Triviality-quantum decoherence of quantum chromodynamics $SU(\infty)$ in the presence of an external strong white-noise electromagnetic field

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We analyze the triviality-quantum decoherence of Euclidean quantum chromodynamics in the gauge invariant quark current sector in the presence of a very strong external white-noise electromagnetic (strength) field within the context of QCD in the 't Hooft limit of a large number of colors.

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I. INTRODUCTION

For a long time, a very interesting (and conceptually) important problem in quantum field theory has been the correct understanding of the triviality phenomena of interacting fields as a kind of "phase-transition" phenomena depending on external parameters including the famous space–time dimensionality. The basic formalism used to understand such an important phenomea is—until present time—the rewriting of the given interacting quantum field generating functional in terms of the famous Symanzik loop space (even at the lattice) [1–3].

The purpose of this paper is to point out quantum field triviality phenomena in another context, however in a more complicated quantum field theory than those analyzed in the literature which is quantum chronodynamics (QCD) at a large number of colors but in the presence of an external random Abelian field. The main idea is to show that exactly such a triviality result for QCD $[SU(\infty)]$ will be the systematic use of the loop space representation for QCD which, by its turn, allows us to exactly integrate out the external random Abelian field when one is analyzing the QCD $[SU(\infty)]$ on the physical sector (observable) of Abelian quark currents (form factors).

In Sec. II we present our ideas and a complete loop analysis of QCD $[SU(\infty)]$ triviality in the presence of randomness. In Sec. III, we present a path-integral renormalization analysis of the resulting effective random surface theory. Finally in Sec. IV, we apply the previous QCD loop analysis to the important case of nonrelativistic (many-body) field theories.

II. THE TRIVIALITY-QUANTUM DECOHERENCE ANALYSIS

In order to show such a triviality-quantum decoherence on bosonic QCD(∞) let us consider the Euclidean generating functional of the Abelian (for simplicity) quarks currents in the presence of an external white-noise electromagnetic field $B_\mu(x)$, simulating a kind of "dissipative" vacuum structure or quantum external resorvoir acting on the system (see the second reference in Ref. [2]),

$$Z[J_\mu(x),B_\mu(x)]=\langle \det_f^{N_c}(\not{D}(A_\mu,B_\mu,J_\mu)\times\not{D}^*(A_\mu,B_\mu,J_\mu))\rangle_{A_\mu}. \quad (1)$$

Here the Euclidean Dirac operator is explicitly given by

$$\not{D}(A_\mu,B_\mu,J_\mu)=i\gamma_\mu(\partial_\mu+eB_\mu+J_\mu+gA_\mu) \quad (2)$$

with gA_μ denoting the Yang-Mills non-Abelian quantum field configurations averaged in Eq. (1) by means of the usual Yang-Mills path integral, $J_\nu(x)$ is the auxiliary source field associated to the Abelian quark currents and $B_\mu(x)$ is a random external electromagnetic field with a strength field $F_{\mu\gamma}(B)$ satisfying a Gaussian statistics with randomness of intensity $\lambda>0$,

$$E_F\{F_{\mu\gamma}(B)F_{\alpha\beta}(B)(y)\}=\lambda\,\delta^{(D)}(x-y)\cdot(\delta_{\mu\alpha}\delta_{\gamma\beta}-\delta_{\mu\beta}\delta_{\gamma\alpha}). \quad (3)$$

Here E_F denotes the stochastic average on the ensemble ofthe external strength Abelian field $F(B)$.[1]

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[1]It is worthwhile to consider that this Abelian external (divergenceless) white-noise field comes mathematically from a standard Stratonovich-Hubbard parametrization of a nonlocal charged piece of quarks Lagrangean arising from interaction with an external apparatus, namely,

$$\exp\left\{-\frac{(e^2\lambda)}{2}\int_{R^D}dx[\partial^\nu(\bar\psi\gamma^\mu\psi)(x)\cdot\Box^{-2}(x,y)\partial_\nu(\bar\psi\gamma^\mu\psi)(y)]\right\}$$
$$=\int D[F_{\mu\nu}]\exp\left\{-\frac{1}{2\lambda}\int_{R^D}dx(F_{\mu\nu})^2(x)\right\}$$
$$\times\delta^{(F)}(\partial_\beta B_\beta=0)\,\delta^{(F)}[(\partial_\mu F_{\mu\nu})-\Box B_\nu)]$$
$$\times\exp\left\{ie\int_{R^D}(\bar\psi\gamma^\mu\psi)\cdot B_\mu)(x)\right\}.$$

