Born-Infeld gravity in any dimension

J. A. Nieto*
Facultad de Ciencias Físico-Matemáticas de la Universidad Autónoma de Sinaloa, Av. de las Américas y Blvd. Universitarios, Ciudad Universitaria, C.P. 80010, Culiacán Sinaloa, México
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We develop a Born-Infeld type theory for gravity in any dimension. We show that in four dimensions our formalism allows a self-dual (or anti-self-dual) Born-Infeld gravity description. Moreover, we show that such a self-dual action is reduced to both the Deser-Gibbons and the Jacobson-Smolin-Samuel action of Ashtekar formulation. A supersymmetric generalization of our approach is outlined.

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I. INTRODUCTION

More than 80 years ago, Eddington [1] introduced the action

$$S_{EDD}=\int d^4x\sqrt{\det(R_{\mu\nu}(\Gamma))}, \tag{1}$$

as a possibility to incorporate affine invariance in a gravitational context. Here $R_{\mu\nu}=R_{\nu\mu}$ is the Ricci tensor and $\Gamma^{\alpha}_{\mu\nu}=\Gamma^{\alpha}_{\nu\mu}$ becomes the Christoffel symbol, after using the Palatini method.

In 1934 Born and Infeld (BI) [2] for different reasons than Eddington proposed a similar action for electrodynamics

$$S_{BI}=\int d^4x\sqrt{\det(g_{\mu\nu}+F_{\mu\nu})}, \tag{2}$$

where $g_{\mu\nu}$ is the space-time metric and $F_{\mu\nu}$ is the electromagnetic field strength. It turns out that the action S_{BI} reduces to Maxwell action for small amplitudes. Some years ago, the action (2) became of very much interest because its relation with SUSY (see [3] and references therein) and string theory (see [4–7] and references therein) and D-brane physics [8]. At present, the BI theory has been generalized to the supersymmetric non-Abelian case (see [9] and references therein) and it has been connected with noncommutative geometry [10] and Cayley-Dickson algebras [11]. Moreover, it turns out intriguing that the BI action can be derived from matrix theory [12] and D9-brane [13]. For interesting comments and observation about the BI theory the reader is referred to the works of Schwarz [14] and Ketov [15].

Inspired by the interesting properties of the BI theory, Deser and Gibbons [16] proposed in 1998 the analogue of S_{BI} for a gravity,

$$S_{DG}=\int d^4x\sqrt{\det(ag_{\mu\nu}+bR_{\mu\nu}+cX_{\mu\nu})}. \tag{3}$$

Here, $X_{\mu\nu}=X_{\nu\mu}$ stands for higher order corrections in curvature and a,b and c are appropriate coupling constants. Deser and Gibbons determined some possible choices for $X_{\mu\nu}$ by imposing three basic criteria to the action (3), namely freedom of ghosts, regularization of singularities and supersymmetrizability. In fact, they found that the expression $X_{\mu\nu}\sim R^{\alpha}_{\mu}R_{\alpha\nu}$ may provide one interesting choice for $X_{\mu\nu}$, allowing the action (3) to describe gravitons but no ghosts.

From the point of view of string theory both the photon as well as the graviton should be part of the spectrum of a string. Therefore one should expect that just as S_{BI} is used to regularize p-branes the S_{DG} action may be used with similar purpose. However, for this possibility to be viable it is necessary to generalize the S_{DG} action to higher dimensions. Moreover, Wohlfarth [17] has pointed out that the supersymmetrizability requirement presumably implied by M-theory, may provide another reason to be interested in such a higher-dimensional extension of S_{DG}.

In this work, we use similar technics as the one used in MacDowell-Mansouri formalism [18] (see also [19] and references therein) in order to generalize the S_{DG} action to higher dimensions. In four dimensions, our approach is reduced to the theory of Deser-Gibbons. We also show that in four dimensions our action admits self-dual (anti-self-dual) generalization which is reduced to the Ashtekar action as proposed by Jacobson-Smolin-Samuel [20]. Since Born-Infeld type action has shown to be also useful to study different aspects of the brane world scenario, our work may appear interesting in this area of research.

The plan of this paper is as follows: In Sec. II, using a MacDowell-Mansouri's type formalism we establish our proposed action for Born-Infeld gravity in any dimension. In Sec. III, we discuss the relationship between our proposed action and the Deser and Gibbons action. In Sec. IV, we consider a self-dual (anti-self-dual) version of our proposed action in four dimensions and we show that it is reduced to the Jacobson-Smolin-Samuel action for the Ashtekar formalism. Finally, in Sec. V we outline a possible supersymmetric generalization of our formalism and we make some final remarks.

