Spontaneous decompactification

Steven B. Giddings*
Department of Physics† and Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-9530, USA

Robert C. Myers‡
Perimeter Institute for Theoretical Physics, 35 King Street North, Waterloo, Ontario N2J 2W9, Canada and Department of Physics, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

(Received 13 May 2004; published 25 August 2004)

Positive vacuum energy together with extra dimensions of space imply that our four-dimensional Universe is unstable, generically to decompactification of the extra dimensions. Either quantum tunneling or thermal fluctuations carry one past a barrier into the decompactifying regime. We give an overview of this process, and examine the subsequent expansion into the higher-dimensional geometry. This is governed by certain fixed-point solutions of the evolution equations, which are studied for both positive and negative spatial curvature. In the case where there is a higher-dimensional cosmological constant, we also outline a possible mechanism for compactification to a four-dimensional de Sitter cosmology.

DOI: 10.1103/PhysRevD.70.046005 PACS number(s): 11.25.Mj, 04.50.+h, 98.80.Cq, 98.80.Jk

I. INTRODUCTION

We now have good reason to believe that we live in an accelerating Universe; this point has particularly been brought home with the recent Wilkinson microwave anisotropy probe (WMAP) results [1], combined with earlier cosmological observations. It is also widely believed that our fundamental description of nature should involve extra small dimensions of space. These two statements alone lead one to a very general argument that the Universe as we know it is unstable to a catastrophic transition [2].

The generic instability is for the extra dimensions of space to begin to grow, and our world to evolve into a higher-dimensional one. However, depending on the details of the potential for the shape and size moduli of the extra dimensions, there may also be basins of attraction with negative potential, which lead to equally catastrophic big crunches. The instability towards expansion of the extra dimensions was first argued for in the context of string theory by Dine and Seiberg [3]. Their arguments were based on supersymmetry. However, Ref. [2] points out that the underlying mechanism is a simple dynamical one, driven by the dynamics of long-distance gravity, and is independent of the existence of supersymmetry.

Examples of potentials exhibiting this instability are now being widely studied in string theory. Flat directions in moduli space have been a long-standing problem in string theory. Recent developments in compactifications with fluxes and branes [4–14] have provided examples of dynamics that fixes these moduli [15,16]. In particular, Ref. [16] showed that by adding an anti-D3 brane to the solutions of Ref. [15], one can lift the vacuum energy and find locally stable minima with positive cosmological constant. Subsequent work on other approaches to vacua with positive cosmological constant has included Refs. [17–19].

These developments have led to much discussion, as well as criticism [20,21], of the resulting picture of a ‘‘landscape’’ of stringy vacua. This picture has led to a forceful resurrection [22] of the idea that constants of nature—particularly the cosmological constant—are determined anthropically; the large number of possible fluxes and resulting vacua [23,24] (for a review see Ref. [25]) together with the observation that in a sufficiently large distribution one expects to find a small enough cosmological constant [26], have given strong fuel to this possibility.

One of the criticisms from Ref. [20] is potentially relevant to our discussion, and so deserves comment. Banks and Dine argue that an effective potential description of the landscape is not justified and cannot be trusted. Underlying this argument is the realization that there are regions in the combined field space of the moduli and metrics where the dynamics becomes strongly coupled. While it is certainly not inconceivable that this could render the entire picture inconsistent, we take a more sanguine perspective. It may well be that there are dragons off in the mountains of the landscape. However, the valley we find ourselves in seems perfectly tame, and we expect that dynamics nearby is likewise tame. Of course we would very much like to understand the strongly coupled dynamics to understand how we arrived to our present vacuum, but for now we shall take the perspective that the dynamics of the full quantum wave function has somehow deposited us here, and our problem is to see what happens next. We provisionally accept that the effective potential is a useful tool in this investigation.

In the next section we review the derivation of the effective potential for the radial dilaton modulus. In particular, this analysis shows that modular landscapes have a generic feature, much like the ‘‘front range’’ of Colorado—the mountains taper off to a semi-infinite plain. As we roll into this region, the extra dimensions of space expand. Section II also discusses examples of potentials that can be obtained

*Email address: giddings@physics.ucsb.edu
†Present address.
‡Email address: rmyers@perimeterinstitute.ca

from fluxes and branes, for example in string theory. Section III turns to the problem of analyzing the asymptotic dynamics of solutions that have escaped a metastable minimum and are running to infinitely expanded extra dimensions. Asymptotically these solutions become fixed-point solutions, whose form we derive from the equations of motion. Section IV discusses mechanisms for escape from the metastable minimum—thermal excitation over or tunneling through the barrier. Finally, in Sec. V, we assemble these results with a general discussion of the decompactification transition out of (our?) metastable de Sitter (dS) space and into the decompactifying regime, with the resulting higher-dimensional evolution. We also briefly discuss a scenario in which transitions can take place back and forth between a higher-dimensional dS space and the four-dimensional one; in this case, the dominant configuration should be determined by the relative entropies, providing a possible mechanism for compactification. The Appendix provides a more detailed phase-plane analysis, for both positive and negative spatial curvatures, of the solutions arising in our discussion of decompactification,

Spontaneous decompactification has also been recently discussed for simple compactifications with fluxes in Ref. [27], which appeared while our paper was being written up.

II. RADIAL DILATON DYNAMICS: GENERALITIES

A. Dimensional reduction

We begin by discussing dynamics of the radial dilaton, following Ref. [2]. Specifically, suppose that we begin with $(d+4)$-dimensional action,

$$S=\int d^{d+4}X\sqrt{-G}[M_P^{d+2}\mathcal{R}+\mathcal{L}(\psi)+\hat{\mathcal{L}}(\psi,R)], \quad (2.1)$$

where X and G are the coordinates and metric of the full $(d+4)$-dimensional spacetime, M_P is the $(d+4)$-dimensional Planck mass, $\mathcal{R}$ is the Ricci scalar, $\mathcal{L}(\psi)$ is the Lagrangian representing the leading contribution of generic matter sources in a derivative expansion, possibly including localized sources such as D branes, and $\hat{\mathcal{L}}(\psi,\mathcal{R})$ summarizes possible corrections to the leading Lagrangian that involve higher powers of the curvature and/or higher derivatives of matter fields. This action may be the effective action for string theory, or for some other fundamental theory of gravity. For our present purposes we assume that all the moduli except the overall volume modulus of the internal space are fixed, e.g., as in Ref. [15]; we will consider the coupled dynamics of this modulus and the four-dimensional metric. (It has been argued in Ref. [28] that in the case of multiple moduli with exponential potentials, a single term dominates, leading to an obvious generalization of our analysis to the multimoduli case.) Thus we assume that Eq. (2.1) has solutions of the form[1]

[1]Here we suppress a possible warp factor, which should not change our general picture.

