Pair production at the focus of two equal and oppositely directed laser beams: The effect of the pulse shape

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We calculate the probability that an $e^- - e^+$ pair is created at rest from vacuum at the focus of two equal and oppositely directed pulsed laser beams. The effects of the finite duration and of the temporal behavior of the resulting laser pulse are taken into account perturbatively and the production probability is compared with the corresponding quantity calculated in the presence of an infinite laser beam with a constant pulse shape. By inserting theoretically achievable numerical values of the pulse parameters it is shown that the induced correction to the production probability cannot be neglected.

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I. INTRODUCTION

In his seminal work [1] Schwinger determined the probability that an $e^- - e^+$ pair is created from vacuum in the presence of a constant and uniform electric field with strength E. Schwinger used the effective Lagrangian technique to obtain the production probability per unit volume and time and the same result has been obtained in [2] by using the causal Green function of the Dirac equation. In the weak field regime $E \ll E_{cr} = m^2c^3/(\hbar e)$ with m and $-e < 0$ the mass and the electric charge of the electron, the probability resulted proportional to the so-called Schwinger factor $\exp(-\pi E_{cr}/E)$. Now, the numerical value of $E_{cr} = 1.3 \times 10^{16}$V/cm is so high that, also today, it is impossible to check experimentally the effect of pair creation because of the technical inability to produce (even locally) a constant and uniform electric field with strength of the order of E_{cr}. For this reason, since the early 1970s many studies have been carried out about the possibility that pairs are created from vacuum in other physical situations such as in the electric field of two colliding heavy ions [3] or at the focus of a laser beam [4]. Concerning this last physical situation, it seems that the x-ray free electron lasers facilities that today are being built at SLAC and DESY laboratories may give the possibility to observe the production from vacuum of $e^- - e^+$ pairs (see [5] and, in particular, Refs. [31–35] therein). As a consequence, many theoretical papers have been devoted to the study of pair production at the focus of a laser beam [6–13]. Actually, in [1] it was also shown that the creation of a pair from vacuum is forbidden in the field of a plane monochromatic wave without the presence of a third body. In fact, by going into details, in [6,12] the production of pairs was studied in the presence of a heavy nucleus, while in [7,13] in the presence of an existing electron-positron plasma. Instead, Fried *et al.* in [8] calculated, by means of the Fradkin functional formulation, the pair production probability per unit volume and unit time at the focus of two crossed lasers at right angle while, finally, the authors of Refs. [9–11] chose the configuration with the two laser beams oppositely directed. In particular, in [9,11], by solving numerically the coupled system of Maxwell and Vlasov equations it is estimated that a few hundred pairs can be created in the spot of two laser beams with central wavelength $\lambda = 0.15$ nm and peak electric field $E = 1.3 \times 10^{15}$ V/cm. Instead, in [10] the pair production probability is calculated by using both analytical and numerical tools and starting from the one-particle Dirac equation.

In all the cited papers the analytical calculations and the theoretical predictions have been done by assuming an infinite duration of the laser pulse and a constant pulse shape. This is, of course, a good approximation when the laser pulse duration is much larger than the laser period. Nevertheless, today, very short extreme ultraviolet and soft x-ray laser pulses of the order of a few hundred attoseconds (1 as $= 10^{-18}$ s) have been produced [14,15] and theoretical estimates suggest that single attosecond pulses may be realized [16–19]. In this respect, it is reasonable to imagine that the effects of the finite duration of the laser pulse and of the temporal behavior of the pulse-shape function may enter the game and may be revealed. In the present paper we want to calculate the probability that an $e^- - e^+$ pair is created at rest from vacuum at the focus of two equal and oppositely directed pulsed laser beams by taking into account the finite duration of the resulting laser pulse. The theoretical model and the approximations that we used and that are described in the next section, allowed us to reduce the problem to solve the equation of a two-level system in the presence of the external laser pulse. This system has been largely used in the study of high order harmonic generation by atoms [20–25]. In particular, some results we have already obtained in [25] will be used here also. Finally, the production probability so calculated has been compared with the analogous quantity obtained in the presence of an infinite laser pulse with a constant pulse shape

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and, as we will remark in the conclusion [see Sec. III], the difference between the two results is appreciable and, hopefully, measurable.

We point out that natural units with $\hbar = c = 1$ will be used throughout.

