$B \to M_1 M_2$: Factorization, charming penguins, strong phases, and polarization

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Using the soft-collinear effective theory we derive the factorization theorem for the decays $B \to M_1 M_2$ with $M_{1,2} = \pi, K, \rho, K^*$, at leading order in Λ/E_M and Λ/m_b. The results derived here apply even if $\alpha_s(E_M \Lambda)$ is not perturbative, and we prove that the physics sensitive to the $E\Lambda$ scale is the same in $B \to M_1 M_2$ and $B \to M$ form factors. We argue that $c\bar{c}$ penguins could give long-distance effects at leading order. Decays to two transversely polarized vector mesons are discussed. Analyzing $B \to \pi\pi$ we find predictions for $B^0 \to \pi^0 \pi^0$ and $|V_{ub}| f_+^{B\to\pi}(0)$ as a function of γ.

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Decays of B mesons to two light mesons are important for the study of CP violation in the standard model. In [1] it was suggested that since $m_b, E_M \gg \Lambda, m_M$ the amplitudes should factorize into simpler nonperturbative objects, and the proposed factorization theorem was checked at one-loop. This approach is often referred to as "QCD factorization" (QCDF). Factorization has also been considered in the "perturbative QCD" (pQCD) approach [3]. These approaches rely on a perturbative expansion in $\alpha_s(E_M \Lambda)$. The results obtained from factorization are quite predictive and may allow us to answer fundamental questions about the standard model. At the current time several important issues remain to be answered. These include (i) the extent to which the results are model-independent consequences of QCD (since QCD is a predictive theory any model-independent limit must give the same answer in different approaches). A complete proof of a factorization theorem will answer this question. (ii) Unambiguous definitions of any nonperturbative hadronic parameters which appear are required. This allows the universality of parameters to be understood, as well as making clear the extent to which predictions rely on model dependent assumptions about parameter values. (iii) Does the power expansion converge? If power suppressed contributions really compete with leading order contributions as some studies [4,5] suggest then the expansion cannot be trusted. In this case the only hope is a systematic modification of the power counting to promote these effects to leading order, or an identification of certain observables that are free from this problem.

The soft collinear effective theory (SCET) [6,7] provides the necessary tools to address these issues. A first study of SCET factorization for $B \to \pi\pi$ has been made in [8]. In this paper we go beyond Refs. [1,3,8] in several ways. We first reduce the SCET operator basis to its minimal form and extend it to allow for all $B \to M_1 M_2$ decays (including two vectors). Our results show that all of the so-called "hard spectator" contributions are already present in the form factors, just with different hard Wilson coefficients. We also derive a form of the factorization theorem which does not rely on a perturbative expansion in $\alpha_s(E_M \Lambda)$, and show that the nonperturbative parameters are still the same as those in the $B \to M$ form factors. In our analysis long distance $c\bar{c}$ penguins [9,10] are investigated, but are left unfactorized. For the values of m_b and m_c realized in nature, we give an argument why these contributions can be leading order. This is contrary to expectations that they are power suppressed [1,5], but in agreement with expectations in [5,9,10]. The presence of these contributions could introduce large LO nonperturbative strong phases. Even in observables that are free from charming penguins our results differ phenomenologically from Ref. [1]. In particular while the power counting in Ref. [1] assumes a hierarchy in parameters $\zeta_J^{B\pi} \ll \zeta^{B\pi}$, we show that SCET allows for other possibilities such as $\zeta_J^{B\pi} \sim \zeta^{B\pi}$ [ζ and ζ_J are defined through Eqs. (12), (18), and (24)]. We demonstrate that the LO SCET results are in agreement with current $B \to \pi\pi$ data, and find that current central values favor $\zeta_J^{B\pi} \gtrsim \zeta^{B\pi}$, albeit with fairly large uncertainties.

We set $M = P$ when discussing pseudoscalars, $M = V$ for vectors, and use an M to denote either. The decays $B \to M_1 M_2$ are mediated in full QCD by the weak $\Delta B = 1$ Hamiltonian, which for $\Delta S = 0$ reads

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \Big(C_1 O_1^p + C_2 O_2^p + \sum_{i=3}^{10,7\gamma,8g} C_i O_i \Big), \tag{1}$$

where the Cabibbo-Kobayashi-Maskawa (CKM) factor is $\lambda_p^{(f)} = V_{pb} V_{pf}^*$ with $f = d$. The standard basis of $f = d$ operators are (with $O_1^p \leftrightarrow O_2^p$ relative to [11])

$$
\begin{aligned}
O_1^p &= (\overline{p}b)_{V-A}(\overline{d}p)_{V-A},\\
O_2^p &= (\overline{p}_\beta b_\alpha)_{V-A}(\overline{d}_\alpha p_\beta)_{V-A},\\
O_{3,4} &= \{(\overline{d}b)_{V-A}(\overline{q}q)_{V-A}, (\overline{d}_\beta b_\alpha)_{V-A}(\overline{q}_\alpha q_\beta)_{V-A}\},\\
O_{5,6} &= \{(\overline{d}b)_{V-A}(\overline{q}q)_{V+A}, (\overline{d}_\beta b_\alpha)_{V-A}(\overline{q}_\alpha q_\beta)_{V+A}\},\\
O_{7,8} &= \frac{3e_q}{2}\{(\overline{d}b)_{V-A}(\overline{q}q)_{V+A}, (\overline{d}_\beta b_\alpha)_{V-A}(\overline{q}_\alpha q_\beta)_{V+A}\},\\
O_{9,10} &= \frac{3e_q}{2}\{(\overline{d}b)_{V-A}(\overline{q}q)_{V-A}, (\overline{d}_\beta b_\alpha)_{V-A}(\overline{q}_\alpha q_\beta)_{V-A}\},\\
O_{7\gamma,8g} &= -\frac{m_b}{8\pi^2}\overline{d}\sigma^{\mu\nu}\{eF_{\mu\nu}, gG^a_{\mu\nu}T^a\}(1+\gamma_5)b.
\end{aligned}
\tag{2}
$$

Here the sum over $q = u, d, s, c, b$ is implicit, α, β are color indices and e_q are electric charges. The $\Delta S = 1$ H_W is obtained by replacing $(f = d) \to (f = s)$ in Eqs. (1) and (2). The coefficients in Eq. (1) are known at next-to-leading-log order [11]. In the naive dimensional regularization scheme, taking $\alpha_s(m_Z) = 0.118$ and $m_b = 4.8$ GeV gives $C_{7\gamma}(m_b) = -0.317$, $C_{8g}(m_b) = -0.149$ and

$$
\begin{aligned}
C_{1-10}(m_b) = \{&1.080, -0.177, 0.011, -0.033, 0.010,\\
&-0.040, 4.9\times10^{-4}, 4.6\times10^{-4},\\
&-9.8\times10^{-3}, 1.9\times10^{-3}\}.
\end{aligned}
\tag{3}
$$

The relevant scales in $B \to M_1M_2$ are m_b, m_c, the jet scale $\sqrt{E\Lambda}$ and Λ. Varying Λ between 100–1000 MeV the jet scale is numerically in the range $\sqrt{E\Lambda} \simeq 0.5$–$1.6$ GeV. Integrating out $\sim m_b$ fluctuations, the effective Hamiltonian in SCET$_\mathrm{I}$ [12] can be written as

$$
\begin{aligned}
H_W = \frac{2G_F}{\sqrt{2}}\sum_{n,\bar n}\Bigg\{&\sum_i\int[d\omega_j]_{j=1}^3 c_i^{(f)}(\omega_j)Q_{if}^{(0)}(\omega_j)\\
&+\sum_i\int[d\omega_j]_{j=1}^4 b_i^{(f)}(\omega_j)Q_{if}^{(1)}(\omega_j) + \mathcal{Q}_{c\bar c} + \ldots\Bigg\},
\end{aligned}
\tag{4}
$$

where $c_i^{(f)}$ and $b_i^{(f)}$ are Wilson coefficients, the ellipses are higher order terms in Λ/Q, $Q = \{m_b, E\}$, and $\mathcal{Q}_{c\bar c}$ denotes operators appearing in long distance charm effects as in Fig. 1. Penguin contractions with light quark loops are included in matching onto $Q_{if}^{(0,1)}$ since their long distance contributions are power suppressed [1]. The long-distance contributions occur when one or both of the quark lines in the penguin loop become soft or collinear. In matching onto SCET these quark lines are left uncontracted and give rise to higher order operators which are power suppressed. An example which gives rise to a six quark operator is given in Fig. 2.

