Layered Higgs phase as a possible field localization on a brane

P. Dimopoulos*
Dipartimento di Fisica, INFN-Rome2 Universita di Roma "Tor Vergata," I-00133, Rome, Italy

K. Farakos†
Physics Department, National Technical University, 15780 Zografou Campus, Athens, Greece
(Received 14 May 2004; published 17 August 2004)

So far it has been found by using lattice techniques that in the anisotropic five-dimensional Abelian Higgs model, a layered Higgs phase exists in addition to the expected five-dimensional one. The exploration of the phase diagram has shown that the two Higgs phases are separated by a phase transition from the confining phase. This transition is known to be first order. In this paper we explore the possibility of finding a second-order transition point in the critical line that separates the first-order phase transition from the crossover region. This is shown to be the case only for the four-dimensional Higgs layered phase while the phase transition to the five-dimensional broken phase remains first order. The layered phase serves as the possible realization of four-dimensional spacetime dynamics, which is embedded in a five-dimensional spacetime. These results are due to gauge and scalar field localization by confining interactions along the extra fifth direction.

DOI: 10.1103/PhysRevD.70.045005 PACS number(s): 11.15.Ha

I. INTRODUCTION: MOTIVATION

Since the mid-1980s lattice gauge models with anisotropic couplings defined in higher D-dimensional spaces have been proposed. These models may exhibit, through a phase transition, a phase that is Coulombic in $(D-1)$-dimensions and shows confinement along the remaining dimension. In fact, this was the result of Fu and Nielsen using mean field techniques in a five-dimensional pure U(1) gauge theory with anisotropic couplings [1]. This new phase was called layered.

The Monte Carlo analysis that followed [2] supported the mean field results and helped to get a more precise picture of the phase diagram [3]. Also in Ref. [4] the orders of the phase transitions have been analyzed.[1]

In addition, as may have been expected, consideration of the interaction with a scalar particle leads to a richer phase diagram. Actually, the exploration of the phase diagram of the model for various sets of lattice parameters values provides strong evidence that the layer phase is stable and appears either in a Higgs phase for the U(1) case [7,8], or in a Coulomb phase for a SU(2) adjoint Higgs model[2] [11].

Since gauge theories defined on a $D>4$ spacetime are known to be nonrenormalizable, an explicit cutoff Λ has to be introduced [12]. Therefore the theory is to be considered as an effective theory that emerges from a more fundamental renormalizable theory (for example, the string theory). For the U(1) gauge field the introduction of the cutoff Λ leads to the admission of the strong coupling phase to be the interesting phase for the five-dimensional theory. As a consequence the lattice methods have to be used as the unavoidable nonperturbative tool for the study of the system.

Up to now the Monte Carlo results show that the transition between the five-dimensional strong coupling phase and the layered Higgs phase is first order. A multilayer structure arises that supports the idea of the confinement along the extra dimension [8,11]. A crucial question may arise: is there any possibility for this phase transition to be of second order? We work on this possibility and we look for a second-order ending point along the first-order critical line.[3] This would give evidence for the layer mechanism to be more realistic and useful in scenarios concerning the localization of the fields on the four-dimensional subspace.

Before proceeding to the lattice model let us present the action of the U(1) Higgs model in five dimensions, which in principle could inspire the lattice action used in the sequel for the numerical simulation. We assume a five-dimensional anti–de Sitter space (AdS_5) with one warped extra dimension. In general the metric reads

$$ds^2=\alpha^2(z)[dx_0^2-d\vec{x}^2]-dz^2. \tag{1.1}$$

We consider $\eta_{\mu\nu}$ to be the four-dimensional Minkowski metric and $\alpha(z)$ the warp factor. We do not need to define explicitly the form of the warp factor. We only require that it goes to zero as $z\to\infty$ ([16–19]). Hence the five-dimensional metric is written

*Email address: dimopoulos@roma2.infn.it
†Email address: kfarakos@central.ntua.gr

[1]It has to be noted that for non-Abelian gauge theories the layer phase exists in six dimensions [1,2]. For the lattice realization of the four-dimensional confining phase in a five-dimensional non-Abelian gauge theory in the context of a *compactified* extra dimension the reader may refer to Refs. [5,6].

[2]Recently a paper appeared [9] that presents a nonperturbative study of the Dvali-Shifman mechanism [10] of the gauge localization on a brane. For that reason a SU(2) gauge theory with an adjoint scalar, whose mass parameter is space dependent, is employed in 3D.

[3]A similar behavior has been seen in the U(1) Higgs model in 4D [13], in the SU(2) Higgs model in 3D [14], and in the SU(2) adjoint Higgs model in 3D [15].

$$g^{MN}=\left(\frac{1}{\alpha^2(z)}\,\eta^{\mu\nu},-1\right). \tag{1.2}$$

We consider now that in such a space we define a five-dimensional Abelian Higgs model, the action of which reads

$$\begin{aligned}S&=S_{\text{gauge}}+S_{\text{scalar}}=-\frac{1}{4g_5^2}\int d^5x\sqrt{g}\;\;F_{MN}F_{KL}g^{MK}g^{NL}\\&\quad+\int d^5x\sqrt{g}\;\;[D_M\Phi^*D_N\Phi g^{MN}-V(\Phi)]\\&=\int d^4x dz\bigg[-\frac{1}{4g_5^2}F_{\mu\nu}F_{\kappa\lambda}\eta^{\mu\kappa}\eta^{\nu\lambda}\\&\quad-\frac{\alpha^2(z)}{2g_5^2}F_{\mu5}F_{\nu5}\eta^{\mu\nu}\bigg]\\&\quad+\int d^4x dz[\alpha^2(z)D_\mu\Phi^*D_\nu\Phi\,\eta^{\mu\nu}\\&\quad-\alpha^4(z)D_z\Phi^*D_z\Phi-\alpha^4(z)V(\Phi)].\end{aligned} \tag{1.3}$$

