Different contributions in $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays

Ayse Kucukarslan*
Middle East Technical University, Physics Department, 06531 Ankara, Turkey

Saime Solmaz†
Balikesir University, Physics Department, 10100 Balikesir, Turkey
(Received 21 May 2004; published 8 September 2004)

We examine the radiative $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays in a phenomenological framework. We consider the vector meson dominance mechanism, chiral loops, intermediate a_0-meson and $\rho - \omega$ mixing. We find the values of the decay width coming from the different amplitudes and compare the results with other studies. We observe that the a_0-meson intermediate state is very important in the case of the $\rho \to \pi^0\eta\gamma$ decay and small in the other case for which vector meson dominance contribution is dominant.

DOI: 10.1103/PhysRevD.70.053004 PACS numbers: 12.20.Ds, 12.40.Vv, 13.20.Jf, 13.40.Hq

I. INTRODUCTION

Radiative decays of low-mass vector mesons into a single photon and a pair of neutral pseudoscalars have attracted continuous attention. The studies of such decays have been a case for tests of vector meson dominance (VMD), through the sequential mechanism $V \to PV \to PP\gamma$ [1,2]. They also offer the possibility of obtaining information on the nature of low-mass scalar mesons. In particular, the nature and the quark substructure of the two scalar mesons, isoscalar $f_0(980)$ and isovector $a_0(980)$, have not been established yet. Several proposals have been made about the nature of these states: $q\bar{q}$ states in quark model [3], $K\bar{K}$ molecules [4] or multiquark $q^2\bar{q}^2$ states [5,6].

Theoretical study of ω and ρ meson decays into a single photon and pseudoscalar π^0 and η mesons as well as other radiative vector meson decays was initiated by Fajfer and Oakes [7]. They described these decays by the gauged Wess-Zumino terms in a low-energy effective Lagrangian and calculated the branching ratios for these decays in which scalar meson contributions were neglected. In their study, they obtained the following branching ratios for $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays: $BR(\omega \to \pi^0\eta\gamma) = 6.26 \times 10^{-6}$, $BR(\rho \to \pi^0\eta\gamma) = 3.98 \times 10^{-6}$. The contributions of intermediate vector mesons to the decays $V^0 \to P^0P^0\gamma$ were later considered by Bramon *et al.* [2] using standard Lagrangians obeying SU(3) symmetry. Their results for the decay rates and the branching ratios of the decays $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ were $\Gamma(\omega \to \pi^0\eta\gamma) = 1.39$ eV, $BR(\omega \to \pi^0\eta\gamma) = 1.6 \times 10^{-7}$, and $\Gamma(\rho \to \pi^0\eta\gamma) = 0.061$ eV, $BR(\rho \to \pi^0\eta\gamma) = 4 \times 10^{-10}$. Their results were not compatible with those by Fajfer and Oakes [7] even if the initial expressions for the Lagrangians were the same. Later, Bramon *et al.* [8] studied these decays within the framework of chiral effective Lagrangians enlarged to include on-shell vector mesons using chiral perturbation theory, and they calculated the branching ratios for $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays as well as other radiative vector meson decays of the type $V^0 \to P^0P^0\gamma$ at the one loop level. They showed that the one loop contributions are finite and to this order no counterterms are required. In this approach, the decays $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ proceed through the intermediate vector meson states and the charged kaon-loops. They obtained the contributions of charged kaon-loops to the decay rates of these decays as $\Gamma(\omega \to \pi^0\eta\gamma)_K = 0.013$ eV and $\Gamma(\rho \to \pi^0\eta\gamma)_K = 0.006$ eV with the pion-loop contributions vanishing in the good isospin limit. Their analysis showed that kaon-loop contributions are one or 2 orders of magnitude smaller than the VMD contributions and the dominant pion-loops are forbidden in these decays due to isospin symmetry. These decays were also investigated by Prades [9]. Using chiral Lagrangians and the extended Nambu-Jona-Lasinio model, he calculated the branching ratios for these decays. The branching ratios for the radiative $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays were found as $BR(\omega \to \pi^0\eta\gamma) = 8.3 \times 10^{-8}$ and $BR(\rho \to \pi^0\eta\gamma) = 2.0 \times 10^{-10}$.

