

Factorization, power corrections, and the pion form factor

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This paper is an investigation of the pion form factor utilizing recently developed effective field theory techniques. The primary results reported are both the transition and electromagnetic form factors are corrected at order Λ/Q due to time ordered products which account for deviations of the pion from being a state composed purely of highly energetic collinear quarks in the lab frame. The usual higher twist wave function corrections contribute only at order Λ^2/Q^2 , when the quark mass vanishes. In the case of the electromagnetic form factor the Λ/Q power correction is enhanced by a power of $1/\alpha_s(Q)$ relative to the leading order result of Brodsky and Lepage, if the scale $\sqrt{\Lambda Q}$ is nonperturbative. This enhanced correction could explain the discrepancy with the data.

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I. INTRODUCTION

Making predictions for hadronic observables is extremely difficult given the complexity of the theory of strong interactions (QCD). In the absence of a solution to the theory, we are forced to accept reduced predictive power. We can make predictions only after we have extracted some crucial related information from the data. Moreover, the necessary information is usually in the form of nonlocal matrix elements and not just fixed couplings. Even with these lowered expectations, it is still highly nontrivial to find observables which we can predict from first principles. The primary tool at our disposal is factorization [1]. Observables which are “factorizable” can be separated into long and short distance contributions. Asymptotic freedom allows for a calculation of the short distance piece via a perturbative expansion in the coupling, while the long distance piece is in principle calculable but in practice must be extracted from experiment. The predictive power lies in the fact that factorization implies universality. That is, the same nonperturbative factor can appear in predictions for disparate processes.

Proving that a certain observable is factorizable is a highly nontrivial process. The pioneering works on the subject [2–4] were based upon diagrammatic techniques and can be quite intricate. Using these techniques, factorization was shown to leading in $1/Q$, where Q is the large energy scale in the process. For example, the photon-pion transition form factor which is defined via ($q = p_\pi - p_\gamma$)

$$\langle \pi^0(p_\pi) | J_\mu(0) | \gamma(p_\gamma, \epsilon) \rangle = -ie F_{\pi\gamma} p_\pi^\nu \epsilon^\rho q^\sigma \epsilon_{\nu\rho\sigma}, \quad (1)$$

can be written to leading order (in $1/Q = 1/\sqrt{|q^2|}$) as

$$F_{\pi\gamma} = \frac{2f_\pi}{Q^2} \int_0^1 dx C_1(x, Q/\mu) \phi_\pi(x, \mu), \quad (2)$$

while the pion electromagnetic (EM) form factor defined as

$$\langle \pi(p) | J_\mu(0) | \pi(p') \rangle = F_\pi(Q^2)(p_\mu + p'_\mu), \quad (3)$$

may be written as

$$F_\pi = (Q_u - Q_d) \frac{f_\pi^2}{Q^2} \times \int_0^1 dx dy T_1(x, y, Q/\mu) \phi_\pi(x, \mu) \phi_\pi(y, \mu). \quad (4)$$

In these expressions ϕ_π carries the universal, nonperturbative, information about the structure of the pion and is defined via

$$-if_\pi \delta^{ab} \phi_\pi(x, \mu) = \int \frac{dz}{2\pi} e^{-ixz\bar{n}p} \langle \pi_{n,p}^a | \bar{q}(z) \Gamma_\pi^b \times W(z, -z) q(-z) | 0 \rangle, \quad (5)$$

where W is a lightlike Wilson line needed for manifest gauge invariance. C_1 and T_1 are the perturbatively calculable, process dependent, high energy Wilson coefficients. At asymptotically large values of Q , the pion wave function is dominated by its first moment and approaches the form $\phi_\pi(x) \propto x(1-x)f_\pi$. Experimentally, the prediction for the transition form factor agrees at the 10% level, while the EM form factor is off by a factor of order one. Thus, the relevant question is, how large are the subleading corrections in each case? This paper undertakes the task of systematically categorizing the power corrections to these processes, including the contribution from the so-called “end-point region.” The issue of the breakdown of factorization will also be discussed.

