Leptonic *CP* violation phases using an ansatz for the neutrino mass matrix and application to leptogenesis

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We further study the previously proposed ansatz, $\text{Tr}(M_\nu) = 0$, for a prediagonal light Majorana type neutrino mass matrix. If *CP* violation is neglected this enables one to use the existing data on squared mass differences to estimate (up to a discrete ambiguity) the neutrino masses themselves. If it is assumed that only the conventional *CP* phase is present, the ansatz enables us to estimate this phase in addition to all three masses. If it is assumed that only the two Majorana *CP* phases are present, the ansatz enables us to present a one parameter family of solutions for the masses and phases. This enables us to obtain a simple "global" view of lepton number violation effects. Furthermore using an SO(10) motivation for the ansatz suggests an amusing toy (clone) model in which the heavy neutrinos have the same mixing pattern and mass ratios as the light ones. In this case only their overall mass scale is not known (although it is constrained by the initial motivation). Using this toy model we make a rough estimate of the magnitude of the baryon to photon ratio induced by the leptogenesis mechanism. Solutions close to the *CP* conserving cases seem to be favored.

I. INTRODUCTION

Remarkably, the recent KamLAND [1], SNO [2] and K2K [3] experiments have added so much to the results obtained from earlier solar neutrino, atmospheric neutrino and accelerator experiments [4] that our knowledge about the neutrino masses and presumed lepton mixing matrix is almost as great as our knowledge of the corresponding quantities in the quark sector. Still there is an uncertainty about the interpretation due to the results of the LSND experiment [5]. However, this experiment will be checked soon by the miniBoone Collaboration so one can wait for confirmation before considering whether there is really a problem with the usual picture of three massive neutrinos. In any event, there is a strong presumption that this knowledge will play an important role in going beyond the standard model of electroweak interactions.

One detail is, of course, lacking compared to the quark case. Since the neutrino oscillation experiments measure only the differences of the neutrino squared masses, the neutrino masses themselves are not known. According to the latest analysis [6] the best fit to these differences is:

$$m_2^2 - m_1^2 = 6.9 \times 10^{-5} \text{ eV}^2,$$
$$|m_3^2 - m_2^2| = 2.6 \times 10^{-3} \text{ eV}^2. \tag{1}$$

Now, there is a simple *complementary* ansatz for the 3×3 neutrino mass matrix, M_ν which, with some assumptions, enables one to obtain the neutrino masses themselves from Eq. (1); it requires:

$$\text{Tr}(M_\nu) = 0. \tag{2}$$

It should be remarked that M_ν is to be regarded as the prediagonal neutrino mass matrix. Furthermore, in the relation $m_1 + m_2 + m_3 = 0$ which evidently results if the neutrino mass matrix is taken to be real symmetric, the individual masses may be either positive or negative. The negative masses can be converted to positive ones by adding appropriate factors of i in the diagonalizing matrix.

Equation (2) was motivated in [7] from the grand unified model, SO(10) [8] and in [9] by noting that it would hold if M_ν is the commutator of two other matrices, as may occur in certain models. If *CP* violation is neglected there are essentially two possible solutions of the ansatz: either m_1 and m_2 have the same sign and are approximately equal to each other and to $-m_3/2$ or else m_1 and m_2 have the opposite sign and are approximately equal to each other in magnitude but much larger than the magnitude of m_3.

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In the present paper we will take the point of view that the ansatz, Eq. (2), is motivated from SO(10). However, the analysis is of course not dependent on the motivation. The SO(10) motivation arises from the observation that Eq. (2) is, although it seems at first different, essentially the same as the characteristic prediction of grand unification:

$$m_b = r m_\tau, \tag{3}$$

relating the mass of the b quark with the mass of the tau lepton ($r \approx 3$ takes account of the running of masses from the grand unification scale to the low energy hadronic scale of about 1 GeV [10]). Note that in SO(10) the neutrino mass matrix takes the form:

$$M_\nu = M_L - M_D^T M_H^{-1} M_D, \tag{4}$$

where M_L, M_H, and M_D are, respectively, the mass matrices of the light neutrinos, heavy neutrinos and heavy-light mixing (or "Dirac matrix"). To start with, M_ν is an arbitrary symmetric matrix. If it is real we have CP invariance. Generally the second, seesaw [11] term is considered to dominate. However, as explained in [7], the present model is based on the assumption that the first term dominates. That might not be unreasonable since a rough order of magnitude estimate for the second term would be $m_t^2/10^{17}$ or about 3×10^{-4} eV. (The quantity 10^{17} includes a factor $r^2 \approx 10$). Thus the second term could be negligible if neutrino masses are appreciably greater than this value.

In [7] the complementary ansatz was mainly studied for the case of real M_ν. Here we will be primarily interested in the more general complex case which allows for nonzero CP violation. Furthermore, the input squared mass differences were not taken to be very similar to those in Eq. (1) but were based on a least squares fit [12] of many different experiments *including* LSND. Here we shall adopt the more conventional values given in Eq. (1). A related analysis of Eq. (2) was recently made in [13].

For an understanding of the interesting leptogenesis mechanism [14] of baryogenesis it is important to also study the properties of the heavy neutrinos which appear there. In the present SO(10) motivated framework this task turns out to be remarkably simple; the heavy neutrino mass matrix is given by

$$M_H = c M_\nu, \tag{5}$$

where c is a numerical constant. This means that the eigenvalues of M_H, to be denoted as M_1, M_2, M_3 are simple multiples of the light neutrino masses m_1, m_2, m_3. In addition the unitary matrix, U which brings M_ν to diagonal form via

$$U^T M_\nu U = \text{diag}(m_1, m_2, m_3) = \hat{M}_\nu, \tag{6}$$

also diagonalizes M_H. In other words the heavy neutrinos are clones of the light neutrinos in this picture. The result

follows from the choice of Higgs fields in SO(10). Trilinear Yukawa terms which supply fermion masses can contain Higgs fields in the 10, 120, and 126 dimensional representations. To get the result just mentioned we need to require that there is only one "126" representation present, although any number of "10's" and "120's" are allowed. Of course we are also assuming the second term in Eq. (4) to be negligible for the purpose of generating the light neutrino masses.

