lambda^a(x) + \bar{\xi}(x \\ &+ a\hat{\mu})\gamma_\mu\lambda^a(x + a\hat{\mu})],\end{aligned} \tag{A3}$$

which reduces to the continuum SUSY transformation $\delta A_\mu^a(x) = 2i\bar{\xi}\gamma_\mu\lambda^a(x)$ in the continuum limit $a \to 0$. Because in this paper we fix $N_c = 2$, some simplifications appear:

$$\begin{aligned}\mathrm{Tr}\{T^aT^bT^cT^d\} &= \tfrac{1}{8}(\delta_{ab}\delta_{cd} - f_{abe}f_{cde}), \\ f_{abe}f_{cde} &= (\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}).\end{aligned} \tag{A4}$$

Using Eq. (A3), it is possible to determine the different pieces of the WTi in Eq. (3.11), i.e., the contact terms, the gauge fixing term, and the Faddeev-Popov term. They are necessary in order to calculate the Feynman rules for one-loop order calculation. In Appendix B, the vertices coming from these pieces are presented together with the ones coming from the supercurrent.

APPENDIX B: VERTICES

Let us determine the contact terms $\{(\delta\mathcal{O})/[\delta\bar{\xi}(x)]\}|_{\xi=0}$. First of all, the variation of the operator insertion, $\mathcal{O} = A_\nu^a(y)\bar{\lambda}^b(z)$, is

$$\delta\mathcal{O} = \delta A_\nu^a(y)\bar{\lambda}^b(z) + A_\nu^a(y)\delta\bar{\lambda}^b(z). \tag{B1}$$

Substituting (A3) into (B1), after some algebra, we obtain

$$\left\langle \frac{\delta \mathcal{O}}{\delta \bar{\xi}(x)} \bigg|_{\xi=0} \right\rangle = i\delta(x-y)\gamma_\nu \langle \lambda^a(y)\bar{\lambda}^b(z)\rangle + i\delta(x-y-a\hat{\nu})\gamma_\nu\langle\lambda^a(y+a\hat{\nu})\bar{\lambda}^b(z)\rangle + \frac{i}{2}ag_0 f_{dac}\delta(x-y)\gamma_\nu\langle\lambda^c(y)A_\nu^d(y)\bar{\lambda}^b(z)\rangle - \frac{i}{2}ag_0 f_{dac}\delta(x-y-a\hat{\nu})\gamma_\nu\langle\lambda^c(y+a\hat{\nu})A^d_\nu(y)\bar{\lambda}^b(z)\rangle - \frac{i}{24}a^2g_0^2(2\delta_{ae}\delta_{cd}-\delta_{ec}\delta_{ad}-\delta_{ed}\delta_{ac})\delta(x-y)\langle A^c_\nu(y)A^d_\nu(y)\lambda^e(y)\bar{\lambda}^b(z)\rangle - \frac{i}{24}a^2g_0^2(2\delta_{ae}\delta_{cd}-\delta_{ec}\delta_{ad} - \delta_{ed}\delta_{ac})\delta(x-y-a\hat{\nu})\langle A^c_\nu(y)A^d_\nu(y)\lambda^e(y+a\hat{\nu})\bar{\lambda}^b(z)\rangle + \delta(x-y)\langle A^a_\nu(y)\sigma_{\rho\tau}\mathcal{G}^b_{\rho\tau}(z)\rangle, \tag{B2}$$

where $\mathcal{G}_{\rho\tau}(z) = -ig_0\mathcal{G}^b_{\rho\tau}(z)T^b$.

