Compact dimensions and their radiative mixing

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For one and two dimensional field theory compactifications we compute in the DR scheme the full dependence on the momentum scale (q) of the one-loop radiative corrections to the 4D gauge coupling. Imposing the discrete shift symmetry of summing the infinite towers of associated Kaluza-Klein (KK) modes, it is shown that higher (dimension) derivative operators are radiatively generated as one-loop counterterms for the case of two (but not for one) compact dimension(s). They emerge as a ''radiative mixing'' of effects (Kaluza-Klein infinite sums) associated with both compact dimensions. Particular attention is paid to the link of the one-loop corrections with their counterparts computed in infrared regularized 4D N=1 heterotic string orbifolds with N=2 sectors. The correction from these sectors usually ignores higher order terms in the IR string regulator ($\lambda_s \to 0$) of type $\lambda_s \ln \alpha'$ ($\alpha' \neq 0$), but these become relevant in the field theory limit. Such terms ultimately re-emerge in pure field theory calculations of $\Pi(q^2)$ as higher dimension one-loop counterterms. We stress the importance of such terms for the unification of gauge couplings and for the predicted value of the string scale.

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I. RADIATIVE CORRECTIONS TO GAUGE COUPLINGS

One-loop radiative corrections to the 4D gauge couplings induced by compact dimensions were extensively studied in the past. In general in a 4D renormalizable model such as the Standard Model (SM) or the Minimal Supersymmetric Standard Model (MSSM), the one-loop ''running'' of the gauge couplings is logarithmic. If these models are considered as low energy limits of higher dimensional models, additional corrections to this ''running'' exist. These are associated with compact dimensions and induced by the corresponding Kaluza-Klein (KK) states which are charged under the gauge group of the model. Such corrections were analyzed in effective field theory (see, for example, Refs. [1–4]) and in string theory models [5–7].

In an effective field theory model with one or two additional compact dimensions one can compute the one-loop correction to the 4D gauge coupling by summing up individual contributions of the Kaluza-Klein states in the loop (Fig. 1). The correction is usually evaluated on-shell ($q^2=0$) and this is particularly true for the string calculations, which in a more general setup also include the additional effect of the winding modes (if present). The coupling corrected by this one-loop threshold correction depends on the UV regulator/cutoff which provides an indication of the UV behavior of the theory. Effective field theory calculations of the one-loop correction $\Pi(q^2=0)$ [8–10] show remarkable quantitative agreement with heterotic string results at ''large'' compactification radii. See however Ref. [11] for a further discussion on the link between these approaches.

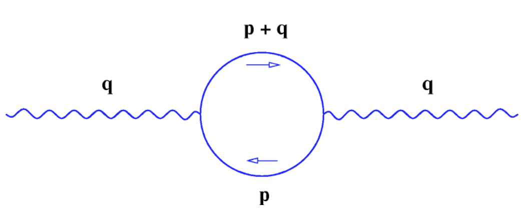

FIG. 1. One-loop diagram contributing to the gauge couplings, with a fermion of mass M_n and its associated Kaluza-Klein tower in the loop. Its expression $\Pi_{\mu\nu}(q^2)=\Pi(q^2)(q_\mu q_\nu - g_{\mu\nu}q^2)$ for $q^2 \neq 0$ can be read from Eq. (1) for one or two compact dimensions.

The 4D gauge coupling obtained as above [hereafter denoted $\alpha(0)$] is usually regarded as the coupling at some ''high'' (compactification) scale [1]. Below the compactification scale it is usually assumed that a 4D theory and corresponding logarithmic ''running'' (in q^2) apply. This is indeed the case under the assumption that the massive Kaluza-Klein states decouple at a momentum scale q above or of the order of the compactification scale(s). In general such decoupling is true for a finite number of states. However, in the case of evaluating the contribution of many *infinite*-level towers of Kaluza-Klein states such situation may turn out to be slightly different.[1] To illustrate this we use an effective field theory model to analyze the more general case of $\Pi(q^2 \neq 0)$ for the one-loop correction (Fig. 1). This will reveal a new effect, present when summing over infinite towers of KK modes. In such case it turns out that higher dimensional operators are radiatively generated as *one-loop counterterms* for the case of two (but not for one) compact dimension(s). This is a result of a (one-loop) ''mixing'' of the two contributions associated each with one compact dimension. Such counterterms are not present if the KK towers are truncated to any large number of modes. We discuss in detail the link of such higher dimension operators in our field theory approach with one-loop heterotic string calculations and their (dis)agreement. Special attention is paid below to the regularization of the divergent series of integrals involved, performed in a gauge invariant way.

To begin with, let us consider the general structure of the one-loop correction in two simple 4D toy models which have one and two additional compact dimensions, respectively.

[1] At the technical level and from a 4D point of view this is related to whether all the series which sum Kaluza-Klein radiative effects from compact dimensions are (uniformly) convergent and can be integrated term by term.

