

Vector-pseudoscalar two-meson distribution amplitudes in three-body B meson decays

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We study three-body nonleptonic decays $B \rightarrow VVP$ by introducing two-meson distribution amplitudes for the vector-pseudoscalar pair, such that the analysis is simplified into the one for two-body decays. The twist-2 and twist-3 ϕK two-meson distribution amplitudes, associated with longitudinally and transversely polarized ϕ mesons, are constrained by the experimental data of the $\tau \rightarrow \phi K \nu$ and $B \rightarrow \phi K \gamma$ branching ratios. We then predict the $B \rightarrow \phi K \gamma$ and $B \rightarrow \phi \phi K$ decay spectra in the ϕK invariant mass. Since the resonant contribution in the ϕK channel is negligible, the above decay spectra provide a clean test for the application of two-meson distribution amplitudes to three-body B meson decays.

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Viewing the experimental progress on three-body nonleptonic B meson decays [1,2], it is urgent to construct a corresponding framework. In [3] we have proposed a formalism based on the collinear factorization theorem in perturbative QCD (PQCD), in which new nonperturbative inputs, the two-meson distribution amplitudes, were introduced [4]. On one hand, a direct evaluation of hard kernels for three-body decays, which contain two virtual gluons at lowest order, is not practical due to the enormous number of diagrams. On the other hand, the region with the two gluons being hard simultaneously is power suppressed and not important. Therefore, the new nonperturbative inputs are necessary for catching dominant contributions in a simple manner. In our formalism the collinear factorization formula for a $B \rightarrow h_1 h_2 h_3$ decay amplitude is written, in general, as

$$\mathcal{M} = \Phi_B \otimes H \otimes \Phi_{h_1 h_2} \otimes \Phi_{h_3}, \quad (1)$$

where Φ_{B, h_3} are the B, h_3 meson distribution amplitudes, $\Phi_{h_1 h_2}$ the $h_1 h_2$ two-meson distribution amplitude, and \otimes represents the convolution in longitudinal momentum fractions x . $\Phi_{h_1 h_2}$ and Φ_{h_3} include not only the twist-2 (leading-twist), but two-parton twist-3 (next-to-leading-twist) components. The computation of the hard kernel H , basically the same as in two-body B meson decays, is restricted to leading order in the coupling constant α_s , so far.

There are two types of factorization theorems: collinear factorization [5–9] and k_T factorization [10,11]. For a comparison of the two types of theorems, refer to [12,13]. Collinear factorization works, if it does not develop an end-point singularity from $x \rightarrow 0$. If it does, collinear factorization breaks down, and k_T factorization is more appropriate. It has been known that collinear factorization of charmed and charmless two-body B me-

son decays suffers the end-point singularities [14]. This is the motivation to develop the PQCD formalism for two-body B meson decays based on k_T factorization [15–17]. This approach has been shown to be infrared finite, gauge invariant, and consistent with the factorization assumption in the heavy-quark limit [18–20]. For three-body B meson decays, the end-point singularities are smeared by the two-meson invariant mass [3], and collinear factorization in Eq. (1) holds. Moreover, it has been demonstrated that both nonresonant contributions and resonant contributions through two-body channels can be included by means of an appropriate parametrization of $\Phi_{h_1 h_2}$ [3].

One of the challenges in the studies of three-body heavy meson decays is the evaluation of the matrix elements for heavy meson transition into two hadrons. There are already several theoretical approaches to this subject in the literature. The naive factorization [21] for three-body B meson decays has been adopted in [22], in which the B meson transition into two hadrons was simply parametrized by a power-law behavior and then fit to experimental data. The matrix elements for the above transition were calculated using the pole model [23–25], in which intermediate-state decays into two hadrons were described by effective weak and strong Lagrangians. The naive factorization has been improved in a so-called QCD-factorization framework [26]. However, only the current-produced amplitudes, i.e., those which can be expressed as products of two form factors in the factorization limit, were studied. The challenging subject of the B meson transition into two hadrons was not addressed [26]. Compared to the above methods, our approach does not rely on the naive factorization, since the nonfactorizable contribution is taken into account through nonfactorizable hard kernels. It is complete in the sense that various topologies of amplitudes, such as the B meson transition into two hadrons and the current-induced one, are analyzed in the same framework. It is also more systematic, because subleading corrections can be evaluated order by order in α_s and power by power in the

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ratios w/m_B and m_{h_3}/m_B , where w is the invariant mass of the two-meson system, and m_B (m_{h_3}) the B (h_3) meson mass.