$$ds^2=ds_4^2+R^2(x)g_{mn}(y)dy^m dy^n. \quad (2.2)$$

The action governing solutions with $R(x)=e^{D(x)}$ varying slowly on the compactification scale follows from dimensional reduction of Eq. (2.1). The Einstein-Hilbert term gives a four-dimensional (4D) effective action

$$S_{EH}=M_P^{d+2}V_d\int d^4x\sqrt{-g_4}[e^{dD(x)}\mathcal{R}_4+d(d-1)\times(\nabla D)^2e^{dD(x)}+e^{(d-2)D}\mathcal{R}_d], \quad (2.3)$$

where V_d and $\mathcal{R}_d$ are the volume and curvature of the d-dimensional compact metric g_{mn}. However, in this frame the 4D effective Planck mass varies with R. To make contact with usual treatments of 4D dynamics, one should choose a new set of units by performing the Weyl rescaling,

$$g_{4\mu\nu}\rightarrow e^{-dD}g_{4\mu\nu}; \quad (2.4)$$

the 4D Planck mass then becomes

$$M_4^2=M_P^{d+2}V_d. \quad (2.5)$$

The additional terms in Eq. (2.1) then contribute an additional potential for D. The net result is an action of the form

$$S=M_4^2\int d^4x\sqrt{-g_4}\left\{\mathcal{R}_4-\frac{1}{2}d(d+2)(\nabla D)^2-V(D)\right\}. \quad (2.6)$$

A central observation of Ref. [2] is that any physics that stabilizes the radial dilaton from runaway to infinite volume, $D\rightarrow\infty$, must do so only locally—the Weyl rescaling (2.4) introduces an inverse power of the volume-squared into the potential. This means that to stabilize the radial dilaton, the higher-dimensional dynamics would have to produce an energy *density* growing at least as fast as the internal *volume* for large volume. This does not appear realizable in any realistic physical theory.

Therefore if there is a positive potential minimum representing the present de Sitter phase of our Universe, this minimum must be metastable, and the potential should generically appear as in Fig. 1 or Fig. 2. Figure 2 produces an instability to a big crunch spacetime [29,30]. Our focus will be the more generic case of runaway to infinitely extended extra dimensions, as follows from Fig. 1.

Of course, in full generality there will be a nontrivial multidimensional moduli space of compact manifolds, and a potential function on this space. This has recently been explored in the case of string theory for the region of string configuration space corresponding to flux compactifications, and the resulting configuration space dubbed [22] ‘‘the landscape.’’ The argument given in Ref. [2] implies that the landscape has a general feature similar to the geology of the front range of the Rocky Mountains in North America: the mountains and valleys of the landscape roll off into a flat plain, extending to infinity.

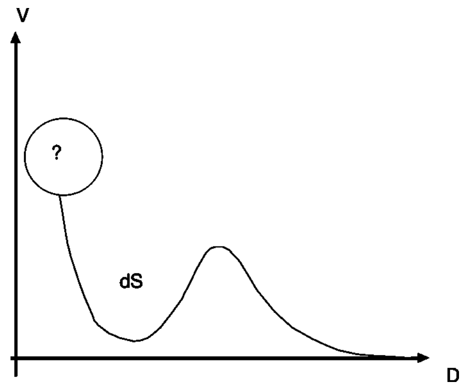

FIG. 1. A sketch of a potential with a metastable de Sitter region, and runaway to infinite dilaton.

B. Dilaton potential—examples

In general the dilaton potential receives contributions from nontrivial field configurations in the extra dimensions. For illustration, it is particularly fruitful to consider examples provided from string theory, particularly branes and fluxes. There may well also be other effects, both perturbative and nonperturbative in the couplings.

Some of the examples from string theory were enumerated in Ref. [2]. A q-form flux field with action

$$S_p \propto -\int d^{d+4}X\sqrt{-G}\frac{F_q^2}{q!} \tag{2.7}$$

gives a potential

$$V_F \propto R^{-d-2q} \tag{2.8}$$

and a space-filling p brane with action

$$S_p = -\frac{\mu_p}{g_s}\int_{M_4\times\mathcal{C}_{p-3}} dV_{p+1} \tag{2.9}$$

gives a potential

$$V_p \propto R^{p-3-2d}. \tag{2.10}$$

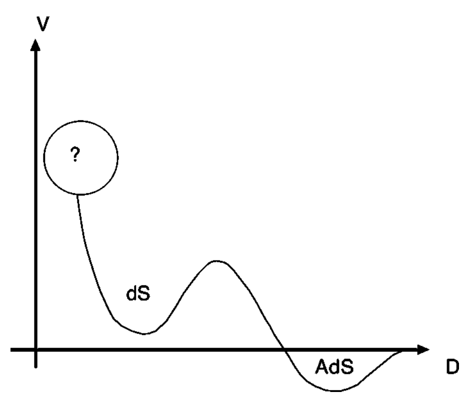

FIG. 2. A sketch of a potential with a metastable de Sitter region, and a minimum with negative cosmological constant. If the Universe fluctuates into the anti–de Sitter (AdS) basin of attraction, it evolves to a big crunch singularity.

From the action (2.3), curvature on the internal manifold can give a potential

$$V_R \propto R^{-d-2}. \tag{2.11}$$

Nonperturbative effects give other contributions, e.g., Ref. [16] included a potential of the form

$$V = Be^{-2aR^4}/R^s, \tag{2.12}$$

with different constants a and s arising from Euclidean D3-brane instantons or gluino condensation. Finally α' corrections were argued in Ref. [31] to produce potential contributions of the form

$$V_K \propto \frac{1}{R^{18}}. \tag{2.13}$$

III. ASYMPTOTIC DILATON DYNAMICS

A. Fixed-point dynamics

Quantitative details of the de Sitter decay will depend on the contribution of these and other possible terms to the potential. However, we will be particularly interested in the asymptotic structure of the expanding region of higher-dimensional space. The dynamics inside this region is governed by the asymptotics of the potential at large dilaton, and for this in general we need only to focus on the leading term. Since this will typically be of the form $V \propto R^n$, let us analyze more closely the dilaton dynamics with a single such term in the potential. With canonically normalized dilaton,

$$\phi = \sqrt{d(d+2)}D, \tag{3.1}$$

we find an action of the form

$$S = M_4^2\int d^4x\sqrt{-g}\left[\mathcal{R}_4 - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] \tag{3.2}$$

with

$$V(\phi) = \frac{A}{2}e^{-\alpha\phi}. \tag{3.3}$$

In these conventions,

$$\begin{aligned}\alpha &= \sqrt{\frac{d}{d+2}}\left(1+\frac{2q}{d}\right) \quad q \ \text{flux}\\ &= \sqrt{\frac{d}{d+2}}\left(1+\frac{d+3-p}{d}\right) \quad p \ \text{brane}\\ &= \sqrt{\frac{d+2}{d}} \quad \text{internal curvature.}\end{aligned} \tag{3.4}$$

Friedman-Robertson-Walker (FRW) solutions for actions of this form, with exponential potentials, have been investigated previously in the literature, see, e.g., Refs. [32–39]. Recently, there has been a great deal of discussion of analogous solutions in string and M theory, see, e.g., Refs. [40–53,27].