We assume each model has a gauge group G with 4D tree level gauge coupling α, and that they are compactified on a one- and two-dimensional orbifolds, respectively. For our discussion the exact details of compactification are somewhat unimportant and one can work in the setup presented in Ref. [1]. 4D $N=1$ supersymmetry is a necessary ingredient to ensure only wave-function-induced corrections to the 4D gauge coupling. To illustrate the main point one can use the QED action in 5D and 6D, respectively, to perform a one-loop calculation of the vacuum polarization diagram in Fig. 1 with a fermion in the loop and its associated tower of KK states. The result obtained is more general and applies to the non-Abelian case too. We use the dimensional regularization scheme (DR) for the UV divergences. Following standard calculations (see Appendix A in Ref. [1]), after performing the traces over the Dirac γ-matrices and with the notation $\Pi_{\mu\nu}(q^2)=\Pi(q^2)(q_\mu q_\nu - g_{\mu\nu}q^2)$ one has[2]

$$\Pi(q^2)=\alpha(2\pi)^\epsilon \frac{\beta}{4\pi}\sum_{\underline{n}}{}' \int_0^1 dx\, 6x(1-x)\Gamma(\epsilon/2) \times\left(\frac{\mu^2}{\pi[M_{\underline{n}}^2+x(1-x)q^2]}\right)^{\epsilon/2}. \tag{1}$$

Here β is the one-loop beta-function coefficient of a state in the loop associated with a KK tower, α is the gauge coupling; μ is the usual finite, nonzero mass scale introduced by the dimensional regularization scheme. Equation (1) is just the familiar 4D result [12] for a state of mass $M_{\underline{n}}$ in the loop, with an additional sum over the KK levels $\underline{n}$. The "primed" sum over $\underline{n}$ runs over all integers $\underline{n}=n\in\mathbf{Z}$ with $n\neq 0$ for one compact dimension and $\underline{n}=(n_1,n_2)$ with $n_{1,2}$ integers and $(n_1,n_2)\neq(0,0)$ for two compact dimensions. We thus exclude this "zero-mode" contribution since we are only interested in the effect of the *massive* Kaluza-Klein modes on the gauge coupling and their decoupling at q^2 smaller than the compactification scales.[2] We also assumed that a discrete "shift" symmetry of the Kaluza-Klein modes/levels $n\to n+1$ holds true, and this imposes the summation over the whole, infinite KK tower(s). One has from Eq. (1),

$$\Pi(q^2)=\alpha(2\pi)^\epsilon \frac{\beta}{4\pi}\int_0^1 dx\, 6x(1-x) \times\sum_{\underline{n}}{}' \int_0^\infty \frac{dt}{t^{1-\epsilon/2}} e^{-\pi t[M_{\underline{n}}^2+x(1-x)q^2]/\mu^2} \tag{2}$$

which simplifies if $q^2=0$,

$$\Pi(0)=\alpha(2\pi)^\epsilon \frac{\beta}{4\pi}\sum_{\underline{n}}{}' \int_0^\infty \frac{dt}{t^{1-\epsilon/2}} e^{-\pi t M_{\underline{n}}^2/\mu^2}. \tag{3}$$

Equation (2) gives the general structure of $\Pi(q^2)$ in models with compact dimensions. The UV region $t\to 0$ is DR regularized. If $M_{\underline{n}}=0$ for some level $\underline{n}$, the exponent in (2) vanishes at $x=0,1$ and then an IR regulator at $t\to\infty$ is also needed. This is introduced by an "infrared" mass shift $\lambda^2\to 0$ of masses $M_{\underline{n}}^2$, ensuring the integral over t is exponentially suppressed at $t\to\infty$ for any $x\in[0,1]$.

$\Pi(0)$ was evaluated in many effective field theory models using UV cutoff regularization, see for example, Refs. [1,4], but such regularizations are not gauge invariant. For generic models with two compact dimensions with/without Wilson lines, $\Pi(0)$ was computed in Refs. [9–11] where the quantitative agreement with its heterotic string counterpart [6,7] was discussed in detail.[3] For one compact dimension $\Pi(q^2)$ was computed in the DR scheme in Ref. [3]. At this point we discuss separately the cases of one and two compact dimensions for $\Pi(q^2)$ to reveal an important difference.

II. ONE COMPACT DIMENSION

Our calculation of $\Pi(q^2)$ for one compact dimension is different from that in Ref. [3], and is performed here in a manner suitable to a later comparison with the case of two compact dimensions. To evaluate $\Pi(q^2)$ we need to know the 4D Kaluza-Klein mass spectrum. This depends on compactification details, but for our purpose we use its most general structure,

$$M_n=\frac{1}{R^2}(n+\rho)^2+\lambda^2, \tag{4}$$

R is the radius of compactification and ρ depends on the orbifold twist or on some additional effects such as Wilson lines vacuum expectation values (vev's). λ may be due to massive initial 5D matter fields. This formula applies, for example, to models with compactification on S^1/Z_2, $S^1/(Z_2\times Z_2)$. In some models λ may actually vanish and if M_n also vanishes for some value of n (if ρ is an integer), the whole exponent in Eq. (2) vanishes for $x=0,1$. Mathematical consistency of Eq. (2) then requires a mass shift of the *whole* tower (zero mode included) by an infrared mass regulator, so we would need to introduce $\lambda\neq 0$ and then take $\lambda\to 0$. For appropriate redefinitions of the parameters ρ, λ, and R, most cases of models with one extra dimension can be recovered. Here we keep R,ρ,λ as arbitrary parameters.