In [3] we have applied Eq. (1) to the modes, in which both h_1 and h_2 are pseudoscalar mesons P . The modes with h_1 being a vector meson V and h_2 a pseudoscalar meson P have been observed recently [27]. Hence, we shall extend our formalism to three-body decays involving the $B \rightarrow VP$ transition, taking $B \rightarrow \phi\phi K$ as an example. We shall first define the ϕK two-meson distribution amplitudes, which are more complicated than the PP ones. A simple parametrization is then proposed, and constrained by the experimental data of the $\tau \rightarrow \phi K \nu$ and $B \rightarrow \phi K \gamma$ branching ratios. Afterwards, we predict the decay spectra of the $B \rightarrow \phi K \gamma$ and $B \rightarrow \phi\phi K$ modes in the ϕK invariant mass. The resonant contribution through the ϕK channel is expected to be negligible: the $K_1(1650)$, $K_2(1770)$, and $K(1830)$ mesons decay into the ϕK pair with the branching ratios not yet available in [28]. Therefore, the above spectra provide a clean test for the application of two-meson distribution amplitudes to three-body B meson decays.

Label the momenta of the ϕ and K mesons from the B meson transition as P_1 and P_2 , respectively. The B meson

momentum P_B and the total momentum of the ϕK pair, $P = P_1 + P_2$, are chosen, in the light-cone coordinates, as

$$P_B = \frac{m_B}{\sqrt{2}}(1, 1, \mathbf{0}_T), \quad P = \frac{m_B}{\sqrt{2}}(1, \eta, \mathbf{0}_T), \quad (2)$$

with the variable $\eta = w^2/m_B^2$. Define $\zeta = P_1^+/P^+$ as the ϕ meson momentum fraction and $r_\phi = m_\phi/m_B$ as the ϕ meson- B meson mass ratio, in terms of which the other kinematic variables are expressed as

$$\begin{aligned} P_2^+ &= (1 - \zeta)P^+, & P_1^- &= [(1 - \zeta)\eta + r_\phi^2]P^+, \\ P_2^- &= (\zeta\eta - r_\phi^2)P^+, \\ P_1^x &= -P_2^x = \sqrt{(\zeta w^2 - m_\phi^2)(1 - \zeta)}, \\ (P_1^x)^2 &= (P_2^x)^2 = P_T^2. \end{aligned} \quad (3)$$

The polarization vectors $\epsilon(\phi)$ of the ϕ meson are obtained from the orthogonality $\epsilon(\phi) \cdot P_1 = 0$ and from the normalization $\epsilon(\phi)^2 = -1$. The exact expressions are given, in the light-cone coordinates $\epsilon = (\epsilon^+, \epsilon^-, \epsilon^x, \epsilon^y)$, by

$$\begin{aligned} \epsilon_L(\phi) &= \frac{1}{r_\phi} \left(\frac{\zeta[\zeta + (1 - \zeta)\eta + r_\phi^2] - 2r_\phi^2}{\sqrt{2}\sqrt{[\zeta + (1 - \zeta)\eta + r_\phi^2]^2 - 4r_\phi^2}}, \frac{[(1 - \zeta)\eta + r_\phi^2][\zeta + (1 - \zeta)\eta + r_\phi^2] - 2r_\phi^2}{\sqrt{2}\sqrt{[\zeta + (1 - \zeta)\eta + r_\phi^2]^2 - 4r_\phi^2}}, \right. \\ &\quad \left. \times \frac{[\zeta + (1 - \zeta)\eta + r_\phi^2]\sqrt{(\zeta\eta - r_\phi^2)(1 - \zeta)}}{\sqrt{[\zeta + (1 - \zeta)\eta + r_\phi^2]^2 - 4r_\phi^2}}, 0 \right), \\ \epsilon_T^{(1)}(\phi) &= \left(-\frac{\sqrt{2}\sqrt{(\zeta\eta - r_\phi^2)(1 - \zeta)}}{\sqrt{[\zeta + (1 - \zeta)\eta + r_\phi^2]^2 - 4r_\phi^2}}, \frac{\sqrt{2}\sqrt{(\zeta\eta - r_\phi^2)(1 - \zeta)}}{\sqrt{[\zeta + (1 - \zeta)\eta + r_\phi^2]^2 - 4r_\phi^2}}, \frac{[\zeta - (1 - \zeta)\eta - r_\phi^2]}{\sqrt{[\zeta + (1 - \zeta)\eta + r_\phi^2]^2 - 4r_\phi^2}}, 0 \right), \\ \epsilon_T^{(2)}(\phi) &= (0, 0, 0, 1). \end{aligned} \quad (4)$$

The terms proportional to r_ϕ will be neglected eventually. The kaon is treated as a massless particle. The ϕ meson emitted from the weak vertex then carries the momentum $P_3 = (m_B/\sqrt{2})(0, 1 - \eta, \mathbf{0}_T)$. Another equivalent, but more general, representation of $\epsilon(\phi)$ is given by

$$\begin{aligned} \epsilon_L^\pm(\phi) &= \frac{P_T^2 \pm P_1^+(P_1^+ - P_1^-)}{\sqrt{2}m_\phi p}, \\ \epsilon_L^x(\phi) &= \frac{P_T(P_1^+ + P_1^-)}{\sqrt{2}m_\phi p}, & \epsilon_T^{(1)\pm}(\phi) &= \frac{\mp P_T}{\sqrt{2}p}, \\ \epsilon_T^{(1)x}(\phi) &= \frac{P_1^+ - P_1^-}{\sqrt{2}p}, \end{aligned} \quad (5)$$

with $p = \sqrt{P_T^2 + (P_1^+ - P_1^-)^2/2}$.