In Sec. II, we give our conventions for the lepton mixing matrix, including one conventional and two Majorana type CP violation phases. An approximate equation relating the complementary ansatz to the parameters of the mixing matrix and the physical light neutrino masses is written down. The solutions for the neutrino masses in the CP conserving case, based on the results of neutrino oscillation experiments, are reviewed. In Sec. III, the ansatz equation is solved on the assumption that only the conventional CP phase, δ is nonzero. It is found that the only solutions correspond to maximal phase, $\sin^2 \delta = 1$ and neutrino masses close to the ones obtained in the CP conserving case. In Sec. IV we investigate the more complicated but very interesting case when only the two Majorana CP violation phases are nonzero. In this case there is a family (modulo a discrete ambiguity) of solutions. We choose the mass of the third light neutrino, m_3 as our free parameter and calculate the remaining neutrino masses and the Majorana phases as functions of m_3. The model gives a lower bound for m_3 and the cosmology criterion on the sum of neutrino masses effectively yields an upper bound. The results for the full range are scanned numerically and a simple analytic interpretation of the pattern is presented. The neutrinoless double beta decay parameter, $|m_{ee}|$ is also calculated for each value of m_3. In Sec. V we make a rough estimate of the baryon to photon ratio based on the leptogenesis mechanism. In order to do this it is necessary to make some statements on the masses and mixings of the heavy neutrinos. Our motivation for the original ansatz suggests a "clone" model in which the heavy neutrinos have the same mass ratios and mixing matrix as the light ones. The only new parameter besides m_3 is the overall mass scale, which however is constrained by the original motivation to be somewhat on the large side. Then it is relatively easy to calculate the lepton asymmetry parameters, ϵ_i for the heavy neutrino decays as functions of mainly m_3. We combine these quantities in a semiquantitative way with criteria from previous treatments of the Boltzmann evolution equations for the decaying neutrinos. It is found that the most plausible scenarios for leptogenesis involve small CP violating Majorana phases and light neutrino masses close to the ones predicted for the CP conserving cases. Finally Sec. VI contains a brief discussion and a brief summary.

II. RELATING THE ANSATZ TO EXPERIMENT

Here we will obtain an approximate equation which will be useful for relating the complementary ansatz to experimental information on neutrino squared mass differences and mixing angles in the general case where *CP* violation is allowed. The notation is the same as in Sec. III of [7] which contains more details. For convenience, we will use what seems to be the most common convention for the part of the leptonic mixing matrix, K_{exp}, which is measured in the usual neutrino oscillation experiments. This part can be constructed as a product of elementary transformations in the (12), (23), and (13) subspaces. For example in the (12) subspace one has:

$$K_{exp} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \quad (9)$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. Since the neutrinos are of Majorana type in this model, there are expected also to be physical *CP* violating effects due to the Majorana phases [15–18]. These may be introduced via a unimodular diagonal matrix of phases,

$$\omega_0(\tau) = \text{diag}(e^{i\tau_1}, e^{i\tau_2}, e^{i\tau_3}), \qquad \tau_1 + \tau_2 + \tau_3 = 0. \quad (10)$$

The full lepton mixing matrix is then expressed as

$$K = K_{exp}\omega_0^{-1}(\tau), \quad (11)$$

which has three mixing angles and three independent *CP* violating phases. We shall use this form in what follows. As an aside, though, we remark that the full matrix could also be written [7] in an unconventional, but more symmetrical, way as:

$$K = \omega_{23}(\theta_{23}, \phi_{23})\omega_{13}(\theta_{13}, \phi_{13})\omega_{12}(\theta_{12}, \phi_{12}). \quad (12)$$

As a final preliminary we need the leptonic *W* interaction term :

$$\mathcal{L} = \frac{ig}{\sqrt{2}} W_\mu^- \bar{e}_L \gamma_\mu K \nu + \text{H.c.}, \qquad K = \Omega^\dagger U, \quad (13)$$

where *U* is defined in Eq. (6) and $\Omega^\dagger$ is a unitary matrix which is needed to diagonalize the charged lepton mass matrix. At this point we shall make the common approximation that Ω can be replaced by essentially the unit matrix. This is certainly not perfect but it seems reasonable for a start. This is a natural approximation in our framework, as will be discussed in Sec. VI. Then *U* may be replaced by *K*, for which some elements are already well known. This enables us to present the ansatz in the form:

$$\text{Tr}[\hat{M}_\nu K_{exp}^{-1} K_{exp}^* \omega_0(2\tau)] = 0, \quad (14)$$

$$\omega_{12}(\theta_{12}, \phi_{12}) = \begin{pmatrix} \cos\theta_{12} & e^{i\phi_{12}}\sin\theta_{12} & 0 \\ -e^{-i\phi_{12}}\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (7)$$

with clear generalization to the (23) and (13) transformations.

Then the usual convention corresponds to the choice:

$$K_{exp} = \omega_{23}(\theta_{23}, 0)\omega_{13}(\theta_{13}, -\delta)\omega_{12}(\theta_{12}, 0), \quad (8)$$

with three mixing angles and the *CP* violation phase δ. Multiplying out yields:

where Eqs. (6) and (11) were used. In writing Eq. (14) we assumed that an additional phase matrix given in Eq. (5.2) of [7] is set equal to the unit matrix. This can be accomplished by an allowed rephasing of the charged lepton fields when $\Omega = 1$. An alternate justification of our treatment is to assume that the ansatz holds in a special basis in which the charged lepton mass matrix is diagonal. With the parameterized mixing matrix of Eq. (9) the ansatz reads:

$$m_1 e^{2i\tau_1}[1 - 2i(c_{12}s_{13})^2 \sin\delta e^{-i\delta}] +$$
$$m_2 e^{2i\tau_2}[1 - 2i(s_{12}s_{13})^2 \sin\delta e^{-i\delta}] +$$
$$m_3 e^{2i\tau_3}[1 + 2i(s_{13})^2 \sin\delta e^{i\delta}] = 0. \quad (15)$$

In this equation we can choose the diagonal masses m_1, m_2, m_3 to be real positive. However it will be a little more convenient in the *CP* conserving case to allow some of them to be negative as well as positive. We shall, for definiteness, mainly use the following best fit values [6] for the mixing angles:

$$s_{12}^2 = 0.30, \qquad s_{23}^2 = 0.50, \qquad s_{13}^2 = 0.003. \quad (16)$$

It should be remarked that the precise value of s_{13} is not well known, in contrast to the other two.

Equation (15) contains three unknown masses and three unknown *CP* phases. It can be written as two real equations and augmented by two equations for two neutrino mass squared differences. Thus there are four equations for six unknowns. By assuming some special simplifications we can make the analysis tractable.

For orientation let us first review the case when the theory is *CP* conserving so that all the three independent *CP* phases vanish. Then we will have three equations for three unknowns. The ansatz now reads $m_1 + m_2 + m_3 = 0$. Define:

$$A = (m_2)^2 - (m_1)^2 \qquad B = (m_3)^2 - (m_2)^2 \qquad (17)$$

It can be deduced [19] from the experimental data that A is positive while the sign of B is not yet known. Their magnitudes are given in Eq. (1). Thus there are two separate cases to be considered. First consider both A and B positive. Then solving as in [7] gives the type I solution:

$$m_1 = 0.0291 \text{ eV}, \qquad m_2 = 0.0302 \text{ eV},$$
$$m_3 = -0.0593 \text{ eV}. \qquad (18)$$

Next consider the type II solution where B is negative; it gives:

$$m_1 = 0.0503 \text{ eV}, \qquad m_2 = -0.0510 \text{ eV},$$
$$m_3 = 0.000\,68 \text{ eV}. \qquad (19)$$

Here m_1 and m_2 are still almost degenerate but differ in sign. However m_3 is now relatively small compared to the others.