We note that the uppercase indices refer to the 5D space, $M,N,K,L=0,\ldots,4$ and the lowercase Greek indices to the 4D space, i.e., $\mu,\nu,\kappa,\lambda=0,\ldots,3$. It is obvious that the scalar field Φ depends on the five-dimensional space (x,z). Then we use the rescaling $\alpha(z)\Phi=\varphi$ for the scalar field. In the rather general case where the quartic scalar potential is considered, the scalar action takes the form

$$\begin{aligned}S_{\text{scalar}}&=\int d^4x dz[D_\mu\varphi^*D^\mu\varphi-\alpha^2(z)D_z\varphi^*D_z\varphi\\&\quad-M(z)^2\varphi^*\varphi-\lambda(\varphi^*\varphi)^2],\end{aligned} \tag{1.4}$$

where $M^2(z)=\alpha^2(z)m^2+[\alpha'(z)]^2+\frac{1}{2}[\alpha^2(z)]''$.[4]

It is a trivial matter for the action to be analytically continued to the Euclidean space from which the lattice action can be defined after following the usual methods for discretization. Therefore we take

$$\begin{aligned}S_L&=S_{\text{gauge}}+S_{\text{scalar}}\\&=\beta_g\sum_x\sum_{1\leqslant\mu<\nu\leqslant4}[1-\cos U_{\mu\nu}(x)]+\sum_x\sum_{1\leqslant\mu\leqslant4}\beta_g'\\&\quad\times[1-\cos U_{\mu5}(x)]+\beta_h\sum_x\sum_{1\leqslant\mu\leqslant4}[\varphi_L(x)\\&\quad-U_{\hat\mu}(x)\varphi_L(x+a\hat\mu)]^*[\varphi_L(x)-U_{\hat\mu}(x)\varphi_L(x+a\hat\mu)]\\&\quad+\beta_h'\sum_x[\varphi_L(x)-U_{\hat5}(x)\varphi_L(x+a\hat5)]^*[\varphi_L(x)\\&\quad-U_{\hat5}(x)\varphi_L(x+a\hat5)]+\sum_x m_L^2\varphi_L^*(x)\varphi_L(x)\\&\quad+\beta_R(\varphi_L^*(x)\varphi_L(x))^2],\end{aligned} \tag{1.5}$$

We denote by $\varphi_L(x)$ the lattice scalar field and

$$\begin{aligned}U_{\mu\nu}(x)&=U_\mu(x)U_\nu(x+a\hat\mu)U_\mu^\dagger(x+a\hat\nu)U_\nu^\dagger(x),\\U_{\mu5}(x)&=U_\mu(x)U_5(x+a\hat\mu)U_\mu^\dagger(x+a\hat5)U_5^\dagger(x)\end{aligned} \tag{1.6}$$

are the plaquettes on the four-dimensional space and along the fifth direction, respectively The U's are the links for the gauge field on the lattice.[5] They are explicitly given by $U_M=e^{iaA_M}$ (with $M=1,\ldots,5$). The primed couplings refer to the interactions along the extra dimension. Moreover, as it can be noticed from the corresponding continuous action, the couplings obey certain relationships, which depend on the warp factor.[6] Hence we have

$$\beta_g'=\alpha^2(x_T)\beta_g,\quad \beta_h'=\alpha^2(x_T)\beta_h,\quad \lambda=\frac{4\beta_R a}{\beta_h^2}, \tag{1.7}$$

$$a^2M^2(x_T)=\frac{2}{\beta_h}m_L^2. \tag{1.8}$$

Therefore due to the assumed form for the warp factor, the interactions for both the gauge and scalar fields are strongly coupled along the extra direction.

Since a brane is defined as any three-dimensional submanifold to which ordinary matter is trapped [20] so that it cannot escape to the bulk, a possible realization of the trapping mechanism is to assume the existence of confinement along the extra dimension. On the lattice this situation can be realized using a lattice model with anisotropic couplings. This is sufficient to lead to the formation of the layered phase through a phase transition. In our context we consider this layered phase on the lattice as a possible paradigm on how a localization of the fields, obeying nonperturbative interactions, may be carried out on the brane due to confining interactions in the bulk.

In this paper we study a simplified realization of the lattice action given by Eq. (1.5) below. This is inspired by Eq. (1.7), i.e., to set the fifth (transverse) direction couplings to a strong coupling regime while we neglect the explicit role of the warp factor in the lattice action. Therefore the lattice action (which leads to the five-dimensional Higgs model in the naive continuum limit [7,8] reads in standard notation

[4]Assuming that $m^2<0$ on the brane $(z=0)$, we note that depending from the exact form of the warp factor the mass term may turn to be positive after a certain distance or at least tends to zero asymptotically along the transverse direction. So we meet the situation of two degenerate minima near the brane and only one minimum far away from it.

[5]Notice also that here we use the symbol x for the whole discretized five-dimensional space. The extra direction now is denoted by x_T.