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II. SCET

The results in this paper are derived utilizing recent developments in the so-called Soft-Collinear Effective Theory (SCET) [5]. In this approach, factorization proofs simplify because modes with varying virtualities are separated at the level of the Lagrangian [6]. Proving factorization becomes tantamount to determining if the theory properly accounts for the IR physics of the process under consideration. Perhaps the true power of the effective field theory approach, as applied to exclusive processes, is that it enables one to account for power corrections in a systematic fashion including the so-called end-point contributions.¹

To build the proper effective field theory one isolates the degrees of freedom responsible for the nonanalytic behavior of the low energy theory. Usually this corresponds to simply integrating out massive fields, but there are cases [7] where one wishes to explicitly separate certain subset of fluctuations of the fields. This separation allows for manifest power counting, which in turn makes the classification of power corrections relatively simple. In SCET the relevant modes in the IR are collinear $p_c = (p_c^+, p_c^-, p_c^\perp) = (n \cdot p_c, \bar{n} \cdot p_c, p_c^\perp) \sim Q(\lambda^2, 1, \lambda)$, soft $p_s \sim Q(\lambda, \lambda, \lambda)$ and ultrasoft (usoft) $p_{us} \sim Q(\lambda^2, \lambda^2, \lambda^2)$, where $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$, and $\lambda \ll 1$ is the expansion parameter. The relevant modes are fixed by the external momenta and use of the Coleman-Norton theorem. In [6], the authors chose $\lambda \propto \Lambda/Q$ as this fixes the transverse momenta of the external lines to be of order Λ , as it should be physically. This also means that the usoft modes have virtuality less than Λ , that is, their wavelength is longer than the confinement radius. These “hyper-confining modes” should not contribute to physical processes. In all the cases discussed in [6], it was shown that these modes do indeed cancel in the matrix elements. There is one additional mode which may contribute in the case when there are at least two collinear directions, namely, the so-called “Glauber region,” whose momenta scale as $(\lambda^2, \lambda^2, \lambda)$. While these modes typically are not relevant in exclusive processes, this has yet to be shown within the context of SCET.

A. The Need for an Intermediate Theory

New subtleties in the theory arise when one considers power corrections to exclusive processes due to what is known as the end-point region, which is defined as the contribution to the process where the hadron is in an asymmetric configuration. That is, one or more of the constituents carries a parametrically small part of the longitudinal momentum.² In the context of effective field

¹These contributions are sometimes also referred to as the “Feynman” or “soft” regions.

²We will always be discussing processes/frames where the hadron is nearly lightlike.

theory language, this configuration corresponds to a sub-leading fluctuation. To see this note that to preserve manifest power counting, we take the pion to be an eigenstate of the leading order Hamiltonian of the effective theory, which as will be discussed below, only contains collinear fields. As such, the overlap of the pion with a field which contains a net soft quantum number³ will be nil. Thus, in the effective field theory formalism these soft regions of the wave function show up via time ordered products wherein a collinear field fluctuates to a soft field and then back to a collinear field. These time ordered products must involve an operator which couples a collinear field with a field whose momenta scale as $(\Lambda, \Lambda, \Lambda)$. However, momentum conservation forbids the coupling of such a field with a collinear field whose momentum scale is $(Q, \Lambda^2/Q, \Lambda)$.

This issue was addressed in [9], where it was pointed out that such interactions will be properly accounted for, if one considers working in two stages. In the initial theory, labeled SCETI, the scaling parameter λ is order $\sqrt{\Lambda/Q}$; this theory is valid at scales below Q but above $\sqrt{\Lambda Q}$. In this theory interactions between collinear $(1, \lambda^2, \lambda)$ and ultrasoft $(\lambda^2, \lambda^2, \lambda^2)$ gluons are permitted. At the scale $\sqrt{\Lambda Q}$, we match onto a second theory SCETII, where λ now scales as Λ/Q . In doing this matching we integrate out modes with invariant mass $q^2 \geq \Lambda Q$. In SCETII, there are no interactions between collinear and soft modes.⁴