Furthermore, the radiative $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays were also considered by Gokalp *et al.* [10] taking into account the contributions of intermediate a_0-meson and intermediate vector meson states. The decay rates and the branching ratios for $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays that they obtained were $\Gamma(\omega \to \pi^0\eta\gamma) = 1.62$ eV, $BR(\omega \to \pi^0\eta\gamma) = 1.92 \times 10^{-7}$ and $\Gamma(\rho \to \pi^0\eta\gamma) = 0.43$ eV, $BR(\rho \to \pi^0\eta\gamma) = 2.9 \times 10^{-9}$. They concluded that although a_0-meson intermediate state amplitude makes a small contribution to $\omega \to \pi^0\eta\gamma$ decay, it makes a substantial contribution to $\rho \to \pi^0\eta\gamma$ decay. Recently, the radiative decays of the ρ and ω mesons into two neutral mesons, $\pi^0\pi^0$ and $\pi^0\eta$, including the mecha-

*Electronic address: akucukarslan@metu.edu.tr
†Electronic address: skerman@balikesir.edu.tr

TABLE I. The branching ratios of the $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays in the literature.

WORK	$\omega \to \pi^0\eta\gamma$	$\rho \to \pi^0\eta\gamma$
[7]	6.26×10^{-6}	3.98×10^{-6}
[2]	1.6×10^{-7}	4.0×10^{-10}
[8]	1.6×10^{-7}	4.0×10^{-10}
[9]	8.3×10^{-8}	2.0×10^{-10}
[10]	1.92×10^{-7}	2.9×10^{-9}
[11]	3.3×10^{-7}	7.5×10^{-10}
this work	5.72×10^{-7}	2.3×10^{-8}
experiment	$<3.3 \times 10^{-5}$	$\cdots$

nisms of sequential vector meson decay, $\rho - \omega$ mixing, and chiral loops, have been studied by Palomar *et al.* [11]. They obtained the branching ratios for the decays $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ as $BR(\omega \to \pi^0\eta\gamma) = 3.3 \times 10^{-7}$, $BR(\rho \to \pi^0\eta\gamma) = 7.5 \times 10^{-10}$ and noted that the dominant contribution is the one corresponding to the sequential mechanism for both cases. Indeed, in their study the $\rho - \omega$ mixing was found non-negligible for $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays.

We collect the results of the above analysis and present them in a chronological order in Table I, which are to be compared with our results in the conclusion part.

Theoretically, the effects of the $\rho - \omega$ mixing in the $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays have not been studied extensively up to now. One of the rare studies of these decays was by Palomar *et al.* [11]. In their study the chiral loops were obtained using elements of $U\chi PT$ which lead to the excitation of the scalar resonances without the need to include them explicitly in the formalism. However, in our work the effect of $a_0(980)$ meson in the decay mechanisms of $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays is included as resulting from a_0-pole intermediate state. We study these decays within the framework of a phenomenological approach in which the contributions of intermediate vector meson states, chiral loops, $\rho - \omega$ mixing and of a scalar $a_0(980)$ intermediate meson state are considered. Expressions related with branching ratios for $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays are presented in conclusion.

II. VMD CONTRIBUTIONS

In our calculation we use the Feynman diagrams, corresponding to this mechanism, shown in Fig. 1(a) for $\omega \to \pi^0\eta\gamma$ decay and in Fig. 2(a) for $\rho \to \pi^0\eta\gamma$ decay. The Lagrangian for the $\omega\rho\pi$-vertex takes the following form:

$$\mathcal{L}^{\text{eff}}_{\omega\rho\pi} = g_{\omega\rho\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\partial_\alpha\vec{\rho}_\beta \cdot \vec{\pi}. \quad (1)$$

