Static quantities of a neutral bilepton in the 331 model with right-handed neutrinos

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A neutral vector boson can possess static electromagnetic properties provided that the associated field is non-self-conjugate. This possibility is explored in the $SU_C(3) \times SU_L(3) \times U_N(1)$ model with right-handed neutrinos, which predicts a complex neutral gauge boson Y^0 in a nontrivial representation of the electroweak group. In this model the only nonvanishing form factors are the CP-even ones, which arise from both the quark and gauge sectors, and contribute to the magnetic dipole and the electric quadrupole moments of this neutral particle.

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I. INTRODUCTION

The electromagnetic properties of neutral particles have been the source of great interest since they are generated at the loop level, thereby opening up the possibility for the detection of new physics effects. Considerable attention has been paid to the electromagnetic properties of neutrinos and the neutral Z boson of the standard model (SM). In particular, the impact of new physics effects on the trilinear couplings of the Z boson has been studied in a model-independent manner using the effective Lagrangian technique [1]. As far as neutral fermions are concerned, it was long realized that the off-shell electromagnetic vertex of a massless Dirac neutrino is a gauge-dependent quantity [2]. On the other hand, a massive Dirac neutrino does have static electromagnetic properties which characterize its magnetic and electric dipole moments. This is to be contrasted with the case of a Majorana neutrino, which only has off-shell electromagnetic properties [3], which in turn is a consequence of the fact that a Majorana neutrino is identical to its antiparticle. A more recent model-independent study of the electromagnetic form factors of Majorana particles with higher spin was presented in Ref. [4]. The situation for neutral spin-1 particles is similar as for neutrinos: a neutral vector boson characterized by a self-conjugate field, for which the particle is identical to its antiparticle, cannot have static electromagnetic properties. This fact has been already discussed in the case of the neutral Z boson [5]. On the contrary, a non-self-conjugate field can have static electromagnetic properties.

The possibility that neutral particles have nonzero static electromagnetic properties was explored in a general context using arguments of gauge invariance and transformation under the discrete symmetries C, P, and T [6]. Several extensions of the SM, such as grand unified theories (GUTs), predict the existence of at least one new complex neutral gauge boson with nonzero content of quantum numbers from the global or local symmetries of the theory. The purpose of this work is to present a calculation in a specific version of the 331 model [7] which predicts the existence of a non-self-conjugate neutral gauge boson in a nontrivial representation of the electroweak group.

The 331 model is based on the simplest non-Abelian extension of the SM group, namely, $SU_C(3) \times SU_L(3) \times U_N(1)$ [7]. This model is appealing and has been the source of interest recently [8] because it requires that the number of fermion families be a multiple of the quark color number in order to cancel anomalies, which suggests a path to the solution of the flavor problem. Another important feature of this model is that the $SU_L(2)$ group is totally embedded in $SU_L(3)$. As a consequence, after the first stage of spontaneous symmetry-breaking (SSB), when $SU_L(3) \times U_N(1)$ is broken down to $SU_L(2) \times U_Y(1)$, a pair of massive gauge bosons associated with four broken generators of $SU_L(3)$ emerge in a doublet of the electroweak group. Contrary to what happens in other theories, the couplings between the new and the SM gauge bosons do not involve any mixing angle, which means that they are expected to be similar in magnitude to the ones existing between the SM gauge bosons themselves.

Apart from the minimal 331 model, another version including right-handed neutrinos has been considered in the literature more recently [9,10]. Its main feature is that it requires a more economic Higgs sector to break the gauge symmetry and generate the fermion masses. This model predicts the existence of a singly-charged boson $Y^{\pm}$ along with a non-self-conjugate neutral boson Y^{0*}. Both of these new gauge bosons can be classified as bileptons since they carry lepton-number $L = \pm 2$, and thus are responsible for lepton-number violating interactions [11]. The neutral bilepton is a very promising candidate in accelerator experiments since it may be the source of neutrino oscillations [12]. The dynamical behavior of the Y^0 boson is somewhat similar to that of the W gauge boson, due to the nontrivial quantum number assignment. For instance, the $Y^0 Y^+ W^+$ coupling resem-

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bles those existing between the electroweak gauge bosons. In the fermionic sector, the Y^0 also couples to the quark pairs (d, D_1), (s, D_2), and (t, T), with D_1, D_2 and T three new quarks predicted by the model. These couplings induce nonzero static electromagnetic properties for the neutral bilepton.

This presentation has been organized as follows. In Sec. II, we present a brief review of the 331 model with right-handed neutrinos, with special emphasis on the current and Yang-Mills sectors. Sec. III is devoted to the calculation of the on-shell vertex $Y^0 Y^{0*}\gamma$. In Sec. IV we analyze the behavior of the Y^0 form factors, and the conclusions are presented in Sec. V.