[6]For the transition from the continuous to the lattice action we have assumed the following rescaling for the scalar field: $2^{1/2}a^{3/2}\varphi=\beta_h\varphi_L$.

$$
\begin{aligned}
S_L &= S_{\text{gauge}} + S_{\text{scalar}} \\
&= \beta_g \sum_x \sum_{1\le\mu<\nu\le 4} [1-\cos U_{\mu\nu}(x)] + \sum_x \sum_{1\le\mu\le 4} \beta_g' [1-\cos U_{\mu 5}(x)] + \beta_h \sum_x \operatorname{Re}\Bigg[4\varphi_L^*(x)\varphi_L(x) \\
&\quad - \sum_{1\le\mu\le 4} \varphi_L^*(x) U_{\hat\mu}(x)\varphi_L(x+a\hat\mu)\Bigg] + \sum_x \beta_h' \operatorname{Re}\Big[(\varphi_L^*(x)\varphi_L(x) - \varphi_L^*(x)U_{\hat 5}(x)\varphi_L(x+a\hat 5)\Big] \\
&\quad + \sum_x \{(1-2\beta_R - 4\beta_h - \beta_h')\varphi_L^*(x)\varphi_L(x) + \beta_R[\varphi_L^*(x)\varphi_L(x)]^2\}.
\end{aligned}
\tag{1.9}
$$

Apart from the resulting simplicity in the context of the phase diagram analysis, a connection of this work with previous studies of the layered phase can be achieved. Moreover, our impression is that the full lattice model is likely to produce physically similar results with the present simplified version. This was also the case for the pure U(1) gauge model. The "static" representation of the model for which the gauge couplings were fixed by hand gave equivalent results with the model in which the warp factor was used for the scaling of the gauge couplings [4].

II. THE ORDER PARAMETERS AND THE CHOICE OF COUPLINGS

We study the Abelian Higgs model on the lattice by using numerical methods. The action is given explicitly by Eq. (1.5). We define five-order parameters, making also the distinction between spacelike and transverselike ones. These are the following.

Spacelike plaquette:

$$P_S \equiv \left\langle \frac{1}{6N^5} \sum_x \sum_{1\le\mu<\nu\le 4} \cos U_{\mu\nu}(x) \right\rangle,$$

Transverselike plaquette:

$$P_T \equiv \left\langle \frac{1}{4N^5} \sum_x \sum_{1\le\mu\le 4} \cos U_{\mu 5}(x) \right\rangle,$$

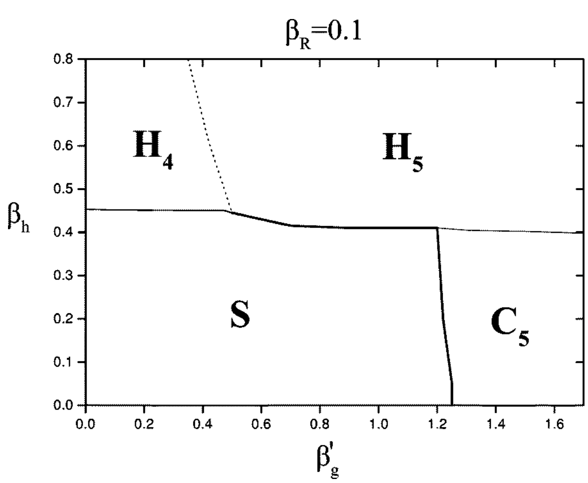

FIG. 1. Phase diagram of the 5D Abelian Higgs model with the spacelike gauge coupling set to the strong coupling $\beta_g = 0.5$ (taken from Ref. [8]).

Space-like link:

$$L_S \equiv \left\langle \frac{1}{4N^5} \sum_x \sum_{1\le\mu\le 4} \cos[\chi(x+\hat\mu) + A_{\hat\mu}(x) - \chi(x)] \right\rangle,$$

Transverse-like link:

$$L_T \equiv \left\langle \frac{1}{N^5} \sum_x \cos[\chi(x+\hat 5) + A_{\hat 5}(x) - \chi(x)] \right\rangle,$$

Higgs field measure squared:

$$R^2 \equiv \frac{1}{N^5} \sum_x \rho^2(x).$$

We have assumed the polar form for the scalar field, i.e., $\varphi_L = \rho(x) e^{i\chi(x)}$.

In Ref. [8] this model has been already studied and a first exploration for the phase diagram is available. In that work, since the parameter space is very large, consisting of five lattice parameters, the choice has been made to fix β_g to 0.5, β_h' to 0.001 and consider two values of β_R (0.1 and 0.01) and explore the parameter space (β_g', β_h). Under these conditions the analysis of the order parameters defined above yielded a phase diagram consisting of the three expected phases, which are the confining phase (S), the Coulomb phase (C_5), and the Higgs phase (H_5), each of them defined in five dimensions. In addition a fourth phase is present: a Higgs phase in four dimensions (H_4) (see Fig. 1). The distinction between H_4 and H_5 can be achieved due to the different behavior of the transverselike order parameters within the two phases. Details and conclusions on the existence of this layer Higgs phase can be found in Ref. [8]. Let us refer also to the fact that the identification for the order of the phase transitions was possible and has lead to the conclusion that (for the two values of β_R used) both H_4 and H_5 are separated from the confining phase by a first-order phase transition. We reproduce the phase diagram for $\beta_R = 0.1$, as it was depicted in Ref. [8] (Fig. 1).