As we shall see in the next section, such FRW solutions, with spatial curvature $k=0, \pm 1$, are useful in studying the decompactification transition. These have the form

$$ds_4^2 = -dt^2 + a^2(t)d\chi^2, \quad \phi = \phi(t), \tag{3.5}$$

where $d\chi^2$ is the appropriate spatial metric with curvature $k=0, \pm 1$. The equations of motion take the form

$$\frac{\dot a^2}{a^2} = \frac{\rho_\gamma}{6} + \frac{\dot\phi^2}{12} + \frac{V}{6}, \tag{3.6}$$

$$\left(\frac{\dot a}{a}\right)^{\bullet} + \frac{\gamma\rho_\gamma}{4} = -\frac{\dot\phi^2}{4}, \tag{3.7}$$

$$\ddot\phi = -3\left(\frac{\dot a}{a}\right)\dot\phi - \frac{dV}{d\phi}. \tag{3.8}$$

Here we have included the possibility of a stress tensor with a barotropic equation of state,

$$p_\gamma = (\gamma - 1)\rho_\gamma, \tag{3.9}$$

for which the density evolves as

$$\dot\rho_\gamma = -3\gamma\frac{\dot a}{a}\rho_\gamma. \tag{3.10}$$

In the case $k=\pm 1$, the dominant effective contribution to ρ at large a is that summarizing the spatial curvature, which gives

$$\rho_{2/3} = -\frac{6k}{a^2}. \tag{3.11}$$

Massive matter ($\gamma=1$) or radiation ($\gamma=\frac{4}{3}$) will make subdominant contributions as $a\to\infty$ in these cases, but if $k=0$ their contribution can be relevant to this asymptotic behavior.

The asymptotic behavior of solutions of these equations is typically governed by fixed-point "tracker" solutions. The fixed points relevant to the cases $k=0, -1$ have been found via phase space analyses in Refs. [35,37]. A simple extension of this work covers the case $k=+1$ as well, as described in the Appendix. Let us understand how these solutions arise. The starting point is the observation that for a scale factor that increases as a power of time (which will be justified shortly),

$$a = a_0(ct)^\beta, \tag{3.12}$$

and for an exponential potential (3.3), Eq. (3.8) drives e^ϕ to infinity as a power:

$$e^{\alpha\phi} = (ct)^2, \tag{3.13}$$

where the constant c is fixed to be

$$c = \frac{\alpha}{2}\sqrt{\frac{A}{3\beta - 1}}. \tag{3.14}$$

Note that Eq. (3.8) can also be integrated to give the ϕ-dependent terms in the Freedman equation:

$$\frac{\dot\phi^2}{12} + \frac{V}{6} = \frac{\beta}{\alpha^2 t^2}. \tag{3.15}$$

Thus, using Eq. (3.10), the Freedman equation (3.6) becomes

$$\frac{\dot a^2}{a^2} = \frac{c'}{t^{3\gamma\beta}} + \frac{\beta}{\alpha^2 t^2} \tag{3.16}$$

for some constant c'. First consider the cases $k=0, -1$, where c' is positive. Which term gives the dominant fixed-point asymptotic behavior as $t\to\infty$ depends on the relative magnitudes of α and γ. A phase plane analysis of the dynamics appears in the Appendix, but the basic features can be understood straightforwardly directly from the equations. For $3\gamma\beta > 2$, the first term becomes subdominant as $t\to\infty$. Then, using the power-law form of the solution, Eq. (3.12), we find $\beta = 1/\alpha^2$. This means the condition for this fixed point to dominate is $\alpha^2 < 3\gamma/2$. In contrast, for $\alpha^2 > 3\gamma/2$ both terms are relevant. Correspondingly, the power-law form (3.12) now implies $\beta = 2/3\gamma$. The complete phase plane analysis (see Refs. [35,37] and the Appendix) shows that these are indeed the fixed points to which the generic solution is attracted—in the Appendix, fixed point (a) for $\alpha^2 < 3\gamma/2$ and (b) for $\alpha^2 > 3\gamma/2$ (see Fig. 4). Any other fixed points are either unstable nodes or saddle points.

The case of positive spatial curvature $k=+1$ allows negative c' and is somewhat more subtle. For $\alpha^2 < 3\gamma/2 = 1$, depending on initial conditions, the solution may reach a turning point and reflect to a collapsing universe, $\dot a < 0$, rather than reaching the asymptotic $a\to\infty$ region. If this happens, other terms in the Freedman equation (3.6), e.g., due to matter/radiation fields, become important in the dynamics, and a general analysis is not possible. However, if the solution asymptotically expands, the analysis is similar to the $k=-1$ case. For $\alpha^2 > 1$, solutions generically reach a turning point and recollapse; this behavior is not well described by the power-law form (3.12). These features are well illustrated by the corresponding phase plot in the Appendix—see Fig. 5.

We can summarize the resulting asymptotically expanding solutions by

$$ds^2 = -dt^2 + a_0^2[c(t-t_0)]^{2\beta}d\chi^2, \tag{3.17}$$

and

$$e^\phi = [c(t-t_0)]^{2/\alpha}, \tag{3.18}$$

with c given in Eq. (3.14). The constant a_0 may be fixed by Eq. (3.6) for $k=\pm 1$ and the expansion exponent β is given by

$$\beta = \max\left(\frac{2}{3\gamma}, \frac{1}{\alpha^2}\right). \tag{3.19}$$

B. Specific cases

Consider first the case of nonzero spatial curvature, $k=\pm 1$. At large radius, the curvature dominates over the matter contributions to the Friedmann equations, and curvature corresponds to $\gamma=\frac{2}{3}$. Thus for $\alpha>1$, which is generically the case for string-theory induced potentials, for $k=-1$ we have $\beta=1$, giving a nonaccelerating solution with a linearly growing scale factor;[2] for $k=+1$ we have collapse. For example, consider a flux-induced potential. We can easily see from Eq. (3.4) that

$$\alpha_{q\ \text{flux}}^2=\left(1+2\frac{q}{d}\right)\left(1+2\frac{q-1}{d+2}\right), \tag{3.20}$$

which gives $\alpha^2>1$. Similarly for internal curvature, we find

$$\alpha_{\text{curvature}}^2=1+\frac{2}{d} \tag{3.21}$$

so again $\alpha^2>1$. The case of a brane-induced potential gives, from Eq. (3.4),

$$\alpha_{p\ \text{brane}}^2=\left(1+\frac{d+3-p}{d}\right)\left(1+\frac{d+1-p}{d+2}\right). \tag{3.22}$$

For $p\leqslant d+2$, i.e., a brane of codimension 1 or higher, we therefore also find $\alpha^2>1$. An exceptional case is that of a completely space-filling brane, $p=d+3$, or equivalently higher-dimensional cosmological constant, which gives

$$\alpha=\sqrt{\frac{d}{d+2}}<1. \tag{3.23}$$

The corresponding solution then has $\beta=1/\alpha^2=1+2/d$ and is accelerating.

Other string theory potentials typically fall off even more rapidly at infinity, removing them further from this accelerating case.

Next consider spatially flat universes, $k=0$. In these cases, the asymptotic density of matter or radiation, and in particular the dominant value of γ, is relevant for determining which attractor governs the dynamics. Note, however, that one still only achieves acceleration for $\alpha<1$.