The part of the lattice action corresponding to the gauge fixing is defined as

$$S_{GF} = \frac{a^2}{2}\sum_x\Big(\sum_\rho[A^c_\rho(x) - A^c_\rho(x-a\hat{\rho})]\Big)^2 = \frac{a^4}{2}\sum_x\Big[\sum_\rho \nabla^{\text{back}}_\rho A^c_\rho(x)\Big]^2 = \frac{a^4}{2}\sum_x\Big[\sum_\rho\nabla^{\text{back}}_\rho A^c_\rho(x)\Big]\Big[\sum_\tau\nabla^{\text{back}}_\tau A^c_\tau(x)\Big], \tag{B3}$$

where $\nabla^{\text{back}}_\rho f(x) = \frac{1}{a}[f(x) - f(x-a\hat{\rho})]$. The variation of the gauge fixing term (B3) can be written as

$$\delta S_{GF} = a^4\sum_x\Big[\sum_\rho\nabla^{\text{back}}_\rho\delta A^c_\rho(x)\Big]\Big[\sum_\tau\nabla^{\text{back}}_\tau A^c_\tau(x)\Big]. \tag{B4}$$

This results in the contribution of the gauge fixing term into the WTi as

$$-\left\langle \mathcal{O}\frac{\delta S_{GF}}{\delta\xi(x)}\bigg|_{\xi=0}\right\rangle = i\gamma_\rho\bigg[\langle\lambda^c(x)\nabla^{\text{forw}}_\rho\nabla^{\text{back}}_\tau[A^c_\tau(x) + A^c_\tau(x-a\hat{\rho})]A^a_\nu(y)\bar{\lambda}^b(z)\rangle + \frac{i}{2}ag_0f_{ecf}\gamma_\rho\langle\lambda^f(x)[A^e_\rho\nabla^{\text{forw}}_\rho\nabla^{\text{back}}_\tau A^c_\tau(x) - A^e_\rho(x-a\hat{\rho})\nabla^{\text{forw}}_\rho\nabla^{\text{back}}_\tau A^c_\tau(x-a\hat{\rho})]A^a_\nu(y)\bar{\lambda}^b(z)\rangle - \frac{i}{24}a^2g_0^2\gamma_\rho(2\delta_{ec}\delta_{fd}-\delta_{ef}\delta_{cd}-\delta_{ed}\delta_{fc}) \times\langle\lambda^e(x)[A^f_\rho A^d_\rho\nabla^{\text{forw}}_\rho\nabla^{\text{back}}_\tau A^c_\tau(x) + A^f_\rho(x-a\hat{\rho})A^d_\rho(x-a\hat{\rho})\nabla^{\text{forw}}_\rho\nabla^{\text{back}}_\tau A^c_\tau(x-a\hat{\rho})]A^a_\nu(y)\bar{\lambda}^b(z)\rangle\bigg], \tag{B5}$$

where $\nabla^{\text{forw}}_\rho f(x) = \frac{1}{a}[f(x-a\hat{\rho}) - f(x)]$.

Finally, the expansion of the Faddeev-Popov action can be written as

$$S_{FP} = a^2\sum_{x,\rho\lesseqgtr 0}\bigg[\bar{\eta}^a(x)\Big(\delta_{ab} + \frac{1}{2}ag_0A^c_\rho f^{acb} + \frac{1}{12}a^2g_0^2A^c_\rho A^d_\rho f^{ace}f^{edb}\Big)\eta^b(x) - \bar{\eta}^a(x)\Big(\delta_{ab} - \frac{1}{2}ag_0A^c_\rho f^{acb} + \frac{1}{12}a^2g_0^2A^c_\rho A^d_\rho f^{ace}f^{edb}\Big)\eta^b(x+a\hat{\rho})\bigg], \tag{B6}$$