III. THE TRANSITION FORM FACTOR

A. Leading Order

In the effective theory each mode is interpolated by a distinct field and scales homogeneously in λ . For instance, collinear modes with large light-cone momentum in the n direction are interpolated by ξ_n and A_n^μ for fermions and gauge bosons, respectively. These fields have support only over momenta of order λ^2 , as their large light-cone and transverse momenta have been scaled out. The leading order Lagrangian for these collinear fields with momenta in the n direction, is given by

$$\mathcal{L}_c^{(0)} = \bar{\xi}_n \left[in \cdot D + i \not{D}_\perp^c \frac{1}{i\bar{n} \cdot D_c} i \not{D}_\perp^c \right] \frac{\not{n}}{2} \xi_n + \mathcal{L}_{cg}^{(0)}, \quad (6)$$

with

$$\begin{aligned} i\bar{n} \cdot D_c &= \bar{\mathcal{P}} + g\bar{n} \cdot A_n, & iD_c^\perp &= \mathcal{P}^\perp + gA_n^\perp, \\ in \cdot D &= in \cdot \partial + gn \cdot A_{us} + gn \cdot A_n. \end{aligned} \quad (7)$$

The operators $\bar{\mathcal{P}}$ and \mathcal{P}^\perp are derivative-like operators

³That is, an operator which has no overlap with a purely collinear state.

⁴One can think of the ultrasoft modes in SCETI as becoming the soft modes in SCETII.

whose eigenvalues are the large light-cone and transverse momenta, respectively. The gluon action, $\mathcal{L}_{cg}^{(0)}$, can be found in [5].

Let us start by considering the pion transition form factor. The leading order matching for this process was performed in [6]. This process involves the scattering of a highly virtual photon and a quark-antiquark constituent pair of an on-shell photon in the n direction. We will work to leading order in α_{em} .⁵ By integrating out the hard off-shell intermediate state in the $\gamma^* + \gamma \rightarrow q + \bar{q}$ process, we generate a two quark operator in the SCETI. The most general spin structures for currents with two collinear particles moving in the same or opposite directions are [6]

$$\bar{\xi}_n \{ \not{n}, \not{n} \gamma_5, \not{n} \gamma_\perp^\mu \} \xi_n; \quad \bar{\xi}_{\bar{n}} \{ 1, \gamma_5, \gamma_\perp^\mu \} \xi_n. \quad (8)$$

From this result we can see that, for the case at hand, there is only one relevant operator structure which interpolates for the pion, namely, $\not{n} \gamma_5$, thus the leading order matching result is of the form

$$O_{\omega_1, \omega_2}^{(0)} = i \epsilon_{\mu\nu}^\perp (\bar{\xi}_n W)_{\omega_1} \not{n} \gamma_5 (W^\dagger \xi_n)_{\omega_2}, \quad (9)$$

where the isospin structure has been suppressed, and $\epsilon_{\mu\nu}^\perp = \frac{1}{2} \bar{n}^\rho n^\sigma \epsilon_{\mu\nu\rho\sigma}$. Here the W 's are the Fourier transforms of lightlike Wilson lines

$$W(y, \infty) = P \exp \left[i \int_y^\infty \bar{n} \cdot A_n(\lambda \bar{n}) d\lambda \right], \quad (10)$$

and $\omega_{1,2}$ the total collinear momenta of the jetlike structure $W^\dagger \xi$ and $\bar{\xi} W$ respectively. Typically each operator is accompanied by label subscripts, such labels are implied if they are not explicit. There is an implied sum over all label momenta such as $\omega_{1,2}$, which will be restricted by momentum conservation when we take a matrix element. We may then decouple the usoft modes from the collinear modes in the Lagrangian via field redefinitions [10]

$$\hat{\xi}_{(n, \bar{n})} = Y_{(n, \bar{n})}^\dagger \xi_{(n, \bar{n})}, \quad \hat{A}_{(n, \bar{n})} = Y_{(n, \bar{n})}^\dagger A_{(n, \bar{n})} Y_{(n, \bar{n})}, \quad (11)$$

where Y is an usoft Wilson line defined as

$$Y_n = P \exp \left[i \int_y^\infty n \cdot A(\lambda n) d\lambda \right]. \quad (12)$$