In the bosonic loop space framework [3] we can express the quark functional determinant, Eq. (1)—which was obtained as an effective generating functional for the color singlet quark current after integrating out the Euclidean quark action—as a functional on the bosonic loop space composed of all trajectories $C_{xx}=\{X_\mu(\sigma),X_\mu(0)=X_\mu(T)=x;0\leqslant\sigma\leqslant T\}$,

$$Z[J_\mu(x),B_\mu(x)]=\left\langle \exp-\left\{N_c\sum_{C_{xx}}[\Phi[C_{xx},B_\mu]\times\Phi[C_{xx},J_\mu]Tr_c(W[C_{xx},A_\mu])]\right\}\right\rangle_{A_\mu}, \tag{4}$$

where $\Phi[C_{xx},B_\mu]$ is the usual Wilson-Mandelstan loop variable defined by the random external electromagnetic field $B_\mu(x)$, and $W[C_{xx},A_\mu]$ is the same loop space object, however with a sum path order and defined by the non-Abelian Yang-Mills quantum Euclidean field $A^a_\mu(x)\lambda_a$. Namely,

$$\Phi[C_{xx},B_\mu]=\exp\left(ie\oint_{C_{xx}}B_\mu(X_\beta(\sigma)dX_\mu(\sigma))\right), \tag{5}$$

$$W[C_{xx},A_\mu]=P\left[\exp\left(i\oint_{C_{xx}}A_\mu(X_\beta(\sigma))dX_\mu(\sigma)\right)\right]. \tag{6}$$

The sum over the closed loops C_{xx} with end-point x is given by the proper-time bosonic path integral below

$$\sum_{C_{xx}}=\int_0^\infty\frac{dT}{T}\int d^Dx\int_{X(0)=x=X(T)}D^F[X(\sigma)]\times\exp\left\{-\frac{1}{2}\int_0^T\dot{X}^2(\sigma)d\sigma\right\}. \tag{7}$$

In Ref. [3], the factorization of the color gauge invariant averages of the products of Wilson loops associated to the Yang-Mills fields A_μ at $SU(\infty)$ was presented on the basis of a diagrammatic analysis. As a consequence of this result the nontrivial dynamical content of the generating functional of Abelian quark currents is entirely given by the fermionic functional determinant written in the $SU(\infty)$ bosonic loop space functional with a factorized form in relation to the loop fields entering its (loop space) structural form as given below,

$$-\ln Z[J_\mu(x),B_\mu(x)]_{SU(\infty)}=\left\{\sum_{C_{xx}}\Phi[C_{xx},B_\mu]\Phi[C_{xx},J_\mu]\langle Tr_cW[C_{xx},A_\mu]\rangle_{SU(\infty)}\right\}. \tag{8}$$

In order to show the triviality quantum decoherence of the bosonic loop space generating functional, Eq. (8), when averaging over the quark currents dependence on the external white-noise Abelian field $B_\mu(x)$, we consider the stochastic average of the Wilson-Mandelstan phase factor defined by the Abelian random field with the following result:

$$E_F\{\Phi[C_{xx},B_\mu]\}=E_F\left\{\exp ie\int_{\Sigma(C_{xx})}F_{\mu\upsilon}(x)d\sigma^{\mu\upsilon}(x)\right\}=\left\{-\frac{(e^2\lambda)}{2}\int_{\Sigma(C_{xx})}d\sigma^{\mu\nu}(x)\times\delta^{(D)}(x-y)d\sigma^{\mu\nu}(y)\right\}. \tag{9}$$

Let us analyze the behavior of the loop space functional, Eq. (9), in terms of the metric properties of the surface $\Sigma(C_{xx})$ bounded by the loop $C_{xx}(\sigma)$. In order to analyze such a geometrical behavior of Eq. (9) we consider an explicit parametrization of the (fixed) surface $\Sigma(C_{xx})$ possessing as a boundary the loop C_{xx},

$$\sum(C_{xx})=\{\varphi_\mu(s,\sigma),0\leqslant s\leqslant 2\pi;0\leqslant\sigma\leqslant T\}. \tag{10}$$

In terms of this two-dimensional surface vector parametrization we rewrite the loop functional, Eq. (9), in the coordinate invariant parametrization form, suitable to analyze its geometrical content

$$\ln(E\{\Phi[C_{xx},B_\mu]\})=-\frac{(e^2\lambda)}{2}\int ds\,d\sigma\int ds'\,d\sigma'\sqrt{h(s,\sigma)}\sqrt{h(s',\sigma')}\times\tau^{\mu\nu}(\varphi_\beta(s,\sigma))\tau^{\mu\nu}(\varphi_\beta(s',\sigma'))\times(\delta^{(D)}(\varphi_\beta(s,\sigma)-\varphi_\beta(s',\sigma'))). \tag{11}$$