III. SEARCHING FOR A SECOND-ORDER PHASE TRANSITION

At this point the question arises whether it could be possible for the H_4 layer phase to appear via a second-order phase transition. Following Ref. [8], we consider the system being in the confining regime by setting $\beta_g = 0.5$ and fixing

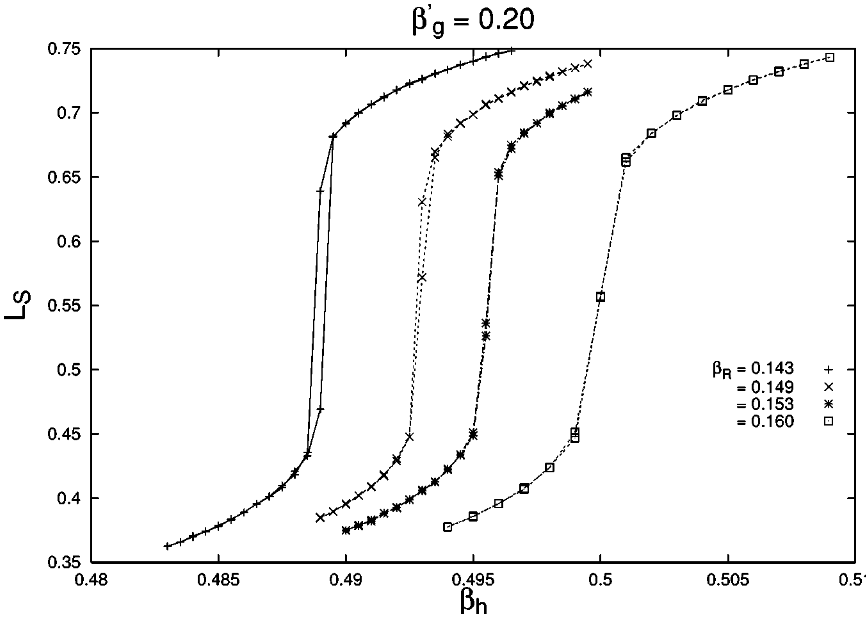

FIG. 2. Hysteresis loops showing that the loop for the link-space order parameter disappears for β_R values larger than 0.153.

β_h' to the very small value 0.001. We expect the phase transitions to the Higgs phases to be weaker as the Higgs self-coupling β_R increases. We explore the order of the S-H_4 phase transition by setting the transverse gauge coupling β_g' to 0.2 while we increase β_R. In advance, it should be noted that, as we move to larger values of β_R, the relative positions of the phases in the phase diagram are substantially similar to what is shown in Fig. 1 for $\beta_R = 0.1$. So, setting β_g'

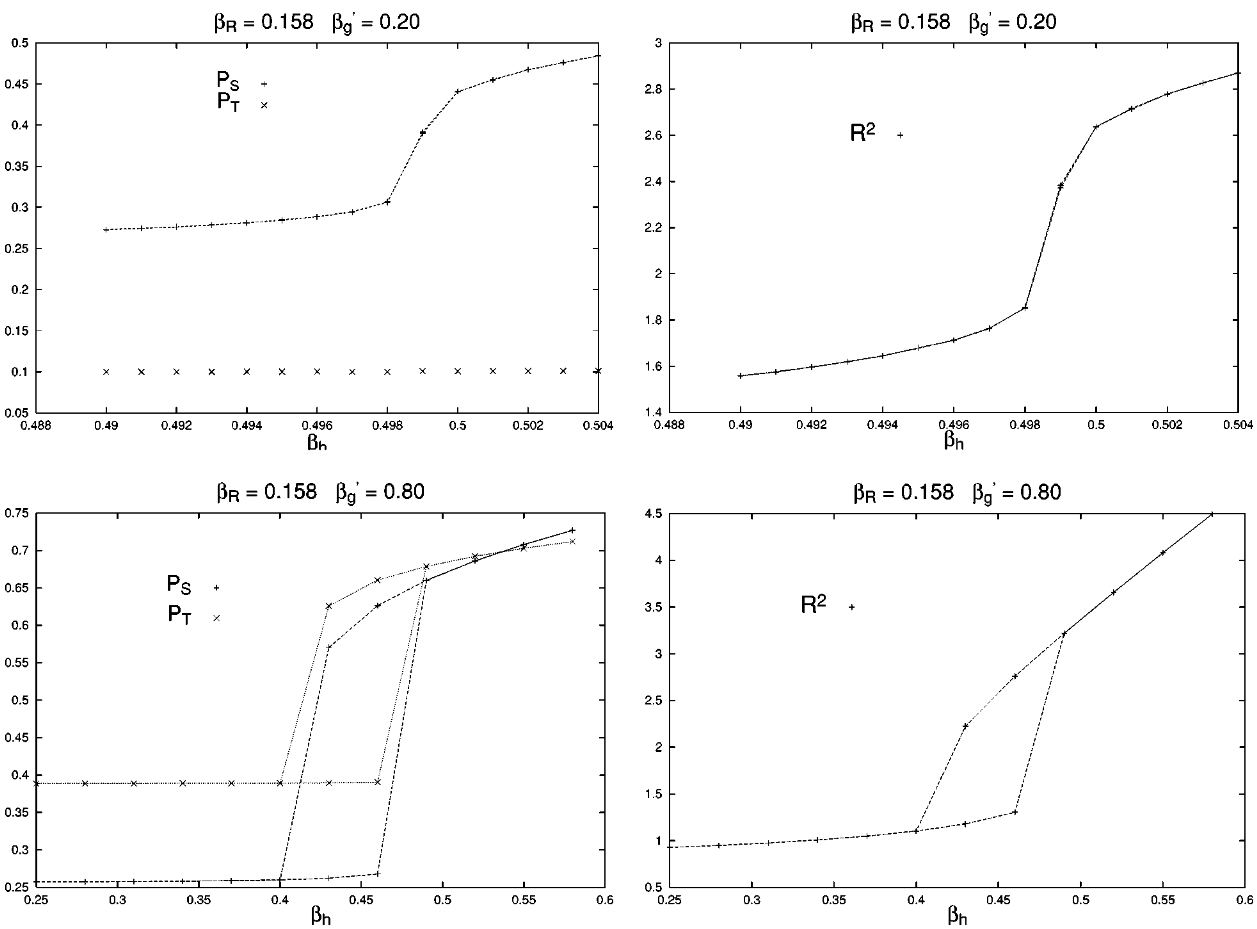

FIG. 3. Characteristic examples showing the obviously different order of the phase transition for the cases S-H_4 and S-H_5 at the same value of β_R, for two different values of β_g'.