II. THE 331 MODEL WITH RIGHT-HANDED NEUTRINOS

331 models are based on the $SU_C(3) \times SU_L(3) \times U_N(1)$ gauge group. In the version with right-handed neutrinos [9] the leptons are arranged as

$$f_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \\ (\nu_L^c)^i \end{pmatrix} \sim (1, 3, -1/3), \qquad e_R^i \sim (1, 1, -1), \qquad i = 1, 2, 3, \tag{1}$$

where i stands for the family index. In the quark sector, a new quark for each family is necessary. The first two quark families transform as

$$Q_{aL} = \begin{pmatrix} d_{aL} \\ -u_{aL} \\ D_{aL} \end{pmatrix} \sim (3, \bar{3}, 0), \qquad u_{aR} \sim (3, 1, 2/3), \qquad d_{aR} \sim (3, 1, -1/3), \qquad D_{aR} \sim (3, 1, -1/3), \tag{2}$$

for $a = 1, 2$, whereas the third family transforms differently

$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ T_L \end{pmatrix} \sim (3, 3, 1/3), \qquad u_{3R} \sim (3, 1, 2/3), \qquad d_{3R} \sim (3, 1, -1/3), \qquad T_R \sim (3, 1, 2/3). \tag{3}$$

As far as the scalar sector is concerned, only three triplets of $SU_L(3)$ are required to achieve the SSB mechanism:

$$\chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix} \sim (1, 3, -1/3), \qquad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho'^+ \end{pmatrix} \sim (1, 3, 2/3), \qquad \eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta'^0 \end{pmatrix} \sim (1, 3, -1/3). \tag{4}$$

In contrast, the minimal version requires three triplets and one sextet. The vacuum expectation values $<\chi>^T = (0, 0, w/\sqrt{2})$, $<\rho>^T = (0, u/\sqrt{2}, 0)$, and $<\eta>^T = (v/\sqrt{2}, 0, 0)$ yield the following SSB pattern

$$SU_C(3) \times SU_L(3) \times U_N(1) \overset{w}{\longrightarrow} SU_C(3) \times SU_L(2) \times U_Y(1) \overset{u,\, v}{\longrightarrow} SU_C(3) \times U_e(1). \tag{5}$$

Notice that in order to break $SU_C(3) \times SU_L(3) \times U_N(1)$ into $SU_C(3) \times SU_L(2) \times U_Y(1)$, only the scalar triplet χ is required. The covariant derivative in the triplet representation is given by

$$\mathcal{D}_\mu = \partial_\mu - ig\frac{\lambda^a}{2}A_\mu^a - ig_N N\frac{\lambda^9}{2}N_\mu, \tag{6}$$

where $\lambda^9 = 2\mathrm{diag}\{1, 1, 1\}/3$ and $\lambda^a (a = 1 \ldots 8)$ are the Gell-Mann matrices. The generators are normalized as $\mathrm{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$ and $\mathrm{Tr}(\lambda^9 \lambda^9) = 2$. In the first stage of SSB, five generators of $SU_L(3)$ along with the one associated with $U_N(1)$ are broken, *i.e.*, $\lambda^a < \chi>_0 \neq 0$, for $a = 4, \ldots, 9$. The linear combination $Y = (3\sqrt{2}N\lambda^9 - \lambda^8)/\sqrt{3}$ annihilates the vacuum and can be identified with the hypercharge operator. In this stage the three exotic quarks and the gauge bosons associated with the broken generators of the 331 group Y^0, $Y^\pm$, and Z' acquire mass. The exotic quarks have the same electric charge as the SM quarks, namely, $Q_{D_{1,2}} = -1/3$ and $Q_T = 2/3$. As for the massive gauge bosons, both Y^0 and $Y^\pm$ are complex, whereas Z' is a real field with no quantum numbers from the electroweak group.

At the Fermi scale, when $SU_C(3) \times SU_L(2) \times U_Y(1)$ is broken down to $SU_C(3) \times U_e(1)$, the masses of the heavy particles receive new contributions. The diagonalization of the complete Higgs kinetic-energy sector leads to the following mass-eigenstate fields:

$$Y_\mu^{0(*)} = \frac{1}{\sqrt{2}}(A_\mu^4 \mp iA_\mu^5), \tag{7}$$

$$Y_\mu^{\mp} = \frac{1}{\sqrt{2}}(A_\mu^6 \mp iA_\mu^7), \tag{8}$$

$$W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu), \quad (9)$$

with $m^2_{Y^0} = g^2(w^2 + u^2)/4$, $m^2_{Y^{\pm}} = g^2(w^2 + v^2)/4$, and $m^2_W = g^2(u^2 + v^2)/4$. The symmetry-breaking hierarchy yields a splitting between the bilepton masses:

$$|m^2_{Y^0} - m^2_{Y^{\pm}}| \leq m^2_W. \quad (10)$$

It is straightforward to obtain the explicit Lagrangian for the current sector. We will concentrate only on those terms involving the complex field Y^0, which in the lepton sector only couples to neutrinos, whereas in the quark sector it couples to both SM and exotic quarks as follows:

$$\mathcal{L}^{\rm NC}_{Y^0} = \frac{g}{\sqrt{2}}\left(-\sum_{i=1,2} \bar{d}_{iL}\gamma^\mu D_{iL} + \bar{u}_{3L}\gamma^\mu T_L\right) Y^0_\mu + \text{H.c.} \quad (11)$$

This is the only term of the fermion sector that contributes to the one-loop induced $Y^0Y^{0*}\gamma$ vertex, whereas in the bosonic sector there are contributions from both gauge and charged scalar fields. In this work we will not consider those contributions arising from the latter and concentrate only on the Yang-Mills sector.