III. CONVENTIONAL *CP* VIOLATION

A fully predictive simple case would correspond to keeping δ as the only *CP* violation phase. Then the real part of the ansatz equation, (15) reads

$$m_1 + m_2 + m_3 - 2s_{13}^2 \sin^2\delta(c_{12}^2 m_1 + s_{12}^2 m_2 + m_3) = 0, \qquad (20)$$

while the imaginary part yields

$$s_{13}^2 \sin 2\delta(c_{12}^2 m_1 + s_{12}^2 m_2 - m_3) = 0. \qquad (21)$$

Note that the m_i's are being taken real here, although they will be allowed to be either positive or negative. A negative m_i is not a source of *CP* violation even though it corresponds to a Majorana phase τ_i of $\pi/2$ when the masses are taken positive [20]. Now Eqs. (17), (20) and (21) constitute four equations for the three m_i's and δ.

However it turns out that, except for the special case when $s_{13}^2 \sin 2\delta = 0$, there is no consistent solution of this set of four equations for four unknowns. To see this, first consider solving simultaneously the three Eqs. (17) and (21) when the special case does not hold. The numerical solution is seen to require $B < 0$ and is found to be (with $A > 0$):

$$m_1 = -0.0548 \text{ eV}, \qquad m_2 = 0.0554 \text{ eV},$$
$$m_3 = -0.0217 \text{ eV}. \qquad (22)$$

We must now check to see if this is consistent with the remaining Eq. (20). That leads to the requirement:

$$s_{13}^2 \sin^2\delta = \frac{1}{4}\left(1 + \frac{m_1 + m_2}{m_3}\right) \approx 0.25, \qquad (23)$$

which, given the numerical value of s_{13}^2 in Eq. (16) clearly leads to the contradiction $\sin^2\delta \approx 80$. This contradiction

will persist even if the upper bound (about 0.044) rather than the best fit for s_{13}^2 is used. The result is also not changed if the signs of all the m_i's are reversed.

Thus the only possibility for pure δ type *CP* violation in the present scheme is the special case where $\sin^2\delta = 1$. Then we must solve simultaneously the three equations consisting of Eq. (20) in which this substitution has been made for $\sin^2\delta$ as well as Eqs. (17). This results in the equation for, say m_2,

$$\text{sign}(m_1)(m_2^2 - A)^{1/2}(1 - 2s_{13}^2 c_{12}^2) +$$
$$m_2(1 - 2s_{13}^2 s_{12}^2) + (B + m_2^2)^{1/2}(1 - 2s_{13}^2) = 0, \qquad (24)$$

where sign(m_3) has been arbitrarily taken positive. Knowing m_2, the other two masses may of course be obtained from Eqs. (17).

Taking, for definiteness, the mixing angle from Eq. (16), one finds essentially two different solutions. These are quite similar to the type I and type II solutions given above in the *CP* conserving case. The type I solution, with $B > 0$ is

$$m_1 = -0.0289 \text{ eV}, \qquad m_2 = -0.0301 \text{ eV},$$
$$m_3 = 0.0592 \text{ eV}. \qquad (25)$$

The type II solution, with $B < 0$, reads

$$m_1 = 0.0503 \text{ eV}, \qquad m_2 = -0.0510 \text{ eV},$$
$$m_3 = 0.000\,81 \text{ eV}. \qquad (26)$$

The very close similarity between the *CP* conserving solutions and the solutions with $\sin^2\delta = 1$ is understandable due to the small value of s_{13}^2.

IV. *CP* VIOLATION DUE TO MAJORANA TYPE PHASES

Since, as we have just seen, there is only one particular allowed value for the conventional *CP* phase, δ if it is considered as the only source of *CP* violation in the present scheme, it is of great interest to investigate the Majorana phases. Clearly, it seems sensible to study these phases with the simplification of putting δ to zero. From Eqs. (7) and (8) it is seen that the same effect is accomplished by setting $s_{13} = 0$. Then the ansatz Eq. (15) takes the form

$$m_1 e^{2i\tau_1} + m_2 e^{2i\tau_2} + m_3 e^{2i\tau_3} = 0. \qquad (27)$$

For our present case it is convenient to take all three m_i's to be real and positive (note that a phase angle $\tau_i = \pi/2$ corresponds to what was taken as a shorthand to be a negative value of m_i). Together, Eq. (27) and Eq. (17) comprise four real equations for five unknowns (three masses and two independent τ_i's). To proceed we shall thus assume a value for m_3 so that we have four equations for four unknowns. In addition there is the twofold ambiguity due to the unknown sign of B. Finally we shall

allow m_3 to vary to obtain a global picture of the situation.

Now, once we have assumed a value for m_3, we can immediately find m_1 and m_2 from Eqs. (17). Furthermore, Eq. (27) can be pictured in the complex plane as a triangle formed from vectors with lengths m_i, having angles $2\tau_i$ as measured from the positive horizontal axis. Then let μ_i be the interior angle opposite side m_i as illustrated by the choice of triangle in Fig. 1.

The problem reduces to one from elementary plane geometry. Given the three sides (m_i), of a triangle, find the three interior angles (μ_i). We may start, for example, by using the law of cosines to get

$$\cos\mu_1 = \frac{-m_1^2 + m_2^2 + m_3^2}{2m_2 m_3}, \qquad (28)$$

and continue similarly to get the others. Finally the parameters τ_i which appear in the actual parameterization of Eq. (10) are found from Fig. 1 as

$$\tau_1 = \frac{1}{6}(\pi - \mu_1 - 2\mu_2),$$

$$\tau_2 = \frac{1}{6}(\pi + 2\mu_1 + \mu_2), \qquad (29)$$

$$\tau_3 = \frac{1}{6}(-2\pi - \mu_1 + \mu_2).$$

In particular, the quantities

$$\begin{aligned} \sin[2(\tau_1 - \tau_2)] &= -\sin(\mu_1 + \mu_2), \\ \sin[2(\tau_1 - \tau_3)] &= \sin\mu_2, \\ \sin[2(\tau_2 - \tau_3)] &= -\sin\mu_1, \end{aligned} \qquad (30)$$

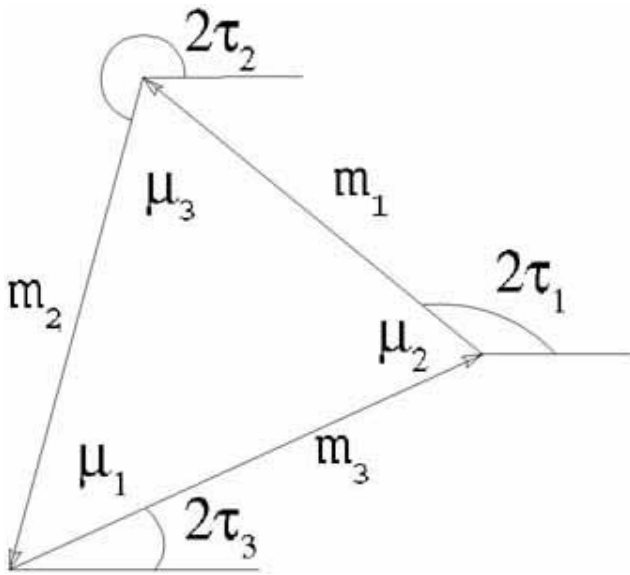will turn out to be of interest. Actually, given the three

FIG. 1. Vector triangle representing Eq. (27).

interior angles μ_i of a triangle we do not get a unique choice of phase differences $(\tau_i - \tau_j)$. While a rotation in the plane of the triangle will not change these phase differences, it is straightforward to see that the reflection of the triangle about any line in the plane will reverse the signs of all the phase differences. Thus there is another solution in which an extra minus sign appears on each right-hand side of (30).