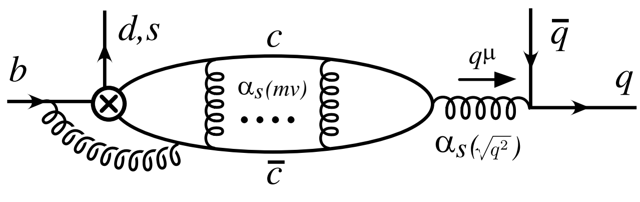

FIG. 1. Example of long distance charming penguins. The mv gluons are nonperturbative and LO soft gluons are exchanged by the b, c, $\bar c$ and spectator quark which is not shown.

In penguin contractions with charm quarks the situation is different due to the threshold region. For the $c\bar c$ system the offshellness depends on the value of $q^2 = m_b^2 x$, and long distance contributions from $x \to 0$ or $x \to 1$ are suppressed [4]. However, for $q^2 \sim 4m_c^2$ the charm quarks are moving nonrelativistically. This region corresponds to momentum fractions $x \simeq 4m_c^2/m_b^2 \simeq 0.4$ in the middle of the distribution $\phi_M(x)$. These contributions have one $\alpha_s(2m_c)$, but cannot be calculated perturbatively. Using nonrelativistic QCD power counting they are "suppressed" by $\mathcal{O}(v)$ with $v \simeq 0.4$–0.5. The velocity v can be treated in principle as an independent expansion parameter. Thus we conclude that these contributions may be leading order, and comparable in size to other penguin terms such as those from the small Wilson coefficients C_{3-6}. A rigorous account of these long distance $c\bar c$ penguin contractions can only be obtained by deriving a factorization theorem for them, however we do not attempt to do so here, and therefore do not write down operators for $\mathcal{Q}_{c\bar c}$.

In Eq. (4) the $\mathcal{O}(\lambda^0)$ operators are (sum over $q = u, d, s$)

$$
\begin{aligned}
Q_{1d}^{(0)} &= [\bar u_{n,\omega_1}\bar{\not n}P_L b_v][\bar d_{\bar n,\omega_2}\not n P_L u_{\bar n,\omega_3}],\\
Q_{2d,3d}^{(0)} &= [\bar d_{n,\omega_1}\bar{\not n}P_L b_v][\bar u_{\bar n,\omega_2}\not n P_{L,R} u_{\bar n,\omega_3}],\\
Q_{4d}^{(0)} &= [\bar q_{n,\omega_1}\bar{\not n}P_L b_v][\bar d_{\bar n,\omega_2}\not n P_L q_{\bar n,\omega_3}],\\
Q_{5d,6d}^{(0)} &= [\bar d_{n,\omega_1}\bar{\not n}P_L b_v][\bar q_{\bar n,\omega_2}\not n P_{L,R} q_{\bar n,\omega_3}],
\end{aligned}
\tag{5}
$$

with $Q_{is}^{(0)}$ obtained by swapping $\bar d \to \bar s$. In Eq. (5) the "quark" fields with subscripts n and $\bar n$ are products of collinear quark fields and Wilson lines with large momenta ω_i. For example,

$$
\bar u_{n,\omega} = [\bar\xi_n^{(u)} W_n \delta(\omega - \bar n\cdot\mathcal{P}^\dagger)],
\tag{6}
$$

where $\bar\xi_n$ creates a collinear quark moving along the n

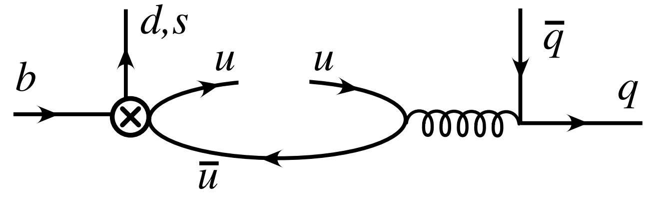

FIG. 2. Example of a long distance light quark penguin which matches onto a power suppressed operator. The $\bar q$ goes in the $\bar n$ direction, the q goes in the n direction, the broken u quark line is soft or collinear and the $\bar u$ and gluon remain hard.

direction, or annihilates an antiquark. The b_v field is the standard usoft heavy quark effective theory field with Lagrangian $\mathcal{L}_h = \bar{b}_v iv \cdot D b_v$. For a complete basis we also need operators with octet bilinears. We take these to be $Q_i^{(0)}$ with $T^A \otimes T^A$ color structure, for example

$$Q_{\overline{1d}}^{(0)} = [\bar{u}_{n,\omega_1} \bar{n\!\!\!/} P_L T^A b_v][\bar{d}_{\bar{n},\omega_2} n\!\!\!/ P_L T^A u_{\bar{n},\omega_3}]. \tag{7}$$

These $\overline{id}$ and $\overline{is}$ operators do not contribute to the decays $B \to M_1 M_2$ at leading order, but will in power corrections. Our basis of $Q_{id}^{(0)}$ operators can be directly related to the one derived in [8], except that we also included $Q_{3d}^{(0)}$ which makes the basis sufficient to accommodate all electroweak penguin effects.

We also need the $\mathcal{O}(\lambda)$ operators for the LO factorization. Defining

$$ig\mathcal{B}_{n,\omega}^{\perp\mu} = \frac{1}{(-\omega)}\{W_n^\dagger [i\bar{n} \cdot D_{c,n}, iD_{n,\perp}^\mu] W_n \delta(\omega - \bar{\mathcal{P}}^\dagger)\} \tag{8}$$

they are

$$\begin{aligned}
Q_{1d}^{(1)} &= \frac{-2}{m_b}[\bar{u}_{n,\omega_1} ig\mathcal{B}_{n,\omega_4}^\perp P_L b_v][\bar{d}_{\bar{n},\omega_2} n\!\!\!/ P_L u_{\bar{n},\omega_3}],\\
Q_{2d,3d}^{(1)} &= \frac{-2}{m_b}[\bar{d}_{n,\omega_1} ig\mathcal{B}_{n,\omega_4}^\perp P_L b_v][\bar{u}_{\bar{n},\omega_2} n\!\!\!/ P_{L,R} u_{\bar{n},\omega_3}],\\
Q_{4d}^{(1)} &= \frac{-2}{m_b}[\bar{q}_{n,\omega_1} ig\mathcal{B}_{n,\omega_4}^\perp P_L b_v][\bar{d}_{\bar{n},\omega_2} n\!\!\!/ P_L q_{\bar{n},\omega_3}],\\
Q_{5d,6d}^{(1)} &= \frac{-2}{m_b}[\bar{d}_{n,\omega_1} ig\mathcal{B}_{n,\omega_4}^\perp P_L b_v][\bar{q}_{\bar{n},\omega_2} n\!\!\!/ P_{L,R} q_{\bar{n},\omega_3}],\\
Q_{7d}^{(1)} &= \frac{-2}{m_b}[\bar{u}_{n,\omega_1} ig\mathcal{B}_{n,\omega_4}^{\perp\mu} P_L b_v][\bar{d}_{\bar{n},\omega_2} n\!\!\!/ \gamma_\mu^\perp P_R u_{\bar{n},\omega_3}],\\
Q_{8d}^{(1)} &= \frac{-2}{m_b}[\bar{q}_{n,\omega_1} ig\mathcal{B}_{n,\omega_4}^{\perp\mu} P_L b_v][\bar{d}_{\bar{n},\omega_2} n\!\!\!/ \gamma_\mu^\perp P_R q_{\bar{n},\omega_3}].
\end{aligned} \tag{9}$$

Our basis in Eq. (9) is simpler than the one in [8] for several reasons. Terms with a $\mathcal{B}_n^\perp$ or $D_n^\perp$ in the $\bar{n}$ bilinear can be reduced to Eq. (9) by Fierz transformations. This shows that hard-spectator and form factor contributions are related. Second, $\mathcal{P}_\perp Q_{if}^{(0)} = 0$, so integration by parts allows a basis for $Q_{if}^{(1)}$ with no n-covariant derivatives, so only field strengths $\mathcal{B}_n^\perp$ appear, plus $[\bar{u}_n \gamma_\perp^\mu P_L b_v]\mathcal{P}_\perp^\mu[\bar{d}_{\bar{n}} n\!\!\!/ P_L u_{\bar{n}}]$ terms which give vanishing contributions. We suppress $Q^{(1)}$'s with octet bilinears that do not contribute at LO. The operators $Q_{5,6}^{(0,1)}$ only contribute to SU(3)$_{\bar{n}}$ singlet production and are not used below.