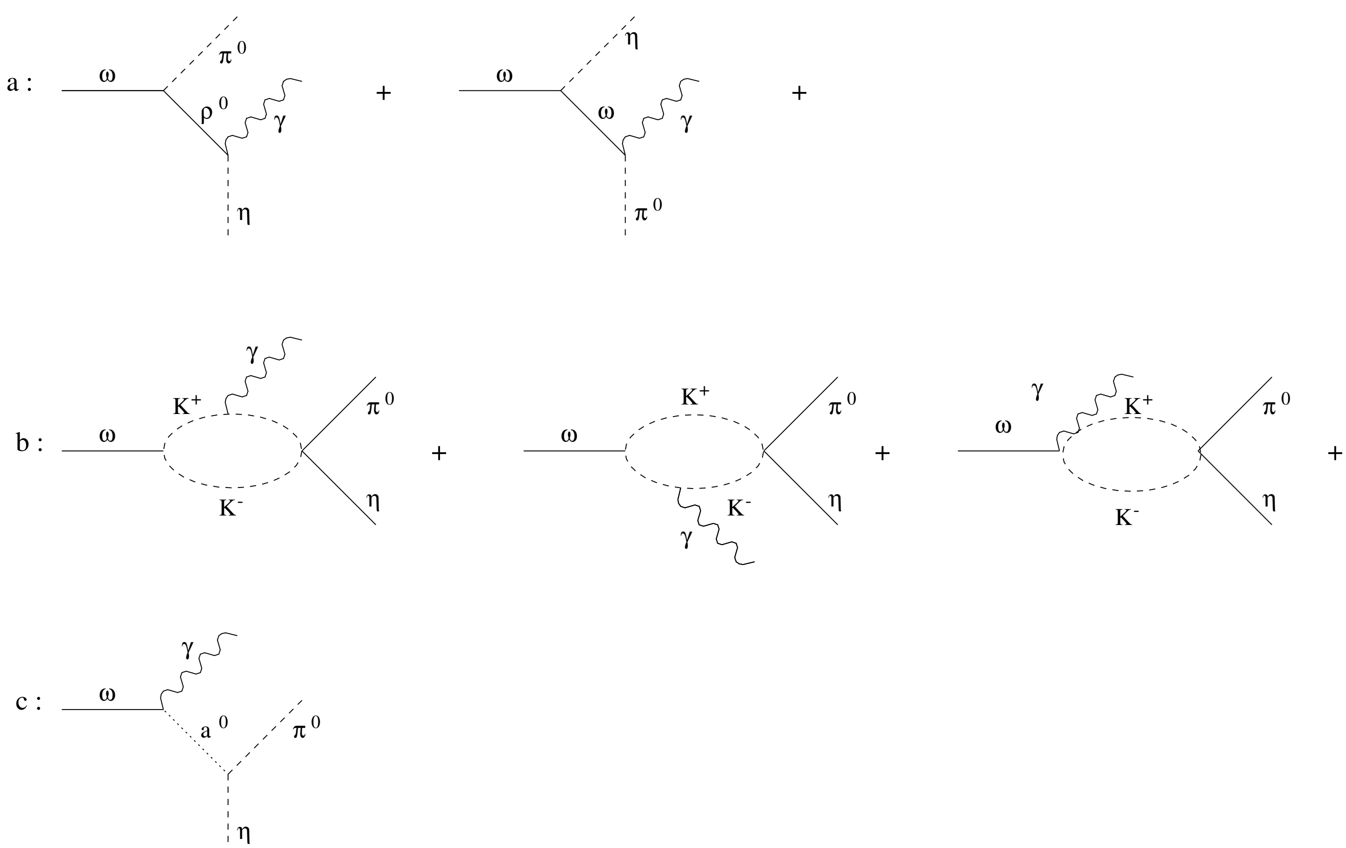

FIG. 1. Feynman diagrams for the decay $\omega \to \pi^0\eta\gamma$.

Since the coupling constant $g_{\omega\rho\pi}$ cannot be determined directly from experiments, theoretically it is extracted from some models and obtains values between 11 GeV^{-1} and 16 GeV^{-1}. We use the value as 15 GeV^{-1} for this coupling constant in this work. The $V\varphi\gamma$-vertices come from the Lagrangians

$$\mathcal{L}^{\rm eff}_{V\varphi\gamma} = g_{V\varphi\gamma}\epsilon^{\mu\nu\alpha\beta}\partial_\mu V_\nu \partial_\alpha A_\beta \varphi, \tag{2}$$

where V_ν is the vector meson field ω_ν or ρ_ν, φ is the pseudoscalar field π^0 or η, and A_β is the photon field. Using the experimental partial widths for $\Gamma(V \to \pi^0\gamma)$ and $\Gamma(V \to \eta\gamma)$ [12], we determine the coupling constants as $g_{\rho\pi\gamma} = 0.696$, $g_{\rho\eta\gamma} = 1.171$, $g_{\omega\pi\gamma} = 1.821$, and $g_{\omega\eta\gamma} = 0.400$. For the $VV\eta$-vertex we use the following effective Lagrangian:

$$\mathcal{L}^{\rm eff}_{VV\eta} = g_{VV\eta}\epsilon^{\mu\nu\alpha\beta}\partial_\mu V_\nu V_\alpha \partial_\beta \eta. \tag{3}$$

Utilizing the experimental decay widths of the $\omega \to 3\pi$ and $\phi \to 3\pi$ decays, Klingl *et al.* [13] obtained the coupling constant $g_{VV\eta}$ as $g_{\omega\omega\eta} = g_{\rho\rho\eta} = 2.624$ GeV^{-1}.

We also use the following momentum dependent width, as discussed by O'Connell *et al.* [14] for $V = \rho$ or ω meson:

$$\Gamma_V(q^2) = \Gamma_V \frac{M_V}{\sqrt{q^2}}\left(\frac{q^2 - 4M_\pi^2}{M_V^2 - 4M_\pi^2}\right)^{3/2}\theta(q^2 - 4M_\pi^2). \tag{4}$$

Since Γ_ω is small, this effect is negligible for $\rho \to \pi^0\eta\gamma$ decay.