Here the surface area tensor is given by

$$d\sigma^{\mu\nu}(x_\beta)|_{x_\beta=\Phi_\beta(s,\sigma)}=(\sqrt{h(s,\sigma)}\tau^{\mu\nu}(\varphi_\beta(s,\sigma))ds\,d\sigma) \tag{12}$$

with

$$\sqrt{h(s,\sigma)}=(\sqrt{\det(\partial_a\varphi_\beta\partial_b\varphi^\beta)(s,\sigma)}), \tag{13}$$

$$\tau^{\mu\nu}(\Phi_\beta(s,\sigma)=(\varepsilon^{ab}\partial_a\varphi^\mu\partial_b\varphi^\nu/\sqrt{h}(s,\sigma)). \tag{14}$$

By introducing a regularization form to the singular delta function appearing on the surface function [Eq. (11)]

$$\delta^{(D)}_{(\varepsilon)}(\varphi^{\beta}(s,\sigma)-\varphi^{\beta}(s',\sigma'))=\int_{|k|<1/\varepsilon} d^Dk\,\exp(ik_\alpha(\varphi_\alpha(s,\varphi)-\varphi_\alpha(s',\sigma'))),\tag{15}$$

one obtains as the leading geometrical functional associated to the trivial surface self-intersecting case $(\sigma,s)=(\sigma',s')$, the well-known Nambu-Goto area surface functional [4] (see Sec. III of this work),

$$-\ln\{E(\Phi[C_{xx},B_\mu])\}=\overline{c}(e^2\lambda)\int ds\,d\sigma(\sqrt{h}h^{ab}\partial_a\varphi^\mu\partial_b\varphi^\mu)(s,\sigma).\tag{16}$$

Here $\overline{c}$ is a positive R^D-dimensional constant related to the renormalization parameters ε used on the regularization form, Eq. (15), and somewhat related tho the analogous expected phenomena of dimensional transmutation on QCD $[SU(\infty)]$. Note that we have used the normalization condition of the surface area tensor to obtain the area functional, Eq. (16),

$$\tau^{\mu\nu}(\varphi_\beta(s,\sigma))\tau^{\mu\nu}(\varphi_\beta(s,\varphi))=1.\tag{17}$$

At this point, it is straightforward to see that for a large white-noise external Abelian field $\lambda\to\infty$ [2], the noise averaged Wilson loop on Eq. (1) is somewhat vanishing for any loop C_{xx}. It is worth calling the reader's attention to the fact that for a given fixed noise strength $\lambda\neq 0$, all loops C_{xx} bounding large minimal area surfaces $\Sigma[C_{xx}]$ are suppressed on the bosonic loop path integral, Eq. (8), and leads to a dynamics of Gluon condensates [3].

Note that the same loop C_{xx} appearing in Eq. (9) enters in the definition of all loop space objects in Eq. (8). This result in turns show us that at the very large noise strength limit $\lambda\to+\infty$, we have the strong triviality of $SU(\infty)$ quantum chromodynamics in the sector of the Abelian quark currents, since all closed loops $C_{xx}(\sigma)$ degenerate to the loop base point x, namely,

$$\lim_{\lambda\to\infty} E_B\{Z(J_\mu(x),B_\mu(x))\}=\lim_{\lambda\to\infty}\exp\left\{-\sum_{(C_{xx}(\sigma)\to x)} e^{-\overline{c}\lambda e^2\,\mathrm{Area}(\Sigma[C_{xx}])}\Omega[C_{xx},J_\mu]\langle Tr_C(W[C_{xx},A_\mu])\rangle\right\}=\exp(0)=1.\tag{18}$$

This is the first main conclusion of our paper about the QCD $[SU(\infty)]$ triviality-quantum decoherence.

A second result we wish to present is related to the somewhat different situation of our Abelian random field, Originating from a source described by a manifold of random currents obeying a pure-white-noise statistics in a physical dimensional space–time R^4,

$$\Delta B_\mu(x)=j_\mu(x),\tag{19}$$

with the white-noise (spaghetti-vacuum [4]) current source correlation function

$$E_j\{j_\mu(x)j_\nu(y)\}=\lambda\,\delta^{(4)}(x-y)\,\delta_{\mu\nu}.\tag{20}$$

In order to see the area behavior for the Abelian phase factor $\Phi[C_{xx},B_\mu]$ in Eq. (4), we probe the system vacuum energy by considering a static pair of quark-antiquark interacting with the random electromagnetic field Eqs. (19) and (20).