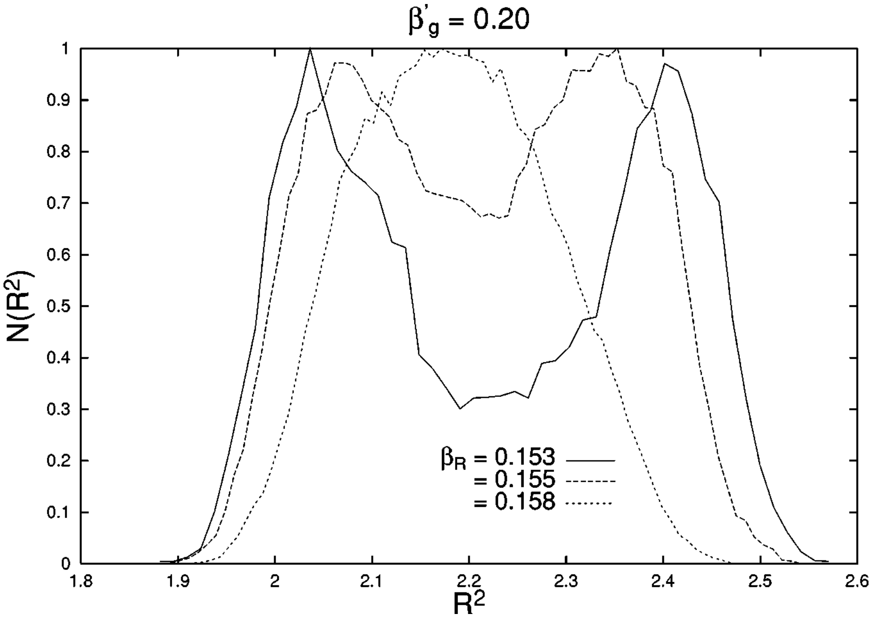

FIG. 4. Histograms of R^2 over a spacelike volume for three values of $\beta_R=0.153$, 0.155, 0.158 and lattice length $N=14$.

to 0.2, we always explore the S-H_4 phase transition.

In the sequel we give strong evidence that the S-H_4 first-order phase transition line ends at a second-order point followed by a crossover region. At the same moment the S-H_5 phase transition remains first order. This additional fact confirms the special nature of the four-dimensional layer Higgs phase.

We give now information for the simulating process. We used a 4-hit Metropolis algorithm for updating the fields. In addition we implemented the global radial algorithm and the overrelaxation algorithm for the updating of the Higgs field. We used four lattice volumes, 8^5, 10^5, 12^5, 14^5, and we performed 20 000–30 000 measurements for each point which we analyzed in the parameter space. We studied a large number of β_R values before concentrating our study to the interval [0.140,0.165] in which the first-order phase transition turns to be a weaker one before it passes to the crossover region.

In the subsequent paragraphs we present our results which are based upon using the hysteresis loop technique, the finite volume size scaling, the susceptibility, and the study of the correlation functions for the Higgs field measure squared.

A. Hysteresis loop technique results

The first tool for the exploration of the phase diagram with β_R is the hysteresis loop technique. Although this technique gives results that have to be taken into account with caution quantitatively, they prove to be very useful as qualitative ones. To this end we use the hysteresis loop results as a general guide to get a crude estimate on the β_R interval within which the phase transition is converted from a first order to a higher-order one. In Fig. 2 we depict the hysteresis loop results for the four-dimensional gauge invariant quantity L_S and for four values of β_R, namely $\beta_R=0.143$, 0.149, 0.153, 0.160. The lattice volume in this example is 8^5. One can see from the figure that while there is a well-formed loop for $\beta_R=0.143$ indicating a first-order phase transition, this changes to a smaller one for $\beta_R=0.149$, and it seems to disappear for $\beta_R=0.153$. Although this value should not be taken too seriously, one should keep in mind that around the value $\beta_R=0.153$ a weaker phase transition is still present. Furthermore we have to mention that the transverse link quantity, L_T (not shown in the figure) remains almost unaffected by the phase transition, being stuck to a very small value close to zero (for details see Ref. [8]).

In Fig. 3 we give an example of the different phase transition orders of the S-H_4 and S-H_5 transitions, both for $\beta_R=0.158$ and lattice volume 8^5. In Fig. 3(a) we present the hysteresis loop results on P_S and P_T for $\beta_g'=0.20$. The behavior of P_S indicates a phase transition though a smooth one since there is no hysteresis loop, while the P_T is almost constant and equals 0.1, in accord with the strong coupling prediction $\beta_g'/2$. This figure should be compared with Fig. 3(b), which refers to $\beta_g'=0.80$. The hysteresis loop results shows a very strong first-order phase transition, exhibited by both P_S and P_T.[7] This behavior refers to the S-H_5 phase transition. Figures 3(c) and 3(d) show the behavior for R^2 which illustrates the fact that for both cases the system passes to a broken phase. In other words, increasing β_g' one finds two different Higgs phases (see, for example, Fig. 1), a four-dimensional and a five-dimensional one both separated from the five-dimensional confining phase by phase transitions of different orders.