A. The Yang-Mills sector of 331 models

In order to calculate the gauge-sector contributions to the $Y^0Y^{0*}\gamma$ vertex, it is necessary to introduce the gauge-fixing term. We found it convenient to use the unitary gauge for our calculation. Since the Yang-Mills sector was discussed to a certain extent in the case of the minimal version of the model [8], we refrain from presenting a more detailed discussion and focus on those points relevant for the present discussion. The Yang-Mills sector associated with the group $\text{SU}_L(3) \times \text{U}_N(1)$ is given by

$$\mathcal{L}_{\rm YM} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a - \frac{1}{2}N_{\mu\nu}N^{\mu\nu}, \quad (12)$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu$ and $N_{\mu\nu} = \partial_\mu N_\nu - \partial_\nu N_\mu$, f^{abc} being the structure constants of the group $\text{SU}_L(3)$. We can write this Lagrangian as

$$\mathcal{L}_{\rm YM} = \mathcal{L}^{\rm SM}_{\rm YM} + \mathcal{L}^{\rm SM-NP}_{\rm YM} + \mathcal{L}^{\rm NP}_{\rm YM}, \quad (13)$$

where the first term represents the Yang-Mills sector associated with the electroweak group:

$$\mathcal{L}^{\rm SM}_{\rm YM} = -\frac{1}{4}F^i_{\mu\nu}F^{\mu\nu}_i - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \qquad i = 1, 2, 3. \quad (14)$$

The term $\mathcal{L}^{\rm SM-NP}_{\rm YM}$ represents the interactions between the SM gauge fields and the heavy ones. It can be written in the following $\text{SU}_L(2) \times \text{U}_Y(1)$-invariant form

$$\begin{aligned}\mathcal{L}^{\rm SM-NP}_{\rm YM} = &-\frac{1}{2}(D_\mu Y_\nu - D_\nu Y_\mu)^\dagger (D^\mu Y^\nu - D^\nu Y^\mu) \\ &- iY^\dagger_\mu (g\mathbf{F}^{\mu\nu} + g'\mathbf{B}^{\mu\nu})Y_\nu \\ &- \frac{ig}{2}\frac{\sqrt{3 - 4s^2_W}}{c_W} Z'_\mu [Y^\dagger_\nu (D^\mu Y^\nu - D^\nu Y^\mu) \\ &- (D^\mu Y^\nu - D^\nu Y^\mu)^\dagger Y_\nu], \end{aligned} \quad (15)$$

where $Y^\dagger_\mu = (Y^{0*}_\mu, Y^+_\mu)$ is a doublet of the electroweak group with hypercharge -1 and $D_\mu = \partial_\mu - ig\mathbf{A}_\mu + ig'\mathbf{B}_\mu$ is the covariant derivative associated with this group. In addition, we have introduced the definitions $\mathbf{F}_{\mu\nu} = \sigma^i F^i_{\mu\nu}/2$, $\mathbf{A}_\mu = \sigma^i A^i_\mu/2$, and $\mathbf{B}_\mu = YB_\mu/2$, with σ^i the Pauli matrices. Finally, the last term in Eq. (13) is also invariant under the electroweak group and comprises the interactions between the heavy gauge fields:

$$\begin{aligned}\mathcal{L}^{\rm NP}_{\rm YM} = &-\frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} + \frac{g^2}{4}\left(Y^\dagger_\mu \frac{\sigma^i}{2} Y_\nu - Y^\dagger_\nu \frac{\sigma^i}{2} Y_\mu\right) \\ &\times \left(Y^{\dagger\mu}\frac{\sigma^i}{2}Y^\nu - Y^{\dagger\nu}\frac{\sigma^i}{2}Y^\mu\right) \\ &+ \frac{3g^2}{16}(Y^\dagger_\mu Y_\nu - Y^\dagger_\nu Y_\mu)(Y^{\dagger\mu}Y^\nu - Y^{\dagger\nu}Y^\mu) \\ &- \frac{3g^2}{4}Z'_\mu Y^\dagger_\nu (Z^\mu_2 Y^\nu - Z^\nu_2 Y^\mu) \\ &- \frac{ig}{2}\frac{\sqrt{3 - 4s^2_W}}{c_W} Y^\dagger_\mu Y_\nu Z'^{\mu\nu}. \end{aligned} \quad (16)$$

From these Lagrangians we have derived the Feynman rules shown in Table I, which are necessary for the calculation of the gauge boson contribution to the $Y^0Y^{0*}\gamma$ vertex. These results are in agreement with Ref. [10]

TABLE I. Feynman rules necessary for the calculation of the gauge boson contribution to the $Y^0Y^{0*}\gamma$ vertex. $V^\pm$ stands for $Y^\pm$ or $W^\pm$. All the 4-momenta are directed inward.