The binding electromagnetic energy between such static probing charges e, separated by a distance R, is computed by evaluating the energy of the Abelian white-noise field $B_\mu(x)$ in the presence of these static quark sources and given explicitly by the following Wilson loop average [3]:

$$V(R)=\lim_{T\to\infty}-\frac{1}{T}lgE_j\left\{\exp ie\oint_{C(R,T)}B_\mu(x,[j])dX_\mu\right\},\tag{21}$$

where the quark-antiquark static space-time trajectory is given by a rectangle $C_{(R,T)}=\{-T/2\leqslant t\leqslant +T/2;-R/2<\sigma<R/2\}$ and E_j denotes the stochastic average over the vacuum current sources, Eq. (20).

The evaluation of the binding energy $V(R)$ can be more invariantly accomplished by writing it in momentum space and using the dimensional regularization of Bollini and Giambiagi [5], after evaluating explicitly the source average in Eq. (21),

$$V(R)=\lim_{T\to\infty}-\frac{1}{2T}\left[\int\frac{d^Dk}{(2\pi)^D}f_\mu(k;C_{(R,T)})\times\frac{\lambda\,\delta_{\mu\nu}}{(k^2)^2}f_\nu(-k,C_{(R,T)})\right]\tag{22}$$

with the rectangle form factor written as follows:

$$f_\mu(k,C_{(R,T)})=ie\oint_{C_{(R,T)}}e^{-ik_\alpha(\sigma)X_\alpha(\sigma)}\frac{dX_\mu(\sigma)}{d\sigma}.\tag{23}$$

As the rectangles $C_{(R,T)}$ is contained in a two-dimensional subspace of the space-time R^D, we can decompose the vector $\vec{k}$ as $\vec{k}=k_0\vec{e}_0+k_1\vec{e}_1+\hat{k}$, where $\hat{k}$ is the projection of $\hat{k}$ over the subspace perpendicular to the subspace $\{\vec{e}_0,\vec{e}_1\}$ containing $C_{(R,T)}$. In addition, the space coordinate system is chosen so that the x-axis direction coincides with the one defined by the spatial sides of the rectangles $C_{(R,T)}$. This coordinate choice leads us to the solutions

$$f_0(k,C_{(R,T)})=-\frac{4e}{k_0}\sin\left(\frac{k_0T}{2}\right)\sin\left(\frac{k_1R}{2}\right),$$
$$f_1(k,C_{(R,T)})=+\frac{4e}{k_1}\sin\left(\frac{k_0T}{2}\right)\sin\left(\frac{k_1R}{2}\right). \tag{24}$$

After substituting Eq. (24) into Eq. (22), we face the problem of evaluating the following dimensionally regularized integral limit of $T\to\infty$. We get as a result,

$$V(R)=\lim_{T\to\infty}\frac{8(e^2\lambda)}{T}\left\{\int_{-\infty}^{+\infty}\frac{dk_1}{(2\pi)}\frac{\sin^2\left(\frac{k_1R}{2}\right)}{(k_1)^2}\left[\int\frac{d^{\nu-2}\hat{k}}{(2\pi)^{\nu-2}}\left(\int_{-\infty}^{+\infty}\frac{dk_0}{(2\pi)}\frac{(k_0^2+k_1^2)}{k_0^2}\frac{\sin^2\left(\frac{k_0T}{2}\right)}{(k_0^2+k_1^2+\hat{k}^2)^2}\right)\right]\right\}. \tag{25}$$

By using the elementary improper integral formula for the evaluation of the k_0-integrand in Eq. (25),

$$\lim_{b\to\infty}\frac{1}{b}\left\{\int_{-\infty}^{+\infty}\left(1+\frac{a^2}{x^2}\right)\frac{\sin^2(bx)}{(x^2+c^2)^2}\right\}=\frac{2\pi a^2}{c^4}. \tag{26}$$