and the contribution of the Faddeev-Popov term into the WTi is

$$-\left\langle\mathcal{O}\frac{\delta S_{FP}}{\delta\bar{\xi}(x)}\bigg|_{\xi=0}\right\rangle = -\frac{ig_0}{2a}f^{gch}\sum_\rho\gamma_\rho\langle\{\bar{\eta}^g(x)\lambda^c(x)[\eta^h(x)+\eta^h(x+a\hat{\rho})] + \bar{\eta}^g(x-a\hat{\rho})\lambda^c(x)[\eta^h(x-a\hat{\rho})+\eta^h(x)] - \bar{\eta}^g(x+a\hat{\rho})\lambda^c(x)[\eta^h(x+a\hat{\rho}) + \eta^h(x)] + \bar{\eta}^g(x)\lambda^c(x)[\eta^h(x)+\eta^h(x-a\hat{\rho})]\}A^a_\nu(y)\bar{\lambda}^b[z]\rangle - \frac{ig_0^2}{4}f^{gch}f^{dcf}\sum_\rho\gamma_\rho\langle\{\bar{\eta}^g(x)\lambda^f(x)A^d_\rho(\eta^h(x)+\eta^h(x+a\hat{\rho})) - \bar{\eta}^g(x-a\hat{\rho})\lambda^f(x)A^d_\rho(x-a\hat{\rho})(\eta^h(x-a\hat{\rho})+\eta^h(x)) - \bar{\eta}^g(x+a\hat{\rho})\lambda^f(x)A^d_\rho(\eta^h(x+a\hat{\rho})+\eta^h(x)) + \bar{\eta}^g(x)\lambda^f(x)A^d_\rho(x-a\hat{\rho})(\eta^h(x)+\eta^h(x-a\hat{\rho}))]A^a_\nu(y)\bar{\lambda}^b(z)\rangle - \frac{ig_0^2}{12}f^{gce}f^{edh}\sum_\rho\gamma_\rho\langle\{\bar{\eta}^g(x)[\lambda^c(x)A^d_\rho + A^c_\rho\lambda^d(x)][\eta^h(x)-\eta^h(x+a\hat{\rho})] + \bar{\eta}^g(x-a\hat{\rho})[\lambda^c(x)A^d_\rho(x-a\hat{\rho}) + A^c_\rho(x-a\hat{\rho})\lambda^d(x)][\eta^h(x-a\hat{\rho})-\eta^h(x)] + \bar{\eta}^g(x+a\hat{\rho})\times[\lambda^c(x)A^d_\rho(x)+A^c_\rho(x)\lambda^d(x)][\eta^h(x+a\hat{\rho})-\eta^h(x)] + \bar{\eta}^g(x)[\lambda^c(x)A^d_\rho(x-a\hat{\rho}) + A^c_\rho(x-a\hat{\rho})\lambda^d(x)][\eta^h(x)-\eta^h(x-a\hat{\rho})]\}A^a_\nu(y)\bar{\lambda}^b(z)\rangle, \tag{B7}$$

where $A^c_\rho \equiv A^c_\rho(x + \frac{a\hat{\rho}}{2})$. It is possible to calculate the vertices and the corresponding Feynman diagrams, up to order g_0^2, from Eqs. (B2), (B5), and (B7) in FT.

Regarding the contact terms in Eq. (B2), all the contributions to order g_0^2 are zero except for the last line of Eq. (B2). The corresponding nonzero Feynman diagrams are shown in Fig. 2. The vertices used here are the two-gluons vertex,

$$\begin{aligned}\tilde{G}^{abc}_{2\tau\rho}(k_1, k_2) = -\frac{1}{2}g_0 f_{abc}\Big\{\sigma_{\rho\tau}\Big[\cos\Big(\frac{k_{1\rho}a}{2} + \frac{k_{2\rho}a}{2}\Big)\cos\Big(\frac{k_{1\rho}a}{2}\Big)\cos\Big(\frac{k_{1\tau}a}{2} + k_{2\tau}a\Big) + \cos\Big(\frac{k_{1\tau}a}{2} + \frac{k_{2\tau}a}{2}\Big)\cos\Big(\frac{k_{2\tau}a}{2}\Big)\\ \times\cos\Big(\frac{k_{2\rho}a}{2} + k_{1\rho}a\Big) - \sin\Big(\frac{k_{1\rho}a}{2} + \frac{k_{2\rho}a}{2}\Big)\sin\Big(\frac{k_{1\rho}a}{2}\Big)\cos\Big(\frac{k_{1\tau}a}{2}\Big) - \sin\Big(\frac{k_{1\tau}a}{2} + \frac{k_{2\tau}a}{2}\Big)\sin\Big(\frac{k_{2\tau}a}{2}\Big)\\ \times\cos\Big(\frac{k_{2\rho}a}{2}\Big)\Big] - \delta_{\rho\tau}\sum_\alpha \sigma_{\alpha\tau}\sin\Big(\frac{k_{1\tau}a}{2} + \frac{k_{2\tau}a}{2}\Big)[\sin(k_{1\alpha}a) - \sin(k_{2\alpha}a)]\Big\},\end{aligned} \tag{B8}$$

and the three-gluons vertex, which we do not report here and gives a zero contribution to the last diagram of Fig. 2.