B. Finite volume size scaling

As it has been discussed in Ref. [8] one of the main features of the S-H_4 phase transition is the multilayer structure. This means that since the system undergoes a transition to a four-dimensional phase rather than a five-dimensional one,

[7]Notice that the unbroken phase is a confining one due to the fact that P_S and P_T follow the strong coupling limits $\beta_g/2$ and $\beta_g'/2$, respectively (for more on that see Ref. [8]).

some special signal should appear. Besides a first-order phase transition this consists of a multipeak structure in the finite lattice volume histograms for the gauge invariant observables, instead of the expected behavior of the two-peak structure. Furthermore, it has been shown that every spacelike gauge invariant quantity defined on each spacelike volume (i.e., a four-dimensional layer) ''feels'' the phase transition for different pseudocritical values of the lattice parameters. Since this is a consequence of the finite lattice volume used for Monte Carlo simulations in combination with the four-dimensional dynamics when the layer phase arises, we justify the choice of analyzing the results on the four-dimensional subspace.

In Fig. 4 we depict the histograms of the Higgs field measure squared, R^2 for $\beta_g'=0.20$ and three values of β_R. All the three histograms refer to β_h values in the critical region. The lattice volume in this figure is 14^5. The R^2 histograms refer to four-dimensional (spacelike) volume. The two peak structure is more pronounced for the smaller value of β_R (i.e., 0.153), where the two peaks are totally separated. For $\beta_R=0.155$ the two-peak structure is less emphasized while for $\beta_R=0.158$ it has already disappeared. In order for someone to use this method with more safety the lattice volume dependence of the two-peak structure should be taken into account. This is provided in Fig. 5. In Fig. 5(a) it is easily seen that the two peaks become well separated as the lattice length increases from 10 to 14, which serves as an indication of a first-order phase transition for the case of $\beta_R=0.153$. This has to be compared with the really inversed behavior for $\beta_R=0.158$ shown in Fig. 5(c). The $\beta_R=0.155$ case, Fig. 5(b), for which the peak separation does not change significantly as the lattice length goes from 10 to 14, gives an estimate of a first-order phase transition becoming much weaker and probably of higher order.

Let us now present more quantitative results by giving the results for the susceptibility of R^2 on the layers for various values of β_R. This is defined by

$$S(R^2)=V_s[\langle (R^2)^2\rangle - \langle R^2\rangle^2],$$

where V_s denotes the spacelike lattice volume. The results are depicted in Fig. 6. The errors have been calculated by using the Jackknife method. It is known that a first-order phase transition is signaled by a linear increase of the maximum of the susceptibility with the volume. This is actually the case for $\beta_R=0.149$ and 0.153. The situation changes for $\beta_R=0.155$ where the linear behavior is apparently absent. In addition, for the bigger values $\beta_R=0.158$ and 0.160 there is not a clear increase with the volume. This case corresponds to a crossover behavior. Therefore, the conclusion is that in the vicinity of $\beta_R=0.155$ we meet with the well-known situation, where a first-order phase transition line ends to a second-order phase transition point followed by a crossover.

C. Correlation functions

In this section we present the behavior of two correlation functions, one defined on the whole five-dimensional space and the other on the spacelike, four-dimensional one. These

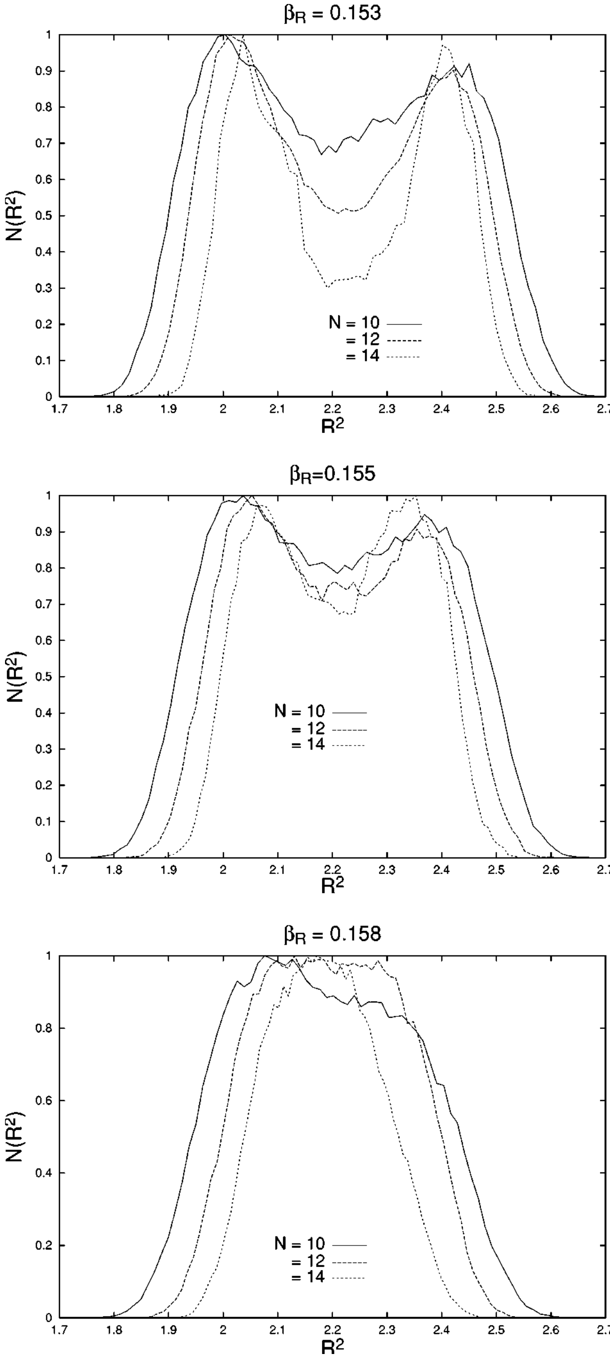

FIG. 5. The histograms for R^2 as the lattice length increases for the three values of $\beta_R=0.153,\ 0.155,\ 0.158$.