II. PROPOSED BORN-INFELD-GRAVITY (BIG) ACTION

Consider the extended curvature

$$\mathcal{R}^{ab}_{\mu\nu}=R^{ab}_{\mu\nu}+\Sigma^{ab}_{\mu\nu}, \tag{4}$$

where

*Electronic address: nieto@uas.uasnet.mx

$$R_{\mu\nu}^{ab}=\partial_\mu\omega_\nu^{ab}-\partial_\nu\omega_\mu^{ab}+\omega_\mu^{ac}\omega_{\nu c}{}^{b}-\omega_\nu^{ac}\omega_{\mu c}{}^{b} \tag{5}$$

and

$$\Sigma_{\mu\nu}^{ab}=-(e_\mu^a e_\nu^b-e_\nu^a e_\mu^b). \tag{6}$$

Here, ω_μ^{ab} is a $SO(d-1,1)$ connection and e_μ^a is a vierbein field. It turns out that Eq. (4) can be obtained using the MacDowell-Mansouri formalism [18] (see also Ref. [19] and references therein). In fact, in such a formalism the gravitational field is represented as a connection one-form associated to some group which contains the Lorentz group as a subgroup. The typical example is provided by the $SO(d,1)$ de Sitter gauge theory of gravity. In this case, the $SO(d,1)$ gravitational gauge field $\omega_\mu^{AB}=-\ \omega_\mu^{BA}$ is broken into the $SO(d-1,1)$ connection ω_μ^{ab} and the $\omega_\mu^{da}=e_\mu^a$ vierbein field, with the dimension d fixed. Thus the de Sitter (or anti–de Sitter) curvature

$$\mathcal{R}_{\mu\nu}^{AB}=\partial_\mu\omega_\nu^{AB}-\partial_\nu\omega_\mu^{AB}+\omega_\mu^{AC}\omega_{\nu C}{}^{B}-\omega_\nu^{AC}\omega_{\mu C}{}^{B} \tag{7}$$

leads to the curvature (4).

Let us now introduce the definition

$$\mathcal{R}_\mu^a\equiv e_b^\nu\mathcal{R}_{\mu\nu}^{ab}, \tag{8}$$

where e_b^ν is the inverse vierbien field.

Our proposed action is

$$S=-\frac{1}{d!}\int d^dx\varepsilon^{\mu_1\cdots\mu_d}\epsilon_{a_1\ldots a_d}\mathcal{R}_{\mu_1}^{a_1}\ldots\mathcal{R}_{\mu_d}^{a_d}, \tag{9}$$

where $\varepsilon^{\mu_1\cdots\mu_d}$ is the completely antisymmetric tensor associated to the space-time, with $\varepsilon^{0\cdots d-1}=1$ and $\varepsilon_{0\ldots d-1}=1$, while $\epsilon_{a_1\ldots a_d}$ is also the completely antisymmetric tensor but now associated to the internal group $S(d-1,1)$, with $\epsilon_{0\ldots d-1}=-1$. We assume that the internal metric is given by $(\eta_{ab})=\mathrm{diag}(-1,\ldots,1)$. So, we have $\epsilon^{0\cdots d-1}=1$.

Using Eq. (6) we get

$$e_b^\nu\Sigma_{\mu\nu}^{ab}=\lambda e_\mu^a, \tag{10}$$

where $\lambda=1-d$. Therefore, Eqs. (4) and (8) and (10) lead to

$$\mathcal{R}_\mu^a=R_\mu^a+\lambda e_\mu^a, \tag{11}$$

where

$$R_\mu^a\equiv e_b^\nu R_{\mu\nu}^{ab}. \tag{12}$$

Substituting Eq. (11) into Eq. (9) we obtain

$$S=-\frac{1}{d!}\int d^dx\varepsilon^{\mu_1\cdots\mu_d}\epsilon_{a_1\ldots a_d}(R_{\mu_1}^{a_1}+\lambda e_{\mu_1}^{a_1})\ldots(R_{\mu_d}^{a_d}+\lambda e_{\mu_d}^{a_d}). \tag{13}$$