We will discuss the D-dimensional interpretation of these solutions in the following section.

IV. DECOMPACTIFICATION TRANSITION

A. Escaping inflation

We next turn to the study of decay of a metastable de Sitter minimum in the potential for the radial dilaton. In particular, we assume that the relevant dynamics for our discussion is that of the radial modulus, and that other possible moduli of the internal manifold are fixed, although as we have pointed out this analysis should extend to the multimoduli case where that is dominated by an exponential potential due to a single modulus, as in Ref. [28]. Thus we consider the Lagrangian (3.2), where $V(\phi)$ has a form sketched in Fig. 1. For simplicity, we work in units where $M_4=1$ in the following.

Any given region of the de Sitter universe corresponding to the metastable minimum will ultimately decay to a decompactifying universe. The time scale depends on the parameters of the potential.[3] If the minimum corresponds to our presently inflating phase, we should have $V_0=V(\phi_0)\sim 10^{-120}\ll 1$. For a general potential, without any further fine tuning, we expect that the height of maximum of the barrier at ϕ_1 is $V_1\sim 1$ and that the width of the barrier is also $\Delta\phi\sim 1$. There are then two basic mechanisms for inducing spontaneous decompactification. The first is for thermal excitations of de Sitter space to take one over the top of the barrier. The second is quantum tunneling through the barrier, as described by Coleman and de Luccia [29].

The thermal activation rate is given by the action of the Hawking-Moss instanton [54], as has been argued from the stochastic approach to inflation [55–61]. This gives the probability of a thermal fluctuation to take one to the top of the barrier. If the change in entropy for this fluctuation is $\Delta\mathbf{S}$, the probability is given by the usual formula $P=\exp\{\Delta\mathbf{S}\}$. This probability is

$$P_{\text{thermal}}=e^{S(\phi_0)-S(\phi_1)}, \tag{4.1}$$

where $S(\phi_0)=-24\pi^2/V_0$ is the action of the de Sitter instanton, i.e., the four-sphere solution, with $\phi=\phi_0$, the metastable minimum. Similarly, $S(\phi_1)$ is the solution with $\phi=\phi_1$, the maximum of the barrier. The de Sitter recurrence time [62] is given by $T_r=\exp\{S(\phi_0)\}$. The thermal decay lifetime is thus shorter than this by an exponential factor:

$$\tau_{\text{thermal}}=e^{-24\pi^2/V_1}T_r\,. \tag{4.2}$$

The tunneling probability is given by [29,16]

$$P_{\text{tunnel}}=\exp\left\{\frac{S(\phi_0)}{[1+(4V_0/3T^2)]^2}\right\}, \tag{4.3}$$

where the tension of the bubble wall is

$$T=\int_{\phi_0}^{\infty}d\phi\sqrt{2V(\phi)}. \tag{4.4}$$

For the parameters that we generically expect, we see that $V_0\ll T^2$. The probability (4.3) therefore yields a decay time

$$\tau_{\text{tunnel}}\sim e^{-64\pi^2/T^2}T_r\,. \tag{4.5}$$

Comparing Eqs. (4.2) and (4.5) shows that which process is dominant clearly depends on the relative magnitudes of the tension T and the barrier height V_1.

[2] However, solutions approaching this fixed point may be eternally accelerating [40,48,50].

[3] For a more detailed review of parameters and dynamics see Ref. [16].

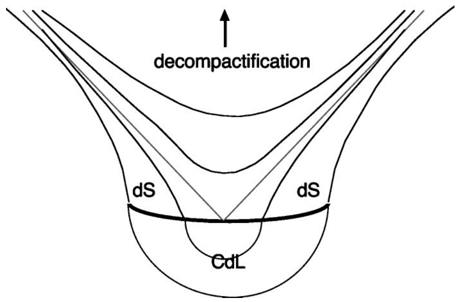

FIG. 3. A representation of the Coleman–de Luccia instanton, describing tunneling from a metastable de Sitter region into a bubble of space in which the extra dimensions of space are expanding. The lower half of the diagram is the Euclidean solution of Ref. [29], for the potential $V(\phi)$. This matches onto the Lorentzian solution pictured in the upper half of the diagram. The straight lines correspond to the $a=0$ surface of the resulting expanding $k=-1$ cosmology; the surfaces above these are surfaces of constant t in this cosmology. The boundary of this decompactifying region is a bubble wall (circular below, hyperbolic above) which asymptotically expands at the speed of light into the metastable de Sitter region.

Once the field excites past the barrier, it classically evolves towards decompactification. In the case of thermal activation, one expects a picture where an entire horizon volume thermally excites over the barrier. The boundary conditions then start the solution near the maximum of the potential, and the resulting classical solution can roll into the decompactification region. However, fluctuations are expected to be important in this case, and so one cannot clearly identify the resulting dynamics as a $k=0$, ± 1 universe.[4] However, these may serve as rough indicators for the dynamics, bearing in mind that different regions may have different spatial curvatures. One also expects to have some excited matter fields in the resulting solution; this is important to avoid certain collapsing solutions in the case of vanishing spatial curvature, $k=0$, but more generally it seems plausible that collapse could occur in some regions, and expansion in others.

In the case of tunneling, one can be a little more specific about the resulting boundary conditions, by matching the Coleman–de Luccia instanton onto a Lorentzian geometry at the turning point. The O(4) symmetry of the instanton continues to an SO(3, 1) symmetry of the subsequent classical solution, implying [29,63] that one has evolution on spatial sections with $k=-1$. These features are illustrated in Fig. 3. The boundary conditions on the "initial slice" of the cosmology are

$$\dot{\phi}=0, \quad a=0, \quad \dot{a}=1, \quad \text{and} \quad \phi=\phi_t, \tag{4.6}$$

where ϕ_t is the value at the turning point.

[4]We thank A. Linde for a discussion on this point.

B. Expansion to higher dimensions

In either case, even if we know the potential explicitly it is typically hard to find an explicit solution. However, the fixed-point behavior of these systems tells us that this is not necessary to understand the asymptotic behavior. Specifically, we expect the asymptotic behavior of the decompactifying solution to be governed by the leading term in the potential as $a\to\infty$. This is expected to generically be one of the exponentials we have considered, and so decompactification is asymptotically described by one of the fixed-point solutions of the preceding section, described by (in the $k=0$, -1 cases) the solution (3.17)–(3.19).