We use Eq. (4) in Eq. (2) and the following result[4] in DR (see Appendix A of Ref. [8]):

$$\int_0^\infty \frac{dt}{t^{1+\epsilon}} \sum_{m\in\mathbf{Z}}{}' e^{-\pi t[\tau(m+\rho)^2+\delta]} = \frac{1}{\epsilon}-\ln\frac{|2\sin\pi[\rho+i(\delta/\tau)^{\frac{1}{2}}]|^2}{\pi e^\gamma \tau(\rho^2+\delta/\tau)}, \quad \delta\geqslant 0, \quad \tau>0. \tag{5}$$

[2]In the 't Hooft gauge [1].

[3]See Ref. [8] for a general field theory computation of $\Pi(0)$ in DR, proper-time and zeta-function regularizations.

[4]Adding a zero-mode contribution to Eq. (5) would cancel the pole $1/\epsilon$ and the $\ln[\pi e^\gamma\tau(\rho^2+\delta/\tau)]$ term.

With the notation $h(x)=x(1-x)$, $\sigma^2\equiv q^2R^2$, and $\nu\equiv\lambda R$ we find from Eq. (2) to order $\mathcal{O}(\epsilon)$,

$$\Pi(q^2)=\alpha\frac{\beta}{4\pi}\Bigg[-\frac{2}{\epsilon}-\ln(4\pi e^{-\gamma})+6\int_0^\infty dx\, h(x)\times\Bigg(\ln\frac{\rho^2+\nu^2+h(x)\sigma^2}{(R\mu)^2}-2\pi[\nu^2+h(x)\sigma^2]^{1/2}-\ln|1-e^{2i\pi\rho}e^{-2\pi[\nu^2+h(x)\sigma^2]^{1/2}}|^2\Bigg)\Bigg]. \tag{6}$$

The dependence of the couplings on q^2 is then

$$\alpha^{-1}(q^2)-\alpha^{-1}(0)=[\Pi(q^2)-\Pi(0)]\alpha^{-1}(0). \tag{7}$$

The first two integrals in (6) give logarithmic and linear terms in qR, depending on the relative size of the parameters involved. The first integral may be regarded as the contribution from a single state of mass equal to that of the zero mode (M_0).

For our later comparison with the two compact dimensions case it is important to notice that the divergence $1/\epsilon$ cancels out in the difference $\Pi(q^2)-\Pi(0)$, to leave a dependence of the one-loop correction on the parameters q, R, and λ only. There are no terms in $\Pi(q^2)$ proportional to q^2/ϵ, which means that higher dimensional (derivative) operators are not generated as one-loop counterterms.[5] The result for the change of the couplings with q^2 is then

$$\alpha^{-1}(q^2)-\alpha^{-1}(0)=\frac{\beta}{4\pi}(\mathcal{J}_1+\mathcal{J}_2+\mathcal{J}_3),$$

$$\mathcal{J}_1\equiv\frac{4}{w}-\frac{5}{3}+2(w-2)(w+4)^{1/2}w^{-3/2}\times\ln\{[(4+w)^{1/2}-w^{1/2}]/2\},$$

$$\mathcal{J}_2\equiv-\frac{3\pi\sigma}{2}\Bigg\{\left(\frac{\nu}{\sigma}\right)^3-\frac{7}{12}\left(\frac{\nu}{\sigma}\right)+\frac{1}{8}\Bigg[3+8\left(\frac{\nu}{\sigma}\right)^2-16\left(\frac{\nu}{\sigma}\right)^4\Bigg]\arctan\frac{\sigma}{2\nu}\Bigg\},$$

$$\mathcal{J}_3\equiv-6\int_0^1 dx\, h(x)\ln\left|\frac{1-e^{2i\pi\rho-2\pi[\nu^2+h(x)\sigma^2]^{\frac{1}{2}}}}{1-e^{2i\pi\rho-2\pi\nu}}\right|^2, \tag{8}$$