Now let us discuss the solutions of the complementary ansatz equation for various assumed values of m_3. In Table I the three real positive masses as well as the corresponding values of the two independent internal angles μ_1, μ_2 of the triangle are listed. Of course, $\mu_1 + \mu_2 + \mu_3 = \pi$. The solution with $B > 0$ (type I with $m_3 > m_2$) will be listed when it exists as well as the type II solution ($B < 0$ or $m_2 > m_3$).

Let us start with large values of m_3 and go down. Just from the ansatz there is no upper bound on the value of m_3. However there is a recent cosmology bound [21] which requires,

$$|m_1| + |m_2| + |m_3| < 0.7 \text{ eV}. \qquad (31)$$

Thus values of m_3 greater than about 0.3 eV are physically disfavored. Table I shows that at this value both type I and type II solutions exist and correspond to almost equilateral triangles. This is true also for higher values of m_3. Notice that since the triangles are close to being equilateral, they have large interior angles and hence [see, for example, Eq. (30)] large *CP* phases. The picture remains very similar down to around $m_3 = 0.1$ eV but as one gets closer to the value, roughly 0.0593 eV, where the real type I solution of Eq. (18) exists, there is a marked change. It is seen that the interior angles of the type I solution become small as it prepares to go to the degenerate triangle corresponding to the real solution. We may get as small *CP* phases as we like by tuning close to the real solution; this is illustrated in Table I for a particular value of m_3. If one further lowers m_3, it is found that the type I solution no longer exists. On the other hand the type II solution persists and does not change much until m_3 approaches the small value of roughly 0.000 68 eV. There are no solutions for m_3 smaller than this value. We can also tune m_3 as illustrated in the table to get as small *CP* phases as we like for the type II case. It should be remarked that the precise numbers in Table I are based on the assumption that the best fit numbers given in Eq. (1) are exact and hence are meaningful to the accuracy given only in the sense of comparing the various solutions with each other, not with experiment. It is straightforward to give an analytic interpretation of the pattern of solutions just observed. First note that *CP* violation corresponds to a non degenerate triangle. Note also that the orientation of the triangle in the complex plane is just obtained by imposing the unimodularity condition for $\omega(\tau)$ in Eq. (10). Hence the internal angles,

TABLE I. Panorama of solutions as m_3 is lowered from about the highest value which is experimentally reasonable to the lowest value imposed by the model. In the type I solutions m_3 is the largest mass while in the type II solutions m_3 is the smallest mass. For each value of m_3, the values of the model predictions for m_1 and m_2 as well as the internal angles μ_1 and μ_2 are given. The model prediction for the neutrinoless double beta decay quantity $|m_{ee}|$ is next shown. Finally, the last column shows the estimated lepton asymmetries due to the decays of the heavy neutrinos. Note that the reversed sign of lepton asymmetry is also possible, as discussed in the text.

type	m_1, m_2, m_3 in eV	μ_1, μ_2 in radians	$\|m_{ee}\|$ in eV	$\epsilon_1, \epsilon_2, \epsilon_3$
I	0.2955, 0.2956, 0.3000	1.038, 1.039	0.185	0.342, 0.433, 0.017
II	0.3042, 0.3043, 0.3000	1.055, 1.056	0.187	0.330, 0.426, -0.0172
I	0.0856, 0.0860, 0.1000	0.946, 0.952	0.058	0.138, 0.060, 0.001 37
II	0.1119, 0.1122, 0.1000	1.106, 1.111	0.065	0.194, 0.088 $-$ 0.0024
I	0.0305, 0.0316, 0.0600	0.258, 0.268	0.030	0.00982, 0.00422, 0.00004
II	0.0783, 0.0787, 0.0600	1.172, 1.186	0.043	0.094, 0.041, $-$ 0.0011
I	0.0291, 0.0302, 0.059 271 564 9	0.000 552, 0.000 574	0.030	1.96×10^{-6}, 0.84×10^{-6}, 0.71×10^{-7}
II	0.0774, 0.0782, 0.059 271 564 9	1.174, 1.188	0.042	0.047, 0.020, $-$0.0011
II	0.0643, 0.0648, 0.0400	1.243, 1.268	0.033	0.052, 0.023, $-$ 0.000 681
II	0.0541, 0.0548, 0.0200	1.355. 1.417	0.024	0.018, 0.0078, $-$ 0.000 335
II	0.0506, 0.0512, 0.0050	1.386, 1.658	0.021	0.0057, 0.0025, $-$ 0.000 082 4
II	0.0503, 0.0510, 0.0010	0.814, 2.313	0.021	0.00073, 0.000 31, $-$ 0.000 012 2
II	0.0503, 0.0510, 0.000 681	0.051 361, 3.089 536	0.021	0.000 034 8, 0.000 015 0, -0.601×10^{-6}

μ_i are really the intrinsic carriers of CP violation. The determinant of whether one has CP violation is the non vanishing of the quantity:

$$\mathcal{A} = [m(m - m_1)(m - m_2)(m - m_3)]^{1/2},$$
$$m = \frac{1}{2}(m_1 + m_2 + m_3), \tag{32}$$

which just expresses the area, $\mathcal{A}$ of a triangle in terms of the lengths of its sides. This area may be rewritten in the convenient form:

$$\mathcal{A} = \frac{1}{4}\left([(m_1 + m_2)^2 - m_3^2][m_3^2 - (m_1 - m_2)^2]\right)^{1/2}. \tag{33}$$

Now we may see that the vanishing of the first factor corresponds to the type I real solution while the vanishing of the second factor corresponds to the type II real solution. Furthermore, for a solution to exist, the argument of the square root should be positive. With the second factor, that establishes the minimum allowed value of m_3 while with the first factor, that establishes the minimum value of m_3 which allows a type I solution.