Next we determine the most general structure of the $p^2 \sim E\Lambda$ contributions in SCET$_\mathrm{I}$. We decouple the usoft modes by making the field redefinitions [6] $\xi_{n'} \to Y_{n'}\xi_{n'}$, $A_{n'} \to Y_{n'} A_{n'} Y_{n'}^\dagger$, with $Y_{n'}$ a Wilson line of $n' \cdot A_{us}$ gluons and $n' = n$ or $\bar{n}$. In $Q_{if}^{(0,1)}$ all Y's cancel except for $(Y_n^\dagger b_v)$

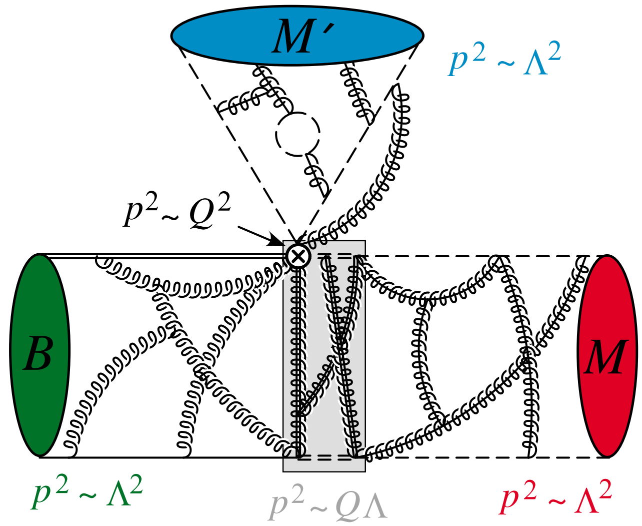

FIG. 3 (color online). Factorization of $B \to MM'$ in SCET.

[8], and the operators factor into (n, v) and $\bar{n}$ parts,

$$Q_{if}^{(0,1)} = \tilde{Q}_{if}^{(0,1)} Q_{if}^{\bar{n}}. \tag{10}$$

In Fig. 3 the M' meson only connects to the rest of the diagram at the scale $p^2 \sim Q^2$, through $Q_{if}^{\bar{n}} = \bar{q}_{\bar{n},\omega_2}\Gamma q'_{\bar{n},\omega_3}$ for some flavors q, q' and Dirac structure Γ. The shaded $p^2 \sim E\Lambda$ region is required to generate the collinear M, similar to the $B \to M$ form factors [12]. At LO it is given by T products of the remaining parts of the operators in Eq. (10), $\tilde{Q}_{if}^{(0,1)}$, with one Lagrangian $\mathcal{L}_{q\xi}^{(j)}$ inserted on the spectator quark to swap it from usoft to collinear:

$$\begin{aligned}
T_1 &= \int d^4y\, d^4y'\, T[\tilde{Q}_{if}^{(0)}(0), i\mathcal{L}_{\xi_n q}^{(1)}(y), i\mathcal{L}_{\xi_n\xi_n}^{(1)}(y')\\
&\quad + i\mathcal{L}_{cg}^{(1)}(y')] + \int d^4y\, T[\tilde{Q}_{if}^{(0)}(0), i\mathcal{L}_{\xi_n q}^{(1,2)}(y)],\\
T_2 &= \int d^4y\, T[\tilde{Q}_{if}^{(1)}(0), i\mathcal{L}_{\xi_n q}^{(1)}(y)].
\end{aligned} \tag{11}$$

Here $\mathcal{L}_{\xi_n q}^{(1)} = \bar{q}_{us} Y ig\mathcal{B}_n^\perp W^\dagger \xi_n$ + H.c. [13], and the form of our other $\mathcal{L}$'s can be found in [14].

Now we match SCET$_\mathrm{I}$ onto SCET$_\mathrm{II}$. A complete treatment of T_1 is an open question due to end point singularities [12,15,16], but $\langle V_\perp | T_1 | B\rangle = 0$ and the nonzero matrix elements can be parametrized as

$$\langle P|T_1|B\rangle = m_B \zeta^{BP}, \qquad \langle V_\parallel|T_1|B\rangle = m_B\zeta^{BV_\parallel}. \tag{12}$$

For T_2 the most general perturbative matching at $\mu^2 \sim E\Lambda$ generates a set of operators with Wilson coefficients given by jet functions J and $J_\perp$ whose form is constrained by reparametrization invariance, chirality, power counting and dimensional analysis [$\omega_1 = z\omega$, $\omega_4 = (1-z)\omega$, $\bar{x} = 1 - x$, $\chi_{n,\omega} = (W^\dagger\xi_n)_\omega$],

$$T[(\bar{\xi}_n W)_{\omega_1} ig\mathcal{B}^{\perp\alpha}_{n,\omega_4} P_{R,L}]^{ia}(0)[ig\mathcal{B}^{\perp}_{n} W^\dagger \xi_n]^{jb}_0(y)$$
$$= i\delta^{ab}\delta(y^+)\delta^{(2)}(y_\perp)\frac{1}{\omega} \times \int_0^1 dx \int \frac{dk^+}{2\pi} e^{+ik^+y^-/2}\Big\{-J_\perp(z,x,k_+) \times \Big(\frac{\not{n}}{2} P_{R,L}\gamma^\alpha_\perp\gamma^\beta_\perp\Big)_{ji}[\bar{\chi}_{n,x\omega}\bar{\not{n}}\gamma^\perp_\beta\chi_{n,-\bar{x}\omega}] + J(z,x,k_+) \times (\not{n}P_{L,R}\gamma^\alpha_\perp)_{ji}[\bar{\chi}_{n,x\omega}\bar{\not{n}}P_{L,R}\chi_{n,-\bar{x}\omega}]\Big\}, \quad (13)$$

where $\{i, j\}$ and $\{a, b\}$ are spin and color indices. At tree level we find that $J(z, x, k_+) = J_\perp(z, x, k_+) = \delta(x - z)\pi\alpha_s(\mu)C_F/(N_c\bar{x}k_+)$. The remaining pieces of T_2 are purely usoft and match directly onto soft operators in $\mathrm{SCET_{II}}$, giving

$$-\frac{2i}{m_b}\int d^4y[\bar{q}_s Y]^j(y)[b_v]^i(0), \quad (14)$$

$$-\frac{2i}{m_b}\int d^4y[\bar{q}_s Y]^j(y)[\gamma^\perp_\alpha b_v]^i(0), \quad (15)$$

where here Eq. (14) goes along with the $J_\perp$ term, and Eq. (15) goes along with the J term.

To obtain the final result for amplitudes we combine Eqs. (10)–(14), simplify the Dirac structure between the soft fields, and take matrix elements. First consider final states containing perpendicularly polarized vector mesons, $B \to V_\perp V_\perp$. Kagan [17] has argued that $B \to V_\perp V_\perp$ is power suppressed relative to the longitudinal polarization, $B \to V_\parallel V_\parallel$. At LO in SCET $Q^{(0,1)}_{if}$ for $i = 1$–6 have scalar bilinears and give vanishing contributions to $B \to V_\perp V_\perp$. The operators $\tilde{Q}^{(1)}_{7f}$ and $\tilde{Q}^{(1)}_{8f}$ generate the $J_\perp$ term in Eq. (13) and could contribute. However, chirality conservation in $\mathrm{SCET_I}$ implies that one vector is L and one is R polarized so the $\tilde{Q}^{(1)}_{7f}$ and $\tilde{Q}^{(1)}_{8f}$ contributions also vanish (quark masses flip chirality and in $\mathrm{SCET_I}$ are suppressed by powers of $m_q/\sqrt{\Lambda E}$ [18]). More explicitly the $J_\perp$ term in Eq. (13) vanishes because the soft Dirac structure can be reduced, $\not{n}P_L\gamma^\alpha_\perp\gamma^\beta_\perp = (g^{\alpha\beta}_\perp + i\epsilon^{\alpha\beta}_\perp)\not{n}P_L$, and this tensor vanishes when contracted with the $\bar{n}$-bilinear,

$$(g^{\alpha\beta}_\perp + i\epsilon^{\alpha\beta}_\perp)\bar{d}_{\bar{n},\omega_2}\not{n}\gamma^\perp_\alpha P_R q_{\bar{n},\omega_3} = 0. \quad (16)$$