Vertex	Feynman rule
$Y^0_\alpha(p)W^-_\lambda(k_1)Y^+_\rho(k_2)$	$ig[(p - k_2)_\lambda g_{\rho\alpha} + (k_2 - k_1)_\alpha g_{\lambda\rho} + (k_1 - p)_\rho g_{\alpha\lambda}]/\sqrt{2}$
$A_\mu(q)V^+_\lambda(k_1)V^-_\rho(k_2)$	$-ie[(k_2 - k_1)_\mu g_{\lambda\rho} + (q - k_2)_\lambda g_{\mu\rho} + (k_1 - q)_\rho g_{\mu\lambda}]$
$Y^0_\alpha Y^{0*}_\beta Y^+_\lambda Y^-_\rho$	$ig^2(2g_{\alpha\rho}g_{\beta\lambda} - g_{\alpha\lambda}g_{\beta\rho} - g_{\alpha\beta}g_{\lambda\rho})/2$
$Y^0_\alpha Y^{0*}_\beta W^+_\lambda W^-_\rho$	$ig^2(2g_{\alpha\lambda}g_{\beta\rho} - g_{\alpha\beta}g_{\lambda\rho} - g_{\alpha\rho}g_{\beta\lambda})/2$
$A_\mu Y^0_\alpha Y^+_\lambda W^-_\rho$	$ige(g_{\alpha\lambda}g_{\rho\mu} - 2g_{\alpha\mu}g_{\lambda\rho} + g_{\alpha\rho}g_{\lambda\mu})/\sqrt{2}$

III. THE STATIC ELECTROMAGNETIC PROPERTIES OF THE Y^0 BOSON

We turn now to the calculation of the static electromagnetic properties of the non-self-conjugate neutral boson Y^0. In the usual notation, the most general on-shell $Y^0_\alpha Y^0_\beta A_\mu$ vertex can be written as [6,13]

$$\Gamma_{\alpha\beta\mu} = ie\Big[2\Delta\kappa(q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu}) + \frac{4\Delta Q}{m^2_{Y^0}} p_\mu q_\alpha q_\beta + 2\Delta\tilde{\kappa}\epsilon_{\alpha\beta\mu\lambda}q^\lambda + \frac{4\Delta\tilde{Q}}{m^2_{Y^0}} q_\beta \epsilon_{\alpha\mu\lambda\rho}p^\lambda q^\rho\Big]. \quad (17)$$

Note that the $p_\mu g_{\alpha\beta}$ term, which is present for a charged particle, is absent as it would violate gauge invariance. This term can only arise through the electromagnetic covariant derivative. The magnetic (electric) dipole moment μ_{Y^0} ($\tilde{\mu}_{Y^0}$) and the electric (magnetic) quadrupole moment Q_{Y^0} ($\tilde{Q}_{Y^0}$) are given in terms of the electromagnetic form factors as follows

$$\mu_{Y^0} = \frac{e}{2m_{Y^0}}(2 + \Delta\kappa), \quad (18)$$

$$Q_{Y^0} = -\frac{e}{m^2_{Y^0}}(1 + \Delta\kappa + \Delta Q), \quad (19)$$

$$\tilde{\mu}_{Y^0} = \frac{e}{2m_{Y^0}}\Delta\tilde{\kappa}, \quad (20)$$

$$\tilde{Q}_{Y^0} = -\frac{e}{m^2_{Y^0}}(\Delta\tilde{\kappa} + \Delta\tilde{Q}). \quad (21)$$

The CP-violating form factors $\Delta\tilde{\kappa}$ and $\Delta\tilde{Q}$ are not induced in the 331 model with right-handed neutrinos. In the fermionic sector, $\Delta\tilde{\kappa}$ can be induced at the one-loop level, but it requires that the neutral boson couples to both left- and right-handed fermions simultaneously [14,15].

In order to compute the contributions to the on-shell $Y^0 Y^{0*}\gamma$ vertex, we used the method described in Refs. [8,16], which is a generalization of the Passarino-Veltman reduction scheme [17]. Since the gauge invariant form (17) is obtained once all the contributions are summed over, the absence of the $p_\mu g_{\alpha\beta}$ term and the cancellation of ultraviolet divergences will serve as a test to check the correctness of our results. Below we will present separately the fermionic and gauge boson contributions to the ΔQ and $\Delta\kappa$ form factors.

A. Fermion contribution

The contribution of this sector comes from the Feynman diagrams shown in Fig. 1. There are two triangle diagrams for each quark pair (d, D_1), (s, D_2), and (t, T). We will denote by q the SM quark and by q' the exotic one. Once the reduction scheme described above is applied to solve the loop amplitudes, the contribution from the (q, q') quark pair can be written as

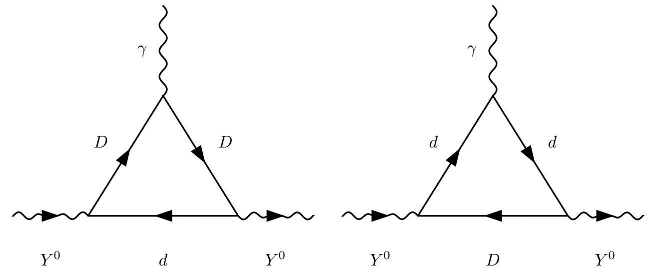

FIG. 1. Feynman diagrams for the fermion contributions to the static quantities of the Y^0 boson.