An important test of the model is the experimental bound on neutrinoless double beta decay. This implies [22]

$$|m_{ee}| < (0.35 \to 1.30) \text{ eV}, \tag{34}$$

where

$$|m_{ee}| = |m_1(K_{exp11})^2 e^{-2i\tau_1} + m_2(K_{exp12})^2 e^{-2i\tau_2} + m_3(K_{exp13})^2 e^{-2i\tau_3}|. \tag{35}$$

Using the parameterization of Eq. (9) and approximating

$s_{13} = 0$ (which is reasonable in the present model since m_3 is never much larger than m_1 or m_2), this can be written simply as:

$$|m_{ee}| = [(c_{12}^2 m_1)^2 + (s_{12}^2 m_2)^2 + 2m_1 m_2 c_{12}^2 s_{12}^2 \cos(\mu_1 + \mu_2)]^{1/2}. \tag{36}$$

Here Eqs. (29) were also used. Reading s_{12}^2 from (16) then enables us to calculate $|m_{ee}|$ for each line of Table I. It is seen that $|m_{ee}|$ decreases smoothly with decreasing m_3 for each of the type I and type II solutions. All the values of m_3 listed are consistent with the present bound. It is interesting that an improvement of the experimental bound by an order of magnitude [23] would provide a good test of the model.

V. ESTIMATE FOR LEPTOGENESIS

When one adopts the SO(10) motivation for the present ansatz, it turns out that the resulting model predicts in a simple way the properties of the heavy neutrinos which are intrinsically contained in the SO(10) theory. This feature may be used in connection with the leptogenesis mechanism [14] of baryogenesis. According to this mechanism, the CP violating and lepton number violating decays of the heavy neutrinos at a high temperature (corresponding to the grand unification scale) in the very early universe establish a lepton asymmetry. As the universe cools further, the $(B + L)$ violating but $(B - L)$ conserving "sphaleron" interaction [24] converts this into a baryon asymmetry which may be compared with the observed ratio of baryons to photons in the universe. There are many interesting discussions of this mechanism in the literature [25–29]. Here, we will estimate the

dependence on neutrino masses and *CP* phases of the predicted baryon asymmetry in the present model.

The starting point of this discussion is the Yukawa term of the Lagrangian density which describes the tree level decay of a heavy Majorana neutrino, N_j (where the subscript denotes a three-valued generation index) to a Higgs doublet member,

$$\Phi^c = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix} \qquad (37)$$

plus the appropriate member of the left-handed lepton doublet,

$$L_i = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix}. \qquad (38)$$

Then the Yukawa term reads:

$$\mathcal{L}_{\text{Yukawa}} = -\sum \bar{L}_i \lambda_{ij} \Phi^c N_j + \text{H.c.}, \qquad (39)$$

where λ_{ij} is the matrix of Yukawa coupling constants. We can simplify this expression, which is supposed to contain the fermion fields in prediagonal bases, in several ways. First, at the high temperature for which the N decays are relevant, the phase transition to spontaneously broken SU(2) × U(1) has not yet taken place. Thus we can consider the light fermions in L_i to be massless and there is no need to insert suitable unitary matrices to bring the light field mass matrices to diagonal form. However the heavy neutrino, N should be related to the physical field $\hat{N}$ with a unitary matrix U as $N = U\hat{N}$. As mentioned in Sec. I, if the SO(10) model contains only a single 126 Higgs type field (although any number of 10's and 120's are allowed) and also if the first (non-seesaw) term in Eq. (4) is dominant, the prediagonal mass matrices for the light and heavy neutrinos must be proportional to each other and the diagonalizing matrix U must be the same one which appears in Eq. (13). Approximating, as we did earlier, Ω to be essentially the unit matrix we can set $U \approx K$. If the model of Sec. IV is adopted, for example, we can specify K, including *CP* phases, to a fair approximation for each assumed value of m_3. Finally we approximate the matrix of Yukawa couplings by:

$$\lambda_{ij} \approx \frac{\delta_{ij} m_i^U}{r'\langle \phi^0 \rangle}, \qquad (40)$$

where m_i^U are the three charge 2/3 quark masses at a low energy scale, $r' \approx 3$ is a suitable factor for running these masses from the grand unified scale to the low energy scale and $\langle \phi^0 \rangle \approx 246/\sqrt{2}$ GeV. Note that $\mathcal{L}_{\text{Yukawa}}$ is the term responsible for generating the neutrino Dirac matrix, M_D in Eq. (4). In the simplest approximation to the SO(10) theory the charge 2/3 quark mass matrix and neutrino Dirac mass matrix are proportional to each other and diagonal (since the quark mixings are after all small).

Putting these things together we arrive at the "effective" term for calculating the heavy neutrino decays (at grand unified scale temperature):

$$\mathcal{L}_{\text{Yukawa}} = -\sum \bar{L}_i h_{ij} \Phi^c \hat{N}_j + \text{H.c.}, \qquad (41)$$

where

$$h_{ij} \approx \frac{m_i^U K_{expij} e^{-i\tau_j}}{\langle \phi^0 \rangle r'}. \qquad (42)$$

The quantities needed for the calculation are the matrix products $(h^\dagger h)_{ij}$. We may further simplify these products by noting that the top quark mass is much heavier than the others so the products approximately become $h_{i3}^\dagger h_{3j}$. Specifically, the diagonal products are:

$$\begin{aligned} (h^\dagger h)_{11} &\approx (s_{12} s_{23}/r')^2, \\ (h^\dagger h)_{22} &\approx (c_{12} s_{23}/r')^2, \\ (h^\dagger h)_{33} &\approx (c_{12}/r')^2, \end{aligned} \qquad (43)$$

where we used the numerical coincidence that $m_t = \langle \phi \rangle$. Furthermore, we set $s_{13} = 0$ in agreement with the model of Sec. IV [See the parameterization of Eq. (9)]. Numerically, with Eq. (16) and $(r')^2 \approx 10$ one obtains $(h^\dagger h)_{11} \approx 1.50 \times 10^{-2}$, $(h^\dagger h)_{22} \approx 3.50 \times 10^{-2}$ and $(h^\dagger h)_{33} \approx 7.00 \times 10^{-2}$. In terms of these diagonal products, the tree level widths of the heavy neutrinos are given by,

$$\Gamma_i = \frac{(h^\dagger h)_{ii} M_i}{8\pi}, \qquad (44)$$

where M_i is the mass of the *i*th heavy neutrino. The off-diagonal products play an important role in determining the lepton asymmetry. They are explicitly given in the model of Sec. IV as:

$$\begin{aligned} (h^\dagger h)_{12} &\approx -s_{12} c_{12} s_{23}^2 e^{i(\tau_1 - \tau_2)}/(r')^2, \\ (h^\dagger h)_{13} &\approx s_{12} s_{23} c_{23} c_{13} e^{i(\tau_1 - \tau_3)}/(r')^2, \\ (h^\dagger h)_{23} &\approx -s_{23} c_{12} c_{23} c_{13} e^{i(\tau_2 - \tau_3)}/(r')^2, \\ (h^\dagger h)_{ij} &= (h^\dagger h)_{ji}^*, \end{aligned} \qquad (45)$$

where the *CP* phases τ_i depend on the choice of m_3 as explained in Sec. IV. Numerically, one has $(h^\dagger h)_{12} \approx -2.29 \times 10^{-2} \exp[i(\tau_1 - \tau_2)]$, $(h^\dagger h)_{13} \approx 2.74 \times 10^{-2} \exp[i(\tau_1 - \tau_3)]$ and $(h^\dagger h)_{23} \approx -4.18 \times 10^{-2} \exp[i(\tau_2 - \tau_3)]$. In arriving at these estimates from Eq. (16) we arbitrarily took all the signs of the trigonometric functions to be positive. This will not lead to any ambiguity since, for the application of interest, the off-diagonal products must be squared.