Thus at LO only $A_{c\bar{c}}$ could give transverse polarized vector mesons so

$$A(\bar{B} \to V_1^\perp V_2^\perp) = \frac{2G_F}{\sqrt{2}}\langle V_1^\perp V_2^\perp|Q_{c\bar{c}}|\bar{B}\rangle. \quad (17)$$

Next consider $B \to V_\parallel V_\parallel$, $B \to V_\parallel P$ and $B \to PP$ decays. Now it is the J term in Eq. (13) that contributes along with possible long distance charming penguins. Because of the form of our operators the J term is identical to the analysis of the $B \to M$ form factors. The LO factorization formula for $A = \langle M_1M_2|H_W|\bar{B}\rangle$ which determines $\bar{B}^0, B^- \to M_1M_2$ with $M_{1,2}$ pseudoscalars or longitudinal vectors is

$$A(\bar{B} \to M_1M_2) = \lambda_c^{(f)}A^{M_1M_2}_{c\bar{c}} + \frac{G_F m_B^2}{\sqrt{2}}\Big\{f_{M_2}\zeta^{BM_1}\int_0^1 du\, T_{2\zeta}(u)\phi^{M_2}(u) + f_{M_1}\zeta^{BM_2}\int_0^1 du\, T_{1\zeta}(u)\phi^{M_1}(u) + \frac{f_B f_{M_1} f_{M_2}}{m_b} \times \int_0^1 du\int_0^1 dx\int_0^1 dz\int_0^\infty dk_+\, J(z,x,k_+)[T_{2J}(u,z)\phi^{M_1}(x)\phi^{M_2}(u) + T_{1J}(u,z)\phi^{M_2}(x)\phi^{M_1}(u)]\phi_B^+(k_+)\Big\}, \quad (18)$$

where $A^{c\bar{c}}$ denote possible long distance charming penguin amplitudes which contribute in channels where $c_4^{(d,s)}$ appear. For each decay mode the set of hard coefficients $T_{i\zeta}$ and T_{iJ} can be obtained from Table I.

A new result from our analysis is that the jet function J in Eq. (18) is the *same* as that appearing in the factorization formula for $B \to M$ form factors [19]. We quote here two of these formulas, one for the standard $B \to P\ell\bar{\nu}$ form factor $f_+(E)$, and one for the form factor $A_\parallel$ for $B \to V_\parallel\ell\bar{\nu}$ decays,

$$A_\parallel(E) = \frac{1}{m_V}\Big[\frac{m_B E A_2(E)}{m_B + m_V} - \frac{(m_B + m_V)}{2}A_1(E)\Big], \quad (19)$$

where

$$E = \frac{m_B^2 + m_M^2 - q^2}{2m_B}. \quad (20)$$

At LO in SCET [12,15,16,19,20]

$$f_+(E) = T^{(+)}(E)\zeta^{BP}(E) + N_0\int_0^1 dz\int_0^1 dx\int_0^\infty dk_+ \times C_J^{(+)}(z,E)J(z,x,k_+,E)\phi^M(x)\phi_B^+(k_+),$$
$$A_\parallel(E) = T^{(A_\parallel)}(E)\zeta^{BV_\parallel}(E) + N_\parallel\int_0^1 dz\int_0^1 dx\int_0^\infty dk_+ \times C_J^{(A_\parallel)}(z,E)J(z,x,k_+,E)\phi^M(x)\phi_B^+(k_+), \quad (21)$$

where $N_0 = f_B f_P m_B/(4E^2)$, $N_\parallel = f_B f_V m_B/(4E^2)$, and the functions $T^{(+,A)}(E), C_J^{(+,A)}(z)$ are combinations of SCET Wilson coefficients and can be found in [19]. In that paper the jet functions $J^{(\perp)}(z, x, k_+)$ in Eq. (13) are denoted by $J_b^{(\perp)}(z, x, k_+)$ and $J_a^{(\perp)}(x, k_+) = \int_0^1 dz\, J_b^{(\perp)}(x, z, k_+)$. At the end point where $E \simeq m_B/2$

TABLE I. Combinations of Wilson coefficients appearing in the factorization formula. Note that these results do not assume isospin symmetry and all VV channels in this table are longitudinal. Because of our basis choice the coefficients $T_{1J,2J}(u,z)$ for all these states are *identical* to $T_{1\zeta,2\zeta}(u)$ with each $c_i^{(f)}(u)$ replaced by $b_i^{(f)}(u,z)$.

M_1M_2	$T_{1\zeta}(u)$	$T_{2\zeta}(u)$	M_1M_2	$T_{1\zeta}(u)$	$T_{2\zeta}(u)$
$\pi^-\pi^+$, $\rho^-\pi^+$, $\pi^-\rho^+$, $\rho_\parallel^-\rho_\parallel^+$	$c_1^{(d)}+c_4^{(d)}$	0	$\pi^+K^{(*)-}$, ρ^+K^-, $\rho_\parallel^+K_\parallel^{*-}$	0	$c_1^{(s)}+c_4^{(s)}$
$\pi^-\pi^0$, $\rho^-\pi^0$	$\frac{1}{\sqrt{2}}(c_1^{(d)}+c_4^{(d)})$	$\frac{1}{\sqrt{2}}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\pi^0K^{(*)-}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}-c_3^{(s)})$	$\frac{1}{\sqrt{2}}(c_1^{(s)}+c_4^{(s)})$
$\pi^-\rho^0$, $\rho_\parallel^-\rho_\parallel^0$	$\frac{1}{\sqrt{2}}(c_1^{(d)}+c_4^{(d)})$	$\frac{1}{\sqrt{2}}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	ρ^0K^-, $\rho_\parallel^0K_\parallel^{*-}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}+c_3^{(s)})$	$\frac{1}{\sqrt{2}}(c_1^{(s)}+c_4^{(s)})$
$\pi^0\pi^0$	$\frac{1}{2}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\frac{1}{2}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\pi^-\bar{K}^{(*)0}$, $\rho^-\bar{K}^0$, $\rho_\parallel^-\bar{K}_\parallel^{*0}$	0	$-c_4^{(s)}$
$\rho^0\pi^0$	$\frac{1}{2}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\frac{1}{2}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\pi^0\bar{K}^{(*)0}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}-c_3^{(s)})$	$-\frac{1}{\sqrt{2}}c_4^{(s)}$
$\rho_\parallel^0\rho_\parallel^0$	$\frac{1}{2}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\frac{1}{2}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\rho^0\bar{K}^0$, $\rho_\parallel^0\bar{K}_\parallel^{*0}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}+c_3^{(s)})$	$-\frac{1}{\sqrt{2}}c_4^{(s)}$
$K^{(*)0}K^{(*)-}$, $K^{(*)0}\bar{K}^{(*)0}$	$-c_4^{(d)}$	0	$K^{(*)-}K^{(*)+}$	0	0

the same parameters ζ^{BM} and jet function J appear in the form factors and in the nonleptonic decays. Since the analysis for J is identical to that in the form factors several important facts can be immediately taken over for $B \to M_1M_2$ decays. In particular to all orders in perturbation theory only the $\phi_B^+(k_+)$ wave function is obtained as proven in Ref. [19]. Also the convolution integrals with J are finite with an identical proof to the one given in Ref. [15]. Finally it is clear that possible messenger fluctuations [21] cannot spoil factorization in $Q_{if}^{(0,1)}$ which have color singlet $\bar{n}$ bilinears, and so their role will be identical to that in the form factors.