where we used the notation $w\equiv q^2/M_0^2=\sigma^2/(\rho^2+\nu^2)$. For $w\ll1$, one has $\mathcal{J}_1=w/5+\mathcal{O}(w^2)$; for $w\gg1$, $\mathcal{J}_1=-5/3+\ln w+\mathcal{O}(1/w)$. Also for $\sigma\ll1$, and ν, fixed, $\mathcal{J}_2=-(\pi/5)\sigma^2/\nu+\mathcal{O}(\sigma^4)$. If σ is fixed and $\nu\ll1$, $\mathcal{J}_2=-9\sigma\pi^2/32+2\pi\nu+\mathcal{O}(\nu^2)$ with the first term giving a "powerlike" (linear) correction in the momentum scale ($\sigma^2\sim q^2$) which is important if $q^2\geqslant1/R^2$. $\mathcal{J}_3$ gives only a mild dependence on the momentum q suppressed for $q^2\geqslant1/R^2$. One may set $\lambda=0$ if the spectrum (4) of the model considered requires it and if ρ is noninteger/nonzero. In such case only the term powerlike in momentum survives in $\mathcal{J}_2$. Equations (8) give the dependence of the couplings on the scale q^2, which is different from that on the UV *cutoff scale* considered in Ref. [1]. The distinctive behavior in q^2 as compared to the 4D case may be used for phenomenology, searches for effects of an extra dimension or unification of gauge couplings in models with a compact dimension. The only parameter in this correction is the scale $1/R$; there is no dependence on the UV regulator/cutoff at one-loop level.

[5]They can however be generated beyond one-loop level.

III. TWO COMPACT DIMENSIONS

The previous analysis can be repeated for two compact dimensions. For the 4D toy-model with two additional compact dimensions the Kaluza-Klein mass spectrum has the general form

$$M^2_{m_1,m_2}=\frac{|m_2-Um_1|^2}{(R_2\sin\theta)^2}, \tag{9}$$

where we introduced the notation $U\equiv U_1+iU_2$ with $U=R_2/R_1\exp(i\theta)$. R_i are the radii of the two compact dimensions. This mass formula can be generalized to T^2/Z_N orbifolds without changing the conclusions below.

An important remark is in place here. The *total* correction $\Pi(q^2)$ includes the contribution of the zero mode (0,0), in addition to that of nonzero modes given by Eq. (2). According to (9) $M_{0,0}=0$ and for x reaching its limits of integration $x=0,1$ the contribution of the zero mode[6] to $\Pi(q^2)$ would have vanishing exponent under the integral over t. This integral would then be divergent in the infrared ($t\to\infty$). A mass shift $M^2_{n_1,n_2}\to M^2_{n_1,n_2}+\lambda^2$ is necessary so that the *total* expression $\Pi(q^2)$ including *massless* modes is well-defined *before* splitting the contributions to $\Pi(q^2)$ into those due to massless and massive modes, respectively. In (2) one sums over massive modes only and the integral over t is indeed well defined for $t\to\infty$ because $M_{m_1,m_2}\neq0$ if $(m_1,m_2)\neq(0,0)$. However, the above discussion requires us to keep the IR regulator in the massive sector as well. In the following the exponential in (2) will therefore be changed to include the (dimensionless) IR regulator λ_0 required by the massless modes,

$$e^{-\pi t(M^2_{m_1,m_2}+x(1-x)q^2)/\mu^2}\to e^{-\pi t[(M^2_{m_1,m_2}+x(1-x)q^2)/\mu^2+\lambda_0^2]},\quad \lambda_0\to0,\quad \lambda\equiv\mu\lambda_0 \tag{10}$$

with λ the infrared mass scale associated with the regulator λ_0. This observation is important because the UV and IR

[6]This is of the form given in Eq. (2) without the sum over the KK levels.

regularization limits, $\epsilon\to 0$ and $\lambda_0\to 0$, respectively, may not "commute" in Eq. (2), even though this equation only sums nonzero modes which have IR-finite contribution.

To evaluate Eq. (2) we use the following result in DR:

$$\int_0^\infty \frac{dt}{t^{1+\epsilon}} \sum_{m_1,m_2}{}' \; e^{-\pi t[\tau|U m_1 - m_2|^2+\delta]} = \frac{1}{\epsilon} + \frac{\pi\delta}{\epsilon}\frac{1}{\tau U_2} - \ln(4\pi e^{-\gamma}\frac{1}{\tau}|\eta(U)|^4) + E\left(\frac{\delta}{\tau}\right), \quad \delta\geqslant 0, \quad \tau>0 \tag{11}$$

with $U=U_1+iU_2$. Equation (11) is valid for $0\leqslant\delta|U|^2/(U_2^2\tau)\leqslant 1$, $0\leqslant\delta/(\tau U_2^2)\leqslant 1$ which are sufficient conditions only. The "primed" sum runs over all integers (m_1,m_2) except the level (0,0) and $\eta(U)$, $E(\delta/\tau)$ are functions defined in the Appendix. The function $E(y)$ is vanishing in the limit $y\to 0$. The result has divergences in ϵ from $t\to 0$ but there are no divergences in δ when $\delta\to 0$ because the integrand is always exponentially suppressed at $t\to\infty$ for $(m_1,m_2)\neq(0,0)$. Note the emergence of the term proportional to $\delta/(\tau\epsilon)$ in addition to[7] $1/\epsilon$ and which will play an important role in the following. This is to be compared to the integral in Eq. (5) where no such term is present. The difference is due to the presence of two sums under the integral in Eq. (11) rather than only one as in the one compact dimension case, Eq. (5).