III. CHIRAL LOOP CONTRIBUTIONS

Apart from the VMD contributions, there is another mechanism based on the chiral kaon-loop whose contribution is quite small in the two cases, $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$. In spite of this, we add the kaon-loop contribution for completeness in our calculation. This mechanism has been studied in [8,9,11] for these decays, and here we follow closely results of these studies.

The one loop Feynman diagrams for $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ are of the form shown in Fig. 1(b) and Fig. 2(b), respectively. For the contribution of these diagrams we use the amplitude given in Ref. [8] derived using chiral perturbation theory. The amplitude is

$$\begin{aligned}\mathcal{A}(V \to \pi^0\eta\gamma)_K = &-\frac{eg}{6\sqrt{3}\pi^2 f_\pi^2}(3p^2 - 6k\cdot p - 4M_K^2)\\ &\times[(\epsilon\cdot u)(k\cdot p) - (\epsilon\cdot p)(k\cdot u)]\\ &\times\frac{1}{M_K^2}I(a,b)\end{aligned} \tag{5}$$

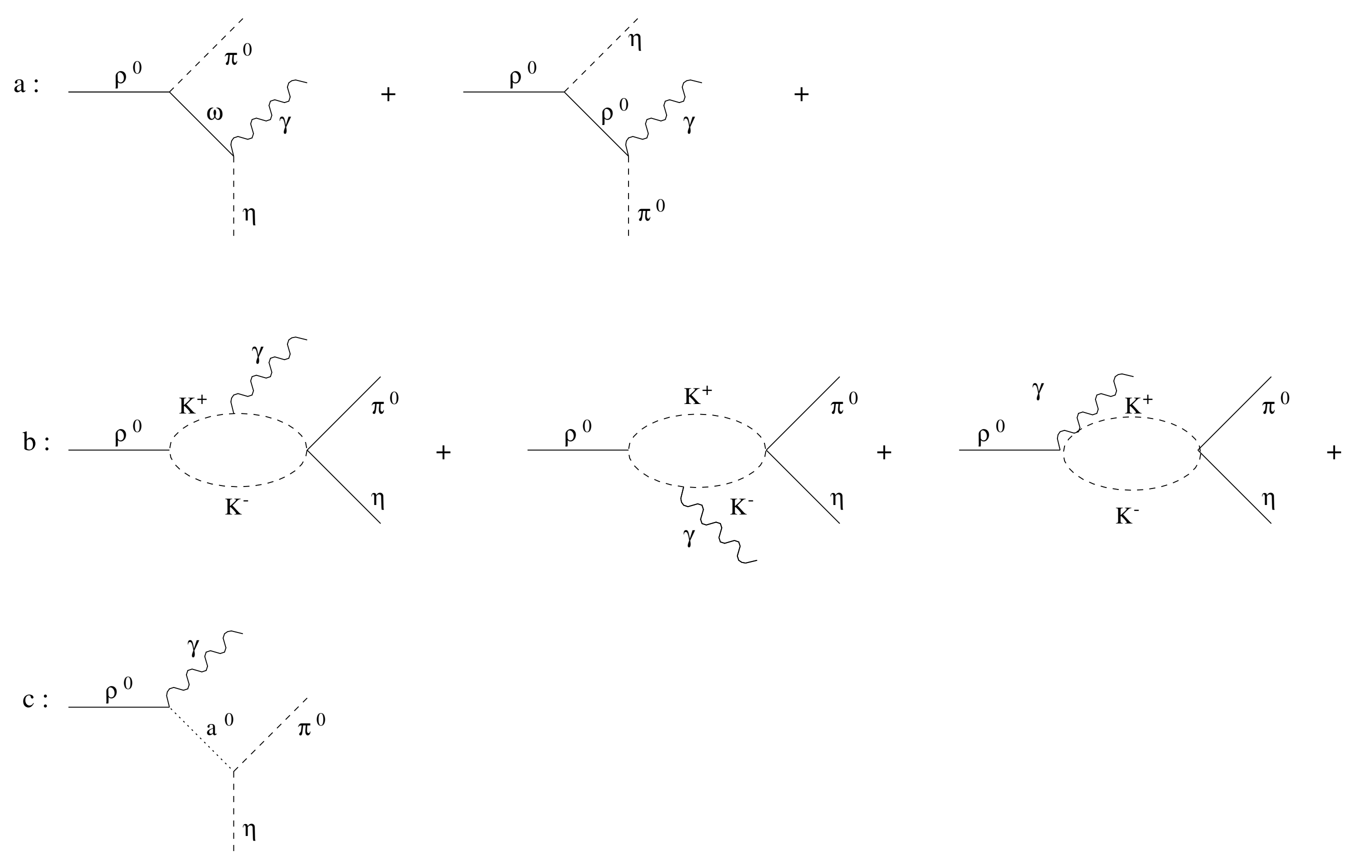

FIG. 2. Feynman diagrams for the decay $\rho^0 \to \pi^0\eta\gamma$.