At this point we compare our result in Eq. (18) with the result in QCDF [1]. From Eq. (25) of [1] the LO factorization theorem is

$$\langle M_1M_2|O_i|\bar{B}\rangle = \tag{22}$$

$$\begin{aligned}&\Big\{F^{B\to M_1}(0)f_{M_2}\int du\, T^{\rm I}_{M_2,i}(u)\phi_{M_2}(u) + (1\leftrightarrow 2)\Big\}\\ &\quad + f_{M_1}f_{M_2}f_B\int du\,dx\,dk^+\\ &\quad \times T_i^{\rm II}(x,u,k_+)\phi_{M_1}(x)\phi_{M_2}(u)\phi_B(k_+),\end{aligned} \tag{23}$$

where the parameters are the QCD form factors $F^{B\to M}(0)$, ϕ_{M_i}, and ϕ_B (other parameters appear when power suppressed terms from annihilation or chirally enhanced corrections are included). In the QCDF power counting the second term is suppressed relative to the first by a factor of α_s. The result in Eq. (22) is quite similar to the SCET formula derived in Eq. (18). However, there are several important differences, which we comment on. The two things that are most important for phenomenology are that QCDF does not allow for a leading order $A_{c\bar{c}}^{\pi\pi}$ contribution, and that the SCET analysis suggests that the contributions from ζ and ζ_J are comparable in size, rather than $\zeta_J^{B\pi} \ll \zeta^{B\pi}$ as in QCDF. As discussed later, current data on $B\to\pi\pi$ seems to support $\zeta_J^{B\pi}\sim\zeta^{B\pi}$, albeit with large uncertainties. This difference has significant phenomenological ramifications, as it implies that even in absence of leading order charming penguin effects the perturbative strong phases predicted in [1] would receive $\mathcal{O}(100\%)$ corrections. Besides these points there are several technical differences between the two formulas. Using $F^{B\to M}(0)$ in Eq. (22) rather than ζ^{BM} does not completely separate out all contributions from the hard scale. Also, in Eq. (22) $T^{\rm I}$ and $T^{\rm II}$ include perturbative contributions from both the $\mu^2 \simeq Q^2$ and $\mu^2 \simeq E\Lambda$ scales [20]. In the result in Eq. (18) these scales are separated in T_{iJ} and J, respectively. If ζ^{BM} is independent of the $\mu^2 \simeq E\Lambda$ scale as argued in Ref. [16] then the scales are also completely separated in the $T_{i\zeta}\zeta^{BM}$ term, otherwise ζ^{BM} still encodes physics at both the jet scale $E\Lambda$ and the scale Λ^2.

The jet function J depends on physics at the intermediate scale, so its perturbative expansion in $\alpha_s(\sqrt{E\Lambda})$ is not as convergent as for the T_{iJ} and $T_{i\zeta}$ which are expanded in $\alpha_s(Q)$. In fact, perturbation theory may fail for J altogether. This can be tested both by experiment [22] and by additional perturbative calculations. Using SCET we can still obtain an expression for $A(\bar{B}\to M_1M_2)$ without expanding J perturbatively:

$$\begin{aligned}A = \frac{G_F m_B^2}{\sqrt{2}}\Big\{&f_{M_1}\int_0^1 du\,dz\,T_{1J}(u,z)\zeta_J^{BM_2}(z)\phi^{M_1}(u)\\ &+f_{M_1}\zeta^{BM_2}\int_0^1 du\,T_{1\zeta}(u)\phi^{M_1}(u)\Big\}\\ &+\{1\leftrightarrow 2\} + \lambda_c^{(f)}A_{c\bar{c}}^{M_1M_2},\end{aligned} \tag{24}$$

where power counting implies $\zeta^{BM}\sim\zeta_J^{BM}\sim(\Lambda/Q)^{3/2}$. Equation (24) defines implicitly the parameter $\zeta_J(z)$. Here the nonperturbative parameters ζ^{BM}, $\zeta_J^{BM}(z)$, and $\phi^M(u)$, still all occur in the $B\to M$ semileptonic and rare form factors. For a model-independent analysis they need to be determined from data. Note that it was possible for us to derive Eq. (24) because in Eq. (18) we separated the scales Q^2 and $E\Lambda$ into T's and J's, respectively. The corresponding results for the form factors in Eq. (21) are

$$f_+ = T^{(+)}(E)\zeta^{BP}(E) + N_0 \int_0^1 dz\, C_J^{(+)}(z)\zeta_J^{BM}(z, E),$$
$$A_\parallel = T^{(A_\parallel)}(E)\zeta^{BV_\parallel}(E) + N_\parallel \int_0^1 dz\, C_J^{(A_\parallel)}(z)\zeta_J^{BV}(z, E). \tag{25}$$

The two form factors in Eq. (21) can be obtained from data on $B \to (P, V_\parallel)\ell\nu$, giving important information on the ζ^{BM}, ζ_J^{BM} appearing in Eq. (18). Note that in Eqs. (18) and (24) the ζ's are evaluated at $E = m_B/2$. Equations (18) and (24) are the main results of our paper.

Using Eq. (24) still requires matching the full theory O_i's onto the $Q_{if}^{(0,1)}$ to determine the Wilson coefficients $c_i^{(f)}$ and $b_i^{(f)}$. For the coefficients of $Q_i^{(0)}$ we find $[f = d, s]$

$$\begin{aligned}
c_1^{(f)} &= \lambda_u^{(f)}\Big(C_1 + \frac{C_2}{N_c}\Big) - \lambda_t^{(f)}\frac{3}{2}\Big(C_{10} + \frac{C_9}{N_c}\Big) + \Delta c_1^{(f)},\\
c_2^{(f)} &= \lambda_u^{(f)}\Big(C_2 + \frac{C_1}{N_c}\Big) - \lambda_t^{(f)}\frac{3}{2}\Big(C_9 + \frac{C_{10}}{N_c}\Big) + \Delta c_2^{(f)},\\
c_3^{(f)} &= -\lambda_t^{(f)}\frac{3}{2}\Big(C_7 + \frac{C_8}{N_c}\Big) + \Delta c_3^{(f)},\\
c_4^{(f)} &= -\lambda_t^{(f)}\Big(C_4 + \frac{C_3}{N_c} - \frac{C_{10}}{2} - \frac{C_9}{2N_c}\Big) + \Delta c_4^{(f)}.
\end{aligned} \tag{26}$$

and for the $Q_i^{(1)}$

$$\begin{aligned}
b_1^{(f)} &= \lambda_u^{(f)}\Big[C_1 + \Big(1 - \frac{m_b}{\omega_3}\Big)\frac{C_2}{N_c}\Big]\\
&\quad -\lambda_t^{(f)}\Big[\frac{3}{2}C_{10} + \Big(1 - \frac{m_b}{\omega_3}\Big)\frac{3C_9}{2N_c}\Big] + \Delta b_1^{(f)},\\
b_2^{(f)} &= \lambda_u^{(f)}\Big[C_2 + \Big(1 - \frac{m_b}{\omega_3}\Big)\frac{C_1}{N_c}\Big]\\
&\quad -\lambda_t^{(f)}\Big[\frac{3}{2}C_9 + \Big(1 - \frac{m_b}{\omega_3}\Big)\frac{3C_{10}}{2N_c}\Big] + \Delta b_2^{(f)},\\
b_3^{(f)} &= -\lambda_t^{(f)}\Big[\frac{3}{2}C_7 + \Big(1 - \frac{m_b}{\omega_2}\Big)\frac{3C_8}{2N_c}\Big] + \Delta b_3^{(f)},\\
b_4^{(f)} &= -\lambda_t^{(f)}\Big[C_4 - \frac{C_{10}}{2} + \Big(1 - \frac{m_b}{\omega_3}\Big)\Big(\frac{C_3}{N_c} - \frac{C_9}{2N_c}\Big)\Big]\\
&\quad + \Delta b_4^{(f)},
\end{aligned} \tag{27}$$

where $\omega_2 = m_b u$, $\omega_3 = -m_b\bar{u} = m_b(u-1)$ and the $\Delta c_i^{(f)}$ and $\Delta b_i^{(f)}$ are perturbative corrections. The $\mathcal{O}(\alpha_s)$ contributions to the $\Delta c_j^{(f)}(u)$ have been calculated in [1] and later in [8]. It is possible that these results will need to be modified by an additional subtraction for the long distance charming penguin. Finally, any full $\alpha_s(m_b)$ analysis requires $\Delta b_j^{(f)}(u, z)$ which are currently unknown, unless the numerical values of ζ, ζ_J are such that $\zeta_J \sim \alpha_s(m_b)\zeta$ so that $\zeta_J^{BM} \ll \zeta^{BM}$ and the $\Delta c_j^{(f)}$ coefficients dominate numerically.