We arrive at the (partial) result

$$V(R)=+\frac{(e^2\lambda)}{(4\pi)^{D/2-1}}\times\left\{\int\frac{dk_1}{(2\pi)}\frac{\sin^2\left(\frac{k_1R}{2}\right)}{k_1^2}\Gamma\left(\frac{6-\nu}{2}\right)|k_1|^{\nu-4}\right\} \tag{27}$$

with the final result on the dimensional regularized form (a general space-time with a continuum dimension ν) and where we have introduced a Coulomb term (by hand) to Eq. (27) associated to a $1/k^2$ propagator—just for completeness,

$$V(R)=V_{\text{Coul}}(R)+V_{\text{Conf}}(R) \tag{28}$$

with

$$V_{\text{Coul}}(R)=\frac{(e^2\lambda)}{(4\pi)^{(\nu/2)-1}}\left\{\Gamma(\nu-3)\times\frac{\sin\left((\nu-4)\frac{\pi}{2}\right)}{2\pi}\Gamma\left(\frac{4-\nu}{2}\right)\right\}(R)^{-\nu+3} \tag{29}$$

and

$$V_{\text{Conf}}(R)=\frac{(e^2\lambda)}{(4\pi)^{(\nu/2)-1}}\left\{\Gamma\left(\frac{6-\nu}{2}\right)\times\frac{\sin\left(\frac{\pi}{2}(\nu-6)\right)}{2\pi}\Gamma(\nu-5)\right\}(R)^{-\nu+5}. \tag{30}$$

At this point one can see that the potential energy term as given by Eq. (30) at the physical four-dimensional space-time leads to the expected "confining" area behavior and to the stochastic Abelian phase factor

$$E_j\left\{\exp ie\oint_{C(R,T)}B_\mu(x,[j]dX_\mu\right\}\sim\exp\exp\{-\bar{c}T\cdot R(e^2\lambda)\} \tag{31}$$

with $\bar{c}$ a positive adimensional constant.

It is worth commenting that Eq. (29) leads to the usual Coulomb law at $D=4$, namely,

$$V_{\text{Coul}}(R)=-\frac{e^2\lambda}{4\pi R}. \tag{32}$$

III. RANDOM SURFACE DYNAMICAL FACTOR IN THE ANALYTICAL REGULARIZATION SCHEME

Sometimes, it is argued in the literature [3,6] that one should consider a dynamical random surface path-integral sum to the surface functional as given by Eq. (11) in the case of the existence of only trivial self-intersections $(\sigma,s)\equiv\xi=(\sigma',s')=\xi'$ on the domain functional

$$Z[\varphi](g_{\text{bare}})=\frac{1}{Z(0)}\int D^F[\varphi(\xi)]\times\exp\left\{-\frac{1}{2}\int d^2\xi(\varphi^\mu(-\Delta)^{+\alpha}\varphi^\mu)(\xi)\right\}\times\exp\left\{-g_{\text{bare}}\int d^2\xi\,\delta^{(D)}(\varphi_\mu(\xi)-\varphi_\mu(\xi'))\right\}. \tag{33}$$

Here α is a regularizing theory's parameter $\alpha\geqslant 1$.

Let us address the problem of renormalization on this self-avoiding random surface functional Eq. (33). First, we point out that one can safely replace the surface self-avoidance on the path-integral interaction weight by an interaction with the tangent plane at the surface point $\varphi_\mu(\xi)$, namely

$$\delta^{(D)}(\varphi_\mu(\xi)-\varphi_\mu(\xi'))=\delta^{(D)}(\varphi_\mu(\xi)-T_\mu(\xi)), \tag{34}$$

where the tangent plane equation is given by

$$T_\mu(\xi')=T_\mu(\xi)=t_\mu^0\cdot\xi_0+t_\mu^{(1)}\xi_1 \tag{35}$$

with $\{t_\mu^{(0)},t_\mu^{(1)}\}$ denoting the surface tangent vectors at $\varphi_\mu(\overline{\xi})$.

By a simple variable change

$$\varphi_\mu(\xi)\rightarrow\varphi_\mu(\xi)-T_\mu(\xi), \tag{36}$$

we obtain as an effective random surface path integral to be analyzed from a renormalization point of view, the self-avoiding random surface interacting with the origin [6],

$$Z[\varphi](g_b)=\frac{1}{Z(0)}\int D^F[\varphi^\mu(\xi)]\times\exp\left\{-\frac{1}{2}\int d^2\xi(\varphi_\mu(-\Delta)^{+\alpha}\varphi_\mu)(\xi)\right\}\times\exp\left\{-g_b\int d^2\xi\,\delta^{(D)}(\varphi_\mu(\xi))\right\}, \tag{37}$$

where g_b denotes the (positive) bare self-avoiding random surface coupling constant.