The three-body B meson decays are dominated by the contribution from the region, in which the ϕK pair pos-

sesses the invariant mass $w^2 \sim O(\bar{\Lambda}m_B)$ [3], $\bar{\Lambda}$ representing a hadronic scale. The orders of magnitude of the components,

$$P^+ \sim O(m_B), \quad P^- \sim O(\bar{\Lambda}), \quad P_T \sim O(\sqrt{\bar{\Lambda}m_B}), \quad (6)$$

are then implied. It is easy to obtain the power counting rules of the polarization vectors from Eq. (4),

$$\begin{aligned} \epsilon_L^+(\phi) &\sim \frac{1}{r_\phi} O(1), & \epsilon_L^x(\phi) &\sim \frac{1}{r_\phi} O(\sqrt{\bar{\Lambda}/m_B}), \\ \epsilon_L^-(\phi) &\sim \frac{1}{r_\phi} O(\bar{\Lambda}/m_B), \end{aligned} \quad (7)$$

$$\epsilon_T^{(1)+}(\phi) \sim \epsilon_T^{(1)-}(\phi) \sim O(\sqrt{\bar{\Lambda}/m_B}),$$

$$\epsilon_T^{(1)x}(\phi) \sim O(1).$$

In the heavy-quark limit the hierarchy $P^+ \gg P_T \gg P^-$ corresponds to a collinear configuration, and suggests the employment of the new nonperturbative inputs, the ϕK two-meson distribution amplitudes. For the PP system, there is only a single twist-2 distribution amplitude associated with the structure γ_μ , and two two-parton twist-3 distribution amplitudes associated with the structures I (the identity) and $\sigma_{\mu\nu}$ [3,4,29]. Here a higher-twist distribution amplitude means that its contribution is suppressed by powers of w/m_B . For the VP system, the relevant structures are more complicated: three twist-2 distribution amplitudes are associated with $\gamma_\mu\gamma_5$ and $\sigma_{\mu\nu}\gamma_5$, and five twist-3 distribution amplitudes with $\gamma_\mu\gamma_5$, $\sigma_{\mu\nu}\gamma_5$, γ_5 , and γ_μ . To decompose the two-meson distribution amplitudes into the components of different twists, we introduce the polarization vectors of the ϕK system,

$$\begin{aligned} \epsilon_L &= \frac{1}{\sqrt{2}\eta}(1, -\eta, 0, 0), & \epsilon_T^{(1)} &= (0, 0, 1, 0), \\ \epsilon_T^{(2)} &= (0, 0, 0, 1). \end{aligned} \quad (8)$$

$$\langle \phi K^+ | \bar{u}(y^-) \gamma_\mu \gamma_5 s(0) | 0 \rangle = P_\mu \int_0^1 dz e^{izP \cdot y} \Phi_{\parallel}(z, \zeta, w), \quad (10)$$

$$\begin{aligned} \langle \phi K^+ | \bar{u}(y^-) \sigma_{\mu\nu} \gamma_5 s(0) | 0 \rangle &= -i \left\{ (\epsilon_{T\mu} P_\nu - \epsilon_{T\nu} P_\mu) \int_0^1 dz e^{izP \cdot y} \Phi_T(z, \zeta, w) + \frac{2}{w} (P_{1\mu} P_{2\nu} - P_{1\nu} P_{2\mu}) \right. \\ &\quad \left. \times \int_0^1 dz e^{izP \cdot y} \Phi_3(z, \zeta, w) \right\}, \end{aligned} \quad (11)$$

$$\langle \phi K^+ | \bar{u}(y^-) \gamma_5 s(0) | 0 \rangle = w \int_0^1 dz e^{izP \cdot y} \Phi_p(z, \zeta, w), \quad (12)$$

$$\langle \phi K^+ | \bar{u}(y^-) \gamma_\mu s(0) | 0 \rangle = i \frac{w}{P \cdot n_-} \epsilon_{\mu\nu\rho\sigma} \epsilon_T^\nu P^\rho n_-^\sigma \int_0^1 dz e^{izP \cdot y} \Phi_\nu(z, \zeta, w), \quad (13)$$

$$\langle \phi K^+ | \bar{u}(y^-) I s(0) | 0 \rangle = 0, \quad (14)$$

where z is the momentum fraction carried by the spectator u quark, and $n_- = (0, 1, \mathbf{0}_T)$ a null vector. We have adopted the convention $\epsilon^{0123} = 1$ for the Levi-Civita tensor $\epsilon^{\mu\nu\rho\delta}$. The above decomposition applies to other VP systems, such as $K^* \pi$, ρK , ...