Since, as shown by Eq. (3.18), the compact manifold expands, the relevant description becomes the higher-dimensional description rather than that of four dimensions. Recall that these differ by the rescaling (2.4), so in terms of the fundamental units of the higher-dimensional theory, the asymptotic metric takes the form

$$ds^2_{4+d}=e^{-dD(t)}[-dt^2+a^2(t)d\chi^2]+e^{2D(t)}ds^2_d. \tag{4.7}$$

In discussing the higher-dimensional form of these solutions, it proves easier to work with the coefficient λ, related to α by

$$\lambda=\sqrt{\frac{d+2}{d}}\alpha; \tag{4.8}$$

thus for fluxes, branes, and internal curvature, we find from Eq. (3.4)

$$\begin{aligned}\lambda&=1+\frac{2q}{d} \quad q \ \text{flux}\\ &=1+\frac{d+3-p}{d} \quad p \ \text{brane}\\ &=1+\frac{2}{d} \quad \text{internal curvature.}\end{aligned} \tag{4.9}$$

Then given the asymptotic solution in Eqs. (3.17) and (3.18), the higher-dimensional metric (4.7) becomes

$$\begin{aligned}ds^2_{d+4}=&[c(t-t_0)]^{-2/\lambda}\{-dt^2+a_0^2[c(t-t_0)]^{2\beta}d\chi^2\}\\ &+[c(t-t_0)]^{4/\lambda d}ds^2_d.\end{aligned} \tag{4.10}$$

For $\lambda>1$, this metric can be simplified by defining a new time coordinate,

$$\tau=\frac{1}{c}\frac{\lambda}{\lambda-1}[c(t-t_0)]^{1-1/\lambda}, \tag{4.11}$$

with which we find

$$ds^2=-d\tau^2+a_0^2(\hat{c}\tau)^{2(\beta\lambda-1)/(\lambda-1)}d\chi^2+(\hat{c}\tau)^{4/d(\lambda-1)}ds^2_d. \tag{4.12}$$

Here the constant $\hat{c}$ is given by

$$\hat{c}=\frac{\lambda-1}{\lambda}c; \qquad (4.13)$$

recall that c was given in Eq. (3.14), and β in Eq. (3.19). Both the three large dimensions and the compact dimensions expand. However, the compact dimensions expand more slowly unless

$$(\beta\lambda-1)d\leqslant 2. \qquad (4.14)$$

Thus for $k=-1$ they do so in the case of flux-induced potentials and of potentials arising from branes of codimension greater than 2.

The special case of a wrapped $p=d+3$ space-filling brane corresponds to $\lambda=1$ and $\beta=1+2/d$. Now we define

$$\tau=\frac{1}{c}\ln[c(t-t_0)], \qquad (4.15)$$

and the metric becomes

$$ds^2=-d\tau^2+e^{4c\tau/d}(a_0^2 d\chi^2+ds_d^2). \qquad (4.16)$$

Thus, locally, the metric looks like a patch of inflating de Sitter space—for a discussion of such asymmetric foliations of de Sitter space, see Ref. [64].

As the size of the compact dimensions become comparable to that of the other three, in general the four-dimensional effective description will break down and one must describe the solutions as fully higher-dimensional solutions. Since the field configurations, e.g., branes and fluxes, on the compact manifold will generically be nonuniform, this means that the resulting higher-dimensional solution is generically nonuniform. Moreover, fluctuations of modes that get light in this limit can become relevant. Nonetheless, in many cases we expect the four-dimensional solutions that we have described to lift to give a higher-dimensional solution, and thus a reasonably accurate picture of the higher-dimensional dynamics.

V. OVERVIEW: THE FALL

Let us assemble the pieces of the picture that we have discussed. Beginning with our Universe in its currently inflating phase, ultimately a fluctuation will carry it out of our metastable minimum. This may either be a thermal fluctuation over the barrier, or a tunneling event through the barrier. In the thermal case, an entire horizon-sized region fluctuates over the barrier. In the tunneling case, a bubble forms, and then expands.

The relevant time scales are extremely long, as they contain a factor of the recurrence time, $T_r\sim\exp\{10^{120}\}$. The decay time, expressed as a fraction of the recurrence time, is given in Eqs. (4.2) and (4.5), and depends on the relative magnitude of the bubble tension and the barrier height.

What happens next depends on which basin of attraction the fluctuation takes us into. There may be accessible basins of attraction with negative effective cosmological constant; in that case the Universe will undergo a big crunch. Alternatively, we know that there is generically an infinite basin of attraction corresponding to decompactification. This is the one that we have considered in this paper. We also expect this basic picture to extend to the case of multimoduli runaway; Ref. [28] has argued that the case of multiple runaway scalars with exponential potentials can be reduced to the single-scalar case. There also may be basins of attraction where some of the dimensions of space decompactify and others remain stabilized. These would be described by straightforward generalization of our analysis.

Once we find ourselves in such a basin of attraction, the Universe decompactifies, with asymptotic dynamics typically given by Eqs. (4.12) or (4.16). In the case of a thermal fluctuation, we expect an entire horizon-sized region to evolve into the region where higher-dimensional dynamics is relevant, but the actual configuration may be highly nonuniform, with collapse in some regions and expansion in others. It is in this context that we imagine that our results for $k=+1$ might be applicable. That is, they might describe the dynamics of a small patch with positive spatial curvature in a larger inhomogeneous solution. The surprising result from the phase plane analysis—see the Appendix—is that for typical potentials, e.g., induced by fluxes or branes, the solutions generically evolve to a big crunch. We expect that such regions collapse into black holes and are thus casually disconnected from the expanding regions.[5]

In the case of tunneling, a bubble of higher-dimensional space forms, and its walls expand into the four-dimensional region at the speed of light. The solution inside the bubble is given by the asymptotic dynamics that we have described. As in old inflation, the majority of space continues to be inflating four-dimensional space [63,65], but any given point will ultimately transition into the higher-dimensional realm.

Now we return to a special case of space-filling brane with $p=d+3$, i.e., a (positive) cosmological constant in $(d+4)$ dimensions. In this case, the landscape as illustrated in Fig. 1 is deceptive. Despite the appearance of the usual infinite plain, there are no solutions rolling towards flat decompactified space in $(d+4)$ dimensions. Rather as we have explicitly shown in Eq. (4.16), the (apparently) metastable dS space transitions to a spacetime which asymptotes to $(d+4)$-dimensional de Sitter space. Recall from the discussion around Eq. (3.23) that this was the special case for which the Universe appeared to be accelerating from a 4D point of view.

Hence this introduces the following interesting possibility. Here we may consider the transition our four-dimensional dS space to a higher-dimensional dS space. However, in this case, we also expect the reverse process to be possible. Moreover, we expect that, if the system can indeed be thought of as a thermal ensemble and the de Sitter entropies as accurate representations of the number of states, the solution that dominates the ensemble should be that with the higher entropy. This could be either the higher-dimensional or lower-dimensional de Sitter space [66], depending on the relative magnitudes of the entropies,

[5]A similar description has been advocated for the crunch singularities appearing in transitions towards AdS minima [22].

$$\mathbf{S}_{d+4} \sim \frac{M_P^{(d+4)(d+2)/2}}{\Lambda_{d+4}^{(d+2)/2}} \quad (5.1)$$

and

$$\mathbf{S}_4 \sim \frac{M_4^4}{\Lambda_4} \sim \frac{M_P^{2(d+2)} V_d^2}{\Lambda_4}, \quad (5.2)$$

where V_d is the volume of the compact manifold at the ‘‘metastable’’ minimum. In fact, it is not difficult to construct examples where the compactified solution has the larger entropy [66]. In particular, we know that a minimum corresponding to the present Universe has an entropy $\mathbf{S}_4 \sim 10^{120}$, and so if the hypothetical higher-dimensional theory had a cosmological constant that is order unity in Planck units, our four-dimensional metastable minimum would be expected to strongly dominate such an ensemble. Hence this scenario could provide a mechanism for compactification from the higher-dimensional theory.