II. THEORETICAL MODEL AND RESULTS

According to what we have said in the introduction, we have to calculate the production of a pair in the field of two identical and oppositely directed pulsed laser beams. On the one hand, without resorting to second quantization, we can describe the production of the pair in the Dirac one-particle framework, that is, as the transition of the electron from a negative energy level to a positive one. On the other hand, since we are dealing at the most with an x-ray laser (wavelength $\sim 10^{-9} - 10^{-8}$ cm), the spatial nonuniformity of the laser beam can be neglected in the creation region which is of the order of the Compton length $\lambda\!\!\!^{-} = 1/m = 3.9 \times 10^{-11}$ cm. For this reason, the one-particle Dirac Hamiltonian of the electron in the presence of two oppositely directed laser beams with central angular frequency ω and peak electric field E can be written as [10]

$$H(t) = \boldsymbol{\alpha} \cdot [\mathbf{p} + e\mathbf{A}(t)] + \beta m \tag{1}$$

where the Dirac matrices $\boldsymbol{\alpha}$ and β are intended in the Dirac representation and where, by assuming linearly polarized lasers along the z direction,

$$\mathbf{A}\,(t) = \frac{E}{\omega} f_T(t) \cos\omega t \hat{\mathbf{k}}. \tag{2}$$

The Hamiltonian (1) with $\mathbf{A}(t)$ given in the previous equation is identical to that used in [10] apart from the pulse-shape function $f_T(t)$. This function, in fact, has been introduced to describe the temporal shape of the laser pulse and it depends on the parameter T which is the duration of the pulse. Because of its physical meaning we can consider the function $f_T(t)$ to be continuous with its derivative and to be even, that is, $f_T(-t) = f_T(t)$. Also, we imagine that the pulse starts at $t = -T/2$ and ends at $t = T/2$, then we assume that $f_T(t)$ is identically zero before $t = -T/2$, it strictly grows for negative times larger than $t = -T/2$, it reaches its absolute maximum at $t = 0$ and $f_T(0) = 1$, and then it strictly decreases for positive times until $t = T/2$ when it becomes again identically zero. Finally, we require that the function $f_T(t)$ is slowly varying with respect to $\cos\omega t$ and that in the limit of very large pulse durations it becomes identically one, that is $\lim_{T\to\infty} f_T(t) \equiv 1$.

Now, in [4] it has been shown that the production probability of a pair decreases exponentially with the electron and the positron linear momenta [see Eqs. (41) and (42) in that paper]. Despite that in [4] only the limit case $f_T(t) \equiv 1$ and $T \to \infty$ was considered, it is reasonable to restrict here our attention to the production of a pair with both the electron and the positron at rest. Also, it is obvious that in the presence of the electromagnetic field described by the vector potential (2) the spin of the electron along the z axis conserves in the transition from the energy level $-m$ to the energy level $+m$ because of the conservation of the total angular momentum. In this way, the electron state $|\psi(t)\rangle$ at a generic time t can be written as the following linear combination of the energy eigenstates:

$$\begin{aligned}|\psi(t)\rangle = c_{-m,\downarrow}(t)|-m,\downarrow\rangle + c_{-m,\uparrow}(t)|-m,\uparrow\rangle \\ + c_{+m,\downarrow}(t)|+m,\downarrow\rangle + c_{+m,\uparrow}(t)|+m,\uparrow\rangle\end{aligned} \tag{3}$$

and the eigenstates with spin-down are never coupled with those with spin-up. In particular, it can be shown that the Dirac equation

$$i\frac{\partial|\psi(t)\rangle}{\partial t} = H(t)|\psi(t)\rangle \tag{4}$$

transforms into the following equation system

$$\begin{cases} i\dot{c}_{-m,\downarrow}(t) = -mc_{-m,\downarrow}(t) - \Omega(t)c_{+m,\downarrow}(t) \\ i\dot{c}_{+m,\downarrow}(t) = +mc_{+m,\downarrow}(t) - \Omega(t)c_{-m,\downarrow}(t) \\ i\dot{c}_{-m,\uparrow}(t) = -mc_{-m,\uparrow}(t) + \Omega(t)c_{+m,\uparrow}(t) \\ i\dot{c}_{+m,\uparrow}(t) = +mc_{+m,\uparrow}(t) + \Omega(t)c_{-m,\uparrow}(t) \end{cases} \tag{5}$$