To compute $\Pi(q^2)$ we apply the substitution (10) in (2) and then use Eq. (11). With the notation $\mathcal{R}^2\equiv R_1 R_2 \sin\theta$ and retaining terms to $\mathcal{O}(\epsilon)$ one finds from (2),

$$\Pi(q^2) = \alpha\frac{-\beta}{4\pi}\left[\frac{2}{\epsilon} + 2\pi\frac{(\lambda\mathcal{R})^2}{\epsilon} + \frac{2\pi}{5}\left(\frac{(q\mathcal{R})^2}{\epsilon} + (q\mathcal{R})^2\ln 2\pi\right) + \ln[4\pi e^{-\gamma}|\eta(U)|^4 U_2(\mu\mathcal{R})^2] + \mathcal{G}(q)\right] \tag{12}$$

with the constraint

$$\lambda^2 + \frac{1}{4}q^2 \leqslant \min\left\{\frac{1}{R_1^2}, \frac{1}{R_2^2}\right\}. \tag{13}$$

This (sufficient) condition is derived from the validity of Eq. (11). In the limit of "removing" the infrared regulator one takes $\lambda\to 0$ or $\lambda^2\ll 1/R_{1,2}^2$ which leaves a condition for the upper value of the momentum scale at which the above result still applies. In (12) the function $\mathcal{G}(q)$ (analytic) also depends on R_1, R_2, λ, but does not depend on the UV regulator ϵ. Its exact expression is not relevant in the following and is given in the Appendix, Eq. (A2). In $\mathcal{G}$ we can safely remove[8] the dependence on the IR regulator λ $(\lambda\to 0)$ to find the result of Eq. (A3).

Note the presence in $\Pi(q^2)$ of the term $(q\mathcal{R})^2/\epsilon$ which does not have a counterpart in the case of one compact dimension. Obviously such term is missed when evaluating only $\Pi(0)$. A somewhat similar term in $\Pi(q^2)$ is $(\lambda\mathcal{R})^2/\epsilon$, since λ^2 and q^2 are on equal footing in $\Pi(q^2)$ in the exponent under the integral over t, see Eq. (2) with the replacement (10). Again, if one had $\lambda=0$ in the (IR finite) massive modes sector, this term would have been missed too.

Following the one compact dimension example, one could in principle write from Eqs. (7) and (12),

$$\alpha^{-1}(q^2) - \alpha^{-1}(0) = \frac{-\beta}{4\pi}\frac{2\pi}{5}\left(\frac{(q\mathcal{R})^2}{\epsilon} + (q\mathcal{R})^2 \times\ln(2\pi) + \frac{5}{2\pi}[\mathcal{G}(q) - \mathcal{G}(0)]\right). \tag{14}$$

Equation (14) shows that the pole $1/\epsilon$ present in both $\Pi(q^2)$ and $\Pi(0)$ cancels out in their difference, similar to the case of one compact dimension. The same applies to the q-independent terms, in particular to the term $(\lambda\mathcal{R})^2/\epsilon$ involving the IR scale λ. One is thus left with the q^2 dependent terms, and of these the most important is that proportional to $(q\mathcal{R})^2/\epsilon$. This term has no equivalent in the case of one compact dimension, see Eqs. (6) and (8). For q^2 close to the compactification $(\text{scales})^2$, $1/R_1^2$ or $1/R_2^2$ the coupling has a pole. Even if $q^2\ll 1/R_1^2$ and $q^2\ll 1/R_2^2$, since $\epsilon\to 0$, one cannot set this term to 0, and a "nondecoupling" effect of the KK modes is manifest. Therefore the limit of scales q well below the compactification scales (hereafter referred to as "infrared") and the UV regularization limit $\epsilon\to 0$ do not commute. As a result a UV-IR "mixing" effect (IR-finite, UV-divergent) exists due to the first term in[9] Eq. (14). The KK level (0,0)—if included—cannot change this picture, because its contribution does not bring in a $\delta/(\tau\epsilon)$ term to Eq. (11) responsible for $(q\mathcal{R})^2/\epsilon$ in Eq. (12).

One concludes that in this regularization setup the Kaluza-Klein nonzero modes give an effect even at momentum scales well below the compactification scale, where one would expect them to be decoupled. The presence of the UV-IR mixing term is a result of considering the effect of an infinite (rather than a "truncated") tower of Kaluza-Klein modes, and as a consequence such "nondecoupling" effect, induced by infinitely many modes, may not be unexpected in the end. It is then puzzling why the term $(q\mathcal{R})^2/\epsilon$ has no counterpart in the one compact dimension case, where we also summed over the whole KK tower. How can we explain this difference? As we discuss later, such term corresponds to

[7]Adding a zero mode (0,0) to Eq. (11) would cancel $1/\epsilon$, but would not cancel the term proportional to δ/ϵ.