This redefinition has the effect of decoupling usoft lines from collinear lines in the action at the cost of introducing Y factors into the operator O_0 . However, we can see that since the Y are usoft and carry no large light-cone momenta they will cancel in O_0 as a consequence of unitarity. The Y 's will, however, show up in subleading Lagrangian and external operators. We will drop the

⁵If we were to treat the photon like a hadron, then the analysis is almost identical to the case of the EM form factor discussed in later paragraphs.

hatted symbol from here on, and thus the reader should assume that all fields have been redefined as in (11).

The matching onto SCETII is trivial at this order. The off-shellness of the external collinear lines is reduced to being less than ΛQ . Taking the matrix element of this operator between the vacuum and one pion state yields the usual leading order result in terms of the pion wave function (2).

B. Power Corrections

Power corrections arise from either matching onto higher order operators or from including corrections from the subleading Lagrangian into time ordered products (TOP's). The order λ action introduces couplings between usoft and collinear fields and is given by [11–13]

$$\begin{aligned} \mathcal{L}_{\xi\xi}^{(1)} &= \bar{\xi}_n i \not{D}_\perp^{us} \frac{1}{i\bar{n} \cdot D_c} i \not{D}_\perp^c \frac{\not{n}}{2} \xi_n + \text{H.c.}, \\ \mathcal{L}_{cg}^{(1)} &= \frac{2}{g^2} \text{tr} \{ [i \mathcal{D}^\mu, i D_c^\perp{}^\nu] [i \mathcal{D}_\mu, i D_\nu^{\perp us}] \} + \text{g.f.}, \quad (13) \\ \mathcal{L}_{\xi q}^{(1)} &= i g \bar{\xi}_n \frac{1}{i\bar{n} \cdot D_c} \not{B}_\perp^c W q_{us} + \text{H.c.} \end{aligned}$$

with $\mathcal{D}^\mu = n^\mu \bar{n} \cdot D_c / 2 + D_c^{\perp \mu} + \bar{n}^\mu n \cdot D / 2$, g.f. denotes gauge fixing terms, and

$$i g B_{\mu\perp}^c \equiv [i\bar{n} \cdot D_c, i D_{\mu c}^\perp] \equiv \bar{n}_\nu (G_n)^{\nu\mu\perp}. \quad (14)$$

The collinear gauge invariant field strength is

$$(G_n)^{\mu\nu} = -\frac{i}{g} [i \mathcal{D}_n^\mu + g A_{n,q}^\mu, i \mathcal{D}_n^\nu + g A_{n,q'}^\nu]. \quad (15)$$

We will also need the order λ^2 Lagrangian

$$\begin{aligned} \mathcal{L}_{\xi\xi}^{(2)} &= \bar{\xi}_n \left(\not{D}_{us}^\perp \frac{i}{\bar{n} \cdot D_c} \not{D}_{us}^\perp - \not{D}_c^\perp \frac{i}{\bar{n} \cdot D_c} \bar{n} \right. \\ &\quad \left. \cdot D_{us} \frac{1}{\bar{n} \cdot D_c} \not{D}_c^\perp \right) \frac{\not{n}}{2} \xi_n. \quad (16) \end{aligned}$$

As discussed above, it is important to understand that time ordered products corrections correspond to perturbations of the states in the effective theory. That is, in the effective theory the pion state is not the physical pion state, it contains only collinear modes in SCETII. The true pion is an eigenstate of the full Hamiltonian, so including perturbations into the time ordered product accounts for this difference in a systematic fashion. There is a direct analogy with heavy quark effective theory [14] which is perhaps illuminating. In the limit where the quark mass is taken to infinity, the B meson is completely static. It is true that in the physical meson the quark has some kinetic energy, but these effects can be included when considering power corrections, via time ordered products with subleading operators. In this way we can build up the full meson state order by order in the

inverse quark mass. For a pedagogical discussion on the subject see [8].