where $I(a, b)$ is the loop function defined as

$$I(a, b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2}\left[f\left(\frac{1}{b}\right) - f\left(\frac{1}{a}\right)\right] + \frac{a}{(a-b)^2}\left[g\left(\frac{1}{b}\right) - g\left(\frac{1}{a}\right)\right] \tag{6}$$

where $a = M_V^2/M_K^2$, $b = (p-k)^2/M_K^2$, $g \simeq 4.2$, and $f_\pi = 132$ MeV. $f(x)$ and $g(x)$ are defined in [15] in which they were evaluated by Lucio and Pestieau.

We will not consider the pion-loops for these decays because it does not contribute to good isospin limit.

IV. SCALAR MESON CONTRIBUTIONS

We add a_0-meson as an intermediate state to the decay mechanism of these decays. The scalar a_0-meson contribution was studied before [10,16] by Gokalp *et al.* within the framework of a phenomenological approach for vector meson decays.

We use the Feynman diagrams for $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays as shown in Fig. 1(c) and Fig. 2(c), respectively. The vertices, $Va_0\gamma$ and $a_0\pi^0\eta$, come from the Lagrangians

$$\mathcal{L}^{\text{eff}}_{Va_0\gamma} = g_{Va_0\gamma}(\partial^\alpha V^\beta \partial_\alpha A_\beta - \partial^\alpha V^\beta \partial_\beta A_\alpha)a_0 \tag{7}$$

$$\mathcal{L}^{\text{eff}}_{a_0\pi\eta} = g_{a_0\pi\eta}\vec{\pi}\cdot\vec{a}_0\eta, \tag{8}$$

where we also define the coupling constants $g_{Va_0\gamma}$ and $g_{a_0\pi\eta}$. Since there are no direct experimental results for $Va_0\gamma$-vertex, we use the values for the coupling constants $g_{Va_0\gamma}$ as $g_{\rho a_0\gamma} = (1.69 \pm 0.39)\ \text{GeV}^{-1}$ and $g_{\omega a_0\gamma} = (0.58 \pm 0.13)\ \text{GeV}^{-1}$, which was determined using the QCD sum rule method in [17]. The decay rate for the $a_0 \to \pi^0\eta$ decay resulting from the above Lagrangian is

$$\Gamma(a_0 \to \pi^0\eta) = \frac{g^2_{a_0\pi\eta}}{16\pi M_{a_0}} \times \sqrt{\left[1 - \frac{(M_{\pi^0} + M_\eta)^2}{M^2_{a_0}}\right]\left[1 - \frac{(M_{\pi^0} - M_\eta)^2}{M^2_{a_0}}\right]}. \tag{9}$$

Using the value $\Gamma_{a_0} = (0.069 \pm 0.011)$ GeV [18], we obtain the coupling constant $g_{a_0\pi\eta}$ as $g_{a_0\pi\eta} = (2.32 \pm 0.18)$ GeV. We use energy-dependent width for the intermediate a_0-meson in the propagators, which leads to an increase of the decay width when compared to the calculation done with a constant width. The energy-dependent width for a_0-meson is

$$\Gamma_{a_0}(q^2) = \Gamma_{a_0}\frac{M^3_{a_0}}{(q^2)^{3/2}} \times \sqrt{\frac{[q^2 - (M_{\pi^0} + M_\eta)^2][q^2 - (M_{\pi^0} - M_\eta)^2]}{[M^2_{a_0} - (M_{\pi^0} + M_\eta)^2][M^2_{a_0} - (M_{\pi^0} - M_\eta)^2]}}. \tag{10}$$

V. THE EFFECTS OF $\rho - \omega$ MIXING

In addition to the VMD contribution given in Section II, the consideration of isospin violation effects enables one to study the mixing of the ρ and ω resonances. Isospin violation effects stem from quark mass differences and electromagnetic corrections. This mixing has been extracted from an analysis of $e^+e^- \to \pi^+\pi^-$ in the $\rho - \omega$ interference region. Guetta and Singer [19] first considered the $\rho - \omega$ mixing in the vector meson decays, and then it was used by Bramon *et al.* [20] and Palomar *et al.* [11]. New contribution coming from the $\rho - \omega$ mixing is to add to the intermediate vector meson diagrams of Fig. 1(a) and Fig. 2(a) for $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$, respectively, expressing the mixing between the isospin states which is described by adding to the effective Lagrangian a term $\mathcal{L} = \prod_{\rho\omega}\omega_\mu\rho_\mu$ leading to the physical states $\rho = \rho(I=1) + \varepsilon\omega(I=0)$ and $\omega = \omega(I=0) - \varepsilon\rho(I=1)$. Then, the full amplitude is written as