[8]This means that the limit $\lambda\to 0$ in $\mathcal{G}$ does not interfere with the ϵ dependence, already isolated in (12).

[9]The term $(\lambda\mathcal{R})^2/\epsilon$ present in $\Pi(q^2)$ or $\Pi(0)$ but not in their difference is itself a similar UV-IR contribution [11].

a counterterm in the action $\mathcal{R}^2 D_M F^{MN} D^K F_{KN}$ which cannot be generated in 5D at one-loop [2] due to Lorentz invariance. At the technical level one can show that $q^2\mathcal{R}^2/\epsilon$ emerges as a one-loop "mixing" of the effects of two compact dimensions: it arises as a mixed contribution between a sum over "original" Kaluza-Klein modes associated with one compact dimension and a "Poisson resummed" (or winding) zero mode[10] of a sum corresponding to the second compact dimension. It is then clear why such term cannot appear in the case of a single compact dimension. This shows explicitly a different behavior of the radiative corrections with respect to the character even/odd of the number of compact dimensions [14] and brings additional effects to those discussed in previous works [1,4].

An immediate question is the regularization dependence of the existence of the term $(q\mathcal{R})^2/\epsilon$. Our comparative analysis shows that the effect exists for two compact dimensions but there is no counterpart for one compact dimension where the same UV regularization was used. This gives some indication that the existence of the term $(q\mathcal{R})^2/\epsilon$ is not the result of a particular UV regularization choice. Further, our previous discussion on the IR regularization does not affect the existence of this term, and finally, the DR scheme used is supposed to provide a UV well-defined and manifestly gauge invariant framework [15]. One may argue that the UV regularization must not affect the IR regime of the theory and that the DR scheme used in this calculation might not respect this condition. However, calculations closely related [11] using an UV regularization with a proper-time cutoff ($t \geq 1/\Lambda^2$) in Eqs. (2), (11) instead of DR, yield a similar UV-IR "mixing" term[11] $(q\mathcal{R})^2 \ln\Lambda$, with the $1/\epsilon$ factor simply replaced by the logarithm of the UV cutoff Λ.

Equations (12) and (14) simply tell us that higher dimension (derivative) operators need to be included for a fully consistent one-loop calculation. This is a significant difference from the previous case of one compact dimension only. Indeed, the presence of the term q^2/ϵ in the effective field theory result shows that for two compact dimensions the DR regularization with minimal subtraction is not sufficient and that higher dimensional operators are radiatively generated/required as *one-loop counterterms*. One such counterterm is $\mathcal{R}^2 D_M F^{MN} D^K F_{KN}$ (for related discussions on this issue see Sec. IV B in Ref. [2]). This is important for it establishes a direct link between the effects of two compact dimensions or their associated infinite KK sums, and the role of higher dimensional operators. In the absence of additional constraints to fix the (otherwise arbitrary) coefficient of such counterterms, the corrections they induce will depend on it with implications for the predictive power of the models. In the case of KK towers "truncated" to a large but finite number of KK modes, such counterterms are not radiatively generated.[12]

We do not address in the following the detailed implications for field theory of such higher dimensional operators, but discuss instead the origin of $(q\mathcal{R})^2/\epsilon$ or equivalently $(q\mathcal{R})^2 \ln\Lambda$ in $\Pi(q^2)$, from a *heterotic* string perspective. This is important because it will show the link between the higher dimensional operators as one-loop counterterms in the field theory approach to $\Pi(q^2)$ and the one-loop radiative effects in string.[13] In doing so we consider that the string provides a "UV completion" of the field theory case, with the latter recovered in the limit $\alpha' \to 0$ of the string, as shown in Refs. [9–11] (also Ref. [8]). A string counterpart of the one-loop correction to gauge couplings considered above is that induced by the $N=2$ sectors of 4D $N=1$ toroidal orbifolds. Such $N=2$ sectors associated with the unrotated two-dimensional torus being one-loop corrections to the gauge couplings due to massive Kaluza-Klein and winding states [5–7]. The (field theory limit of such) string calculation for $\Pi(0)$ does agree with the pure field theory result for $\Pi(0)$ [9] which sums Kaluza-Klein effects only, although the relation between these different approaches is rather subtle [11]. This is particularly true when analyzing the more general case of $\Pi(q^2)$. Let us explain this in detail.

The one-loop string calculation for $\Pi(0)$ [5,6] which sums only massive modes' effects needs itself a regularization, this time in the IR region only. In string theory one ultimately computes a one-loop diagram associated with $\Pi(0)$ rather than $\Pi(q^2)$ which we would need for comparison with Eq. (12). However, since q^2 and λ^2 are on equal footing[14] in $\Pi(q^2)$ of Eq. (12) and also in the exponential in (2) with replacement (10), it is enough to investigate the role of the string counterpart of our λ. This is just the IR regulator in string (hereafter denoted λ_s) which, unlike q^2, is also present in $\Pi(0)$ computed by string, and can still convey some information about $\Pi(q^2 \neq 0)$!