with

$$\Omega(t) = \frac{eE}{\omega} f_T(t) \cos\omega t. \tag{6}$$

Equations (5) are the equations of motions of two independent two-level systems both with energy gap $2m$ and in the presence of an external perturbation $\Omega(t)$ and $-\Omega(t)$, respectively [the sign in front of $\Omega(t)$ is, obviously, irrelevant]. Now, even if the external lasers are x-ray lasers then $\omega \ll 2m$ in such a way the external perturbation can be considered adiabatic. In this respect, the adiabatic treatment used in [25] in the different context of high order harmonic generation can also be used here to calculate the probability $P(T/2)$ that a pair is created at the end of the laser pulse. In fact, by assuming, for example, that $c_{-m,\downarrow}(-T/2) = 1$ and $c_{-m,\uparrow}(-T/2) = 0$, this probability is given by[1]

$$P\left(\frac{T}{2}\right) = 2\left|c_{+m,\downarrow}\left(\frac{T}{2}\right)\right|^2 \tag{7}$$

where the factor 2 takes into account the remaining analogous spin-up case in which $c_{-m,\downarrow}(-T/2) = 0$ and $c_{-m,\uparrow}(-T/2) = 1$ (we remind, as observed in [10], that the state $|\psi(t)\rangle$ is normalized to two). Now, the coeffi-

[1]From the physical meaning of the coefficients in the linear combination (3), $P(T/2)$ is, actually, the probability that the pair is *present* at $T/2$ but with an abuse we will denote it as a "production" or a "creation" probability instead of as a "presence" probability.

cients given in Eqs. (21) and (22) in [25] correspond exactly to the coefficients $c_{-m,\downarrow}(t)$ and $c_{+m,\downarrow}(t)$ [or, equivalently, $c_{-m,\uparrow}(t)$ and $c_{+m,\uparrow}(t)$] calculated up to first order in the time derivative of $\Omega(t)$. By using those coefficients and by adapting the notation, it can easily be shown that up to first order

$$P^{(1)}\left(\frac{T}{2}\right) = 2\left|G\left(\frac{T}{2}\right)\right|^2 \tag{8}$$

where [see Eqs. (9), (23) and (25) in [25]]

$$G\left(\frac{T}{2}\right) \equiv \frac{E}{2E_1}\int_{-T/2}^{T/2} dt \frac{d[f_T(t)\cos\omega t]/dt}{1+(\frac{E}{E_1})^2 f_T^2(t)\cos^2\omega t} \times \exp\left[2im\int_{-T/2}^{t} dt' \sqrt{1+\left(\frac{E}{E_1}\right)^2 f_T^2(t')\cos^2\omega t'}\right] \tag{9}$$

with $E_1 = m\omega/e = \omega/mE_{cr}$. Since we are interested in the square modulus of the quantity $G(T/2)$, we can use the equivalent expression

$$G\left(\frac{T}{2}\right) = i\frac{E}{E_1}\,\mathrm{Im}\left\{\int_0^{\omega T/2} d\eta \frac{d[f_T(\eta/\omega)\cos\eta]/d\eta}{1+(\frac{E}{E_1})^2 f_T^2(\eta/\omega)\cos^2\eta} \times \exp\left[2i\frac{m}{\omega} \times \int_0^{\eta} d\eta' \sqrt{1+\left(\frac{E}{E_1}\right)^2 f_T^2(\eta'/\omega)\cos^2\eta'}\right]\right\} \tag{10}$$

where the change of variable $\eta = \omega t$ has been performed. Now, despite that the previous integral cannot be evaluated exactly, we have said that in our hypotheses $m/\omega \gg 1$ and then the phase of the exponential in Eq. (10) is large. In this way, an asymptotic estimate of the external integral in Eq. (10) can be given by using the steepest descent method [26]. As we have said, this problem has been solved in [4] in the case in which $T \to \infty$ and $f_T(t) \equiv 1$ and our procedure is very similar. In particular, our purpose here is to correct that result by considering long but not infinite pulse durations and a time-dependent pulse-shape function. Now, the pair is most likely to be produced at $t = 0$ when the electric field is maximum instead of at the beginning or at the end of the pulse. For these reasons we write the pulse-shape $f_T(t)$ as [remember that $f_T(t) \leq 1$]