Matching onto SCETI we may generate order λ operators by inserting $B_\mu^{c\perp}$ into $O^{(0)}$,

$$O^{(1)} = i\bar{n}_\nu \epsilon_{\perp\mu\rho} (\bar{\xi}_n W)_{\omega_1} \bar{\not{D}}_\nu \gamma_5 (W^\dagger B_n^{\rho\perp} W)_{\omega_2} (W^\dagger \xi_n)_{\omega_3}. \quad (17)$$

But this operator does not interpolate for the pion.⁶ Operators with insertions of D_\perp^c , can be absorbed into $O^{(1)}$. We may also consider quark mass effects, which may arise from either the expansion of the fields via

$$q = \left[1 + \frac{1}{\bar{n} \cdot iD_c} (i\not{D}_\perp^c - m_q) \frac{\not{n}}{2} \right] \xi_n, \quad (18)$$

or from including mass effects in perturbative matching. Using a simple spurion [10] analysis and the fact that there is only one possible nonvanishing Dirac structure in the effective theory which violates chirality, it is simple to show that matching cannot generate any $O(\lambda^0)$ operators⁷ which are linear in the quark mass to all orders in perturbation theory. However, the introduction of a quark mass generates a new $O(\lambda)$ operator

$$O_m^{(1)} = i\epsilon_{\mu\nu}^\perp (\bar{\xi}_n W)_{\omega_1} \epsilon_{\alpha\beta}^\perp (W [i\not{D}_\perp^\alpha \gamma_\perp^\beta - i\bar{D}_\perp^\alpha \gamma_\perp^\beta] W^\dagger)_{\omega_2} \times (W^\dagger \xi_n)_{\omega_3}. \quad (19)$$

The spontaneous breaking of chiral symmetry would lead these contributions to be numerically enhanced.

The possible order λ^2 operators which are bilinear in the collinear quarks are

$$O_a^{(2)} = i\epsilon_{\mu\nu}^\perp (\bar{\xi}_n W)_{\omega_1} (W_n^\dagger n \cdot DW^\dagger)_{\omega_2} \bar{\not{D}}_\nu \gamma_5 (W^\dagger \xi_n)_{\omega_3}, \quad (20)$$

$$O_b^{(2)} = i\epsilon_{\mu\nu}^\perp (\bar{\xi}_n W)_{\omega_1} (W \not{D}_\perp^c W^\dagger)_{\omega_2} \times (W \not{D}_\perp^c W^\dagger)_{\omega_3} \bar{\not{D}}_\nu \gamma_5 (W^\dagger \xi_n)_{\omega_4}, \quad (21)$$

where the appearance of the momentum subscript implies the existence of nontrivial Wilson coefficients which account for possible insertions of the operator $\bar{n} \cdot D_c$. $O_b^{(2)}$ is a representative of a class of operators with two transverse covariant derivatives acting in all possible ways. The Wilson coefficients of these operators will be related by reparameterization invariance [12]. Other operators involving collinear field strength operators can be expressed in terms of linear combinations of these operators. In addition, it is possible to generate operators with one collinear and one usoft quark, i.e., $\bar{\xi}_n W \not{n} \gamma_5 q_{us}$, but the contribution from this operator will be order λ^3 since an additional insertion of a subleading operator is necessary

⁶We work in a frame where the transverse momentum of the pion vanishes.

⁷Though the operator itself is leading order it would come in suppressed by m_q/Q .

to get a nonvanishing matrix element between pion and vacuum.