Certainly, this is not the entire story. Typically within this framework, compactified anti–de Sitter solutions will also arise, e.g., by adjusting fluxes [66]. It is not clear what their role should be in such a scenario [20,21,30]. However, this aspect of the dynamics seems a fruitful ground for future research.

ACKNOWLEDGMENTS

The authors would like to thank R. Bousso, G. Horowitz, L. Kofman, A. Linde, M. Lippert, L. Susskind, and M. Taylor for very valuable conversations. We also thank Jordan Hovdebo for his advice in preparing the phase plane figures. The work of S.B.G. was supported in part by the Department of Energy under Contract No. DE-FG02-91ER40618. This work was initiated at the Perimeter Institute, which S.B.G. would like to thank for its kind hospitality. Research at the Perimeter Institute is supported in part by funds from NSERC of Canada. Part of this work was also carried out during the Superstring Cosmology workshop at the Kavli Institute for Theoretical Physics, whose support is gratefully acknowledged; research at the KITP was supported in part by the National Science Foundation under Grant No. PHY99-07949.

APPENDIX: PHASE PLANE ANALYSIS FOR $k = \pm 1$

Following the phase plane analysis of Ref. [37], we describe the general solutions of Eqs. (3.6)–(3.8) for the cases where the spatial metric is either positively or negatively curved. Hence in Eqs. (3.6) and (3.7), we have $\gamma = \frac{2}{3}$ and substitute for ρ_γ as in Eq. (3.10). The phase plane dynamics explicitly reveals the presence of fixed-point ‘‘tracker’’ solutions, discussed in the main text, as the generic end points of various solutions.

We begin by defining

$$x = \frac{\dot\phi}{2\sqrt{3}\dot a/a}, \quad y = \frac{\sqrt{V}}{\sqrt{6}\dot a/a}, \quad z = \frac{1}{\dot a}. \quad (A1)$$

With this choice of variables, the Freedman constraint (3.6) becomes

$$x^2 + y^2 - kz^2 = 1 \quad (A2)$$

which can be used to eliminate z. The second-order equations (3.7) and (3.8) can be written in the form

$$x' = -2x(1-x^2) + y^2(\sqrt{3}\alpha - x), \quad (A3)$$

$$y' = y(1-y^2) - xy(\sqrt{3}\alpha - 2x), \quad (A4)$$

where prime denotes $d/d\log a$ and α is the (positive) constant characterizing the exponential potential, as in Eq. (3.3). Note that these equations are independent of k. The feature which then distinguishes these two cases is the range of the variables as determined by the constraint (A2), i.e., $x^2 + y^2 \leqslant 1$ for $k = -1$ and $x^2 + y^2 \geqslant 1$ for $k = +1$. We are also generally interested in solutions describing an expanding universe and so we focus our attention on $y \geqslant 0$. Since Eqs. (A3) and (A4) are symmetric under $y \to -y$, the $y < 0$ dynamics is a simple extrapolation.

These equations exhibit the following fixed points:
(a) $x = x_a = \alpha/\sqrt{3},\ y = y_a = \sqrt{1-\alpha^2/3},\ z^2 = 0$,
(b) $x = x_b = 1/\sqrt{3}\alpha,\ y = y_b = \sqrt{2}/\sqrt{3}\alpha,\ z^2 = k(\alpha^{-2} - 1)$,
(c) $x = 0,\ y = 0,\ z^2 = -k$,
(d) $x = 1,\ y = 0,\ z^2 = 0$,
(e) $x = -1,\ y = 0,\ z^2 = 0$.

Which of these fixed points is physically realized (and their stability properties) depends on the values of parameters k and α. In particular, the fixed point (a) only appears for $\alpha < \sqrt{3}$ in order that y_a is real. While (b) has x_b and y_b real for all values of α, it is only relevant for $\alpha < 1$ with $k = +1$ and $\alpha > 1$ with $k = -1$ so that the fixed point lies in the appropriate domain of the phase plane, i.e., $z^2 \geqslant 0$. Similarly, (c) is only relevant for $k = -1$. Finally the fixed points (d) and (e) always appear relevant independent of the parameters. Figures 4 and 5 illustrate the fixed points and the flows in the phase plane for the various distinct parameter ranges. We now comment briefly on these results.

Consider the case $k = -1$, as described by Fig. 4. First for $\alpha < 1$, (a) is the only stable fixed point and then describes the end-point behavior of generic solutions. As described in the text, amongst the string theory potentials, this situation only seems to be realized with a completely space-filling brane (with $p = d+3$), in which case this fixed point corresponds to an asymptotically de Sitter solution in the full $(d+4)$-dimensional spacetime. For $1 < \alpha < \sqrt{3}$, (a) becomes a saddle point and then disappears (i.e., moves off into the complex plane) for $\alpha > \sqrt{3}$. Hence in the regime $\alpha > 1$ which seems to be the generic case for string theory, (b) is the stable fixed point.[6]

Figure 5 illustrates the flows for $k = +1$. In this case, (a)

[6]Qualitatively, these results apply for the case of any barotropic fluid with a positive energy density [37]—the latter would be the dominant energy contribution for the case $k = 0$.