For the gauge fixing terms in Eq. (B5), we need the vertex with one-gluon-one-gluino, which is similar to Eq. (4.15) in the continuum limit,

$$\widetilde{GF}^{ab}_{1\rho\tau}(p, k) = -\frac{4i}{a^2}\delta_{ab}\gamma_\rho\sin(k_\rho a)\sin\Big(\frac{k_\tau a}{2}\Big), \tag{B9}$$

the vertex with two-gluons-one-gluino,

$$\begin{aligned}\widetilde{GF}^{fce}_{2\tau\rho}(p, q, k) = \frac{2g_0}{a}f_{ecf}\Big\{\gamma_\rho\sin\Big(\frac{k_\rho a}{2} + \frac{q_\rho a}{2}\Big)\sin\Big(\frac{q_\rho a}{2}\Big)\\ \times\sin\Big(\frac{q_\tau a}{2}\Big) - \gamma_\tau\sin\Big(\frac{k_\tau a}{2} + \frac{q_\tau a}{2}\Big)\\ \times\sin\Big(\frac{k_\tau a}{2}\Big)\sin\Big(\frac{k_\rho a}{2}\Big)\Big\},\end{aligned} \tag{B10}$$

and finally the three-gluons-one-gluino vertex (nonsymmetrized),

$$\begin{aligned}\widetilde{GF}^{efdc}_{3\rho\sigma\tau}(p, k, q, t) = -\frac{1}{3}g_0^2(2\delta_{ec}\delta_{fd} - \delta_{ef}\delta_{cd} - \delta_{ed}\delta_{fc})\\ \times\gamma_\rho\delta_{\rho\sigma}\sin\Big(\frac{k_\rho a}{2} + \frac{q_\rho a}{2} + \frac{t_\rho a}{2}\Big)\\ \times\sin\Big(\frac{t_\rho a}{2}\Big)\sin\Big(\frac{t_\tau a}{2}\Big).\end{aligned} \tag{B11}$$

For the Faddeev-Popov terms in Eq. (B7), we need one-gluino-ghost-antighost vertex

$$\begin{aligned}\widetilde{FP}^{cgh}_\rho(p, -q, k) = \frac{4g_0}{a}f^{gch}\sum_\rho \gamma_\rho\cos\Big(\frac{k_\rho a}{2}\Big)\sin\Big(\frac{q_\rho a}{2}\Big)\\ \times\cos\Big(\frac{k_\rho a}{2} - \frac{q_\rho a}{2}\Big)\end{aligned} \tag{B12}$$

and one-gluino-one-gluon-ghost-antighost vertex

$$\begin{aligned}\widetilde{FP}^{cdgh}_{1\rho}(p, t, -q, k) = -\frac{2i}{3}g_0^2(f^{gce}f^{edh}\\ + f^{gde}f^{ech})\gamma_\rho\Big\{\sin\Big(\frac{k_\rho a}{2}\Big)\sin\Big(\frac{q_\rho a}{2}\Big)\\ \times\cos\Big(\frac{k_\rho a}{2} + \frac{t_\rho a}{2} - \frac{q_\rho a}{2}\Big)\Big\}.\end{aligned} \tag{B13}$$

As we can see from Eqs. (B12) and (B13), the vertices are already order g_0 and g_0^2, so plugging into Eq. (4.28) is already more than $O(g_0^2)$. This implies that the Faddeev-Popov terms do not contribute to order g_0^2.