The IR regularized string result for $\Pi(0)$ contains in addition to the well-known one-loop result [6], higher order terms in the IR regulator which in a DR scheme of the IR divergence have for example, the form[15] $\lambda_s \ln\alpha'$. For technical details on how such term can arise in string, from the degenerate orbits of the modular group SL$(2,Z)$, see, for example, Appendix[16] A of Ref. [13] and also the calculation in the Appendix of Ref. [6]. Here the IR string regulator $\lambda_s \to 0$ and $\alpha' \sim 1/M_s^2$ with M_s the string scale. For $\alpha' \neq 0$ the

[10]Poisson resummation in one dimension gives $\Sigma_{n\in\mathbf{Z}} \exp(-\pi t n^2/R^2) = R/\sqrt{t}\,\Sigma_{p\in\mathbf{Z}} \exp(-\pi p^2 R^2/t)$; here n labels original KK modes while p denotes their "Poisson resummed" or dual (winding) modes referred to in the text.

[11]Equation (11) with UV cutoff regularization instead of DR has $\pi\delta/(\epsilon\tau U_2)$ replaced by a term proportional to $\delta\ln\Lambda$ [11].

[12]For more details on the decoupling of infinitely many modes in a $\lambda\phi^4$ theory see Ref. [16].

[13]This can be done even though the string only computes $\Pi(0)$ rather than $\Pi(q^2)$, see later.

[14]By this we mean that in equation (12) there are both $(\lambda\mathcal{R})^2/\epsilon$ and $(q\mathcal{R})^2/\epsilon$ terms.

[15]In a modular invariant IR regularization of the string such α'-dependent terms should be SL$(2,Z)_T$ invariant.

[16]See Eqs. (A1), (A10), and (A12) in Ref. [13]. (A12) brings $\mathcal{O}(\varepsilon)$ term $\varepsilon\ln(T_2U_2)$, $(T_2 \sim R_1R_2/\alpha')$ discussed here with $\varepsilon \to \lambda_s$.

term $\lambda_s \ln \alpha'$ vanishes when $\lambda_s \to 0$ and this explains why it is not kept in the final, infrared regularized string result.

What does this tell us for the pure field theory approach to $\Pi(0)$ or $\Pi(q^2)$ which sums KK effects only? In the field theory limit of the string calculation, one takes $\alpha' \to 0$ (infinite string scale) to suppress string effects (winding modes) but keep those due to *massive* KK states only, considered in field theory. In such case, the value of $\lambda_s \ln \alpha'$ depends on the order of taking the limits of IR regularization $\lambda_s \to 0$ and of field theory $\alpha' \to 0$. This situation applies to other IR regularizations [6,7] of the string as well. We are not aware of any string symmetry which imposes the order to take these limits. The term $\lambda_s \ln \alpha'$ then becomes relevant in the field theory limit. In this limit, λ_s ($\lambda_s \to 0$) is replaced by its field theory counterpart λ^2 ($\lambda^2 \to 0$) while α' plays the role that the UV proper-time cutoff regulator $1/\Lambda^2$ does in the field theory approach. With these replacements, an UV-IR ''mixing'' term (IR finite, UV divergent) should emerge, $(\lambda \mathcal{R})^2 \ln \Lambda$, just as we found in the field theory approach for $\Pi(0)$. But this also tells us something about $\Pi(q^2)$ in field theory. With the observation that λ and q are on equal footing in $\Pi(q^2)$, this ''mixing'' terms implies that one should expect in the field theory limit a term $(q\mathcal{R})^2 \ln \Lambda$ in the proper-time regularization of the UV or $(q\mathcal{R})^2/\epsilon$ in the DR scheme. This is in agreement with our field theory result Eq. (12) where such a term is found, and a strong consistency check of the field theory calculation.

This discussion provides an insight into the role that higher dimension operators play in understanding the link between the *infrared* regularized string result and pure field theory approaches for $\Pi(q^2)$. It implies in addition that corrections to gauge couplings from infrared regularized string calculations should retain the terms of structure $\lambda_s \ln \alpha'$ in the final correction to $\Pi(0)$, if an *exact* agreement with their field theory counterpart is to be maintained.

This discussion has implications for the unification of gauge couplings in 4D supersymmetric models. We refer here to the attempts to match the MSSM unification scale with the (heterotic) string scale value. In MSSM-like models gauge couplings unify at $\sim 2\times 10^{16}$ GeV [17] which is marginally below the predicted string scale $\sim g_{GUT} 5.27 \times 10^{17}$ GeV [5]. Our discussion on the heterotic string shows that for the models addressed the effects of higher dimension counterterms are not included in the one-loop string corrections. As a result the predicted value of the string scale M_s does *not* include the effects from such operators. This finding should be considered when attempting solutions for an *exact* matching of the MSSM unification scale with the heterotic string scale.