Below we present some details of the expansion of the kinematic factors. For Eq. (10), we have applied

$$(P_1 - P_2)_\mu \approx (2\zeta - 1)P_\mu, \quad (15)$$

where the coefficient $2\zeta - 1$ is absorbed into the distribution amplitude Φ_{\parallel} , giving its ζ dependence. Similarly, we have approximated the kinematic factor for the matrix element in Eq. (11),

$$\epsilon_{T\mu}(\phi) P_{1\nu} - \epsilon_{T\nu}(\phi) P_{1\mu} \approx \zeta (\epsilon_{T\mu} P_\nu - \epsilon_{T\nu} P_\mu), \quad (16)$$

where the coefficient ζ is absorbed into Φ_T , and $\epsilon_{T\mu}$ is a

transverse polarization vector of the ϕK system. The contribution from another distribution amplitude Φ_3 can be combined with that from Φ_T via the approximation,

$$\langle \phi [P_1, \epsilon(\phi)] K^+(P_2) | \bar{u}(y^-) \Gamma s(0) | 0 \rangle, \quad (9)$$

using the ϕ meson and kaon distribution amplitudes up to twist 3 [31,32], where Γ represents a structure among I , γ_5 , γ_μ , $\gamma_\mu\gamma_5$, and $\sigma_{\mu\nu}\gamma_5$. The matrix elements can be expressed as the products of the corresponding form factors with the kinematic factors. For example, the matrix element for $\Gamma = \gamma_\mu\gamma_5$ is written as the product of the form factor F_{\parallel} with the kinematic factor $(P_1 - P_2)_\mu$. The kinematic factors are then approximated in terms of the momentum P and the polarization vectors ϵ of the ϕK system according to the power counting rules in Eqs. (6) and (7). The resultant ζ -dependent coefficients in the approximation contribute to the ζ dependence of the ϕK two-meson distribution amplitudes.

We then derive the decomposition up to $O(w/m_B)$,

transverse polarization vector of the ϕK system. The contribution from another distribution amplitude Φ_3 can be combined with that from Φ_T via the approximation,

$$\frac{2}{w} (P_{1\mu} P_{2\nu} - P_{1\nu} P_{2\mu}) \approx 2\sqrt{\zeta(1-\zeta)} (\epsilon_{T\mu}^{(1)} P_\nu - \epsilon_{T\nu}^{(1)} P_\mu), \quad (17)$$

where the coefficient $\sqrt{\zeta(1-\zeta)}$ comes from P_1^x in the $m_\phi \rightarrow 0$ limit. Since the branching ratio is a sum over the transverse polarizations $\epsilon_{T\mu}^{(1)}$ and $\epsilon_{T\mu}^{(2)}$, we omit the coefficient 2, and replace $\epsilon_{T\mu}^{(1)}$ by the two possible $\epsilon_{T\mu}$. We have employed the approximation for the matrix element in Eq. (13),

$$\frac{2}{w} \epsilon_{\mu\nu\rho\sigma} \epsilon_T^\nu(\phi) P_1^\rho P_2^\sigma \approx \frac{w}{P \cdot n_-} \zeta \epsilon_{\mu\nu\rho\sigma} \epsilon_T^\nu P^\rho n_-^\sigma. \quad (18)$$

For this structure, the ϕ meson emitted from the weak vertex must carry a transverse polarization, and a non-vanishing hard kernel demands that the subscript μ denotes a transverse component. The coefficient ζ is then the sum of ζ , $\zeta - 1$, and $1 - \zeta$ from the combinations $[\epsilon_T^{(1)\nu}(\phi) = \epsilon_T^{(1)\perp}(\phi)$, $P^\rho = P^+$, $P_2^\sigma = P_2^-$], $[\epsilon_T^{(1)\nu}(\phi) = \epsilon_T^{(1)\perp}(\phi)$, $P^\rho = P^-$, $P_2^\sigma = P_2^+$], and $[\epsilon_T^{(1)\nu}(\phi) = \epsilon_T^{(1)-}(\phi)$, $P^\rho = P^+$, $P_2^\sigma = P_2^\perp$], respectively. A coefficient 2 for the last combination has been omitted for the same reason.