Concerning the vertices of S_μ for a one-loop calculation, we need the vertices corresponding to one-gluon-one-gluino, the two-gluons-one-gluino, and finally the three-gluons-one-gluino. They can be calculated from (3.3). The vertex one-gluon-one-gluino [using Eq. (4.5)] is

$$\tilde{S}^{abc}_{1\mu,\rho\tau}(q, p) = -\frac{2i}{a}\delta_{ab}\sigma_{\rho\tau}\gamma_\mu\cos\Big(\frac{p_\tau a}{2}\Big)\sin(p_\rho a), \tag{B14}$$

which reduces to the continuum one in the limit $a \to 0$ [see Eq. (4.6)], while the vertex two-gluons-one-gluino is

$$\begin{aligned}\tilde{S}^{abc}_{2\mu,\rho\tau}(q, p_1, p_2) = \frac{1}{2}g_0 f_{abc}\Big\{\sigma_{\rho\tau}\gamma_\mu\Big[\cos\Big(\frac{p_{1\rho}a}{2} + \frac{p_{2\rho}a}{2}\Big)\cos\Big(\frac{p_{1\rho}a}{2}\Big)\cos\Big(\frac{p_{1\tau}a}{2} + p_{2\tau}a\Big) + \cos\Big(\frac{p_{1\tau}a}{2} + \frac{p_{2\tau}a}{2}\Big)\cos\Big(\frac{p_{2\tau}a}{2}\Big)\\ \times\cos\Big(\frac{p_{2\rho}a}{2} + p_{1\rho}a\Big) - \sin\Big(\frac{p_{1\rho}a}{2} + \frac{p_{2\rho}a}{2}\Big)\sin\Big(\frac{p_{1\rho}a}{2}\Big)\cos\Big(\frac{p_{1\tau}a}{2}\Big) - \sin\Big(\frac{p_{1\tau}a}{2} + \frac{p_{2\tau}a}{2}\Big)\sin\Big(\frac{p_{2\tau}a}{2}\Big)\\ \times\cos\Big(\frac{p_{2\rho}a}{2}\Big)\Big] - \delta_{\rho\tau}\sum_\alpha \sigma_{\alpha\tau}\gamma_\mu\sin\Big(\frac{p_{1\tau}a}{2} + \frac{p_{2\tau}a}{2}\Big)[\sin(p_{1\alpha}a) - \sin(p_{2\alpha}a)]\Big\}.\end{aligned} \tag{B15}$$

We do not present here the three-gluons-one-gluino vertex because its contribution to the last Feynman diagram for the supercurrent, in Fig. 1, is zero by color considerations.

APPENDIX C: OFF-SHELL REGIME

In order to separate the contribution of T_μ and S_μ at tree level, we cannot impose $p = q$, which would greatly simplify the calculation. We are forced to use general external momenta p and q [while the momentum transfer of the operator insertion is $(p - q) \neq 0$, see Fig. 1]. Once the external momenta has been extracted from the propagators, in order to get the value of Z_T, the simplifications $p^2 = q^2$ and $p \cdot q = 0$ are used. This is still an off-shell regime which simplifies the dilogarithm functions.

At one-loop order, two propagator integrals are tabulated in [41,42] while three propagator integrals on the lattice are tabulated in [43] in terms of lattice constants plus the following continuum counterparts:

$$I_{0;1\mu;2\mu\nu;3\mu\nu\rho}(p,q) = \frac{1}{\pi^2}\int d^4k \frac{1; k_\mu; k_\mu k_\nu; k_\mu k_\nu k_\rho}{k^2(k+p)^2(k+q)^2}. \tag{C1}$$

With the help of [50,51], one can give the expression for $I_0(p,q)$ and write down recursively $I_{1\mu}(p,q)$, $I_{2\mu\nu}(p,q)$, $I_{3\mu\nu\rho}(p,q)$ in terms of the scalar functions p^2, q^2, $p \cdot q$, and I_0, plus Lorentz structures. As an example [49], $I_0(p,q)$ is a complicated function of p and q, in terms of the dilogarithm as follows:

$$I_0(p,q) = \frac{1}{\Delta}\Big[\mathrm{Li}_2\Big(\frac{p\cdot q - \Delta}{q^2}\Big) - \mathrm{Li}_2\Big(\frac{p\cdot q + \Delta}{q^2}\Big) + \frac{1}{2}\mathrm{Ln}\Big(\frac{p\cdot q - \Delta}{p\cdot q + \Delta}\Big)\mathrm{Ln}\Big(\frac{(q-p)^2}{q^2}\Big)\Big], \tag{C2}$$

where Δ is the triangle function defined as

$$\Delta^2 = (p\cdot q)^2 - p^2 q^2 \tag{C3}$$

and

$$\mathrm{Li}_2(x) = -\int_1^x \frac{\mathrm{Ln}t}{t-1}dt \tag{C4}$$

is the dilogarithm.