$$f_T(t) = 1 - \delta f_T(t) \tag{11}$$

and we assume to deal with times t near 0 such that $\delta f_T(t) \ll 1$. Now, in order to apply the steepest descent method to evaluate the external integral in Eq. (10) we have to determine the stationary points $\tilde{\eta}$ of the exponent in Eq. (10) in the complex plane such that their real parts are nonnegative and less than or equal to $\omega T/2$. These stationary points are determined by the condition

$$1 + \left(\frac{E}{E_1}\right)^2 f_T^2(\tilde{\eta}/\omega)\cos^2\tilde{\eta} = 0 \tag{12}$$

and they will be determined up to first order in $\delta f_T(\tilde{\eta}/\omega)$. If we split $\tilde{\eta}$ as the sum $\tilde{\eta} = \tilde{\eta}_\infty + \delta\tilde{\eta}$ of the zero-order solution $\tilde{\eta}_\infty$ and of the first-order correction $\delta\tilde{\eta}$ we obtain

$$\tilde{\eta}_\infty = \tilde{\eta}_{\infty,n} = \frac{2n+1}{2}\pi + i\,\mathrm{arcsinh}\left(\frac{E_1}{E}\right) \qquad n = 0, 1, \ldots, N \tag{13}$$

as already calculated in [4], and

$$\delta\tilde{\eta} = \delta\tilde{\eta}_n = i\frac{E_1}{\sqrt{E^2+E_1^2}}\delta f_T\left(\frac{\tilde{\eta}_{\infty,n}}{\omega}\right) \qquad n = 0, 1, \ldots, N \tag{14}$$

where N is the largest natural number such that $(2N + 1)\pi/2 \leq \omega T/2$. Now, from an experimental point of view the goal is to build an x-ray free electron laser such that $E \gtrsim 10\, E_1$ (see [5] and Ref. [37] therein) and, for this reason, we decide to work in the strong field regime $E \gg E_1$. On the other hand, we are giving an asymptotic estimate of the probability $P^{(1)}(T/2)$ so we do not need to sum the contributions to the external integral in Eq. (10) of all the stationary points; it is enough to take only the dominant one. It is easy to show that this contribution comes from the stationary point with the smallest imaginary part [at first order in $\delta f_T(\tilde{\eta}/\omega)$]. In turn, by reminding from Eq. (11) that the correction $\delta f_T(t)$ is small at small times and by observing that in the strong field regime $E \gg E_1$ the zero-order stationary points $\tilde{\eta}_{\infty,n}$ are next to real, it is easy to show that the dominant contribution comes from the stationary point with the smallest real part that is from $\tilde{\eta}_0$. For notational simplicity this stationary point will be indicated simply as $\tilde{\eta} = \tilde{x} + i\tilde{y} = \tilde{x}_\infty + \delta\tilde{x} + i(\tilde{y}_\infty + \delta\tilde{y})$ and

$$\tilde{x}_\infty = \frac{\pi}{2}, \qquad \delta\tilde{x} = -\frac{E_1}{\sqrt{E^2+E_1^2}}\,\mathrm{Im}\left[\delta f_T\left(\frac{\tilde{\eta}_\infty}{\omega}\right)\right], \tag{15a}$$

$$\tilde{y}_\infty = \mathrm{arcsinh}\left(\frac{E_1}{E}\right), \qquad \delta\tilde{y} = \frac{E_1}{\sqrt{E^2+E_1^2}}\,\mathrm{Re}\left[\delta f_T\left(\frac{\tilde{\eta}_\infty}{\omega}\right)\right] \tag{15b}$$

with $\tilde{\eta}_\infty \equiv \tilde{\eta}_{\infty,0} = \tilde{x}_\infty + i\tilde{y}_\infty$. As it is clear from the previous equation, although we decided to work in the strong field regime $E \gg E_1$ we will use the expression of $\tilde{\eta}$ without approximation in the ratio E_1/E and we will perform the limit $E \gg E_1$ only at the end of the calculations. We proceed in this way in order to avoid inconsistencies in the approximations. In fact, we are also working in the limit $\delta f_T(t) \ll 1$ and the relative magnitude of the two small parameters E_1/E and $\delta f_T(t)$ is *a priori* unknown.