Our strategy does not provide the z dependence. Assuming the z dependence of each $\Phi_i(z, \zeta, w)$ to be asymptotic, we propose the parametrization,

$$\begin{aligned} \Phi_{\parallel}(z, \zeta, w) &= \frac{3F_{\parallel}(w)}{\sqrt{2N_c}} z(1-z)(2\zeta - 1), \\ \Phi_T(z, \zeta, w) &= \frac{3F_T(w)}{\sqrt{2N_c}} z(1-z)\zeta, \\ \Phi_3(z, \zeta, w) &= \frac{3F_3(w)}{\sqrt{2N_c}} z(1-z), \\ \Phi_p(z, \zeta, w) &= \frac{3F_p(w)}{\sqrt{2N_c}} z(1-z), \\ \Phi_v(z, \zeta, w) &= \frac{3F_v(w)}{\sqrt{2N_c}} z(1-z)\zeta. \end{aligned} \quad (19)$$

The timelike form factors $F_{\parallel,T,3,p,v}(w)$ define the normalization of the ϕK two-meson distribution amplitudes. Note that these form factors are normalized to $F_{\parallel,T,3,p,v}(m_\phi) = 1$ in order to respect the kinematic threshold of decay spectra. Our strategy also reveals the power behaviors of the form factors in the asymptotic region with large w , $F_{\parallel,T}(w) \sim 1/w^2$, and $F_{3,p,v}(w) \sim m_0/w^3$, $m_0 \approx 1.7$ GeV [16,33] being the chiral scale. Therefore, we further parametrize the form factors in the whole range of w for the evaluation of the nonresonant contribution:

$$\begin{aligned} F_{\parallel}(w) &= \frac{m_{\parallel}^2}{(w - m_\phi)^2 + m_{\parallel}^2}, \\ F_T(w) &= \frac{m_T^2}{(w - m_\phi)^2 + m_T^2}, \\ F_3(w) = F_p(w) &= \frac{m_0 m_{\parallel}^2}{(w - m_\phi)^3 + m_0 m_{\parallel}^2}, \\ F_v(w) &= \frac{m_0 m_T^2}{(w - m_\phi)^3 + m_0 m_T^2}, \end{aligned} \quad (20)$$

where the two free parameters $m_{\parallel,T}$, expected to be few GeV [3], are determined by the fit to the measured $\tau \rightarrow \phi K \nu$ and $B \rightarrow \phi K \gamma$ branching ratios [28]. The form factors depending on the parameter m_{\parallel} (m_T) are associated with the longitudinally (transversely) polarized ϕ meson.

We stress that Eqs. (10) and (11) contain not only the twist-2 distribution amplitudes, but the twist-3 ones. The

expansion in Eq. (15) corresponding to the component $\mu = \perp$ generates

$$\begin{aligned} w \epsilon_{T\mu} \int_0^1 dz e^{izP \cdot y} \Phi_a(z, \zeta, w), \\ \Phi_a(z, \zeta, w) = \frac{3F_{\parallel}(w^2)}{\sqrt{2N_c}} z(1-z)\sqrt{\zeta(1-\zeta)}. \end{aligned} \quad (21)$$

Similarly, we extract two twist-3 distribution amplitudes from Eqs. (16) and (17) corresponding to the components $\mu, \nu = +, -$, given by

$$\begin{aligned} (\epsilon_{L\mu} P_\nu - \epsilon_{L\nu} P_\mu) \int_0^1 dz e^{izP \cdot y} [(2\zeta - 1)\Phi_3(z, \zeta, w) \\ - \Phi_t(z, \zeta, w)], \\ \Phi_t(z, \zeta, w) = \frac{3F_T(w^2)}{\sqrt{2N_c}} z(1-z)\sqrt{\zeta(1-\zeta)}. \end{aligned} \quad (22)$$

For the ϕK system, the above twist-3 distribution amplitudes lead to smaller contributions compared to Φ_p and Φ_v , and have been ignored: because of $m_\phi^2/w^2 \sim 1$, the range in Eq. (28) below indicates $\zeta \sim 1$, and that the contribution from Φ_a is suppressed by the factor $\sqrt{1-\zeta}$. There exists a strong cancellation in Eq. (22). For other systems, such as ρK , these twist-3 distribution amplitudes could be numerically important, because of $m_\rho^2/w^2 \ll 1$ in this case.

For the B meson distribution amplitude, we use the model [16],

$$\Phi_B(x) = N_B x^2 (1-x)^2 \exp\left[-\frac{1}{2} \left(\frac{x m_B}{\omega_B}\right)^2\right], \quad (23)$$

with the shape parameter $\omega_B = 0.40 \pm 0.04$ GeV [34], and the normalization constant N_B being related to the decay constant $f_B = 190$ MeV (in the convention $f_\pi = 130$ MeV) via $\int_0^1 \Phi_B(x) dx = f_B/(2\sqrt{2N_c})$. The range of ω_B is determined from a fit to the values of the $B \rightarrow \pi$ form factor from light-cone sum rules [35,36]. The above Φ_B is identified as Φ_+ of the two leading-twist B meson distribution amplitudes Φ_\pm defined in [37,38]. Equation (23), vanishing at $x \rightarrow 0$, is consistent with the behavior required by equations of motion [39]. It has been shown that the B meson distribution amplitude is normalizable in k_T factorization theorem [40], contrary to the conclusion drawn in the framework of collinear factorization theorem [41,42]. Another distribution amplitude $\bar{\Phi}_B$, identified as $\bar{\Phi}_B = (\Phi_- - \Phi_+)/\sqrt{2}$ with a zero normalization, contributes at the next-to-leading power of $\bar{\Lambda}/m_B$ [34]. It has been verified numerically [43] that the contribution to the $B \rightarrow \pi$ form factor from Φ_B is much larger than from $\bar{\Phi}_B$.