We now match onto the lower theory SCETII, where external virtualities are restricted to be less than ΛQ . In doing so, the external states will pick out a subset of the collinear modes whose transverse label momentum is of order Λ . In addition, the usoft modes get relabeled to be soft modes. For Wilson lines this transformation is denoted by $Y \rightarrow S$. $O_a^{(2)}$ will now scale as $O(\Lambda^2/Q^2)$, since $n \cdot D$ picks out the smallest component of the collinear field which now scales in this way. The operators in the class of $O_b^{(2)}$ will have vanishing Wilson coefficients when matching onto SCETII since we set the external transverse momenta to zero. Formally this occurs because

$$\mathcal{P}^\perp \xi^{II} = 0. \quad (22)$$

However, there are a class of identical operators with usoft derivatives as well which will give Λ^2/Q^2 corrections in SCETII. This scaling arises because usoft modes in SCETI match onto soft modes in SCETII which scale as Λ/Q . In addition, we will generate new operators by considering the time ordered products in SCETI

$$T_1 = \int d^4x d^4y T[O^{(0)} \mathcal{L}_i^{(1)}(x) \mathcal{L}_i^{(1)}(y)], \quad (23)$$

$$T_2 = \int d^4x T[O^{(0)} \mathcal{L}_j^{(2)}(x)],$$

where $\mathcal{L}_i^{(1)}$ and $\mathcal{L}_i^{(2)}$ are any of the first and second order Lagrangian corrections, respectively. Since the external states have vanishing transverse momenta as far as the label operators in SCETI are concerned, any operator with an odd number of transverse derivatives vanishes. TOPs with an even number of derivatives need not vanish since there exists nonvanishing Wick contractions.⁸ The TOPs T_1 and T_2 , however, will have nonvanishing matching coefficients and will contribute at order Λ/Q .

To see how this scaling comes about, note that while the power corrections to the Lagrangian $\mathcal{L}_i^{(1,2)}$ will scale down from being order Λ/Q to Λ^2/Q^2 ; this will not happen within the time ordered product. The reason for this is that once we lower the virtuality of the external lines down to $q^2 \sim \Lambda^2$, the collinear field scaling goes from $\sqrt{\Lambda Q}$ to Λ , but internal collinear lines connecting operators at distinct points will still scale as they did in SCETI. This leads to enhancements in time ordered products relative to local operators. For instance, if we consider T_2 , then a Wick contraction of collinear lines between the two operators leads to a fermion line which is off-shell by ΛQ . Thus the effective scaling of the product of these two fields is Λ/Q , whereas if they carried off-shellness Λ^2 , the scaling would be just Λ^2/Q^2 , hence the enhancement. Furthermore, if the scale

⁸Note this is not true for the ‘‘local’’ operators $O_b^{(2)}$.

$\sqrt{\Lambda Q}$ is perturbative we may integrate out these off-shell modes perturbatively.

Thus the correction to the leading order result may be written formally as

$$\begin{aligned} \frac{Q^3}{2i} \delta F_{\pi\gamma} &= \int d^4x d^4y \int d\omega_j C(\omega_j) \\ &\times \langle \pi_{n,p} | O_{\omega_j}^{(0)} \mathcal{L}_i^{(1)}(x) \mathcal{L}_i^{(1)}(y) | 0 \rangle \\ &+ \int d^4x \int d\omega_j C(\omega_j) \langle \pi_{n,p} | \\ &\times | O_{\omega_j}^{(0)} \mathcal{L}_i^{(2)}(x) | 0 \rangle. \end{aligned} \quad (24)$$

Note that the large light-cone momenta (ω_i) flowing out of the leading order operator will now flow into the vertices of the subleading Lagrangian insertions.

It is interesting to note that in the SCET formalism *all* the power corrections are factorable, in the sense that they can be written as products of matrix elements of various types of fields.⁹ That is, factorization is manifest in all the power corrections, since after the field redefinition, none of the various types of fields communicate. As a consequence of this, all of the soft fields may be factored into vacuum matrix elements which are independent of the hadron, since these fields have no overlap with the hadrons. This implies that, under our working assumption that SCET, in particular, SCETII, as formulated in [5,9] is the appropriate effective field theory for the above processes, the soft pieces of all hadrons are universal. Thus, there is hope that we can extract the soft structure functions and use them to make predictions in disparate processes.