$$\Delta Q^{\rm Ferm} = 6aQ\Big\{\frac{2}{\Delta_{qq'}}(x_{q'} - x_q)[1 - 3(x_{q'} + x_q) + 2(x_{q'} - x_q)^2]\mathrm{arccosh}\Big(\frac{x_{q'} + x_q - 1}{2\sqrt{x_{q'}x_q}}\Big) + 4(x_{q'} - x_q) + [x_{q'} + x_q - 2(x_{q'} - x_q)^2] \times \log\Big(\frac{x_{q'}}{x_q}\Big)\Big\}, \quad (22)$$

$$\Delta\kappa^{\rm Ferm} = 9aQ(x_{q'} - x_q)\Big\{\frac{2}{\Delta_{qq'}}[x_{q'} + x_q - (x_{q'} - x_q)^2] \times \mathrm{arccosh}\Big(\frac{x_{q'} + x_q - 1}{2\sqrt{x_{q'}x_q}}\Big) - 2 + (x_{q'} - x_q) \times \log\Big(\frac{x_{q'}}{x_q}\Big)\Big\}, \quad (23)$$

with $a = g^2/(96\pi^2)$, $x_i = m_i^2/m^2_{Y^0}$ and $\Delta^2_{ij} = (x_i + x_j - 1)^2 - 4x_j x_j$. A factor of 3 has been included to account for the quark color number, and Q stands for the quark charge in units of that of the positron. Equations (22) and (23) are to be summed over the (d, D_1), (s, D_2), and (t, T) quark pairs.

Both $\Delta Q^{\rm Ferm.}$ and $\Delta\kappa^{\rm Ferm.}$ are antisymmetric under the interchange of x_q and $x_{q'}$, which means that they vanish when the q and q' quarks are degenerate. Since it is expected that the exotic quarks are heavier than the SM ones ($x_{q'} \gg x_q$), it would be interesting to have analytical expressions for the scenario in which $x_q \sim 0$ and $x_{q'}$ is arbitrary. After some algebra, Eqs. (22) and (23) yield

$$\Delta Q^{\rm Ferm} = 12aQx_{q'}\Big[2 + (2x_{q'} - 1)\log\Big(\frac{|x_{q'} - 1|}{x_{q'}}\Big)\Big], \quad (24)$$

$$\Delta\kappa^{\rm Ferm} = 18aQx_{q'}\Big[1 + x_{q'}\log\Big(\frac{|x_{q'} - 1|}{x_{q'}}\Big)\Big]. \quad (25)$$

In the heavy-mass limit, $\Delta\kappa^{\text{Ferm.}} \to -9aQ$ and $\Delta Q^{\text{Ferm.}} \to 0$. Of course, when $x_{q'} \to 0$, the degenerate fermion case is recovered and both form factors vanish.

B. Gauge boson contribution

We found it convenient to make the calculation for this contribution in the unitary gauge. Although the triangle diagrams give rise to fourth-order tensor integrals due to the longitudinal part of the gauge boson propagators, our calculation scheme is suited to work out this class of terms straightforwardly. The static electromagnetic properties of the Y^0 boson arise from the six Feynman diagrams shown in Fig. 2, whose amplitudes can be constructed out of the Feynman rules presented in Table I. After solving the loop integrals, the full amplitude can be cast in the form of Eq. (17), which leads to

$$\Delta Q^{\text{Bos}} = \frac{a}{2x_Y x_W}(\Delta_{YW}^2 + 12x_X x_W)\Big\{4(x_Y - x_W) + 4[(x_Y + x_W) - 2(x_Y - x_W)^2]\log\Big(\frac{x_Y}{x_W}\Big) + \frac{2}{\Delta_{YW}}(x_Y - x_W)[1 - 3(x_W + x_Y) + 2(x_Y - x_W)^2]\text{arccosh}\Big(\frac{x_Y + x_W - 1}{2\sqrt{x_Y x_W}}\Big)\Big\}, \tag{26}$$

and

$$\Delta\kappa^{\text{Bos}} = \frac{3a}{2x_Y x_W}\Big\{(x_Y - x_W)[1 + (x_Y - x_W)^2 - 2(x_Y + x_W - 6x_Y x_W)] - \{x_Y(1 - x_Y)^2(3 + x_Y) + x_Y x_W[x_Y(8x_Y - 9x_W - 13) + 9] + x_W(1 - x_W)^2(3 + x_W) + x_Y x_W[x_W(8x_W - 9x_Y - 13) + 9]\} \times \log\Big(\frac{x_Y}{x_W}\Big) - 2(x_Y - x_W)\Delta_{YW}[3 - (x_Y - x_W)^2 + 2(x_Y + x_W + 6x_Y x_W)\text{arccosh}\Big(\frac{x_Y + x_W - 1}{2\sqrt{x_Y x_W}}\Big)\Big\}, \tag{27}$$

with $x_W = m_W/m_{Y^0}$ and $x_Y = m_{Y^\pm}/m_{Y^0}$. Because of the mass splitting (10), the bileptons would be nearly degenerate if $m_{Y^\pm} \geq m_W$. Therefore, it is worth obtaining analytical expressions for the form factors in this scenario. Equations (26) and (27) yield the following results for $x_Y = 1$:

$$\Delta Q^{\text{Bos}} = \frac{a}{2}(8 + x_W)\Big\{4(1 - x_W) + [1 + x_W(2x_W - 5)] \times \log(x_W) + \frac{2}{\sqrt{(x_W - 4)x_W}}(1 - x_W)x_W \times (2x_W - 7)\text{arccosh}\Big(\frac{\sqrt{x_W}}{2}\Big)\Big\}, \tag{28}$$

and

$$\Delta\kappa^{\text{Bos}} = \frac{3a}{4}\Big\{2(1 - x_W)(8 + x_W) + [16 + (x_W - 3)x_W \times (12 + x_W)]\log(x_W) - 2(x_W - 1) \times \sqrt{(x_W - 4)x_W}(12 + x_W)\text{arccosh}\Big(\frac{\sqrt{x_W}}{2}\Big)\Big\}. \tag{29}$$