It is instructive to point out that the formal perturbation expansion around the massless two-dimensional (2D) fluctuating surface vector position $\{\varphi_\mu(\xi)\}$ is ill defined in the case of $\alpha=1$ in Eq. (37) due to the severe infrared divergences of the associated Laplacean Green function on R^2. As a consequence of the above-mentioned remark, we start from the beginning with the Riesz-Hadamard expression of the Seeley α-power of the Laplacian as written in the kinetic term of Eq. (37),

$$G_\alpha(\xi_1,\xi_2)=(-\Delta)^{-\alpha}(\xi_1,\xi_2)=\frac{e^{-i\pi\alpha}\Gamma(1-\alpha)}{4^\alpha(\pi)^{1/2}\Gamma(\alpha)}|\xi_1-\xi_2|^{2(\alpha-1)}=\int d^2k\,e^{ik(\xi_1-\xi_2)}|k|^{-2\alpha}. \tag{38}$$

We, thus, renormalize Eq. (33) from Eq. (37) by means of the renormalization prescription at the physical case of $\alpha=1$ (pure Laplacian),

$$g_b=\frac{g_{\text{ren}}}{(1-\alpha)^{D/2}} \tag{39}$$

$$Z_R[\varphi_\mu](g_{\text{ren}})=\lim_{\substack{\alpha\to 1\\ \alpha>1}}Z[\varphi_\mu](g_b(\alpha)). \tag{40}$$

Let us show that Eq. (40) is a well-defined formal power expansion in the renormalized coupling constant g_{ren} as given by Eq. (39).

In order to show this result, we make the power expansion of the α-regularized path integral, Eq. (37),

$$Z_R[\varphi_\mu](g_{\text{ren}})=\sum_{\ell=0}^{\infty}\frac{(-g_b)^N}{N!}\left\{\prod_{j=1}^{N}\int d^2\xi_j\,\det{}^{-D/2}[G_\alpha(\xi_i,\xi_j)]\right\}. \tag{41}$$

The particulars of Eq. (41) for each N under the renormalization prescription, Eq. (39), are a straightforward consequence of the following properties:

First,

$$\lim_{\substack{\alpha\to 0\\ \alpha>1}}(\varphi_\alpha(\xi_1,\xi_2))=\lim_{\substack{\alpha\to 1\\ \alpha>1}}\left\{\frac{e^{-i\pi\alpha}\Gamma(1-\alpha)}{4^\alpha\pi^{1/2}\Gamma(\alpha)}(0)^{2\alpha-1}\right)=0. \tag{42}$$

Second,

$$\lim_{\substack{\alpha\to 1\\ \alpha>1}}\det\begin{bmatrix}G_\alpha(\xi_1,\xi_2) & G_\alpha(\xi_1,\xi_2)\\ G_\alpha(\xi_2,\xi_1) & G_\alpha(\xi_2,\xi_1)\end{bmatrix}=\lim_{\substack{\alpha\to 1\\ \alpha>1}}\left[-\frac{e^{-2\pi i\alpha}}{4^{2\alpha}\pi}\left(\frac{\Gamma(1-\alpha)}{\Gamma(\alpha)}\right)^2\right](|\xi_1-\xi_2|)^{4(\alpha-1)}=\frac{C_2}{(1-\alpha)^2}. \tag{43}$$

Third,

$$\lim_{\substack{\alpha\to 1\\ \alpha>1}}\det\begin{bmatrix}0 & G_\alpha(\xi_1,\xi_2) & G_\alpha(\xi_1,\xi_2)\\ G_\alpha(\xi_2,\xi_1) & 0 & G_\alpha(\xi_2,\xi_1)\\ G_\alpha(\xi_3,\xi_1) & G_\alpha(\xi_3,\xi_2) & 0\end{bmatrix}=\frac{e^{-3\pi i\alpha}}{4^{3\alpha}\pi\frac{3}{2}}\left(\frac{\Gamma(1-\alpha)}{\Gamma(\alpha)}\right)^3(1+1')=\frac{C_3(1)}{(1-\alpha)^3}. \tag{44}$$

Finally,

$$\lim_{\substack{\alpha\to 1 N\times N\\ \alpha>1}} \det[G_\alpha(\xi_1,\xi_2)] = \frac{e^{-\pi i\alpha N}}{4^{N\alpha}\pi^{N/2}}\frac{1}{(1-\alpha)^N}C_N, \qquad (45)$$

with

$$C_N=\det[A_{i,j}]=-(N-1)(-1)^N, \qquad (46)$$

where $[A_{i,j}]$ is the matrix whose entries are

$$[A_{i,j}]=\begin{cases}0 & \text{if } i=j,\\ 1 & \text{if } i\neq j.\end{cases} \qquad (47)$$

As a consequence of the above analysis, we obtain our renormalization result for Eq. (41) at the limit $\alpha\to 1$,

$$Z_R[\varphi_\mu](g_{\text{ren}})=\sum_{\ell=0}^{\infty}\frac{(-g_{\text{ren}})^\ell}{\ell!}C_\ell\cdot A^\ell<\infty \qquad (48)$$

with $A=\int d^2\xi$ denoting the internal random surface area and $C_\ell=e^{-i\pi\ell}/4^\ell pi^{\ell/2}(-1)^\ell(1-\ell)$.