IV. THE ELECTROMAGNETIC FORM FACTOR

A. Leading Order

Let us now consider the EM form factor. As we will see, the existence of two jets will have important ramifications. In this case, the Lagrangian splits into two pieces, one for each type of collinear mode, which do not communicate,

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\bar{n}} + \mathcal{L}_n. \quad (25)$$

The usual leading order Brodsky-Lepage (BL) result was regained in SCET by matching the full QCD current onto four quark operators which are generated at order $\alpha_s(Q)$ [6],

⁹I thank Mark Wise and Iain Stewart for emphasizing this point to me.

$$\begin{aligned} O_1^{(0)} &= (n + \bar{n})_\mu (\bar{\xi}_n W_n)_{\omega_1} \bar{\psi} \gamma_5 (W_n^\dagger \xi_n)_{\omega_2} \\ &\times (\bar{\xi}_{\bar{n}} W_{\bar{n}})_{\omega_3} \bar{\psi} \gamma_5 (W_{\bar{n}}^\dagger \xi_{\bar{n}})_{\omega_4}, \\ O_8^{(0)} &= (n + \bar{n})_\mu (\bar{\xi}_n W_n)_{\omega_1} \bar{\psi} \gamma_5 T^a (W_n^\dagger \xi_n)_{\omega_2} \\ &\times (\bar{\xi}_{\bar{n}} W_{\bar{n}})_{\omega_3} \bar{\psi} \gamma_5 T^a (W_{\bar{n}}^\dagger \xi_{\bar{n}})_{\omega_4}. \end{aligned} \quad (26)$$

The subscripts of these operators denote their color representation.

Scaling the fields by the usoft Wilson lines has no effect. The fields of opposing directions do not couple and, therefore, the matrix element of these operators between pion states of opposing lightlike directions factorizes into the result (5) with the octet contribution vanishing.

B. Power Corrections

When we are concerned with power corrections, we must consider the matching onto operators which may not necessarily have nonvanishing overlap with the pions. A nonzero overlap can be achieved via some TOPs with subleading terms in the Lagrangian. In the EM case we may also match onto a quark bilinear

$$O^{(-2)}(\omega_1, \omega_2) = (\bar{\xi}_{\bar{n}} W_{\bar{n}})_{\omega_1} \Gamma (W_n^\dagger \xi_n)_{\omega_2}, \quad (27)$$

where the possible Dirac structures Γ are given in (8). This operator is enhanced by a factor λ^{-2} relative to the four quark operator since each collinear field scales as λ . However, charge conjugation implies that the only structure with nonvanishing Wilson coefficient is γ_μ^\perp which will not contribute for the case of the pion but will for the case of the ρ . At next order in matching the operators

$$O_a^{(-1)} = \bar{n}_\mu (\bar{\xi}_{\bar{n}} \tilde{\mathcal{D}}_{c,\bar{n}}^\perp W_{\bar{n}})_{\omega_1} (W_n^\dagger \xi_n)_{\omega_2}, \quad (28)$$

$$O_b^{(-1)} = n_\mu (\bar{\xi}_{\bar{n}} W_{\bar{n}})_{\omega_3} (W_n^\dagger \tilde{\mathcal{D}}_{c,n}^\perp \xi_n)_{\omega_4}, \quad (29)$$

are generated at order α_s^0 . Current conservation and reparameterization invariance will relate the Wilson coefficients (C_a, C_b) of these two operators such that $\omega_1 C_a + \omega_4 C_b = 0$. Note that we have introduced here two covariant derivatives with labels n and \bar{n} , referring to the two distinct collinear sectors.

As opposed to the previous subleading operators, $O_{a,b}$ are not invariant under the ultrasoft field redefinitions (11). In the redefined basis we have

$$O_a^{(-1)} = \bar{n}_\mu (\bar{\xi}_{\bar{n}} \tilde{\mathcal{D}}_{c,\bar{n}}^\perp W_{\bar{n}} Y_{\bar{n}})_{\omega_1} (Y_n^\dagger W_n^\dagger \xi_n)_{\omega_2}, \quad (30)$$

$$O_b^{(-1)} = n_\mu (\bar{\xi}_{\bar{n}} W_{\bar{n}} Y_{\bar{n}})_{\omega_3} (Y_n^\dagger W_n^\dagger \tilde{\mathcal{D}}_{c,n}^\perp \xi_n)_{\omega_4}. \quad (31)$$