At this point the application of the steepest descent method to calculate the external integral in Eq. (10) is identical to the case treated in [4].[2] For this reason we give directly the following expression of the probability $P^{(1)}(T/2)$ as

$$P^{(1)}\left(\frac{T}{2}\right) \sim 2\left(\frac{2\pi}{3}\right)^2 \exp(-2A)\cos^2 B \sim \left(\frac{2\pi}{3}\right)^2 \exp(-2A). \tag{16}$$

In this equation the quantities A and B are defined, analogously to Eq. (41) in [4]. It is not necessary to give the exact (and cumbersome) expression of B because, since $B \sim m/\omega \gg 1$, we can approximate $\cos^2 B \sim 1/2$. Instead, the quantity A is given by

$$A = \frac{2m}{\omega} \int_0^{\tilde{y}} dy \,\mathrm{Re}\left\{\sqrt{1 + \left(\frac{E}{E_1}\right)^2 f_T^2\left(\frac{\tilde{x} + iy}{\omega}\right)\cos^2(\tilde{x} + iy)}\right\}. \tag{17}$$

By defining $y/\tilde{y} = \rho$ and by retaining only the terms up to first order in $\delta f_T(t)$ it can easily be shown that the quantity A becomes

$$A = \frac{2E_{cr}}{E} \int_0^1 d\sigma \sqrt{\frac{1-\sigma^2}{1+(E_1/E)^2\sigma^2}} + \frac{\delta\tilde{y}}{\tilde{y}_\infty}\frac{2E_{cr}}{E} \int_0^1 d\sigma \sqrt{\frac{1-\sigma^2}{1+(E_1/E)^2\sigma^2}} + \frac{2E_{cr}}{E}\int_0^1 d\sigma \left\{(1-\sigma^2)\left[1+\left(\frac{E_1}{E}\right)^2\sigma^2\right]\right\}^{-1/2}$$
$$\times\left\{\mathrm{Re}\left[\delta f_T\left(\frac{\tilde{x}_\infty + i\,\mathrm{arcsinh}(E_1/E)\sigma}{\omega}\right)\right]\sigma^2 - \frac{\delta\tilde{y}}{\tilde{y}_\infty}\sigma\,\mathrm{arcsinh}\left(\frac{E_1}{E}\sigma\right)\sqrt{\left(\frac{E}{E_1}\right)^2 + \sigma^2}\right\} \tag{18}$$

where we have introduced the adimensional variable $\sigma = (E/E_1)\sinh(\tilde{y}_\infty \rho)$ and where we remind that $E_{cr} = m^2/e$. In the previous expression we singled out three terms: the first one is the zero-order term and it is the same as in Eq. (44) in [4], while the other two terms are the corrections due to the pulse-shape function. The expression (18) of the quantity A can be further simplified by reminding that our results are valid in the strong field regime $E \gg E_1$. By retaining only the terms up to $(E_1/E)^2$ we obtain

$$\begin{aligned} A &= \frac{\pi}{2}\frac{E_{cr}}{E}\left[1 - \frac{1}{8}\left(\frac{E_1}{E}\right)^2 + \delta f_T\left(\frac{\pi}{2\omega}\right) + a_1 \delta f_T\left(\frac{\pi}{2\omega}\right)\left(\frac{E_1}{E}\right)^2\right] \\ &\simeq \frac{\pi}{2}\frac{E_{cr}}{E}\left[1 - \frac{1}{8}\left(\frac{E_1}{E}\right)^2 + \delta f_T\left(\frac{\pi}{2\omega}\right)\right] \end{aligned} \tag{19}$$

where we did not evaluate the coefficient a_1 because the corresponding term is negligible with respect to the others. Nevertheless, it is worth noting that this term goes essentially as $(\omega T)^{-2} \times [m\omega/(eE)]^2 \sim (E_p/E)^2$ with $E_p = m\omega_p/e$ the quantity analogous to E_1 but with $\omega_p = 2\pi/T$ the typical frequency of the pulse shape instead of the laser frequency ω. Also, we have checked that the next correction proportional to $(E_1/E)^4$ is negligible with respect to $\delta f_T(\pi/2\omega)$ in the physical regime we are interested in.