The lepton asymmetry ϵ_i, due to the decay of the *i*th heavy neutrino is defined as the ratio of decay widths:

$$\epsilon_i = \frac{\Gamma(N_i \to L + \Phi) - \Gamma(N_i \to \bar{L} + \bar{\Phi})}{\Gamma(N_i \to L + \Phi) + \Gamma(N_i \to \bar{L} + \bar{\Phi})}. \qquad (46)$$

In this formula $L + \Phi$ stands for all pairs of the types $e_j^- + \phi^+$ and $\nu_j + \bar{\phi}^0$. This is an effect which violates C and CP conservation, in agreement with the requirement of Sakharov [25]. To get a nonzero value one must include the interference between the tree diagram from Eq. (41) and the one loop diagrams (of both "self-energy" and "triangle" types). If the masses of the heavy neutrinos are well separated the result [14,30] is:

$$\epsilon_i = \frac{1}{8\pi} \sum_{j \neq i} \frac{Im[(h^\dagger h)_{ij}(h^\dagger h)_{ij}]}{(h^\dagger h)_{ii}} f(M_j^2/M_i^2),$$

$$f(x) = \sqrt{x}\left[\frac{1}{1-x} + 1 - (1+x)\ln(1+1/x)\right]. \quad (47)$$

Note that the contribution to the lepton asymmetry of the lightest heavy neutrino is expected to be the most important one for the final calculation of baryon asymmetry [26].

Now let us make numerical estimates for the lepton asymmetries when all CP violation is due to Majorana phases (Sec. IV). From Eq. (5) we relate the heavy neutrino masses to the light neutrino masses simply as:

$$M_i = cm_i, \quad (48)$$

where c is a real, positive constant. This equation has earlier been used [31] for the study of leptogenesis in the framework of a left-right symmetric model. It should be noted that renormalization group effects [32] will modify the exact proportionality of the light and heavy neutrino masses as well as the equality of the corresponding diagonalizing matrices. This should be taken into account for a more accurate treatment. In the model of Sec. IV the third neutrino is typically somewhat further away in mass from the other two, which are always relatively close. For example, in the type II situation, m_3 is the lightest of the light neutrino masses so M_3 will be the lightest of the heavy neutrino masses and the contribution to the lepton asymmetry is ϵ_3. Using Eqs. (47), (48), (45) and (30) we obtain:

$$\epsilon_3 \approx [-4.27 f([m_1/m_3]^2) \sin\mu_2 \\ + 9.94 f([m_2/m_3]^2) \sin\mu_1] \times 10^{-4}. \quad (49)$$

Notice that c has canceled out in this formula and also cancels out in the determination of the angles μ_i. Thus the lepton asymmetry given by this formula does not depend on the overall scale of the heavy neutrino masses.

In the type I case, the heavy neutrino spectrum consists of two nearly degenerate lighter states, (N_1, N_2) and a heavier state, N_3. For the corresponding asymmetries ϵ_1 and ϵ_2, the diagrams involving self-energy type corrections are enhanced since an internal heavy neutrino line will be close to its mass shell. The formulas [30] thus, for greater accuracy, involve the decay widths and we will approximate:

$$\epsilon_1 = \frac{Im[(h^\dagger h)_{12}(h^\dagger h)_{12}]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} \frac{(M_1^2 - M_2^2)M_1\Gamma_2}{(M_1^2 - M_2^2)^2 + M_1^2\Gamma_2^2},$$

$$\epsilon_2 = \frac{Im[(h^\dagger h)_{12}(h^\dagger h)_{12}]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} \frac{(M_1^2 - M_2^2)M_2\Gamma_1}{(M_1^2 - M_2^2)^2 + M_2^2\Gamma_1^2}. \quad (50)$$

Again, we may replace the heavy neutrino masses using Eq. (48) and note that the factor c cancels out. Inserting numbers, we obtain:

$$\epsilon_1 \approx -\frac{1.39 \times 10^{-3} \sin(\mu_1 + \mu_2)m_1 m_2(m_1^2 - m_2^2)}{(m_1^2 - m_2^2)^2 + 1.94 \times 10^{-6}m_1^2 m_2^2}$$

$$\epsilon_2 \approx -\frac{5.96 \times 10^{-4} \sin(\mu_1 + \mu_2)m_1 m_2(m_1^2 - m_2^2)}{(m_1^2 - m_2^2)^2 + 3.56 \times 10^{-7}m_1^2 m_2^2}. \quad (51)$$

The values of all these asymmetries for the range of possibilities are listed in the last column of Table I.

Furthermore it must be noted that, owing to the non-uniqueness of sign for all of Eqs. (30), reversing the signs of all the lepton asymmetries also yields a solution corresponding to our initial ansatz.

Although the scale of the heavy neutrinos has been seen to cancel out of the formulas (49) and (51) for the lepton asymmetries in favor of their ratios (which are the same as those of the light neutrinos in this model), there is nevertheless a consistency condition implied by the SO(10) motivation for the starting ansatz. This arises since Eq. (39) is not only the source of the lepton asymmetry but also provides the seesaw contribution to the light neutrino masses. For our motivation we assumed that this contribution was dominated by the first term of Eq. (4). To make a rough estimate of what this means we assume all matrices of the seesaw term to be diagonal. Then the value of $c^{1/2}$ defined in Eq. (48) should be greater than $m_i^U/(m_i r')$ in order that the first term of Eq. (4) be greater than the second term. In the case of the type I solution with $m_3 \approx 0.06$ eV shown in Table I, this implies that the lightest heavy neutrino should be heavier than about 2.6×10^{13} GeV. For the case of the type II solution with $m_3 \approx 7 \times 10^{-4}$ eV, the lightest heavy neutrino should be heavier than about 4.4×10^{15} GeV.

The goal of the baryogenesis problem is to understand the ratio $\eta_B = n_B/n_\gamma$, the net baryon number density divided by the photon density. Experimentally, this quantity is found [21], from the study of big bang nucleosynthesis, to be

$$\eta_B = (6.5 \pm 0.4) \times 10^{-10}. \quad (52)$$

To obtain nonzero η_B, it is not sufficient, as pointed out by Sakharov [25], just to have nonzero values of the lepton asymmetry ϵ_i, defined in Eq. (46). In addition, the CP violating decays of the heavy neutrinos must occur out of thermal equilibrium. A detailed treatment requires solution of the Boltzmann evolution equations for the system [33]. Here we shall make a rough estimate

which we use to draw what might be a fairly robust conclusion.