Following Ref. [51], where a tensor decomposition of $I_{1\mu}(p,q)$, $I_{2\mu\nu}(p,q)$, $I_{3\mu\nu\rho}(p,q)$ is used, it is shown that all the integrals can be written in terms of I_0 and others scalars:

$$I_{1\mu} = I_1(p,q)p_\mu + I_1(q,p)q_\mu, \tag{C5}$$

where

$$I_1(p,q) = \frac{1}{\Delta^2}\Big[q^2\mathrm{Ln}\Big(\frac{(q-p)^2}{q^2}\Big) - p\cdot q\mathrm{Ln}\Big(\frac{(q-p)^2}{p^2}\Big) + \frac{q^2 p\cdot(q-p)}{2}I_0\Big]. \tag{C6}$$

The integral $I_{2\mu\nu}$ is symmetric in μ and ν as well as under $p \leftrightarrow q$ and, hence, has the following tensor decomposition:

$$I_{2\mu\nu} = \delta_{\mu\nu}I_A + \Big(p_\mu p_\nu - \frac{\delta_{\mu\nu}}{4}p^2\Big)I_B(p,q) + \Big(p_\mu q_\nu + q_\mu p_\nu - \frac{\delta_{\mu\nu}}{2}p\cdot q\Big)I_C + \Big(q_\mu q_\nu - \frac{\delta_{\mu\nu}}{4}q^2\Big) \times I_B(q,p), \tag{C7}$$

where I_A, I_B, and I_C are symmetric under $p \leftrightarrow q$ and tabulated in [51]. In this reference an explicit expression for $I_{3\mu\nu\lambda}$ is presented, which is quite complicated and we do not report here.

The general result for arbitrary p and q using (C2), (C6), and (C7) and the corresponding expression for $I_{3\mu\nu\lambda}$ (in [51]) contains huge quantities or terms (sometimes up to 1000 terms). Therefore a simplification which still leaves us in the off-shell regime is required. Let us rewrite (C3) in the following way:

$$\Delta^2 = -p^2q^2\Big(-\frac{(p\cdot q)^2}{p^2q^2} + 1\Big), \tag{C8}$$

where $p \cdot q = pq\cos\alpha$, where $0 < \alpha < \pi$. This implies that $(p-q)^2 = p^2 + q^2 - 2pq\cos\alpha$.

By using Eq. (C8), it is possible to simplify I_0, I_1, I_2, and I_3. In fact,

$$\Delta = i\sqrt{p^2q^2\Big(1 - \frac{(p\cdot q)^2}{p^2q^2}\Big)} = i\sqrt{p^2}\sqrt{q^2}\sqrt{1-\cos\alpha^2}. \tag{C9}$$

Substituting (C9) in (C2), we have

$$I_0 = \frac{1}{ipq\sqrt{1-\cos\alpha^2}}\Big[\mathrm{Li}_2\Big(\frac{pq\cos\alpha - ipq\sqrt{1-\cos\alpha^2}}{q^2}\Big) - \mathrm{Li}_2\Big(\frac{pq\cos\alpha + ipq\sqrt{1-\cos\alpha^2}}{q^2}\Big) + \frac{1}{2}\mathrm{Ln}\Big(\frac{pq\cos\alpha - ipq\sqrt{1-\cos\alpha^2}}{pq\cos\alpha + ipq\sqrt{1-\cos\alpha^2}}\Big)\mathrm{Ln}\Big(\frac{p^2+q^2-2pq\cos\alpha}{q^2}\Big)\Big]. \tag{C10}$$