IV. FINAL REMARKS AND CONCLUSIONS

For one- and two-dimensional orbifold compactification we considered the general case of evaluating at one-loop level $\Pi(q^2)$ in a manifestly gauge invariant scheme (DR). For these models we discussed comparatively the dependence of the couplings on the momentum scale q^2 and $1/R^2$, and the role of higher dimensional operators as one-loop counterterms. These can be generated when the summation over the *infinite* towers of Kaluza-Klein modes is performed. The analysis showed a different behavior of the one-loop correction with respect to the character even/odd of the number of compact dimensions, with such operators generated for the case of two but not for one compact dimension(s).

For one compact dimension the change of the couplings $\alpha^{-1}(q^2) - \alpha^{-1}(q'^2)$ with respect to the momentum scale is UV regulator independent at *one-loop* level, unlike the case of more common approaches using cutoff regularization [1]. For one compact dimension the results can be used for phenomenology, unification of the gauge couplings and searches for effects from compact dimensions.

For two compact dimensions a similar analysis of the one-loop effects suggests the existence of a correction which couples low (''infrared'') scales below the compactification scales, to UV divergent terms. This implies the existence in this toy-model of some ''nondecoupling'' effects at low energies, due to a ''mixing'' of the two *infinite* towers of Kaluza-Klein states. The emergence of such nondecoupling term in the effective field theory can be reinterpreted and explained simply by the presence—for two compact dimensions—of higher dimensional operators which are required as *one-loop counterterms*.

We investigated in detail the origin of such operators from the heterotic string perspective. The origin of these counterterms can be related to string corrections to $\Pi(0)$ of type $\lambda_s \ln \alpha'$ (with $\lambda_s \to 0$ the IR string regulator) which are usually discarded in the final one-loop string result, since $\alpha' \neq 0$. However, they become relevant in the field theory limit, and also in pure field theory calculations where the two regularization limits (in IR, UV) do not commute. This raises some intriguing issues about the *infrared* problem in heterotic string and its link with higher dimensional one-loop counterterms in field theory.

If the Kaluza-Klein towers are ''truncated'' to a finite number of modes, such operators are not generated. In such case the discrete ''shift'' symmetry of summing over an infinite tower of Kaluza-Klein modes is broken. Under our initial assumption that such symmetry holds, the higher dimensional operators can be seen to account for nonperturbative effects. This is because such operators are ultimately related to effects of a zero ''mode'' of a ''Poisson resummed'' Kaluza-Klein series, i.e., a winding mode (nonperturbative) effect.

It is possible that in fully specified models symmetry arguments may be identified to avoid the presence of such higher dimension operators. Nevertheless we think these findings are important for phenomenology, in particular for the scale of unification of gauge couplings. We argued that one-loop effects from higher dimension counterterms are not included in the (predicted) value of the heterotic string scale and this may be one reason for its (small) mismatch with the MSSM unification scale.

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APPENDIX

The functions $\eta(U)$ and $\mathcal{E}(y)$ used in the text are

$$\eta(U)=e^{\pi i U/12}\prod_{n\geqslant 1}(1-e^{2i\pi n U}),$$

$$\begin{aligned}E(y)=&\frac{\pi y}{U_2}\ln(4\pi e^{-\gamma}\tau U_2^2)-2\ln\frac{\sinh \pi y^{1/2}}{\pi y^{1/2}}\\&+2\pi^{1/2}U_2\sum_{k\geqslant 1}\frac{\Gamma(k+1/2)}{(k+1)!}\left[\frac{-y}{U_2^2}\right]^{k+1}\zeta(2k+1)\\&-\ln\prod_{m_1\geqslant 1}[|1-e^{-2\pi(y+U_2^2m_1^2)^{1/2}}e^{2i\pi U_1 m_1}|^4\\&\times|1-e^{2i\pi U m_1}|^{-4}]\end{aligned} \tag{A1}$$

with $E(y\rightarrow 0)\rightarrow 0$. The function $\mathcal{G}(q)$ used in Eq. (12) is defined as

$$\begin{aligned}\mathcal{G}(q)\equiv&2\ln\pi+2\pi(\lambda\mathcal{R})^2\ln 2\pi+2\int_0^1 dx\,x(1-x)\\&\times\mathcal{E}\{(R_2\sin\theta)^2[\lambda^2+x(1-x)q^2]\}.\end{aligned} \tag{A2}$$

The series of Riemann ζ-functions present in E [uniformly convergent under the conditions of Eqs. (11), (13)] can be integrated termwise. Removing the IR regulator ($\lambda_0\rightarrow 0$ or $\lambda\ll 1/R_{1,2}^2$) gives

$$\mathcal{G}(q)\equiv 2\ln\pi+2\int_0^1 dx\,x(1-x)E[(R_2\sin\theta)^2x(1-x)q^2]. \tag{A3}$$

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