There are several issues in the phenomenological use of the factorization formula. There is a hierarchy due to CKM factors and the C_i's which have to be accounted for in the $c_i^{(f)}$ and $b_i^{(f)}$. For example, C_1 is about a factor of 6 larger than any of the other coefficients, making $c_1^{(d)}$, $b_1^{(d)}$, and $b_2^{(d)}$ large. We will refer to quantities as "contaminated" if $1/m_b$ power corrections could compete with LO results due to the hierarchy in Wilson coefficients. Unless these corrections can be accounted for or proven to be absent, one should assign $\sim$100% uncertainty to predictions for contaminated decays. The determination of whether a quantity is contaminated depends on the relative size of ζ^{BM} and ζ_J^{BM}. If $\zeta^{BM} \gg \zeta_J^{BM}$ as in QCDF then any $f = d$ decay in Table I that is independent of $c_1^{(d)}$ could receive large corrections, making quantities such as $Br(\bar{B}^0 \to \pi^0\pi^0)$ contaminated [4]. Here the most problematic are large power corrections proportional to $C_1\Lambda/E$ which is $\sim C_2$ and $\gg C_{i\geq 3}$. These can arise, for example, from T products involving the $Q_{2f}^{(0)}$ operators. The situation is much better in the case $\zeta^{BM} \sim \zeta_J^{BM}$ since any decay depending on $c_1^{(d)}$, $b_1^{(d)}$, or $b_2^{(d)}$ will not be contaminated and can be expected to have power corrections of normal size, $\sim$20%. Our analysis of $B \to \pi\pi$ below favors this situation, and in this case $Br(\bar{B}^0 \to \pi^0\pi^0)$ is not contaminated.

At leading order in Λ/E there are only two sources of strong phases: the one-loop Δc_i, Δb_i which can become complex [1], and the unfactorized $A_{c\bar{c}}$ charming penguin. Additional final state phases come from power corrections $\sim\Lambda/E$. It is known from $\bar{B}^0 \to D^0\pi^0$ decays that Λ/E corrections produce $\sim 30°$ nonperturbative strong phases in agreement with dimensional analysis [22]. These large phases have nothing to do with a Λ/m_c expansion so we expect strong phases of similar size from power corrections in $B \to M_1M_2$. For contaminated decays, such as $B \to KK$, nonperturbative strong phases $\propto C_1$ could be order unity.

The factorization theorems in Eqs. (18) and (24) can be used to make quantitative predictions for nonleptonic $B \to MM'$ decays. There are many applications; a few of the more important categories are (i) decay modes which are independent of charming penguin contributions are determined by ζ and ζ_J which can be extracted from semileptonic form factors. (ii) SCET implies SU(3) relations beyond those following from H_W in Eq. (1) with full QCD. It also simplifies the structure of SU(3) breaking corrections. (iii) For $B \to VV'$ SCET allows us to analyze polarization effects. (iv) Using isospin SCET makes predictions for matrix elements whose quantum numbers differ from those of the $A_{c\bar{c}}^{M_1M_2}$ amplitudes. In the remainder of the paper we discuss examples in each of these categories. In particular we show that Eq. (24) gives a reasonable fit to the current $B \to \pi\pi$ data.

The parameters ζ^{BM} and ζ_J^{BM} in Eq. (24) for nonleptonic decays are common to those appearing in $B \to M$ form factors Eq. (25). Decays that do not depend on $A_{c\bar{c}}$ include all combinations in Table I that are independent of c_4 and b_4, such as $B^- \to \pi^0\pi^-$ and $B^- \to \rho^0\rho^-$ once isospin is used. For example,

$$\begin{aligned}\sqrt{2}A(B^- \to \pi^-\pi^0) = \frac{G_F m_B^2}{\sqrt{2}} f_\pi &\times\bigg\{\int_0^1 du\, dz (b_1^{(d)} + b_2^{(d)} - b_3^{(d)})(u, z) \\ &\times \zeta_J^{B\pi}(z)\phi^\pi(u) \\ &+ \zeta^{B\pi}\int_0^1 du (c_1^{(d)} + c_2^{(d)} - c_3^{(d)})(u) \\ &\times \phi^\pi(u)\bigg\}, \end{aligned} \tag{28}$$

At tree level the $b_i^{(f)}$'s are independent of z and this relation gives a clean constraint on $\zeta^{B\pi}$ and $\zeta_J^{B\pi} = \int dz\, \zeta_J^{B\pi}(z)$.

Flavor SU(3) symmetry is a powerful tool for studying nonleptonic B decays. In one particular application, Ref. [23] proposed using flavor SU(3) symmetry to determine γ from $B^+ \to K\pi, \pi^+\pi^0$. Corrections to this approach come from SU(3) breaking effects and are typically $\sim$30%. The factorization relation Eq. (24) implies enhanced SU(3) relations beyond those in QCD. For example, in QCD all $B \to PP$ decays to two pseudoscalar octet mesons are parametrized in the SU(3) limit by five complex amplitudes. Using the SCET factorization formula Eq. (18) this number is reduced to one complex amplitude $A_{c\bar{c}}$, one real number ζ and one real function $\zeta_J(z)$. In the language of Ref. [23] the operators in Eq. (4) do not generate the E, A, and PA amplitudes, so these are power suppressed.

In certain cases the SU(3) breaking can be also computed. Such an example is the determination of two SU(3) breaking parameters $R_{1,2}$ appearing in a SU(3) relation used to extract γ [23]

$$\begin{aligned} &A(B^- \to \bar{K}^0\pi^-) + \sqrt{2}A(B^- \to K^-\pi^0) \\ &\quad = \sqrt{2}\frac{|V_{us}|}{|V_{ud}|}(R_1 - \delta_{EW} e^{i\gamma} R_2) A(B^- \to \pi^-\pi^0). \end{aligned} \tag{29}$$

Here δ_{EW} parametrizes the largest electroweak penguin effects and is calculable. The parameters $R_{1,2}$ can be expressed in terms of $\zeta^{B\pi}$, ζ^{BK}, $\zeta_J^{B\pi}(z)$, $\zeta_J^{BK}(z)$ and calculable Wilson coefficients and do not involve $A_{c\bar{c}}^{\pi\pi}$ or $A_{c\bar{c}}^{K\pi}$.

Polarization measurements in decays to two vector mesons have received much attention recently. These decays were studied in Ref. [17], and it was argued that factorization implies $R_T \sim 1/m_b^2$ and $R_\perp / R_\parallel = 1 + \mathcal{O}(1/m_b)$, where $R_{0,T,\perp,\parallel}$ denote the longitudinal, transverse, perpendicular, and parallel polarization fractions ($R_T = R_\perp + R_\parallel$, $R_0 + R_T = 1$). Using SCET we find that R_T is power suppressed in agreement with [17], unless the charming penguin amplitude $A_{c\bar{c}}$ spoils this result. We cannot resolve the validity of the $R_\perp/R_\parallel$ relation working only at LO in $1/m_b$. Experimentally, one finds [24,25]

$$\begin{aligned} R_0(B^+ \to \rho^+\rho^0) &= 0.975 \pm 0.045, \\ R_0(B^0 \to \rho^+\rho^-) &= 0.98^{+0.02}_{-0.08} \pm 0.03, \\ R_0(B^0 \to \phi K^*) &= 0.49 \pm 0.06. \end{aligned} \tag{30}$$

It has been argued that the large transverse polarization observed in the ϕK^* mode might provide a second hint at new physics in $b \to s\bar{s}s$ channels beyond $\sin(2\beta)$ from $B \to \phi K_S$. Unfortunately this conclusion could be spoiled by a contribution from $A_{c\bar{c}}$ at leading order. $A_{c\bar{c}}$ does not contribute to $B^+ \to \rho^+\rho^0$, but can affect $B^0 \to \phi K^*$ and $B^0 \to \rho^+\rho^-$. Until charming penguins are better understood the polarization measurements do not provide a clean signal of physics beyond the standard model (SM). An alternative SM explanation has been offered in Ref. [17] in terms of large power corrections from annihilation.