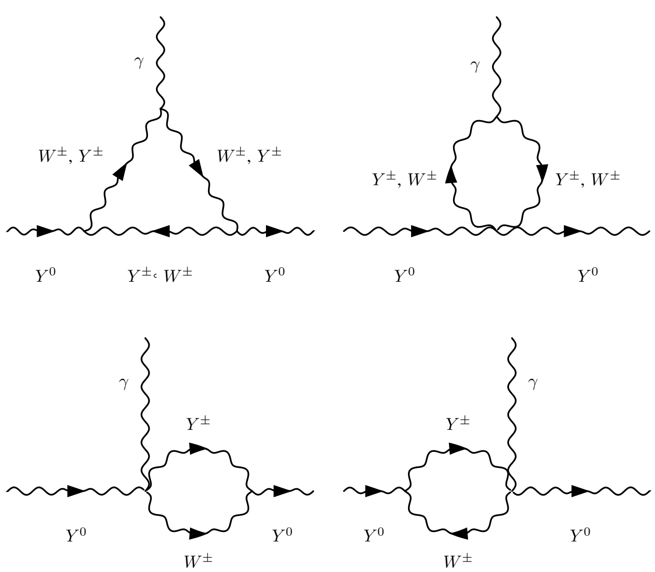

FIG. 2. Feynman diagrams for the gauge boson contributions to the static quantities of the Y^0 boson.

From the previous results, it is easy to see that the contributions to the Y^0 form factors are antisymmetric under the interchange of the masses of the particles circulating in the loop, which means that they vanish when these particles are degenerate, i.e., $m_q = m_{q'}$ and $m_W = m_Y$.

IV. NUMERICAL EVALUATION

We turn now to the numerical analysis of the Y^0 form factors. We would like to emphasize that our main aim is to estimate the size and behavior of the form factors in some illustrative scenarios rather than making a careful study of the allowed parameter space of the model, which is beyond the present work.

In addition to the mass of the Y^0 boson, there are four other unknown parameters which enter into the Y^0 form factors. These are the masses of the three exotic quarks m_{D_1}, m_{D_2}, and m_T, together with the charged bilepton mass $m_{Y^\pm}$. Since the splitting between the bilepton masses is bounded, i.e., $|m_{Y^0}^2 - m_{Y^\pm}^2| \leq m_{W^+}^2$, $m_{Y^\pm}$ is bounded once m_{Y^0} is fixed. Although in the minimal 331 model the bilepton masses are bounded from above at 1 TeV as a result of matching the gauge couplings constants at the Fermi scale, which leads to $\sin\theta_W \leq 1/4$ [18], in the version with right-handed neutrinos the same condition leads to $\sin\theta_W \leq 3/4$, which yields less stringent constraints on the bilepton masses. The most recent bounds indicate that m_{Y^0} is greater than 100 GeV [9,19]. We will thus analyze the form factors in the range 100 GeV $\leq m_{Y^0} \leq$ 500 GeV.

As for the exotic quarks, although there are bounds on the masses of the exotic quarks predicted in other SM extensions, to our knowledge there are no such bounds in the specific case of the 331 model with right-handed neutrinos. However, it is reasonable to assume that the exotic quarks are heavier than the top quark. Therefore, for the corresponding masses we will consider values ranging from 200 to 800 GeV. Furthermore, as will be shown below, the maximal value of the fermionic contribution to the static quantities of the Y^0 boson is reached in this mass range. Below we will evaluate separately the fermion and boson contribution to the Y^0 form factors.

A. Fermion contribution

The general behavior of the fermion contribution to the static quantities of the charged W boson has been discussed to a large extent in the literature [14,15,20,21]. The main peculiarity of the CP-even electromagnetic form factors of a neutral particle is that the contribution arising from a degenerate fermion pair vanishes since the amplitude is antisymmetrical under the interchange $m_q \to m_{q'}$. Although the latter is also true for an arbitrarily charged gauge boson, their CP-even static quantities do not vanish for degenerate fermions since $Q_q \neq Q_{q'}$. In the following analysis we will consider the scenario in which the exotic quarks are degenerate, with a mass m_Q. As already explained, we will consider the range 200 GeV $\leq m_Q \leq$ 800 GeV. In Figs. 3 and 4 we show the $\Delta\kappa$ and ΔQ form factors as a function of m_Q for some illustrative values of the neutral bilepton mass m_{Y^0}, namely, 300, 350 and 400 GeV. We note that the curves displayed in Figs. 3 and 4 are the full contribution from the three quark families. In the range under consideration for m_Q, the form factors are considerable smaller for $m_{Y^0} \leq$ 200 GeV. We can clearly observe that there is a dramatic enhancement in the m_{Y^0} threshold $m_{Y^0} = m_q + m_Q$, which stems from the fact that the respective quark pair (q, Q) can be directly produced from the bilepton provided that $m_{Y^0} \geq m_q + m_Q$. Above the threshold and in the heavy-mass limit, both form factors decrease rapidly and vanish when m_Q is much larger than the mass of the SM quarks. It is interesting to point out that the individual contributions to $\Delta\kappa$ from each fermion pair tend to the constant value $-9a$ in the heavy fermion limit, whereas ΔQ vanishes. This is in accordance with the decoupling theorem [22]: since $\Delta\kappa$ is associated with a term that arises from dimension-four operators, it is expected to be sensitive to nondecoupling effects of heavy physics, whereas ΔQ cannot be sensitive to this class of effects as it is associated with a term generated by a nonrenormalizable dimension-six operator [23]. In spite of the nondecoupling nature of the contributions from each quark family, the full $\Delta\kappa$ vanishes in the heavy fermion limit. It turns out that the partial contributions, which are proportional to the quark charge, become constant and their sum