Simplifying, we have

$$I_0 = \frac{1}{ipq\sin\alpha}\Big\{\mathrm{Li}_2\Big[\frac{p}{q}(\cos\alpha - i\sin\alpha)\Big] - \mathrm{Li}_2\Big[\frac{p}{q}(\cos\alpha + i\sin\alpha)\Big] + \frac{1}{2}\mathrm{Ln}\Big(\frac{\cos\alpha - i\sin\alpha}{\cos\alpha + i\sin\alpha}\Big)\mathrm{Ln}\Big(\frac{p^2}{q^2} + 1 - \frac{2p}{q}\cos\alpha\Big)\Big\}, \tag{C11}$$

and finally

$$I_0 = \frac{1}{ipq\sin\alpha}\left\{\left[\mathrm{Li}_2\left(\frac{p}{q}\exp^{-i\alpha}\right) - \mathrm{Li}_2\left(\frac{p}{q}\exp^{i\alpha}\right)\right] + \frac{1}{2}\mathrm{Ln}(\exp^{-2i\alpha})\mathrm{Ln}\left(\frac{p^2}{q^2} + 1 - \frac{2p}{q}\cos\alpha\right)\right], \quad \text{(C12)}$$

or

$$I_0 = \frac{1}{pq\sin\alpha}\left\{\frac{1}{i}\left[\mathrm{Li}_2\left(\frac{p}{q}\exp^{-i\alpha}\right) - \mathrm{Li}_2\left(\frac{p}{q}\exp^{i\alpha}\right)\right] - \alpha\mathrm{Ln}\left(\frac{p^2}{q^2} + 1 - \frac{2p}{q}\cos\alpha\right)\right\}. \quad \text{(C13)}$$

Using Eq. (C13), we can now simplify the recursive expressions for I_1, I_2, and I_3. Let us define

$$\hat{p}_\mu \equiv \frac{p_\mu}{\sqrt{p^2}}, \quad \text{(C14)}$$

where clearly $|\hat{p}_\mu| = 1$. The simplification in Eq. (4.33) corresponds to $\alpha = \frac{\pi}{2}$ (then, we have $\cos\alpha = 0$ and $\sin\alpha = 1$) and $p^2 = q^2$, which corresponds to the substitution $p \longrightarrow q$ in all the results.[3]

[3]Notice that the substitution $p \longrightarrow q$ is made *after* the external momenta from the diagrams has been extracted from the propagators, and the IR have been dealt, using the Kawai procedure. The calculation of each diagram has been done in a completely off-shell regime, but in order to read the values of the renormalization constants a proper simplification should be done. Only at the very end of the calculation is the substitution $p \longrightarrow q$ made.