correlation functions involve the Higgs field measure squared R^2, defined in Sec. II. The definition of the correlation functions is given by

$$C_{S,T}(n)=\sum_i \frac{\langle (R^2)_i (R^2)_{i+n}\rangle - \langle (R^2)_i\rangle^2}{\langle (R^2)_i^2\rangle - \langle (R^2)_i\rangle^2}, \qquad (3.1)$$

where n takes values from 1 to N (i.e., the lattice size). The indices S and T are used to distinguish the correlators. The one defined in the transverse direction is noted with the index T. The other defined in the spacelike volume is denoted with S.

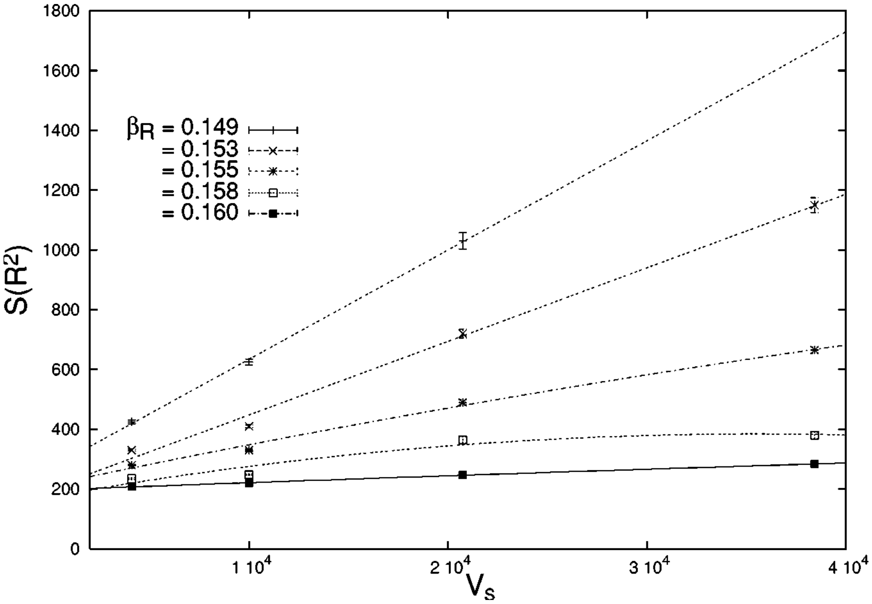

FIG. 6. The susceptibility versus the spacelike volume for five values of β_R in the critical β_R region.

The results for the two correlators are radically different. An example of our results is shown in Fig. 7. This refers to the case of $N=14$ lattice size for three values of β_R. We see that while C_T decreases very fast, reaching zero and fluctuating around it, C_S takes values different from zero. This serves as a clear evidence that a layered phase is formed. The layers are decoupled as a consequence of the strong coupling imposed on the transverse direction, which has the implication of vanishing C_T. Moreover, the rather reasonable behavior of C_S shows that inside the layers a four-dimensional dynamics is still met as it might be expected.

Another very interesting feature of the C_S correlation function is that as the β_R value decreases the curve becomes more flat. We should note that in the case of a second-order phase transition and for infinite volume this should be really flat. This is a fact corresponding to infinite correlation length or vanishing mass for the lightest scalar mode. In other words, by adjusting the β_h value into the critical region we might expect a mass behavior of the type $m_s \propto (\beta_R - \beta_R^c)^{\nu}$. The light scalar mass calculation can be achieved by using a fit of the form const$\times\cosh[m_s(x-N/2)]$ to the correlation functions C_S. The parameter m_s is the dimensionless mass parameter of the scalar mode. An example of the fits is shown in Fig. 7. The results for m_s for the cases considered are shown in Table I. From Table I and for the largest lattice size used we can see that m_s decreases by a factor of 1.7 between $\beta_R=0.160$ and 0.155. A more clear signal for the vanishing m_s would require larger volumes and still higher computer time. Nevertheless, after considering the previous analysis on susceptibility combined with the results from the study of the correlations, we are justified to estimate that at

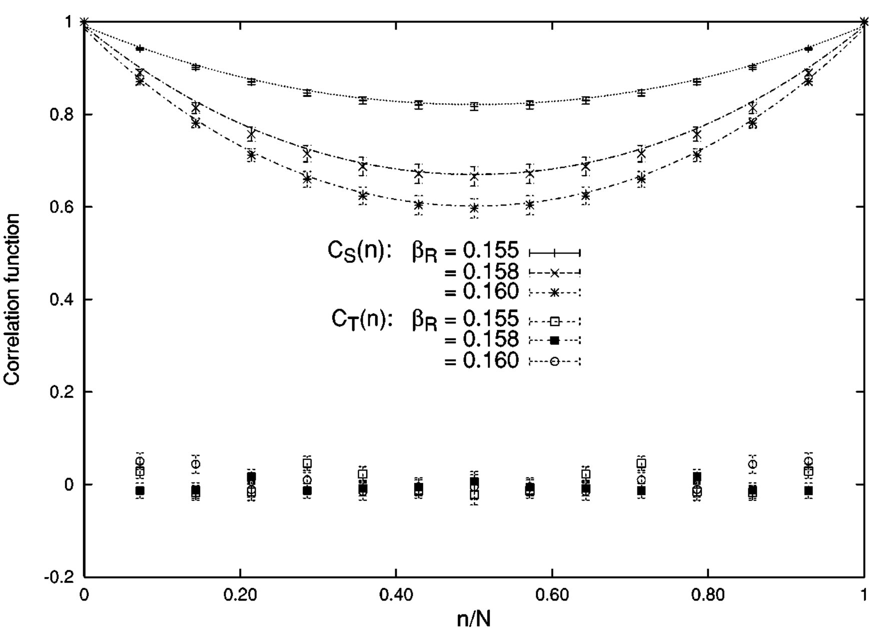

FIG. 7. The spacelike and timelike correlation functions for $L=14$ and for three values of β_R $=0.155$, 0.158, 0.160, in the region of β_h where the susceptibilities show a peak.