Finally, let us complement our studies on the area behavior of the surface functional as given by Eq. (9) in a more physical way. Let us see its area behavior by using distribution theory on surfaces [6]. First, we introduce a R^D vector basis along the coordinate lines $\partial\phi^\mu/\partial\sigma$ and $\partial\phi^\mu/\partial s$. We have, thus, the surface-intrinsic distributional results

$$\begin{aligned}&\delta^{(D)}(\phi_\mu(s,\sigma)-\phi_\mu(s',\sigma'))\\ &\quad=\delta_\varepsilon^{(D-2)}(0)\left(\frac{1}{\sqrt{h((s,\sigma)}}\delta^{(1)}(s-s')\delta^{(1)}(\sigma-\sigma')\right)\end{aligned} \qquad (49)$$

and

$$d\sigma_{\mu\nu}(x)|_{x^\alpha=\phi^\alpha(s,\sigma)}=\sqrt{h(s,\sigma)}\cdot ds\, d\sigma\cdot\tau_{\mu\nu}(\phi_\alpha(s,\sigma)). \qquad (50)$$

Here $\delta_\varepsilon^{(D-2)}(0)$ means a regularized form of the delta function singular value $\delta^{(D-2)}(0)$ and physically related to the nontrivial structure of the nonperturbative phenomenon of the coupling constant dimensional transmutation (see the Appendix of the first reference in Ref. [4]).

After substituting Eqs. (49) and (50) into the random surface term, Eq. (9), we obtain

$$\begin{aligned}\text{Eq. }(11)&=\frac{e^2\lambda}{2}\int_0^T d\sigma\int_0^{2\pi}ds\sqrt{h(\phi^\alpha(s,\sigma))}\int_0^T d\sigma'\\ &\quad\times\int_0^{2\pi}ds'\int_0^{2\pi}ds'\sqrt{h(\phi^\alpha(s',\sigma'))}\\ &\quad\times\left\{\delta_\varepsilon^{(2)}(0)\frac{\delta(\sigma-\sigma')\delta(s-s')}{\sqrt{h(\phi^\alpha(s',\sigma'))}}\right\}\\ &=\frac{e^2\lambda}{2}\int_0^T d\sigma\int_0^{2\pi}d\sigma\sqrt{h(\phi^\alpha(s,\sigma))}\\ &=\text{Area}\left(\sum_{C_{xx}}\right)\frac{e^2\lambda}{2}.\end{aligned} \qquad (51)$$

IV. THE NONRELATIVISTIC CASE

In this section, we apply the analysis presented in Sec. II for quantum chromodynamics at the 't Hooft limit in a nonrelativistic finite-temperature nonlinear Schrödinger theory.

Let us start our analysis by considering the partition functional of the following Schrödinger bosonic many-body field theory with a quartic interaction at the temperature $T=k\beta$ (k denotes the Boltzmann constant) in the physical space R^3 and the partition functional is written in the form of a Bell-Wiegel path integral [7]

$$\begin{aligned}Z[T,\vec B]=&\int_{\psi(r,0)=\psi(r,\beta)}D^F[\psi(r,t)]\int_{\tilde\psi(r,0)=\tilde\psi(r,\beta)}D^F[\overline{\psi}(r,t)]\exp\left\{-\frac12\int_0^\beta dt\int_\Omega d^3r\,\psi^*(r,t)\left[-\frac{\hbar^2}{2m}\left(i\vec\nabla-\frac{e\hbar}{mc}\vec B\right)^2\right.\right.\\ &\left.\left.+\frac{\partial}{\partial t}\right]\psi(r,t)\right\}\exp\left\{-\frac12\int_0^\beta dt\int_\Omega d^3r\,d^3r\cdot|\psi(r,t)|^2V(r-r')|\psi(r',t)|^2\right\}.\end{aligned} \qquad (52)$$

Note the presence of the external random magnetic vector potential supposed to satisfy the white-noise statistics with randomness strength λ,

$$E_B\{(\text{rot}\,\vec B)_i(\vec r)(\text{rot}\,\vec B)_j(\vec r')\}=\lambda\,\delta^{(3)}(\vec r-\vec r')\,\delta_{ij} \qquad (53)$$

and the nonrelativistic field excitations interacting through a short-range pair potential $V(r-r')$.