[1] N. Seiberg and E. Witten, Nucl. Phys. **B426**, 19 (1994); **B430**, 485 (1994); N. Seiberg, Phys. Rev. D **49**, 6857 (1994).
[2] A. Feo, Nucl. Phys. Proc. Suppl. **119**, 198 (2003).
[3] J. R. Hiller, S. S. Pinsky, and U. Trittmann, Nucl. Phys. **B661**, 99 (2003).
[4] J. Nishimura, S.-J. Rey, and F. Sugino, J. High Energy Phys. 02 (2003) 032.
[5] S. Catterall and S. Karamov, Phys. Rev. D **68**, 014503 (2003).
[6] M. Beccaria and C. Rampino, Phys. Rev. D **67**, 127701 (2003).
[7] M. Beccaria, M. Campostrini, and A. Feo, Nucl. Phys. Proc. Suppl. **119**, 891 (2003).
[8] M. Campostrini and J. Wosiek, Phys. Lett. B **550**, 121 (2002).
[9] K. Itoh, M. Kato, H. Sawanaka, H. So, and N. Ukita, J. High Energy Phys. 02 (2003) 033.
[10] D. B. Kaplan, E. Katz, and M. Unsal, J. High Energy Phys. 05 (2003) 037.
[11] A. G. Cohen, D. B. Kaplan, E. Katz, and M. Unsal, J. High Energy Phys. 08 (2003) 024.
[12] J. Giedt, Nucl. Phys. **B668**, 138 (2003).
[13] M. Lüscher, Nucl. Phys. **B549**, 295 (1999).
[14] M. Lüscher, Nucl. Phys. **B568**, 162 (2000).
[15] M. Lüscher, hep-th/0102028.
[16] M. Lüscher, J. High Energy Phys. 06 (2000) 028.
[17] G. Curci and G. Veneziano, Nucl. Phys. **B292**, 555 (1987).
[18] I. Montvay, Nucl. Phys. B, Proc. Suppl. **53**, 853 (1997).
[19] G. Koutsoumbas, I. Montvay, A. Pap, K. Spanderen, D. Talkenberger, and J. Westphalen, Nucl. Phys. B, Proc. Suppl. **63**, 727 (1998).
[20] R. Kirchner, S. Luckmann, I. Montvay, K. Spanderen, and J. Westphalen, Nucl. Phys. B, Proc. Suppl. **73**, 828 (1999).
[21] R. Kirchner, S. Luckmann, I. Montvay, K. Spanderen, and J. Westphalen, Phys. Lett. B **446**, 209 (1999).
[22] DESY-Münster Collaboration, I. Campos *et al.*, Eur. Phys. J. C **11**, 507 (1999).
[23] DESY-Münster-Roma Collaboration, F. Farchioni *et al.*, Eur. Phys. J. C **23**, 719 (2002).
[24] I. Montvay, Int. J. Mod. Phys. A **17**, 2377 (2002).
[25] DESY-Münster Collaboration, A. Feo *et al.*, Nucl. Phys. B, Proc. Suppl. **83**, 661 (2000).
[26] M. Lüscher, Nucl. Phys. **B413**, 481 (1994).
[27] I. Montvay, Nucl. Phys. **B466**, 259 (1996); I. Montvay, Comput. Phys. Commun. **109**, 144 (1998).
[28] A. Donini, M. Guagnelli, P. Hernandez, and A. Vladikas, Nucl. Phys. **B523**, 529 (1998).
[29] DESY-Münster-Roma Collaboration, F. Farchioni *et al.*, Nucl. Phys. B, Proc. Suppl. **106**, 941 (2002).
[30] DESY-Münster-Roma Collaboration, F. Farchioni *et al.*, Nucl. Phys. B, Proc. Suppl. **94**, 791 (2001).
[31] I. Montvay and G. Münster, *Quantum Fields on a Lattice* (Cambridge University Press, Cambridge, England, 1994).
[32] Y. Taniguchi, Phys. Rev. D **63**, 014502 (2000).
[33] T. Galla, Diploma thesis, University of Münster, 1999.
[34] B. de Wit and D. Freedman, Phys. Rev. D **12**, 2286 (1975).
[35] M. Bochicchio, L. Maiani, G. Martinelli, G. Rossi, and M. Testa, Nucl. Phys. **B262**, 331 (1985).
[36] M. Testa, J. High Energy Phys. 04 (1998) 002.
[37] A. Vladikas (private communication), and internal notes from the DESY-Münster-Roma Collaboration.
[38] A. Vladikas (private communication).
[39] A. Casher and Y. Shamir, hep-th/9908074.
[40] T. Galla, internal notes from the DESY-Münster-Roma Collaboration.
[41] H. Kawai, R. Nakayama, and K. Seo, Nucl. Phys. **B189**, 40 (1981).
[42] R. K. Ellis and G. Martinelli, Nucl. Phys. **B235**, 93 (1984); **B249**, 750 (1985).

[43] H. Panagopoulos and E. Vicari, Nucl. Phys. **B332**, 261 (1990).
[44] C. Christou, A. Feo, H. Panagopoulos, and E. Vicari, Nucl. Phys. **B525**, 387 (1998); **B608**, 479(E) (2001).
[45] B. Alles, A. Feo, and H. Panagopoulos, Phys. Lett. B **426**, 361 (1998); **553**, 337 (2003).
[46] B. Alles, M. Campostrini, A. Feo, and H. Panagopoulos, Phys. Lett. B **324**, 433 (1994).
[47] S. Caracciolo, P. Menotti, and A. Pelissetto, Nucl. Phys. **B375**, 195 (1992).
[48] F. Farchioni (private communication).
[49] A. Feo (to be published).
[50] G. 't Hooft and M. J. G. Veltman, Nucl. Phys. **B153**, 365 (1979).
[51] J. S. Ball and T.W. Chiu, Phys. Rev. D **22**, 2542 (1980); Phys. Rev. D **22**, 2550 (1980).