By inserting the previous result in Eq. (16), the probability $P^{(1)}(T/2)$ simply becomes

$$P^{(1)}\left(\frac{T}{2}\right) \sim \left(\frac{2\pi}{3}\right)^2 \exp\left\{-\pi\frac{E_{cr}}{E}\left[1 - \frac{1}{8}\left(\frac{E_1}{E}\right)^2 + \delta f_T\left(\frac{\pi}{2\omega}\right)\right]\right\}. \tag{20}$$

This final expression of the production probability clearly shows the presence of the nonperturbative Schwinger exponential corrected by a "laser frequency" term proportional to $(E_1/E)^2 = (m\omega/eE)^2$ and by a "pulse-shape" term depending in fact on the exact form of the pulse-shape function. Obviously, the correcting "pulse-shape" term in the exponential is positive implying, as expected, that the creation probability is smaller than that in the presence of an infinite beam with pulse shape identically equal to one. In particular, this correcting term is obtained by substituting in the original Schwinger exponential the electric field E with the pulse-shape modulated field $E \times [1 - \delta f_T(\pi/2\omega)]$. This is expected by looking at Eqs. (2) and (11) although the instant $\pi/2\omega$ where the pulse-shape function is evaluated depends on the details of the steepest descent method. Finally, we also want to point out that the quantity (20) is the *total* probability that a pair is created *at rest*. Instead, in the previous cited papers [1,2,4] the authors calculate the pair production probability *per unit volume and unit time* and they obtain the Schwinger factor *by integrating on all the electron and positron momenta.* Nevertheless, the fact that we have also obtained the Schwinger factor is not surprising. In fact, as it is shown in [2,4], every differential probability that the pair is created with the electron momentum between $\mathbf{p}$ and $\mathbf{p} + d\mathbf{p}$ is also proportional to a Schwinger-like factor $\exp[-\pi(1 +$

[2]We only want to point out that the stationary points of the exponent in Eq. (10) are poles of the integrand function of the external integral in such a way the steepest descents cannot pass exactly through these points. Nevertheless, this problem has also been dealt with in [4] and we refer the reader to that paper for a more detailed discussion.

$p_\perp^2/m^2)E_{cr}/E]$ with $p_\perp^2 = p_x^2 + p_y^2$ that becomes just $\exp(-\pi E_{cr}/E)$ for a pair created at rest.

We want to conclude by giving more quantitative estimates and checks of our results. To do this we consider the typical pulse-shape function

$$f_T^{\cos^2}(t) = \cos^2\left(\frac{\pi t}{T}\right) \quad (21)$$

so that [see Eq. (11)]

$$\delta f_T^{\cos^2}(t) \simeq \left(\frac{\pi t}{T}\right)^2 \quad (22)$$

and the probability (20) becomes

$$P^{(1)\cos^2}\left(\frac{T}{2}\right) \sim \left(\frac{2\pi}{3}\right)^2 \exp\left\{-\pi\frac{E_{cr}}{E}\left[1 - \frac{1}{8}\left(\frac{E_1}{E}\right)^2 + \left(\frac{\pi^2}{2\omega T}\right)^2\right]\right\}. \quad (23)$$

First, we shall evaluate a typical correction to the production probability induced by the finite duration of the laser pulse. Now, the relative difference between the probability $P^{(1)\cos^2}(T/2)$ and the analogous one calculated in the presence of an infinite laser beam with pulse-shape function always equal to one is given by

$$\Delta^{\cos^2} \equiv \left|\frac{P^{(1)\cos^2}(\frac{T}{2}) - \lim_{T\to\infty} P^{(1)\cos^2}(\frac{T}{2})}{P^{(1)\cos^2}(\frac{T}{2})}\right| \sim \exp\left[\pi\frac{E_{cr}}{E}\left(\frac{\pi^2}{2\omega T}\right)^2\right] - 1 \quad (24)$$

where the symbol $\sim$ indicates that the previous asymptotic value of the probability $P^{(1)\cos^2}(T/2)$ has been substituted. To do a numerical estimate we use the laser parameters given as "goal" parameters in [5]: photon energy $\omega = 8.3$ keV and peak electric field $E = 2.0 \times 10^{15}$ V/cm (that is, $E = 0.15E_{cr}$ and $E = 9.3\ E_1$). By also considering as the laser pulse duration the "optimistic" value $T = 10$ as $= 10^{-17}$ s, Eq. (24) gives $\Delta^{\cos^2} = 0.032$ that is a correction of the order of 3% which is not negligible and (hopefully) measurable. Incidentally, we observe that by using the previous parameters, the correcting "laser frequency" and "pulse-shape" terms in the exponential in Eq. (23) are of the same order of magnitude, while we have checked that the next "laser frequency" correction can be neglected.