The leading order matrix elements between back to back pions of this operator vanish. To generate a nonzero overlap, all that is needed is the proper insertion of subleading operators which will inject one collinear quark into each jet. Furthermore, to get a nonvanishing Wilson coefficient

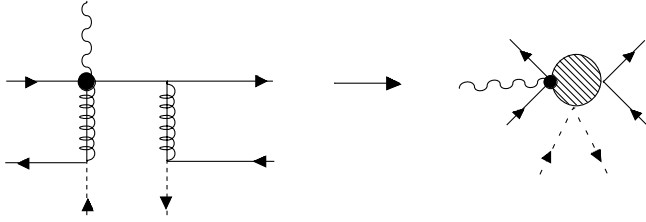


FIG. 1. Integrating out modes with virtuality $\sqrt{\Lambda Q}$ lead to operators which represent the soft piece of the wave function. The solid (hatched) circle represents modes off-shell by Q ($\sqrt{\Lambda Q}$).

cient, the TOP should contain an even number of insertions of \not{D}_c^\perp [9]. This can be accomplished via a usoft (which becomes soft in SCETII) partonic fluctuation. The TOP

$$T_{us}(\omega_i) = \int d^4x_i T\{[C_a(\omega_1, \omega_2)O_a^{(-1)}\mathcal{L}_{\xi\xi}^{n(1)}(x_1) + C_b(\omega_3, \omega_4)O_b^{(-1)}\mathcal{L}_{\xi\xi}^{\bar{n}(1)}(x_1)]\mathcal{L}_{\xi q}^{\bar{n}(1)}(x_2)\mathcal{L}_{\xi q}^{n(1)}(x_3)\}, \quad (32)$$

gives an order Λ/Q contributions to the EM pion form factor of the form

$$Q^2\delta F_\pi = \int d\omega_i \langle \pi_{n,p} | T_{us}(\omega_i) + \text{H.c.} | \pi_{\bar{n},p'} \rangle. \quad (33)$$

This type of correction was anticipated in [2] and is simply a correction to the pion state. Note that each of the operators must be accompanied by a subleading Lagrangian which contains a collinear gluon moving in the opposite direction relative to the gluon in the operators. This insertion is needed to ensure that there are an even number of transverse covariant derivatives in both directions.

If the scale ΛQ were perturbative then we could integrate the intermediate gluons as depicted in Fig. 1. The ultrasoft quark lines, being fluctuations of virtuality Λ^2 , would only be closed once the matrix element is taken. There is an additional operator which gives an identical contribution but with $n \leftrightarrow \bar{n}$. TOPs at this order involving insertions of the quark mass have zero matching coefficients as they involve powers of the external transverse momentum. However, there will be quark mass dependence in the nonperturbative matrix elements in SCETII which will be relevant for SU(3) [15] and are only down by m/Λ .

Notice that the usual leading order BL result is proportional to $\alpha_s(Q)$ while this soft contribution scales as $\alpha_s(\sqrt{Q\Lambda})^2$. The data only reaches $Q^2 = 10 \text{ GeV}^2$, with error bars as large as the signal itself at larger Q [16]. Given that Λ is of order 1 GeV, the scale $\sqrt{\Lambda Q}$ is likely nonperturbative over most of the range of the data, giving the TOP an enhancement of $1/\alpha_s(Q)$ relative to the leading order BL result. Thus, we see that the lack of concordance between theory and data may be due to the enhanced power correction in the EM form factor. Of course, there are other possible reasons for the discrepancy. It could be that using the asymptotic wave function is a poor approximation for these values of Q . However, the fact that the transition form factor seems to agree with the data [17], within theory errors, lends credence to the possibility that the enhanced power correction discussed here could be the real culprit. Finally, the discrepancy could also be due to the extrapolation of the $\gamma^* p \rightarrow \pi n$ data to the pion pole [18].

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