At this point, we rewrite the partition functional by means of Siegert's trick of reducing the nonlocal spatial pair interaction by an independent interaction of each Schrödinger field excitation with a fluctuating external scalar field $\phi(\vec r,t)$ with Gaussian (nonwhite) statistics,

$$E_\phi\{\phi(\vec r,t)\phi(\vec r',t')\}=V(\vec r-\vec r')\,\delta(t-t'). \qquad (54)$$

One finds, thus, the following result for the partition functional written as statistics averages over ensembles of the physical random magnetic field rot $\vec{B}(r,t)$ and the auxiliary scalar field $\phi(\vec{r},t)$. Namely

$$E_B\{Z(T,\vec{B})\}=E_B\left\{E_\phi\left[\det{}^{-1/2}\left(\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\left(i\vec{\nabla}-\frac{e\hbar}{mc}\vec{B}\right)^2+i\phi(\vec{r},t)\right)\right]\right\}. \quad (55)$$

Let us go from the field path integrals on Eq. (55) to the ensemble of spatial loops through a loop expansion for the functional determinant resulting from integrating out the Schrödinger bosonic matter quantum fields. It yields as a result the following functional defined on the bosonic three-dimensional loop space $\{\vec{x}(\sigma),0\leqslant\sigma\leqslant\beta,\vec{x}(0)=\vec{x}(\beta)=\vec{r}\}$:

$$\ln\left[\det{}^{-1/2}\left(\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\left(i\vec{\nabla}-\frac{e\hbar}{mc}\vec{B}\right)^2+i\phi(r,t)\right)\right]$$
$$=+\frac{1}{2}\left\{N\int_\Omega d^3r\left[\int_{\vec{x}(0)=\vec{r}}^{\vec{x}(\beta)=\vec{r}}D^F[\vec{x}(\sigma)]\right.\right.$$
$$\times\exp\left(-\frac{1}{2}m\int_0^\beta(\vec{x}(\sigma))^2\,d\sigma\right)$$
$$\times\exp\left(\frac{ie}{\hbar c}\int_0^\beta\vec{B}(\vec{x}(\sigma))\dot{\vec{x}}(\sigma)d\sigma\right)$$
$$\left.\left.\times\exp\left(-\int_0^\beta\phi(\vec{x}(\sigma),\sigma)d\sigma\right)\right]\right\}, \quad (56)$$

where we have introduced explicitly the integer N, given by the number of different bosonic matter species.

After substituting the purely bosonic loop space, Eq. (56), into the statistics averages as given by Eq. (55) and evaluating them by means of a cummulant expansion (in a generic form) and valid, at least for the limit $N\to 0$ [1],

$$E\{e^{Nf}\}=\exp\{N\langle f\rangle+\tfrac{1}{2}N^2(\langle f^2\rangle-\langle f\rangle^2)+O(N^3)\}, \quad (57)$$

one obtains explicitly that the dominant behavior of the random magnetic field average on Eq. (56) is governed by the three-dimensional analogous of that area-surface functional, Eq. (9),

$$E_{\vec{B}}\left\{\exp\left(\frac{ie}{\hbar c}\int_\Sigma(\mathrm{rot}\,\vec{B})(\Sigma)d\vec{\sigma}\right)\right\}$$
$$=\exp\left\{-\frac{\lambda e^2}{\hbar^2c^2}\int_{\Sigma_r}\int_{\Sigma_{r'}}d\vec{\sigma}(\vec{r})\,\delta^{(3)}(\vec{r}-\vec{r}\,')d\vec{\sigma}(\vec{r}\,')\right\}, \quad (58)$$

where Σ is the "minimal" area surface bounded by the bosonic closed contour (loop) $\vec{x}(\sigma)$ entering on the loop path integral, Eq. (56).

As a consequence of Eq. (58), one can see that for a large white-noise magnetic field strength $\lambda\to\infty$, this averaged phase factor is only nonzero for a surface Σ of zero area, which is equivalent to the suppression of the quantum phenomena and reducing the quantum gas partition functional, Eq. (52), to a classical gas partition functional since all closed quantum trajectories reduce to the loop base point (see theorem 10.1 in second reference of Ref. [7]).

Hence, one can see again that quantum phenomena in fluctuating magnetic field can be viewed as quantum phenomena in a dissipative media that destroys quantum phase coherence and leading to the theory's triviality [8].

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