In summary, we calculate the hard kernels by contracting the quark-level diagrams with the matrix elements,

$$\begin{aligned}
\langle 0|\bar{b}(0)_i d(y^-)_j|B(P_B)\rangle &= \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{-ixP \cdot y} [(\not{P}_B + m_B)\gamma_5]_{ij} \Phi_B(x), \\
\langle \phi K(P, \epsilon_L)|\bar{u}(y^-)_j s(0)_i|0\rangle &= \frac{1}{\sqrt{2N_c}} \int_0^1 dz e^{izP \cdot y} \{(\gamma_5 \not{P})_{ij} \Phi_{\parallel}(z, \zeta, w) + (\gamma_5)_{ij} w \Phi_p(z, \zeta, w)\}, \\
\langle \phi K(P, \epsilon_T)|\bar{u}(y^-)_j s(0)_i|0\rangle &= \frac{1}{\sqrt{2N_c}} \int_0^1 dz e^{izP \cdot y} \left\{ (\gamma_5 \not{\epsilon}_T \not{P})_{ij} [\Phi_T(z, \zeta, w) + \Phi_3(z, \zeta, w) \sqrt{\zeta(1-\zeta)}] \right. \\
&\quad \left. + i \frac{w}{P \cdot n_-} \epsilon_{\mu\nu\rho\sigma} (\gamma^\mu)_{ij} \epsilon_T^\nu P^\rho n_-^\sigma \Phi_v(z, \zeta, w) \right\}, \tag{24}
\end{aligned}$$

which follow Eqs. (10)–(13). The calculation of hard kernels is as simple as of two-body decays. It is observed that the distribution amplitudes $\Phi_{\parallel,T,3}$ give leading contributions, and those from $\Phi_{p,v}$ are suppressed by a power of w/m_B .

The $\tau \rightarrow \phi K \nu$ differential decay rate in the ϕK invariant mass is written as

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_\tau^4}{384\pi^3} |V_{us}|^2 \sqrt{\eta}(1-\eta)^2 F_{\parallel}^2, \tag{25}$$

with m_τ being the τ lepton mass, and V_{us} the Cabibbo-Kobayashi-Maskawa matrix element. The $B \rightarrow \phi K \gamma$ decay spectrum is written as

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_B^4}{256\pi^3} \sqrt{\eta}(1-\eta) \int_{m_b^2/w^2}^1 d\zeta |\mathcal{M}(\zeta, w)|^2, \tag{26}$$

with the amplitude,

$$\begin{aligned}
\mathcal{M}(\zeta, w) &= \frac{e}{4\pi^2} V_{ts} V_{tb} m_b \mathcal{A}(\zeta, w), \\
\mathcal{A}(\zeta, w) &\equiv \langle \phi K | \bar{b} \sigma^{\mu\nu} \epsilon_\mu(\gamma) q_\nu (1 - \gamma_5) s | B \rangle \\
&= 8\pi C_F m_B^2 \epsilon_T(\gamma) \cdot \epsilon_T \int_0^1 dx_1 dz \frac{\Phi_B(x_1)}{x_1 z m_B^2 + P_T^2} \left\{ [(1+z)(\Phi_T(z, \zeta, w) + \Phi_3(z, \zeta, w) \sqrt{\zeta(1-\zeta)}) \right. \\
&\quad \left. + \sqrt{\eta}(1-2z)\Phi_v(z, \zeta, w)] \frac{\alpha_s(t_e^{(1)}) C_{7\gamma}^{\text{eff}}(t^{(1)})}{z m_B^2 + P_T^2} + \sqrt{\eta} \Phi_v(z, \zeta, w) \frac{\alpha_s(t_e^{(2)}) C_{7\gamma}^{\text{eff}}(t^{(2)})}{x_1 m_B^2} \right\}. \tag{27}
\end{aligned}$$

$\epsilon(\gamma)$ and q_ν represent the photon polarization vectors and the photon momentum, respectively. m_b is the b quark mass, and $C_{7\gamma}^{\text{eff}}$ the corresponding effective Wilson coefficient [44]. All the terms of $O(\eta)$ in \mathcal{M} have been neglected for consistency. The requirement $P_T^2 \geq 0$ leads to the bounds of ζ as shown in Eq. (26),

$$m_\phi^2/w^2 \leq \zeta \leq 1. \tag{28}$$

The hard scales are chosen as the maximal virtuality in each quark-level diagram [3,16],

$$t^{(1)} = \sqrt{z m_B^2 + P_T^2}, \quad t^{(2)} = \sqrt{x_1 m_B^2 + P_T^2}. \tag{29}$$

The above collinear factorization formula is well defined, since the invariant mass of the two-pion system, appearing through P_T , smears the end-point singularities from $z \rightarrow 0$. Even if one adopts a model of the B meson distribution amplitude, which vanishes only linearly in x_1 , Eq. (27) is still well defined due to the presence of P_T .