We finally examine in some detail the predictions of this paper for $B \to \pi\pi$ decays, and show that they reproduce the existing data. The present world averages are [26]

$$\begin{aligned} S_{\pi\pi} = -0.74 \pm 0.16, \qquad C_{\pi\pi} &= -0.46 \pm 0.13, \\ Br(B^+ \to \pi^0\pi^+) &= (5.2 \pm 0.8) \times 10^{-6}, \\ Br(B^0 \to \pi^+\pi^-) &= (4.6 \pm 0.4) \times 10^{-6}, \\ Br(B^0 \to \pi^0\pi^0) &= (1.9 \pm 0.5) \times 10^{-6}, \end{aligned} \tag{31}$$

where the branching fractions are CP averages. The amplitudes are naturally divided into two pieces with different CKM factors, as $A \equiv \lambda_u^{(d)} T + \lambda_c^{(d)} P$, where T and P are usually called "tree" and "penguin" amplitudes. The decay amplitudes for $B \to \pi\pi$ can be written in a model-independent way as

$$\begin{aligned} A(\bar{B}^0 \to \pi^+\pi^-) &= \lambda_u^{(d)} T_c (1 + r_c e^{i\delta_c} e^{i\gamma}), \\ A(\bar{B}^0 \to \pi^0\pi^0) &= \lambda_u^{(d)} T_n (1 + r_n e^{i\delta_n} e^{i\gamma}), \\ \sqrt{2}A(B^- \to \pi^0\pi^-) &= \lambda_u^{(d)} T, \end{aligned} \tag{32}$$

where (r_c, δ_c) and (r_n, δ_n) parametrize the ratio of penguin to tree contributions to $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^0\pi^0$, respectively. We have neglected small electroweak penguin contributions. Isospin gives the relations

$$T = T_c + T_n, \qquad T_c r_c e^{i\delta_c} + T_n r_n e^{i\delta_n} = 0, \tag{33}$$

leaving only five independent strong interaction parameters in Eq. (32).

In the first step of the analysis, we assume that β, γ are known, use this to disentangle the tree and penguin amplitudes, and thus extract the five parameters in Eq. (32). In a second step, these parameters are compared with the leading order predictions from SCET, and used to extract the nonperturbative parameters appearing in the factorization formula Eq. (24), working at tree level in matching at the hard scale. The resulting SCET parameters are then used to predict values for $|V_{ub}|f_+(0)$ and $Br(B^0 \to \pi^0\pi^0)$ as functions of γ.

Assuming values for the CKM angles β and γ we can use the five pieces of experimental data given in Eq. (31) to determine the five hadronic parameters in Eq. (32). Using $(\beta, \gamma) = (23°, 64°)$ [26] and the data for the CP asymmetries we find for the penguin parameters r_c and δ_c

$$r_c = 0.75 \pm 0.35, \qquad \delta_c = -44° \pm 12°. \tag{34}$$

This is in good agreement with the recent determinations of these parameters in Ref. [27]. Using the branching ratio data as input, we can determine the tree parameters as well. We find

$$\begin{aligned} |T| &= N_\pi (0.29 \pm 0.02)\Big(\frac{3.9\times 10^{-3}}{|V_{ub}|}\Big), \\ |t| &= 2.07 \pm 0.42, \qquad |t_n| = \begin{cases} 1.15 \pm 0.33 & \text{(I)} \\ 1.42 \pm 0.35 & \text{(II)}, \end{cases} \end{aligned} \tag{35}$$

where $N_\pi = (G_F/\sqrt{2}) m_B^2 f_\pi$ and we defined

$$t = \frac{T}{T_c}, \qquad t_n = \frac{T_n}{T_c}. \tag{36}$$

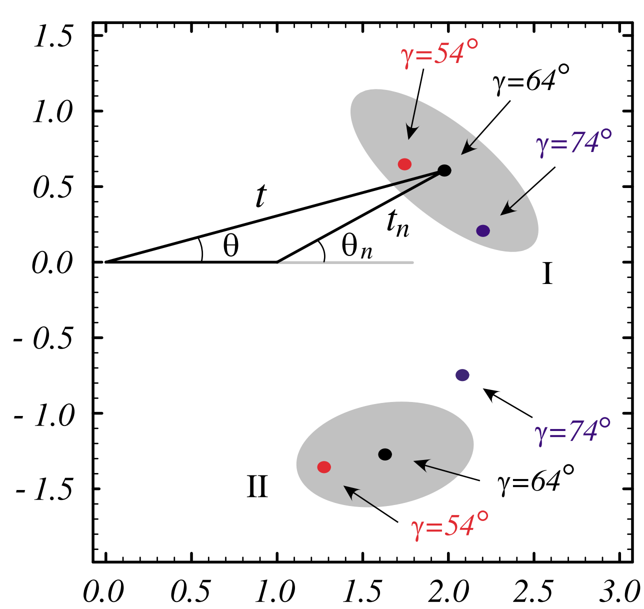

FIG. 4 (color online). Constraints on the triangle of tree amplitudes $T/T_c - T_n/T_c = 1$ from current world averaged data on $B \to \pi\pi$. The shaded regions show the two 1-σ regions for $\gamma = 64°$ including the error correlation between $|t|$ and $|t_n|$. The central values for $\gamma = 54°$ and $\gamma = 74°$ are also shown.

Some of the errors in Eqs. (34) and (35) have sizable correlations. The results for the tree triangle are shown graphically in Fig. 4. The two $\gamma = 64°$ solutions correspond to those in Eq. (35) and the ellipses denote 1σ contours. Also shown in this figure is the isospin tree triangle, which for the reduced tree level amplitudes reads $1 + t_n = t$. There are two strong phases in this triangle which are also shown in the figure, namely θ between T and T_c and θ_n between T_n and T_c.

As a second step the extracted amplitudes are compared with the predictions of this paper at leading order in Λ/m_b and tree level in the SCET Wilson coefficient $c_i^{(d)}$ and $b_i^{(d)}$. At this order our result has only four independent hadronic parameters. The tree amplitudes T, T_c are given by the factorization relation Eq. (18) and depend on the nonperturbative parameters $\zeta^{B\pi}, \zeta_J^{B\pi}$,

$$\begin{aligned} T &= N_\pi \frac{1}{3}(C_1 + C_2)[4\zeta^{B\pi} + (4 + \langle \bar{u}^{-1}\rangle_\pi)\zeta_J^{B\pi}], \\ T_c &= N_\pi\Big[\Big(C_1 + \frac{C_2}{3} + C_4 + \frac{C_3}{3}\Big)\zeta^{B\pi} \\ &\quad + \Big(C_1 + C_4 + (1 + \langle \bar{u}^{-1}\rangle_\pi)\frac{C_2 + C_3}{3}\Big)\zeta_J^{B\pi}\Big], \end{aligned} \tag{37}$$

where $\langle \bar{u}^{-1}\rangle_\pi = \int_0^1 \phi_\pi(u)/(1-u)$, and $\zeta_J^{B\pi} = \int dz\, \zeta_J^{B\pi}(z)$. The penguin amplitude also gets a contribution from the complex $A_{c\bar{c}}^{\pi\pi}$ amplitude, so

$$\begin{aligned} P \equiv -\left|\frac{\lambda_u^{(d)}}{\lambda_c^{(d)}}\right| T_c r_c e^{i\delta_c} &= N_\pi\Big[\Big(C_4 + \frac{C_3}{3}\Big)\zeta^{B\pi} \\ &\quad + \Big(C_4 + (1 + \langle \bar{u}^{-1}\rangle_\pi)\frac{C_3}{3}\Big)\zeta_J^{B\pi} + \frac{1}{N_\pi}A_{c\bar{c}}^{\pi\pi}\Big]. \end{aligned} \tag{38}$$

The amplitude T_n is given by the isospin relation Eq. (33) as $T_n = T - T_c$. At tree level in SCET Wilson coefficients the $B \to \pi$ form factor at $q^2 = 0$ is

$$f_+(0) = \zeta^{B\pi} + \zeta_J^{B\pi}. \tag{39}$$

Neglecting the $O[\alpha_s(m_b)]$ corrections introduces an error of about 10% for the T amplitudes, which is smaller than the expected size of the power corrections $\sim O(\Lambda/E)$.

Equation (37) implies that the tree amplitudes T, T_c are calculable in terms of the ζ, ζ_J parameters, and their relative strong phases are small $\theta, \theta_n \sim O[\alpha_s(m_b), \Lambda/E]$. On the other hand, the penguin amplitude P can have an O(1) strong phase due to the charming penguin amplitude $A_{c\bar{c}}^{\pi\pi}$. The pattern of results in Fig. 4 supports these predictions for the tree amplitudes T, T_c for the upper hand solution. In particular, within the experimental uncertainty the phases θ and θ_n are still consistent with being small and compatible with order $O(\Lambda/E)$ effects.