Finally, since some approximations have been done to obtain the final result Eq. (23) it is useful to compare it with the exact production probability obtained by numerically integrating the Schroedinger equation in the form (5) with the pulse shape (21) and by evaluating Eq. (7). In particular, in Fig. 1 we plot the exact probability (7) and our asymptotic estimate (23) as functions of the peak electric field E expressed in unit of E_1. The exact probabilities show rapid oscillations that have been averaged in $P^{(1)\cos^2}(T/2)$ [see Eq. (16)]. Also, the function $P^{(1)\cos^2}(T/2)$ gives a good average of the behavior of the exact probability. In fact, we have performed a best fit of the exact probabilities by using the test function $c_1 \exp(-c_2/x)$ (in most of the electric field range considered we can neglect the correcting "laser frequency" term in the exponential without appreciable error) and we have obtained the best values $c_1 = 3.6 \pm 1.3$ and $c_2 = 197 \pm 11$ to be compared with [see Eq. (23)] $(2\pi/3)^2 = 4.4$ and with $\pi\{1 + [\pi^2/(2\omega T)]^2\}E_{cr}/E_1 = 194$, respectively. The good agreement (but the error in c_1 is quite large) confirms the validity of our treatment and of the approximations made.

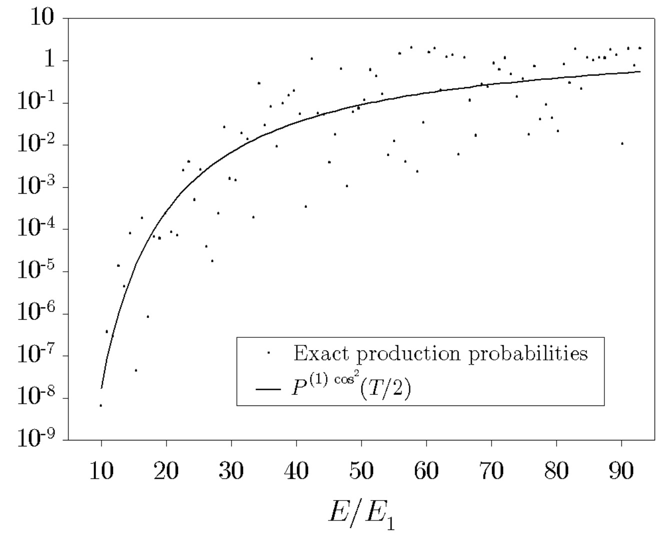

FIG. 1. Semilog plot of the exact production probabilities (dots) calculated by evaluating Eq. (7) after integrating numerically the Schroedinger equation in the form (5) with the pulse shape (21) and of the approximated asymptotic production probabilities (continuous curve) calculated by means of Eq. (23). The probabilities are plotted as functions of the peak laser electric field E expressed in unit of E_1 and the numerical values of all the parameters are those given below Eq. (24).

III. SUMMARY AND CONCLUSIONS

In this paper we have calculated the probability that an $e^- - e^+$ pair is created at rest from vacuum in the field of two identical but oppositely directed laser pulses. Various papers have been devoted to this subject but always considering (from a theoretical point of view) pulses with infinite duration and constant pulse shape. In the present paper we have taken into account the finite duration and the form of the laser pulse by introducing the pulse-shape function $f_T(t)$. Since the pair production probability decreases exponentially with the electron (positron) linear momentum we have evaluated the probability that the electron and the positron making the pair are created at rest. In this way the system has been reduced to two decoupled two-level (both with energies $-m$ and $+m$) systems subject to an external adiabatic perturbation (the laser periods we have in mind are much larger than the typical times during which a pair is created). By using a technique already applied in [25], we have been able to calculate the production probability at the end of the pulse by including perturbatively the effect of the pulse shape. The final result Eq. (20) has been obtained in the strong field limit $E \gg E_1 = m\omega/e$ and it shows the typical Schwinger nonperturbative dependence on the external peak electric field E, a correction proportional to $(E_1/E)^2$ connected to the laser frequency and a correction depending on the pulse-shape function evaluated at $t = \pi/(2\omega)$. As expected, the resulting probability is less than the corresponding one in the presence of an infinite pulse with a constant time profile equal to unit. Finally, by considering the typical $\cos^2$ pulse-shape function and by using values of the pulse parameters that are expected to be accessible experimentally in a few years, we concluded that the corrections to the production probability due to the pulse shape can also be of the order of 3%.

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