Because there exists only an upper bound for the measured $\tau \rightarrow \phi K \nu$ branching ratio, we also consider the $\tau \rightarrow K^* \pi \nu$ branching ratio, when constraining the parameter m_{\parallel} . That is, we assume that m_{\parallel} in the two decay modes, i.e., the timelike ϕK and $K^* \pi$ form factors, do not

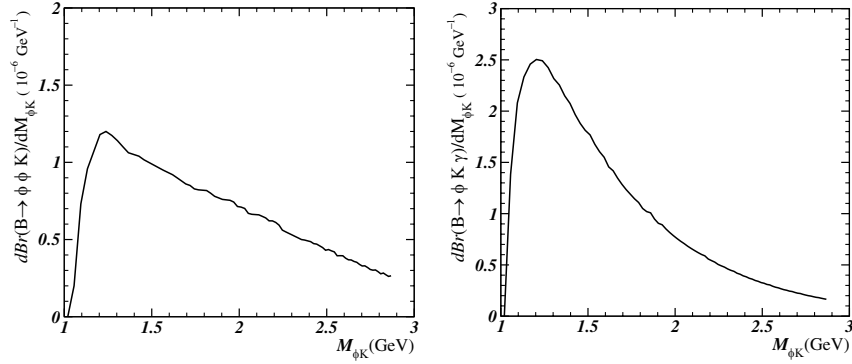
differ much. The experimental data and the theoretical prediction from chiral perturbation theory are (Refs. [28,45,46], respectively)

$$\begin{aligned}
B(\tau \rightarrow \phi K \nu) &< 6.7 \times 10^{-5}, \\
B(\tau \rightarrow K^* \pi \nu) &\approx 2.7 \times 10^{-4}, \\
B(B \rightarrow \phi K \gamma) &= (3.4 \pm 0.9 \pm 0.4) \times 10^{-6}. \tag{30}
\end{aligned}$$

For the application of our formalism to the τ decays, we shall trust it up to the order-of-magnitude accuracy, since the ratio $\eta = w^2/m_\tau^2$ in this case is not a small parameter. With $m_{\parallel,T} \approx 2$ GeV, we obtain from Eqs. (25) and (27)

$$\begin{aligned}
B(\tau \rightarrow \phi K \nu) &= 6.8 \times 10^{-5}, \\
B(\tau \rightarrow K^* \pi \nu) &= 4.5 \times 10^{-4}, \\
B(B \rightarrow \phi K \gamma) &= (2.9_{-0.5}^{+0.7}) \times 10^{-6}, \tag{31}
\end{aligned}$$

consistent with Eq. (30) (up to order of magnitude for the τ decay data as stated above). Our results are stable with respect to the variation of m_{\parallel} and m_τ around few GeV. Therefore, the theoretical error in Eq. (31) comes from the variation of the shape parameter ω_B , which can be regarded as an estimate of the uncertainty from hadronic dynamics. The predicted $B \rightarrow \phi K \gamma$ decay spectrum is

FIG. 1. $B \rightarrow \phi K \gamma$ and $B \rightarrow \phi \phi K$ decay spectra in the ϕK invariant mass.

shown in Fig. 1, which exhibits a maximum at the ϕK invariant mass around 1.3 GeV, consistent with our power counting rules.

After constraining the two-meson distribution amplitudes, we predict the $B \rightarrow \phi \phi K$ decay spectrum in the ϕK invariant mass. For this mode, the amplitude \mathcal{M} is written as

$$\mathcal{M} = f_\phi V_{tb}^* V_{ts} \sum_{i=3}^5 [\mathcal{F}_{Li}^{P(s)} + \epsilon_T \cdot \epsilon_{3T}(\phi) \mathcal{F}_{Ti}^{P(s)}], \quad (32)$$

$$\begin{aligned} \mathcal{F}_{Li}^{P(s)} = & 8\pi C_F m_B^2 \int_0^1 dx_1 dz \frac{\Phi_B(x_1) \Phi_{\parallel}(z, \zeta, w)}{x_1 z m_B^2 + P_T^2} \left\{ [(1 \right. \\ & + z) \Phi_{\parallel}(z, \zeta, w) + \sqrt{\eta}(1-2z) \Phi_p(z, \zeta, w)] \\ & \times \frac{\alpha_s(t_e^{(1)}) a_i^{(s)}(t^{(1)})}{z m_B^2 + P_T^2} + 2\sqrt{\eta} \Phi_p(z, \zeta, w) \\ & \left. \times \frac{\alpha_s(t_e^{(2)}) a_i^{(s)}(t_e^{(2)})}{x_1 m_B^2} \right\}, \quad (33) \end{aligned}$$