First, we should remark that the baryon asymmetry generated by the sphaleron mechanism would be about $-1/3$ [34] (for a review see [26]) of an initial lepton asymmetry. The lepton number violating decays of the ith heavy neutrino are usually roughly taken to be out of equilibrium if the decay rate Γ_i in Eq. (44) is less than the Hubble rate,

$$H \approx 1.7 g_*^{1/2} T^2 / M_P, \qquad (53)$$

where $g_* \approx 100$ is the number of effective light degrees of freedom at the leptogenesis scale, T is the temperature (corresponding to the mass of the decaying heavy neutrino) and $M_P \approx 1.22 \times 10^{19}$ GeV. In the present model this ratio takes the explicit form:

$$K_i = \frac{\Gamma_i}{H} \approx \frac{(h^\dagger h)_{ii} M_P}{427 M_i}, \qquad (54)$$

which is seen to be inversely proportional to M_i. The net baryon asymmetry is estimated as [26],

$$\eta_B \approx -\frac{7}{3 g_*} \sum_i \epsilon_i D_i, \qquad (55)$$

where the D_i are suppression factors to be obtained by numerical solution [33] of the Boltzmann equations. It is generally accepted that only the contributions of the lightest heavy neutrinos should not get washed out; thus we will set $D_i = 0$ for the heavier neutrinos. If $10 < K_i \lesssim 10^6$, the suppression factor is often approximated by the analytic form [35]

$$D_i \approx \frac{0.3}{K_i [\ln(K_i)]^{0.6}}. \qquad (56)$$

When $K_i < 1$, the suppression factor is expected to be of order unity if M_i is not too large. However, as M_i gets larger there is a sizeable washout effect [36].

Glancing at the last column in Table I and comparing with the experimental value of η_B in Eq. (52) as well as Eqs. (55) and (56) suggests that the values of η_B obtained for typical values of the assumed light neutrino mass parameter m_3 would be considerably larger than the experimental baryon asymmetry. However, we can expect to be able to obtain agreement with the experimental value since, as discussed in Sec. IV, we may make the Majorana *CP* phases as small as we like by continuously tuning the independently chosen variable, m_3 so that the triangle of mass vectors gets arbitrarily close to 1 of the two degenerate straight line cases which causes $\mathcal{A}$ in Eq. (33) to vanish. Thus the solutions of the model which would be consistent with the observed baryon asymmetry correspond to neutrino masses more or less close the real cases of either Eq. (18) or Eq. (19).

The qualitative points: (i) that in the present model the value of the free parameter, m_3 can always be tuned to be arbitrarily close to its values for the two real solutions (so that the *CP* violation and hence leptogenesis strength becomes as small as desired) and (ii) that the characteristic lepton asymmetries, ϵ_i for values of m_3 away from these two real solutions are rather large, comprise the main result of our discussion of the application of the $\mathrm{Tr}(M_\nu) = 0$ ansatz to the baryogenesis problem. These points lead to the expectation that the physical value of m_3 is likely to be close to 1 of the two values in Eq. (18) or Eq. (19) and that this conclusion might persist even when our simplifications are not made. A more accurate treatment would include the features: (a) effect of nontrivial charged lepton mixing matrix, (b) renormalization group induced deviations from the clone treatment of the heavy Majorana neutrinos and (c) full integration of the Boltzmann evolution equations. Even though our main conclusion is a qualitative one, it seems nevertheless an interesting exercise to find what values of light and heavy neutrino masses correspond to the correct order of magnitude of the observed baryon asymmetry. Note that the seeming great accuracy of the entries in Table I is not meant for precise comparison with experiment, but for comparison of the results of different m_3 choices with each other.

Specifically, consider the tuned type I solution in Table I with $m_3 \approx 0.05927$ eV. We noted in the discussion after Eq. (51) that this would correspond to heavy neutrino clone masses (M_1, M_2, M_3) greater than about $(2.60, 2.70, 5.27) \times 10^{13}$ GeV, respectively. We assume that the two lighter neutrinos are the important ones and set $D_3 = 0$. The ratios (K_1, K_2) defined in Eq. (54) would then be less than about $(16.5, 37.1)$ and would result in suppression factors (D_1, D_2) greater than $(0.010, 0.0037)$. Using Eq. (55) and Table I for ϵ_1 and ϵ_2 then gives $|\eta_B| = 5.4 \times 10^{-10}$, close to the experimental value in Eq. (52). This can be adjusted by further tuning m_3 or to some extent by varying the overall mass scale of (M_1, M_2, M_3).

For the type II case, first consider the solution in Table I with $m_3 \approx 0.0007$ eV. As discussed before, this would correspond to heavy neutrino clone masses (M_1, M_2, M_3) greater than about $(320\,320\,4.4) \times 10^{15}$ GeV. In this case, M_3 is the lightest of the three heavy neutrinos and is assumed to be the relevant one. We thus set $D_1 = D_2 = 0$. The ratio K_3 given in Eq. (54) is then about 0.45 and indicates that the lightest heavy neutrino is, as desired, decaying out of equilibrium. However, because its mass is considerably higher than that of the type I case just discussed, there is more wash out [36], $D_3 \approx 3.5 \times 10^{-5}$. Reading ϵ_3 from Table I then gives $\eta_B \approx 5 \times 10^{-13}$, about 3 orders of magnitude too small. Thus we must raise the value of m_3 a bit. Backing off a little to the case $m_3 \approx 0.005$ eV in Table I increases the value of ϵ_3 and also allows us [in line with the dominance of the first term in Eq. (4)] to choose the lower bound of M_3 to

be smaller, around 6×10^{14} GeV. This results in an estimate $|\eta_B| \approx 16 \times 10^{-10}$, which is the correct order of magnitude. One might wonder whether the contributions to η_B from ϵ_1 and ϵ_2 are completely washed out in a case like the present. However, even if they were dominant, it would just require us to tune more closely toward small m_3.

Thus, if the model of CP violation with just the Majorana phases is correct, the magnitude of the baryon to photon ratio can be understood when either the sum of the three light neutrino masses is about 0.118 eV and $|m_{ee}| = 0.030$ eV (type I) or the sum of the three light neutrino masses is about 0.107 eV and $|m_{ee}| = 0.021$ eV (type II). In both cases the CP violating Majorana phases are extremely small. That might suggest a possible model in which a small CP violating perturbation due to some separate effect modifies an otherwise CP conserving lepton sector.