TABLE I. The masses in lattice units. We observe that for $L=14$ and $\beta_R=0.155$ the value for the mass parameter has decreased by a factor of 1.7 in comparison with the $\beta_R=0.160$ corresponding value.

Lattice size	$\beta_R=0.155$	$\beta_R=0.158$	$\beta_R=0.160$
10	0.145(3)	0.165(4)	0.181(4)
12	0.112(3)	0.137(3)	0.167(7)
14	0.090(3)	0.131(6)	0.152(7)

$\beta_R=0.155(2)$ a second-order phase transition point should be expected.

IV. CONCLUSIONS

We believe that we have serious evidence that the five-dimensional Abelian Higgs model with strong coupled interactions along the fifth (transverse) direction reveals a four-dimensional dynamics with broken gauge symmetry. This occurs via a second-order phase transition. The existence of the layered phase can be considered as a realization for the localization of the gauge and scalar fields for models defined in a higher-dimensional space with the extra dimensions being warped. Although the lattice volumes and the computer power available is not conclusive for the second-order critical point (so that the calculation of critical exponents is out of consideration for the moment), our results provide an estimate for the value of the Higgs self-coupling at which the line of the first-order transition line ends in a second-order transition point along the four-dimensional space.

ACKNOWLEDGMENT

We thank A. Kehagias and G. Koutsoumbas for reading and comments on the manuscript. We also wish to thank K. Bachas and M. Giovannini for useful discussions. Thanks are also due to the Computer Center of the NTUA for the computer time allocated. The work of P.D. is partially supported by ‘‘Thales’’ project of NTUA and the ‘‘Pythagoras’’ project of the Greek Ministry of Education.

[1] Y.K. Fu and H.B. Nielsen, Nucl. Phys. **B236**, 167 (1984); **B236**, 127 (1985).
[2] D. Berman and E. Rabinovici, Phys. Lett. **157B**, 292 (1985).
[3] C.P. Korthals-Altes, S. Nicolis, and J. Prades, Phys. Lett. B **316**, 339 (1993); A. Hulsebos, C.P. Korthals-Altes, and S. Nicolis, Nucl. Phys. **B450**, 437 (1995).
[4] P. Dimopoulos, K. Farakos, A. Kehagias, and G. Koutsoumbas, Nucl. Phys. **B617**, 237 (2001).
[5] S. Ejiri, J. Kubo, and M. Murata, Phys. Rev. D **62**, 105025 (2000); S. Ejiri, S. Fujimoto, and J. Kubo, *ibid.* **66**, 036002 (2002).
[6] K. Farakos, P. de Forcrand, C.P. Korthals Altes, M. Laine, and M. Vettorazzo, Nucl. Phys. **B655**, 170 (2003).
[7] P. Dimopoulos, K. Farakos, G. Koutsoumbas, C.P. Korthals-Altes, and S. Nicolis, J. High Energy Phys. **02**, 005 (2001).
[8] P. Dimopoulos, K. Farakos, and S. Nicolis, Eur. Phys. J. C **24**, 287 (2002).
[9] M. Laine, H.B. Meyer, K. Rummukainen, and M. Shaposhnikov, J. High Energy Phys. **04**, 027 (2004).
[10] G.R. Dvali and M.A. Shifman, Phys. Lett. B **396**, 64 (1997); **407**, 452(E) (1997).
[11] P. Dimopoulos, K. Farakos, and G. Koutsoumbas, Phys. Rev. D **65**, 074505 (2002).
[12] K.R. Dienes, E. Dudas, and T. Gharghetta, Nucl. Phys. **B537**, 47 (1999).
[13] J.L. Alonso *et al.*, Nucl. Phys. **B405**, 574 (1993).
[14] K. Kajantie, M. Laine, K. Rummukainen, and M. Shaposhnikov, Phys. Rev. Lett. **77**, 2887 (1996); K. Rummukainen, M. Tsypin, K. Kajantie, M. Laine, and M. Shaposhnikov, Nucl. Phys. **B532**, 283 (1998).
[15] A. Hart, O. Philipsen, J.D. Stack, and M. Teper, Phys. Lett. B **396**, 217 (1997).
[16] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999); **83**, 3370 (1999).
[17] A. Kehagias, Phys. Lett. B **469**, 123 (1999); A. Kehagias and K. Tamvakis, *ibid.* **504**, 38 (2001).
[18] M. Giovannini, Phys. Rev. D **64**, 124004 (2001); **65**, 124019 (2002).
[19] M. Shaposhnikov and P. Tinyakov, Phys. Lett. B **515**, 442 (2001); M. Laine, H.B. Meyer, K. Rummukainen, and M. Shaposhnikov, J. High Energy Phys. **01**, 068 (2003).
[20] V.A. Rubakov, Usp. Fiz. Nauk **171**, 913 (2001).