$$\mathcal{A}(V \to \pi^0\eta\gamma) = \mathcal{A}_0(V \to \pi^0\eta\gamma) + \varepsilon\widetilde{\mathcal{A}}(V' \to \pi^0\eta\gamma), \tag{11}$$

where $\mathcal{A}_0$ and $\widetilde{\mathcal{A}}$ include the contributions coming from the different terms, ε is the mixing parameter

$$\varepsilon \equiv \frac{\prod_{\rho\omega}}{M_V^2 - M_{V'}^2 - i(M_V\Gamma_V - M_{V'}\Gamma_{V'})} \tag{12}$$

and it is obtained as $\varepsilon = (-0.006 + i0.036)$ using the experimental values for M_V and Γ_V and $\prod_{\rho\omega} = (-3811 \pm 370)\ \text{MeV}^2$, which was determined by O'Connell *et al.* [14].

Another effect of the mixing is modifying the propagator in $\mathcal{A}_0$ as follows:

$$\frac{1}{D_V(s)} \longrightarrow \frac{1}{D_V(s)}\left(1 + \frac{g_{V'\pi\gamma}}{g_{V\pi\gamma}}\frac{\prod_{\rho\omega}}{D_{V'}(s)}\right) \tag{13}$$

with $D_V(s) = s - M_V^2 + iM_V\Gamma_V$.

We express the invariant amplitude $\mathcal{A}(E_\gamma, E_\pi)$ using the $\rho - \omega$ mixing for the $\omega \to \pi^0\eta\gamma$ as $\mathcal{A}(\omega \to \pi^0\eta\gamma) = \mathcal{A}^0(\omega \to \pi^0\eta\gamma) + \varepsilon\widetilde{\mathcal{A}}(\rho \to \pi^0\eta\gamma)$ where $\mathcal{A}^0$ and $\widetilde{\mathcal{A}}$ are the invariant amplitudes coming from

TABLE II. The decay widths coming from the different contributions to the $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays.

Γ (eV)	VMD	VMD + $(\rho - \omega)$ mixing	K-loop	a_0 meson	Total
$\omega \to \pi^0\eta\gamma$	1.51	1.54	0.0133	0.70	4.83
$\rho \to \pi^0\eta\gamma$	0.08	0.08	0.006	3.43	3.44

TABLE III. The branching ratios coming from the different contributions to the $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays.

BR	VMD	VMD + $(\rho - \omega)$ mixing	K-loop	a_0 meson	Total
$\omega \to \pi^0\eta\gamma$	1.79×10^{-7}	1.82×10^{-7}	1.6×10^{-9}	8.25×10^{-8}	5.72×10^{-7}
$\rho \to \pi^0\eta\gamma$	5.27×10^{-10}	5.27×10^{-10}	4.0×10^{-11}	2.3×10^{-8}	2.3×10^{-8}

the diagrams in Fig. 1(a)–1(c) and in Fig. 2. For the $\rho \to \pi^0\eta\gamma$ decay, we follow the same procedure and we can write the full amplitude as $\mathcal{A}(\rho \to \pi^0\eta\gamma) = \mathcal{A}^0(\rho \to \pi^0\eta\gamma) - \varepsilon\widetilde{\mathcal{A}}(\omega \to \pi^0\eta\gamma)$.

In our calculation, the decay width for these decays can be obtained by integration:

$$\Gamma(V \to \pi^0\eta\gamma) = \int_{E_{\gamma,\min}}^{E_{\gamma,\max}} dE_\gamma \int_{E_{\pi,\min}}^{E_{\pi,\max}} dE_\pi \frac{d\Gamma}{dE_\gamma dE_\pi}. \tag{14}$$

The minimum photon energy is $E_{\gamma,\min} = 0$ and the maximum photon energy is given as $E_{\gamma,\max} = [M_V^2 - (M_\pi + M_\eta)^2]/2M_V$. The minimum and maximum values for pion energy E_π are given by

$$\frac{1}{2(2E_\gamma M_V - M_V^2)}\{-2E_\gamma^2 M_V - M_V(M_V^2 + M_\pi^2 - M_\eta^2) + E_\gamma(3M_V^2 + M_\pi^2 - M_\eta^2) \pm E_\gamma[4E_\gamma^2 M_V^2 + M_V^4 + (M_\pi^2 - M_\eta^2)^2 - 2M_V^2(M_\pi^2 + M_\eta^2) + 4E_\gamma M_V(-M_V^2 + M_\pi^2 + M_\eta^2)]^{1/2}\}. \tag{15}$$

The differential decay probability of $V^0 \to \pi^0\eta\gamma$ decay for an unpolarized V^0-meson ($V^0 = \omega, \rho^0$) at rest is then given in terms of the invariant amplitude $A(E_\gamma, E_\pi)$:

$$\frac{d\Gamma}{dE_\gamma dE_\pi} = \frac{1}{(2\pi)^3}\frac{1}{8M_V}|\mathcal{A}|^2 \tag{16}$$

where E_γ and E_π are the photon and pion energies, respectively. We perform an average over the spin states of the vector meson and a sum over the polarization states of the photon.