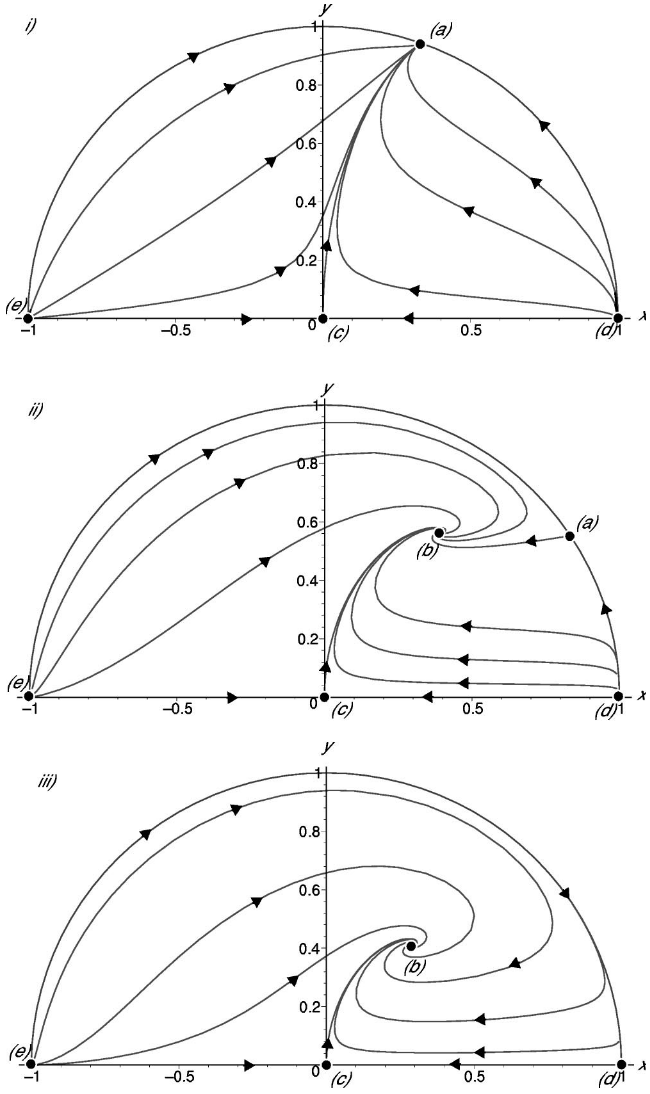

FIG. 4. Fixed points and some typical trajectories for $k=-1$ in the regimes (i) $\alpha<1$, (ii) $1<\alpha<\sqrt{3}$, and (iii) $\alpha>\sqrt{3}$. (The specific plots above were made for $\alpha=1/\sqrt{3}$, $5/2\sqrt{3}$, and 2.)

is again a stable fixed point for $\alpha<1$ and the same comments given above apply here. In fact, (b) only appears in this parameter range as well, but it is an unstable saddle point. Hence for $\alpha>1$, the generic flows run off to infinity in directions bounded by $y^2/x^2\leqslant 2$. This behavior is also generic for a broad range of initial conditions when $\alpha<1$. Physically these flows correspond to expanding cosmologies where the kinetic energy in the scalar field dominates the potential, and the spatial curvature leads to decelerated expansion. As a result $\dot{a}=1/z$ reaches zero at some finite value of a (and t). Hence beyond this point, they begin collapsing. In the phase plane, this transition is realized by mapping the asymptotic flow with (x,y) to $(-x,-y)$ and following the flow backwards with respect to the arrows indicated in the figures. In the present context, the solutions of interest in the present context are those which run off to infinity with $x>0$, i.e., $\dot{a}$, $\dot{\phi}>0$. In this case, for any value of α, they flow towards a big crunch at the fixed point (e). However, as commented in the main text, we expect other terms, e.g., matter and perturbations, to become important in the final stage of this crunch.

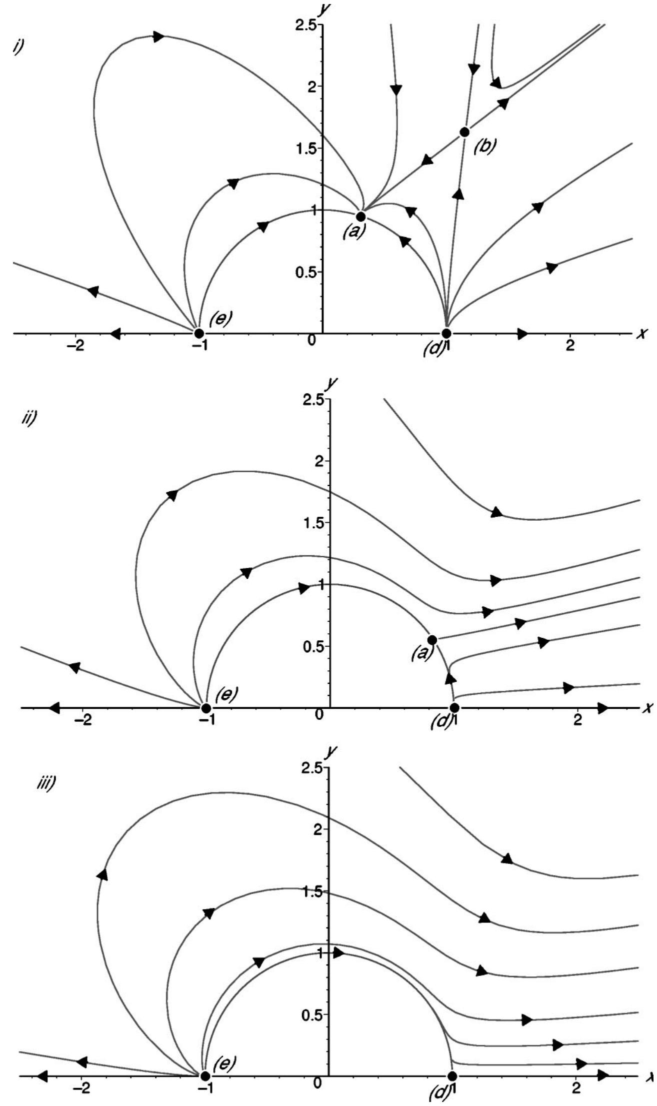

FIG. 5. Fixed points and some typical trajectories for $k=+1$ in the regimes: (i) $\alpha<1$, (ii) $1<\alpha<\sqrt{3}$, and (iii) $\alpha>\sqrt{3}$. (The specific plots above were made for $\alpha=\frac{1}{2}$, $5/2\sqrt{3}$, and 2.)

Note that during the collapse stage above, we have $\dot{a}$, $x<0$ and hence $\dot{\phi}>0$. That is, the internal space continues to expand for these solutions. From a higher-dimensional perspective, however, we would still regard this as a collapsing phase, as given the full metric (4.7), one can easily show that the proper volume element is contracting both in the noncompact four dimensions and in the full $(d+4)$-dimensional solution.

We have focused on the end point of the decompactification trajectories, but it is interesting to consider the full trajectories and follow the flows backwards to an initial fixed point. Generically, for the entire range of parameters, the flows originate at one of the repulsive fixed points (d) or (e). Hence these solutions emerge from a singular big bang. As with the crunch above, the analysis provided here is expected to be incomplete as many other terms will be important near this initial singularity. The generic appearance of these singularities was, of course, observed by Refs. [20,21,30] and used as part of their criticism of string theory landscape picture. However, for tunneling from a metastable dS vacuum, these singularities do not appear as part of the evolution of the system. Another possible viewpoint on this may be to consider an ''extended'' landscape, in which the 4D radius (or even more general metric degrees of freedom) is included; in this space the singular regions may be well separated from the solutions of interest, e.g., by being in different basins of attraction, though this question deserves further exploration.

We might also comment that, in any range of parameters considered above, there are many trajectories which begin with negative x and cross over to positive x. These would fall into the class of M-theory solutions, which were recently discussed [40–49] because they exhibit an accelerating phase. As first noted in Ref. [42], near $x=0$, the scalar potential momentarily dominates the energy density and so cosmic acceleration is seen by four-dimensional observers. However, this is only a momentary phase since, as noted in the main text, generically the M-theory potentials are too steep to generate an (continuously) accelerating solution.