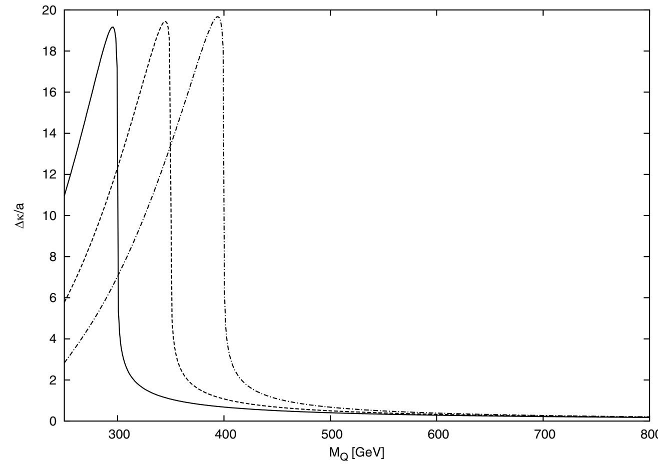

FIG. 3. The fermion contribution to the $\Delta\kappa$ form factor as a function of the mass of the exotic quarks, which are assumed to be degenerate, for different values of the neutral bilepton mass: 300 (continuous line), 350 (dashes), and 400 GeV (dashes and points).

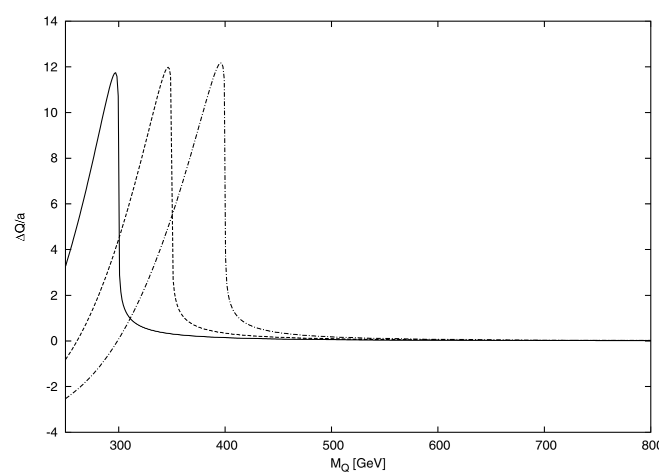

FIG. 4. The same as in Fig. 3 for the ΔQ form factor.

vanishes since it is proportional to $Q_{D_1} + Q_{D_2} + Q_T = 0$. This is to be contrasted with the behavior of the fermion contribution to the $\Delta\kappa$ form factor of the W boson in the heavy fermion limit. In this case the contribution of each quark family is proportional to $Q_u - Q_d = 1$, thus the sum over the three quark families does not vanish.

From Figs. 3 and 4 we can conclude that $\Delta\kappa$ can be of the order of $10a$, whereas ΔQ is about 1 order of magnitude below. This behavior is similar to that observed for the size of the fermion contribution to the electromagnetic form factors of the W boson in the SM [20] and some of its extensions [21]. Although the maximal value of the form factors is reached around the threshold $m_{Y^0} = m_q + m_Q$, there is no reason to expect that such a scenario is realized in nature. The scenarios shown through Figs. 3 and 4 are very illustrative of the behavior of the quark contribution to the static quantities of the Y^0 boson and so we refrain from presenting the most general case in which the exotic quark are nondegenerate.

B. Gauge boson contribution

In Figs. 5 and 6, we show the contributions from the gauge bosons to the electromagnetic form factors of the Y^0 boson as a function of m_{Y^0} when the bileptons are degenerate and also when $m_{Y^\pm}$ reaches its minimal and maximal allowed values: $m_{Y^\pm}^2 = m_{Y^0}^2 - m_W^2$ and $m_{Y^\pm}^2 = m_{Y^0}^2 + m_W^2$. The form factors are restricted to lie in the strip bounded by the extremal lines. Although the form factors seem to increase indefinitely as m_{Y^0} increases, they tend to a constant value for very large m_{Y^0}. There is no contradiction with the decoupling limit, as one cannot make large the internal mass $m_{Y^\pm}$ while keeping fixed the external mass m_{Y^0} due to the bound (10). Furthermore, the quantities which have physical meaning are the magnetic dipole and electric quadrupole moments [See Eqs. (18) and (19)], which do vanish for very large m_{Y^0}. From Figs. 5 and 6, it is evident that $\Delta\kappa$ is 1 order of magnitude larger than ΔQ for each value of m_{Y^0}. The fact that the size of $\Delta\kappa$ is larger than that of ΔQ has been also observed for the case of the electromagnetic form factors of the charged W boson form within all of the theories studied up to now.