$$\begin{aligned} \mathcal{F}_{Ti}^{P(s)} = & 8r_\phi \pi C_F m_B^2 \int_0^1 dx_1 dz \frac{\Phi_B(x_1)}{x_1 z m_B^2 + P_T^2} \\ & \times \left\{ [\Phi_T(z, \zeta, w) + \Phi_3(z, \zeta, w) \sqrt{\zeta(1-\zeta)} \right. \\ & + z\sqrt{\eta} \Phi_v(z, \zeta, w)] \frac{\alpha_s(t_e^{(1)}) a_i^{(s)}(t^{(1)})}{z m_B^2 + P_T^2} \\ & \left. - \sqrt{\eta} \Phi_v(z, \zeta, w^2) \frac{\alpha_s(t_e^{(2)}) a_i^{(s)}(t_e^{(2)})}{x_1 m_B^2} \right\}, \quad (34) \end{aligned}$$

where $\epsilon_{3T}(\phi)$ denote the polarization vectors of the ϕ meson emitted from the weak vertex. The definitions of the Wilson coefficients $a_i^{(q)}(t)$ are referred to [47]. For a similar reason, we have dropped all the $O(\eta)$ terms. Equations (27), (33), and (34) represent the amplitudes of the B meson transition into a VP meson pair associated with different effective operators. We display the predicted $B^\pm \rightarrow \phi \phi K^\pm$ decay spectrum in Fig. 1, which also exhibits a maximum at the ϕK invariant mass

around 1.3 GeV. Integrating the spectrum over η , we obtain the branching ratio without the resonant contribution in the $\phi\phi$ channel,

$$B(B^\pm \rightarrow \phi \phi K^\pm) = (1.3_{-0.3}^{+0.4}) \times 10^{-6}. \quad (35)$$

The uncertainty arises from the variation of the shape parameter ω_B of the B meson distribution amplitude.

We have examined other sources of theoretical uncertainty. The correction to the branching ratios from the neglected $O(\eta)$ terms is about 10%. To investigate the uncertainty from different parametrization of meson distribution amplitudes, we have tried

$$\Phi'_B(x) = N'_B x(1-x) \exp\left[-\frac{1}{2} \left(\frac{x m_B}{\omega'_B}\right)^2\right]. \quad (36)$$

First, the shape parameter $\omega'_B = 0.9$ GeV is determined from the fit to the value of the $B \rightarrow \pi$ transition form factor about 0.3. The model $\Phi'_B(x)$ is then employed to fix the ϕK two-meson distribution amplitudes from the data of the $B \rightarrow \phi K \gamma$ branching ratios. It is observed that the symmetric z dependence in Eq. (19) should be modified into

$$z(1-z) \rightarrow z(1-z)[1 + 0.5(1-2z)], \quad (37)$$

which is reasonable since the ϕ meson is heavier than the kaon. After going through the above procedure, we predict the $B \rightarrow \phi \phi K$ branching ratio using the distribution amplitudes in Eqs. (36) and (37), and find that the result increases only by 8%. We have also checked the sensitivity of our prediction to the parametrization of the timelike form factors. Obeying the normalization and the asymptotic behavior required by PQCD, the models with $(w - m_\phi)^2$ [$(w - m_\phi)^3$] being replaced by $w^2 - m_\phi^2$ [$(w^2 - m_\phi^2)^{3/2}$] are also allowed. Adopting the B meson distribution amplitude in Eq. (23), the $\tau \rightarrow \phi K \nu$ and $B \rightarrow \phi K \gamma$ data just imply a slight increase of the parameters m_{\parallel} and m_T to 3–4 GeV. Then we predict the $B \rightarrow \phi \phi K$ branching ratio using the new parametrization, which is enhanced only by 12%. The above investigations indicate that the PQCD predictions will be insensitive to the

parametrization of meson distribution amplitudes, if the procedure of determining meson distribution amplitudes is followed.

Note that the $B^\pm \rightarrow \phi\phi K^\pm$ branching ratio has been measured to be $B(B^\pm \rightarrow \phi\phi K^\pm) = (2.6_{-0.9}^{+1.1} \pm 0.3) \times 10^{-6}$ for a $\phi\phi$ invariant mass below 2.85 GeV [27]. We suggest that the decay spectrum in the ϕK invariant mass should also be measured (only the spectrum in the $\phi\phi$ invariant mass was presented in [27]), such that the

dynamics of the $B \rightarrow VP$ transition can be explored. To derive the spectrum in the $\phi\phi$ invariant mass, we need to define the VV two-meson distribution amplitudes, which will be discussed in the future.

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