VI. RESULTS AND CONCLUSION

The contributions of different amplitudes to the decay rate and the branching ratio of the decays, $\omega \to \pi^0\eta\gamma$, $\rho \to \pi^0\eta\gamma$, are shown in Table II and Table III, respectively. We consider the intermediate vector meson, chiral loops, intermediate a_0-meson, and $\rho - \omega$ mixing. The dominant contribution is the one corresponding to the vector meson dominance mechanism in two cases except for the $\rho \to \pi^0\eta\gamma$ decay. On the contrary, intermediate a_0-meson is the dominant contribution of the $\rho \to \pi^0\eta\gamma$ decay.

The resulting photon spectra for the decay rate is plotted in Fig. 3 for the decay $\omega \to \pi^0\eta\gamma$ and in Fig. 4 for the decay $\rho \to \pi^0\eta\gamma$. The separate contributions coming from vector meson dominance amplitude, $\omega - \rho$ mixing amplitude, a_0-meson intermediate state amplitude, chiral loop amplitudes and their interference, as well as the contribution of total amplitude, are explicitly shown. As we can see in these two figures, the contribution of the VMD amplitude does not change if we add the

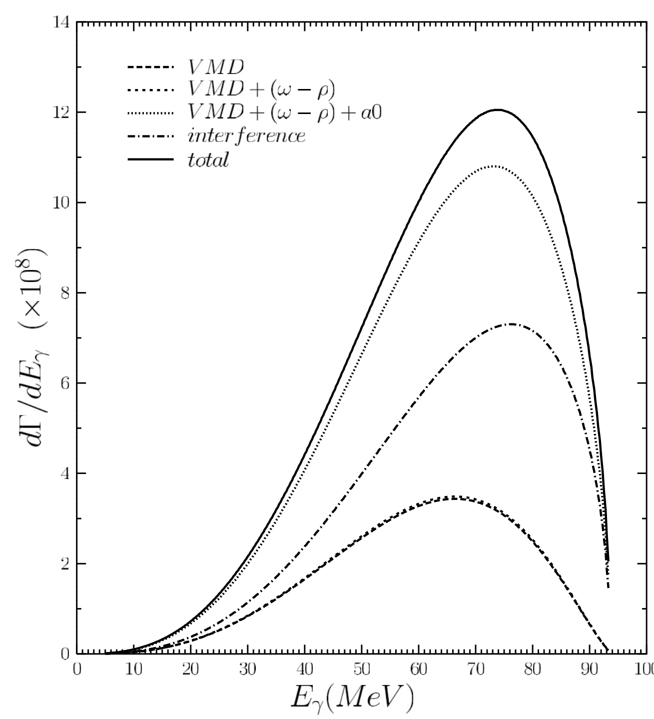

FIG. 3. The photon spectra for the decay width of $\omega \to \pi^0\eta\gamma$ decay. The contributions of different terms resulting from the amplitudes of VMD, chiral loops, a_0-meson intermediate state, and $\rho - \omega$ mixing are indicated.

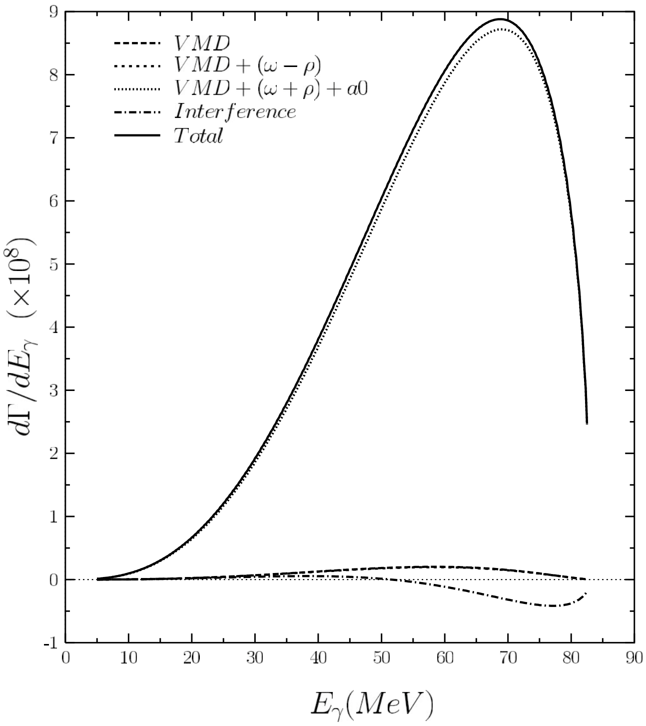

FIG. 4. The photon spectra for the decay width of $\rho \to \pi^0\eta\gamma$ decay. The contributions of different terms resulting from the amplitudes of VMD, chiral loops, a_0-meson intermediate state, and $\rho - \omega$ mixing are indicated.