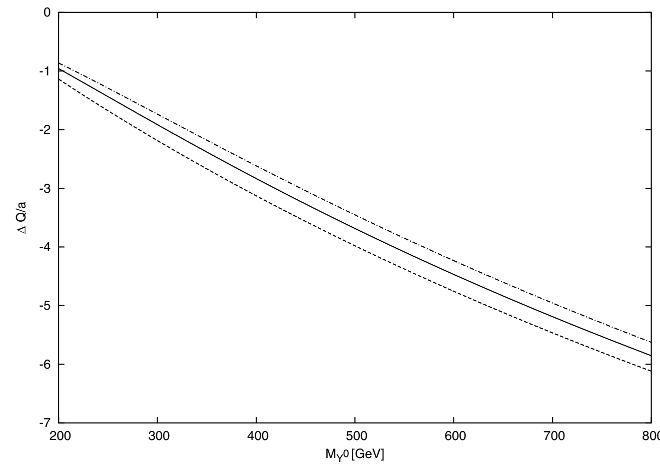

FIG. 6. The same as in Fig. 5 for the ΔQ form factor.

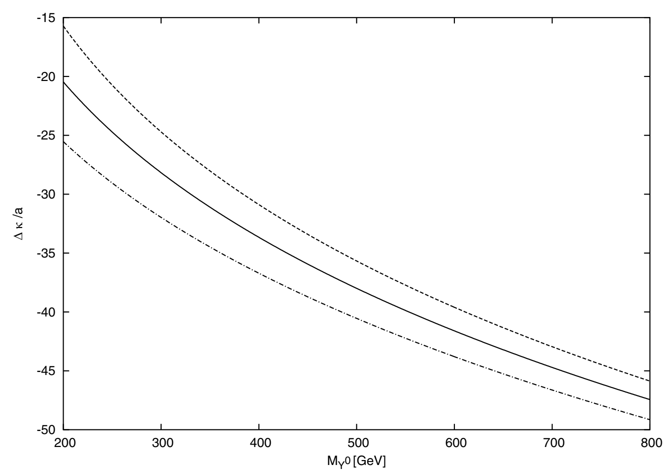

FIG. 5. The gauge boson contribution to the $\Delta\kappa$ form factor of the Y^0 boson as a function of its mass when $m_{Y^\pm} = m_{Y^0}$ (continuous line), $m_{Y^\pm}^2 = m_{Y^0}^2 - m_W^2$ (dashes) and $m_{Y^\pm}^2 = m_{Y^0}^2 + m_W^2$ (dashes and points). The last two curves correspond to the case when the $m_{Y^\pm}$ reaches its maximal and minimal allowed values. $\Delta\kappa$ is restricted to lie in the strip bounded by the extremal lines.

To obtain the total contribution to the Y^0 form factors, it is necessary to sum over the fermion and gauge boson contributions, along with the one arising from the scalar sector of the theory. Apart from the specific details of the model, we do not expect that the size of the scalar contribution is different to that observed in the case of the W form factors. In that case, the scalar sector yields a marginal correction. In fact a very large number of Higgs bosons would be required to yield a large correction.

V. SUMMARY

A neutral vector boson can have static electromagnetic properties provided that the associated field is non-self-conjugate. We have presented the calculation of the static electromagnetic properties of the neutral non-self-conjugate boson Y^0 which arises in the $SU(3)_c \times SU(3)_L \times U(1)_N$ model with right-handed neutrinos. This model is interesting since it requires that the fermion families be a multiple of the quark color number in order to cancel anomalies, thereby suggesting a solution to the family problem. It has been pointed out that the Y^0 boson is a good candidate in high energy experiments since it may be the source of neutrino oscillations as it is responsible of lepton-number violating interactions. The calculation was done in the unitary gauge and the fermion and gauge boson contributions were obtained by a modified version of the Passarino-Veltman reduction scheme. As a crosscheck, the form factors were obtained independently by the Feynman parameter technique and the results, expressed in terms of parametric integrals, were numerically evaluated and compared with the results obtained via the Passarino-Veltman method. A perfect agreement was observed. In this model the Y^0 boson only couples to left-handed fermions and so only the CP-even form factors are induced at the one-loop level. The behavior of both contributions was analyzed. In the fermion sector there is the contribution of the three quark pairs (D_1, d), (D_2, s), and (T, t), with D_1, D_2, and T three exotic quarks whose charge is identical to that of the respective SM quark. As for the gauge boson contribution, there is the contribution of a singly-charged bilepton $Y^{\pm}$. The symmetry-breaking hierarchy yields an upper bound on the splitting between the bilepton masses such that $|m^2_{Y^0} - m^2_{Y^{\pm}}| \leq m^2_W$, which means that the bileptons are nearly degenerate provided that their mass is heavier than m_W. From the numerical analysis we can conclude that the size of the Y^0 form factors is somewhat similar to that observed for the W boson form factors in the SM and some of its extensions.

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