---

[1] D. N. Spergel *et al.*, Astrophys. J., Suppl. **148**, 175 (2003) [astro-ph/0302209].
[2] S. B. Giddings, Phys. Rev. D **68**, 026006 (2003) [hep-th/0303031].
[3] M. Dine and N. Seiberg, Phys. Lett. B **162**, 299 (1985).
[4] J. Polchinski and A. Strominger, Phys. Lett. B **388**, 736 (1996) [hep-th/9510227].
[5] S. Chaudhuri and D. A. Lowe, Nucl. Phys. **B469**, 21 (1996) [hep-th/9512226].
[6] K. Becker and M. Becker, Nucl. Phys. **B477**, 155 (1996) [hep-th/9605053].
[7] J. Michelson, Nucl. Phys. **B495**, 127 (1997) [hep-th/9610151].
[8] K. Becker and M. Becker, J. High Energy Phys. **07**, 038 (2001) [hep-th/0107044].
[9] M. Haack and J. Louis, Phys. Lett. B **507**, 296 (2001) [hep-th/0103068].
[10] J. Louis and A. Micu, Nucl. Phys. **B635**, 395 (2002) [hep-th/0202168].
[11] S. Gukov, C. Vafa, and E. Witten, Nucl. Phys. **B584**, 69 (2000) [hep-th/9906070]; **B608**, 477(E) (2001).
[12] K. Dasgupta, G. Rajesh, and S. Sethi, J. High Energy Phys. **08**, 023 (1999) [hep-th/9908088].
[13] B. R. Greene, K. Schalm, and G. Shiu, Nucl. Phys. **B584**, 480 (2000) [hep-th/0004103].
[14] G. Curio, A. Klemm, D. Lust, and S. Theisen, Nucl. Phys. **B609**, 3 (2001) [hep-th/0012213].
[15] S. B. Giddings, S. Kachru, and J. Polchinski, Phys. Rev. D **66**, 106006 (2002) [hep-th/0105097].
[16] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003) [hep-th/0301240].
[17] C. Escoda, M. Gomez-Reino, and F. Quevedo, J. High Energy Phys. **11**, 065 (2003) [hep-th/0307160].
[18] A. Saltman and E. Silverstein, hep-th/0402135.
[19] M. Becker, G. Curio, and A. Krause, Nucl. Phys. **B693**, 223 (2004).
[20] T. Banks, M. Dine, and E. Gorbatov, hep-th/0309170.
[21] M. Dine, hep-th/0402101.
[22] L. Susskind, hep-th/0302219.
[23] M. R. Douglas, J. High Energy Phys. **05**, 046 (2003) [hep-th/0303194].
[24] S. Ashok and M. R. Douglas, J. High Energy Phys. **01**, 060 (2004) [hep-th/0307049].
[25] M. R. Douglas, hep-th/0401004.
[26] R. Bousso and J. Polchinski, J. High Energy Phys. **06**, 006 (2000) [hep-th/0004134].
[27] L. Jarv, T. Mohaupt, and F. Saueressig, hep-th/0403063.
[28] M. N. R. Wohlfarth, Phys. Rev. D **69**, 066002 (2004) [hep-th/0307179].
[29] S. R. Coleman and F. de Luccia, Phys. Rev. D **21**, 3305 (1980).
[30] T. Banks, hep-th/0211160.
[31] K. Becker, M. Becker, M. Haack, and J. Louis, J. High Energy Phys. **06**, 060 (2002) [hep-th/0204254].
[32] J. D. Barrow, A. B. Burd, and D. Lancaster, Class. Quantum Grav. **3**, 551 (1986).
[33] J. D. Barrow, Phys. Lett. B **187**, 12 (1987).
[34] J. J. Halliwell, Phys. Lett. B **185**, 341 (1987).
[35] B. Ratra and P. J. Peebles, Phys. Rev. D **37**, 3406 (1988).
[36] A. R. Liddle, Phys. Lett. B **220**, 502 (1989).
[37] E. J. Copeland, A. R. Liddle, and D. Wands, Phys. Rev. D **57**, 4686 (1998) [gr-qc/9711068].
[38] A. P. Billyard, A. A. Coley, and R. J. van den Hoogen, Phys. Rev. D **58**, 123501 (1998) [gr-qc/9805085].
[39] A. A. Coley and R. J. van den Hoogen, Phys. Rev. D **62**, 023517 (2000) [gr-qc/9911075].
[40] For a review, see P. K. Townsend, hep-th/0308149, in Proceedings of the 14th International Congress on Mathematical Physics (ICMP 2003) (to be published).
[41] P. K. Townsend and M. N. R. Wohlfarth, Phys. Rev. Lett. **91**, 061302 (2003) [hep-th/0303097].
[42] R. Emparan and J. Garriga, J. High Energy Phys. **05**, 028 (2003) [hep-th/0304124].
[43] L. Cornalba and M. S. Costa, Phys. Rev. D **66**, 066001 (2002) [hep-th/0203031].
[44] N. Ohta, Phys. Rev. Lett. **91**, 061303 (2003) [hep-th/0303238].
[45] S. Roy, Phys. Lett. B **567**, 322 (2003) [hep-th/0304084].
[46] M. N. R. Wohlfarth, Phys. Lett. B **563**, 1 (2003) [hep-th/0304089].
[47] C. M. Chen, P. M. Ho, I. P. Neupane, and J. E. Wang, J. High Energy Phys. **07**, 017 (2003) [hep-th/0304177].
[48] C. M. Chen, P. M. Ho, I. P. Neupane, N. Ohta, and J. E. Wang, J. High Energy Phys. **10**, 058 (2003) [hep-th/0306291].
[49] N. Ohta, Prog. Theor. Phys. **110**, 269 (2003) [hep-th/0304172].
[50] I. P. Neupane, hep-th/0311071.
[51] P. G. Vieira, Class. Quant. Grav. **21**, 2421 (2004).
[52] E. Bergshoeff, A. Collinucci, U. Gran, M. Nielsen, and D. Roest, Class. Quantum Grav. **21**, 1947 (2004) [hep-th/0312102].

[53] J. G. Russo, hep-th/0403010.
[54] S. W. Hawking and I. G. Moss, Phys. Lett. **110B**, 35 (1982).
[55] A. A. Starobinsky, in *Fundamental Interactions* (MGPI, Moscow, 1984), p. 55.
[56] A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990).
[57] S. J. Rey, Nucl. Phys. **B284**, 706 (1987).
[58] M. Mijic, Phys. Rev. D **42**, 2469 (1990).
[59] M. Mijic, Int. J. Mod. Phys. A **6**, 2685 (1991).
[60] A. D. Linde, D. A. Linde, and A. Mezhlumian, Phys. Rev. D **49**, 1783 (1994) [gr-qc/9306035].
[61] A. D. Linde, Phys. Rev. D **58**, 083514 (1998) [hep-th/9802038].
[62] L. Dyson, J. Lindesay, and L. Susskind, J. High Energy Phys. **08**, 045 (2002) [hep-th/0202163].
[63] A. H. Guth and E. J. Weinberg, Nucl. Phys. **B212**, 321 (1983).
[64] F. Leblond, D. Marolf, and R. C. Myers, J. High Energy Phys. **06**, 052 (2002) [hep-th/0202094].
[65] S. W. Hawking, I. G. Moss, and J. M. Stewart, Phys. Rev. D **26**, 2681 (1982).
[66] R. Bousso, O. DeWolfe, and R. C. Myers, Found. Phys. **33**, 297 (2003) [hep-th/0205080].