effect of $\omega - \rho$ mixing. The situation changes in two cases when we include VMD, a_0-meson intermediate state amplitude with $\omega - \rho$ mixing. The interference term between all contribution is constructive for $\omega \to \pi^0\eta\gamma$ decay as shown in Fig. 3. For the decay $\rho \to \pi^0\eta\gamma$, a_0-meson intermediate state amplitude contribution is quite significant in comparison with other contributions as seen clearly in Fig. 4.

The effects of the $\rho - \omega$ mixing are too small for both cases, especially for $\rho \to \pi^0\eta\gamma$ decay in which it is not any contribution, but it modifies the propagator in the vector meson dominance mechanism, so the $\rho - \omega$ mixing should be added to the calculation. The loop contribution for the $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays is found to be small due to the relatively high mass of the kaons, as mentioned also in [8].

As it can be deduced from Table I, all results of the previous studies and our value for the branching ratio of $\omega \to \pi^0\eta\gamma$ decay are well below the experimental upper limit for this decay. Since we do not have any experimental value, the results obtained for the branching ratio of the $\rho \to \pi^0\eta\gamma$ decay cannot be compared with measurements. It should be expected that in the near future experiments related to $\rho \to \pi^0\eta\gamma$ decay will verify or refute our results. For the case of the $\omega \to \pi^0\eta\gamma$ decay, recently the CMD-2 Collaboration obtained the following upper limit, $BR(\omega \to \pi^0\eta\gamma) < 3.3 \times 10^{-5}$ [21]. Therefore, evaluated values are in agreement with the experimental limit for $\omega \to \pi^0\eta\gamma$ decay.

ACKNOWLEDGMENTS

We are grateful to A. Gokalp and O. Yilmaz for useful discussions.

[1] P. Singer, Phys. Rev. **128**, 2789 (1962); **130**, 2441 (1963); **161**, 1694 (1967).
[2] A. Bramon, A. Grau, and G. Pancheri, Phys. Lett. B **283**, 416 (1992).
[3] N. A. Törnqvist and M. Roos, Phys. Rev. Lett. **76**, 1575 (1996).
[4] J. Weinstein and N. Isgur, Phys. Rev. D **41**, 2236 (1990).
[5] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); **17**, 1444 (1978).
[6] N. N. Achasov, Nucl. Phys. A **675**, 279 (2000).
[7] S. Fajfer and R. J. Oakes, Phys. Rev. D **42**, 2392 (1990).
[8] A. Bramon, A. Grau, and G. Pancheri, Phys. Lett. B **289**, 97 (1992).
[9] J. Prades, Z. Phys. C **63**, 491 (1994). **11**, 571 (1994).
[10] A. Gokalp, Y. Sarac, and O. Yilmaz, Eur. Phys. J. C **22**, 327 (2001).
[11] J. E. Palomar, S. Hirenzaki, and E. Oset, Nucl. Phys. A **707**, 161 (2002).
[12] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
[13] F. Klingl, N. Kaiser, and W. Weise, Z. Phys. A **356**, 193 (1996).
[14] H. B. O'Connell, B. C. Pearce, A. W. Thomas, and A. G. Williams, Prog. Part. Nucl. Phys. **39**, 201 (1997).
[15] J. L. Lucio M. and J. Pestieau, Phys. Rev. D **42**, 3253 (1990); **43**, 2447 (1991).
[16] A. Gokalp, A. Kucukarslan, S. Solmaz, and O. Yilmaz, J. Phys. G **28**, 2783 (2002); **28**, 3021 (2002).
[17] A. Gokalp and O Yilmaz, Eur. Phys. J. C **22**, 323 (2001).
[18] S. Teige *et al.*, Phys. Rev. D **59**, 012001 (1998).
[19] D. Guetta and P. Singer, Phys. Rev. D **63**, 017502 (2001).
[20] A. Bramon, R. Escribano, J. L. Lucio M., and M. Napsuciale, Phys. Lett. B **517**, 345 (2001).
[21] R. R. Akhmetshin *et al.*, Phys. Lett. B **580**, 119 (2004).