Using the numbers in Eq. (35) for $|T|$ and $|t|$ and the SCET results in Eqs. (37), we can extract the nonperturbative parameters ζ, ζ_J. Taking leading-log (LL) order for the coefficients ($C_1 = 1.107$, $C_2 = -0.248$, $C_3 = 0.011$, $C_4 = -0.025$ at $\mu = 4.8$ GeV) and $\langle \bar{u}^{-1} \rangle_\pi = 3$ [28], we find

$$\zeta^{B\pi}|_{\gamma=64^\circ} = (0.05 \pm 0.05)\Big(\frac{3.9 \times 10^{-3}}{|V_{ub}|}\Big), \quad \zeta_J^{B\pi}|_{\gamma=64^\circ} = (0.11 \pm 0.03)\Big(\frac{3.9 \times 10^{-3}}{|V_{ub}|}\Big), \tag{40}$$

where the quoted errors are propagated from the experimental errors from $|T|$ and $|t|$ in Eq. (35). Using the results for r_c and δ_c in Eq. (34) and $|V_{cb}| = 0.041$ the penguin amplitude is

$$\frac{P}{N_\pi}\bigg|_{\gamma=64^\circ} = (0.043 \pm 0.013) e^{i(136^\circ \pm 12^\circ)}. \tag{41}$$

The $\zeta^{\pi\pi}$ and $\zeta_J^{\pi\pi}$ terms in Eq. (38) contribute 0.002 to P/N_π, which is only a small part of the experimental result. The perturbative corrections from the $\Delta c_i^{(f)}$'s or particularly the $\Delta b_i^{(f)}$'s can add terms whose rough size is estimated to be $\sim \zeta_J^{B\pi} C_1 \alpha_s(m_b)/\pi \simeq 0.007$. After removing these contributions, the sizable remainder would be attributed to $A_{c\bar{c}}^{\pi\pi}$. Since $A_{c\bar{c}}^{\pi\pi}$ can have a large nonperturbative strong phase, the large phase in Eq. (41) supports the conclusion that this term contributes a substantial amount to P/N_π.

The extraction of the above parameters allows us to make two model-independent predictions with only γ and $|V_{ub}|$ as input. First a prediction for the semileptonic $B \to \pi$ form factor $f_+(0)$ is possible. Combining Eq. (40) with Eq. (39) we find

$$f_+(0)|_{\gamma=64^\circ} = (0.17 \pm 0.02)\Big(\frac{3.9 \times 10^{-3}}{|V_{ub}|}\Big). \tag{42}$$

In Fig. 5 we show results for $\zeta^{B\pi}$, $\zeta_J^{B\pi}$, and $f_+(0)$ for other values of γ, thus generalizing the results in Eqs. (40) and (42). Note that including the correlation in the errors for $\zeta^{B\pi}$ and $\zeta_J^{B\pi}$ has led to a smaller uncertainty for $f_+(0)$. Theory uncertainty is not shown in Eq. (42) or Fig. 5, and the most important source is power corrections which we estimate to be ± 0.03 on $f_+(0)$. One-loop $\alpha_s(m_b)$ corrections are also not yet included. Varying $\mu = 2.4$–9.6 GeV in the LL coefficients changes $f_+(0)$ by only a small amount ∓ 0.01.

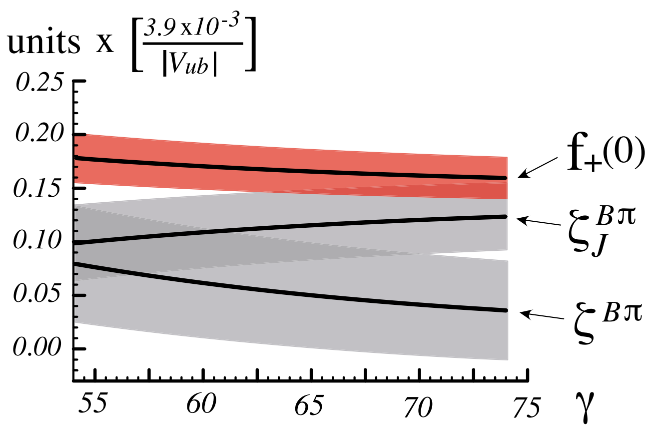

FIG. 5 (color online). Model-independent results for $\zeta^{B\pi}$, $\zeta_J^{B\pi}$, and the $B \to \pi$ form factor $f_+(q^2 = 0)$ as a function of γ. The shaded bands show the 1-σ errors propagated from the $B \to \pi\pi$ data.

It is interesting to note that the central values from our fit to the data give $\zeta_J^{B\pi} \gtrsim \zeta^{B\pi}$ which differs from the hierarchy used in QCDF. Furthermore our central value for $f_+(0)$ is substantially smaller than the central values obtained from both QCD sum rules [29] [$f_+(0) = 0.26$], from form factor model based fits to the semileptonic data [30] [$f_+(0) = 0.21$], or those used in the QCDF analysis [4] [$f_+(0) = 0.28$ or 0.25].

Our analysis can also be used to make a prediction for $Br(B^0 \to \pi^0\pi^0)$. At tree level in SCET $|t_n| = |t| - 1$ which gives

$$\frac{\bar{\Gamma}(B^0 \to \pi^0\pi^0)}{\bar{\Gamma}(B^- \to \pi^0\pi^-)} = \Big(\frac{|t| - 1}{|t|}\Big)^2 + \frac{r_c^2}{|t|^2} - \frac{2r_c}{|t|}\Big(1 - \frac{1}{|t|}\Big)\cos(\delta_c)\cos(\gamma). \tag{43}$$

Thus we predict

$$Br(B^0 \to \pi^0\pi^0) = \begin{cases} (1.0 \pm 0.7) \times 10^{-6}, & \gamma = 54^\circ \\ (1.3 \pm 0.6) \times 10^{-6}, & \gamma = 64^\circ \\ (1.8 \pm 0.7) \times 10^{-6}, & \gamma = 74^\circ. \end{cases} \tag{44}$$

These results are all in reasonable agreement with the current world average. The uncertainty quoted in Eq. (44) is only from the inputs in Eq. (43), and will be directly reduced when the first four measurements in Eq. (31) improve. Since the $\zeta_J^{B\pi}$ term in Eq. (40) is $\gtrsim \zeta^{B\pi}$ our results for $Br(B^0 \to \pi^0\pi^0)$ are not contaminated and we expect that theoretical uncertainty from power corrections plus $\alpha_s(m_b)$ corrections will add a $\sim 20\%$–30% uncertainty to the results in Eq. (44). Note that one can turn the analysis in Eq. (44) around and use the data on $B \to \pi\pi$ in Eq. (31) to give a new method for determining the value of γ, where the theoretical input from factorization is that the tree triangle is flat.

Our values in Eq. (44) are somewhat larger than the central values predicted in QCDF ($\sim 0.3 \times 10^{-6}$ [4]) or pQCD ($\sim 0.2 \times 10^{-6}$ [31]). For $\gamma = 54^\circ$ the first term in Eq. (43) dominates our result, while the r_c^2 penguin term has a large cancellation with the interference term $\propto \cos(\gamma)$. For larger γ's this cancellation becomes less effective and $Br(B^0 \to \pi^0\pi^0)$ increases. In QCDF $\zeta^{B\pi}$ dominates over a small $\zeta_J^{B\pi}$, but has a small coefficient $\propto C_2 + C_1/3$, so the first term in Eq. (43) is small. In pQCD the $M_{a,e}$ terms which are multiplied by C_1 are also small for $B \to \pi^0\pi^0$.

In this paper we have used SCET to derive a factorization theorem for $B \to M_1 M_2$ decays and explored the theoretical and phenomenological implications. Several issues for $B \to M_1 M_2$ still remain to be resolved. A factorization formula for the polarization effects should be investigated beyond leading order. It needs to be shown that the n–$\bar{n}$ factorization is not spoiled by Glauber degrees of freedom. The one-loop Δb_i's need to be computed, as well as a resummation of Sudakov logarithms which are given by the evolution equations for the SCET operators. Charming penguin effects need to be better understood in an effective theory approach, and a full factorization theorem for the $A_{c\bar{c}}$ amplitude should be worked out. Finally, power corrections (including so-called chirally enhanced terms, annihilation contributions, and $C_1 \Lambda/E$ terms) should be studied using SCET.

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