We can also calculate the baryon to photon ratio in the model of Sec. III, where δ is the only CP violating phase. There we noted that the only possible choices of δ consistent with our ansatz satisfy $\sin^2 \delta = 1$. Then we have the type I solution for light neutrino masses given in Eq. (25) and the type II solution given in Eq. (26). The corresponding CP violation factors are now (to first order in the small parameter s_{13}) for the type I case:

$$Im[(h^\dagger h)_{12}(h^\dagger h)_{12}] \approx \frac{s_{13} \sin\delta \sin(2\theta_{12})}{(r')^4} c_{23} s_{23}^3,$$

$$Im[(h^\dagger h)_{13}(h^\dagger h)_{13}] \approx -\frac{s_{13} \sin\delta \sin(2\theta_{12})}{(r')^4} s_{23} c_{23}^3, \quad (57)$$

$$Im[(h^\dagger h)_{23}(h^\dagger h)_{23}] \approx -Im[(h^\dagger h)_{13}(h^\dagger h)_{13}],$$

and for the type II case:

$$Im[(h^\dagger h)_{12}(h^\dagger h)_{12}] \approx -\frac{s_{13} \sin\delta \sin(2\theta_{12})}{(r')^4} c_{23} s_{23}^3,$$

$$Im[(h^\dagger h)_{13}(h^\dagger h)_{13}] \approx \frac{s_{13} \sin\delta \sin(2\theta_{12})}{(r')^4} s_{23} c_{23}^3, \quad (58)$$

$$Im[(h^\dagger h)_{23}(h^\dagger h)_{23}] \approx Im[(h^\dagger h)_{13}(h^\dagger h)_{13}].$$

As in the cases where only the Majorana phases contribute to the CP violation, the predicted lepton asymmetries, ϵ_i will typically lead to a value of the baryon to photon ratio much larger than the experimental one. In the present case it is not possible to fine tune δ. The only possibility would be to fine tune s_{13} to an extremely small value. This seems more artificial since s_{13} is not required to vanish in the CP conserving situation. In any event, an experimental measurement of nonzero s_{13} would, practically speaking, rule out this case as a candidate for leptogenesis.

VI. DISCUSSION AND SUMMARY

In this paper, we investigated an ansatz which correlates information about the four quantities in the light neutrino sector which are not yet known from experiment; namely, the absolute mass of any particular neutrino, the "conventional" CP violation phase and the two Majorana phases. Of course, with input from analyses of neutrino oscillation experiments, the masses of the other two neutrinos can be found, up to a discrete ambiguity, if the mass of one is specified. The results of the present paper can be used for calculating many quantities of experimental interest like the neutrinoless double beta decay amplitude factor m_{ee} (presented in Sec. IV) and various lepton number violating decays.

The ansatz is not completely predictive, unless some assumptions are made. We first reviewed the case of assumed CP conservation (where just the three neutrino masses are obtained). Then we showed that if only the conventional CP violation phase is assumed to be nonzero, its value is fixed by the ansatz to be maximal. A possibly more interesting case appears if we assume that only the two Majorana phases are nonzero. This enables us to scan the limited allowed range of assumed neutrino mass, m_3 (say) and find the other two neutrino masses as well as the two Majorana phases for each value of m_3. The result seems to cut through a "cross section" of interesting possibilities which are described in a simple way. The still more complicated case without setting any of the three CP phases to zero gives a two parameter family of solutions and will be treated elsewhere. Another (common) assumption we made for a first analysis is that the measured lepton mixing matrix is dominated by the neutrino factor. This is consistent with the finding in recent years that the mixing in the neutrino sector is apparently much larger than the mixing in the quark sector (which in models is usually relatively small and similar to that of the charged lepton factor).

It seems relevant to discuss briefly the status of the motivations for the complementary ansatz we are using. One motivation, based on a loop mechanism for generating neutrino masses was discussed recently by He and Zee [9]. Our motivation [7] was based on the grand unification group SO(10). This group is well known to have the elegant feature that it accommodates one generation of elementary fermions as well as an extra (now desired) neutrino field in its fundamental spinor irreducible representation. Naturally, there are many possibilities for doing a detailed calculation using this group. One may ask whether it should be regarded as being derived from a superstring theory, whether it should be supersymmetric, whether the symmetry breakdown should be dynamical, whether the symmetry breakdown should be induced by Higgs fields and if so what kind and how many, etc.? We focus, in our

motivation, on the conventional possibility of using Higgs fields since it seems almost kinematical now (although since no Higgs field has yet been seen one should keep an open mind). Of course, there have been many interesting treatments along these lines [37]. Our ansatz is suggested by a relation involving only the neutrino mass matrix which might be true (or at least approximately correct) in a large number of models. In SO(10), tree level masses from a renormalizable Lagrangian can be obtained by using any number of Higgs mesons belonging to the 10, 120, and 126 dimensional representations. However, examining the form of the predicted mass matrices shows that the following fairly general relation [38] holds:

$$\mathrm{Tr}(M^D - rM^E) \propto \mathrm{Tr}(M_L), \qquad (59)$$

for any number of 10's and 120's but only a single 126 present. Here M^D and M^E are, respectively, the prediagonal mass matrices of the charge $-1/3$ quarks and charge -1 leptons while $r \approx 3$, as previously mentioned. M_L, which arises from the 126 Higgs field Yukawa couplings, is the non-seesaw part of the light neutrino mass matrix which appears in Eq. (4). Taking traces cancels the contributions (antisymmetric matrices) of any 120 Higgs multiplets to the left-hand side. Then, assuming the transformations which bring M^D and M^E to diagonal form to be roughly close to the identity we observe that the left-hand side is approximately equal to $m_b - rm_\tau$, which is about zero. In fact this is a characteristic prediction of grand unification. In turn the right-hand side gives us the starting ansatz when it is assumed that the non-seesaw term dominates in Eq. (4). Of course, if this domination is to hold the masses of the heavy neutrinos should not be too low. The present paper is in effect exploring the range of possibilities which exist when these assumptions are made in SO(10) models. An interesting question is whether this kind of limit or the pure seesaw limit gives a better description of nature, even if both terms are actually required.

We remark that SO(10) also gives another similar relation,

$$\mathrm{Tr}(M^U - r'M_D) \propto \mathrm{Tr}(M_L), \qquad (60)$$

when only one 126 Higgs field exists. Here M^U and M_D respectively denote the prediagonal charge 2/3 quark mass matrix and the prediagonal neutrino ''Dirac'' matrix connecting the heavy and light neutrino fields.

An intriguing way to learn more about *CP* violation in the lepton sector is the study of the leptogenesis mechanism of baryogenesis. We saw that the treatment of this process simplifies when one adopts the present SO(10) motivation. Then the light neutrino mass matrix M_ν and M_L are approximately equal and proportional (due to the assumption of only one 126 field in the theory) to the heavy neutrino mass matrix M_H. The only free parameter for the heavy neutrinos is their overall mass scale and this should not be too small to preserve non-seesaw dominance. We showed in Sec. V that it is easy to estimate the lepton asymmetry parameters ϵ_i for a ''panorama'' of values of the independent variable m_3 since they are actually independent of the overall heavy neutrino mass scale. As far as the resulting baryon to photon ratio, η_B [parameterized in Eq. (55)] is concerned, the typical values of the ϵ_i give η_B much greater than the experimental one for suppression factors D_i of order unity. We observed that if the suppression factors are not too small one can therefore always choose a value of m_3 close enough to one of the two essentially different *CP* conserving solutions so that the Majorana phases are small enough to get experimental agreement for η_B. Using estimates of the suppression factors taken from other earlier studies, we noted that this conclusion seems reasonable. Of course the study of the suppression factors by solving the Boltzmann evolution equations is an important topic which involves many subtleties and would repay further work in the present model. Finally, the possible indication of very small *CP* phases might suggest a model in which the *CP* violation in the lepton sector has a separate identifiable source.

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