From this action we get

$$S=-\frac{\lambda^d}{d!}\int d^dx\varepsilon^{\mu_1\cdots\mu_d}\epsilon_{a_1\ldots a_d}e_{\mu_1}^{a_1}\ldots e_{\mu_d}^{a_d}-\frac{d\lambda^{d-1}}{d!}\int d^dx\varepsilon^{\mu_1\cdots\mu_d}\epsilon_{a_1\ldots a_d}e_{\mu_1}^{a_1}\ldots e_{\mu_{d-1}}^{a_{d-1}}R_{\mu_d}^{a_d}$$
$$-\frac{d(d-1)\lambda^{d-2}}{2!d!}\int d^dx\varepsilon^{\mu_1\cdots\mu_d}\epsilon_{a_1\ldots a_d}e_{\mu_1}^{a_1}\ldots e_{\mu_{d-2}}^{a_{d-2}}R_{\mu_{d-1}}^{a_{d-1}}R_{\mu_d}^{a_d}-\cdots-\frac{1}{d!}\int d^dx\varepsilon^{\mu_1\cdots\mu_d}\epsilon_{a_1\ldots a_d}R_{\mu_1}^{a_1}\ldots R_{\mu_d}^{a_d}. \tag{14}$$

Since

$$\varepsilon^{\mu_1\cdots\mu_d}\epsilon_{a_1\ldots a_d}=-e(e_{a_1}^{[\mu_1}\ldots e_{a_d}^{\mu_d]}), \tag{15}$$

where the bracket $[\mu_1\ldots\mu_d]$ means completely antisymmetric and $e=\det(e_\mu^a)$, we find that the action (14) can be written as

$$S=\lambda^d\int d^dxe+\lambda^{d-1}\int d^dxeR+\frac{\lambda^{d-2}}{2!}\int d^dxe(R^2-R^{\mu\nu}R_{\mu\nu})+\cdots+\int d^dx\det(R_\mu^a). \tag{16}$$

Here, $R\equiv e_a^\mu R_\mu^a$ and $R_{\mu\nu}\equiv e_\nu^a R_{\mu a}$, while $R^{\mu\nu}\equiv g^{\mu\alpha}g^{\nu\beta}R_{\alpha\beta}$, where $g^{\alpha\beta}$ is the matrix inverse of $g_{\alpha\beta}\equiv e_\alpha^a e_\beta^b\eta_{\alpha\beta}$. We recognize in the first and second terms in the action (16) the Einstein-Hilbert action with cosmological constant in d-dimensions.

III. RELATION WITH DESER-GIBBONS TYPE ACTION

Let us define the tensor

$$G_{\mu\nu}\equiv\frac{1}{\lambda^2}\mathcal{R}_\mu^a\mathcal{R}_\nu^b\eta_{ab}. \tag{17}$$

Using Eq. (11) we get

$$G_{\mu\nu}=\frac{1}{\lambda^2}(R^a_\mu+\lambda e^a_\mu)(R^b_\nu+\lambda e^b_\nu)\eta_{ab}$$
$$=g_{\mu\nu}+\Lambda R_{\mu\nu}+\frac{\Lambda^2}{4}R^\alpha_\mu R_{\alpha\nu}, \tag{18}$$

where $\Lambda=2/\lambda$. Here, we used the fact that $R_{\mu\nu}=R_{\nu\mu}$.

From Eq. (17), it is not difficult to see that

$$\det(G_{\mu\nu})\equiv-\frac{1}{\lambda^{2d}}\mathcal{R}^2, \tag{19}$$

where $\mathcal{R}=\det(\mathcal{R}^a_\mu)$. Therefore, our proposed action (9) can be expressed in terms of $G_{\mu\nu}$ as

$$S=-\frac{1}{d!}\int d^dx\varepsilon^{\mu_1\cdots\mu_d}\epsilon_{a_1\ldots a_d}\mathcal{R}^{a_1}_{\mu_1}\ldots\mathcal{R}^{a_d}_{\mu_d}=\int d^dx\mathcal{R}$$
$$=\lambda^d\int d^dx\sqrt{-\det(G_{\mu\nu})}. \tag{20}$$

In virtue of Eq. (18) the action (20) becomes

$$S=\frac{2^d}{\Lambda^d}\int d^dx\sqrt{-\det\left(g_{\mu\nu}+\Lambda R_{\mu\nu}+\frac{\Lambda^2}{4}R^\alpha_\mu R_{\alpha\nu}\right)}, \tag{21}$$

which is a Born-Infeld type action for gravity in any dimension. In four dimensions, we recognize that the action (21) corresponds to the one proposed by Deser and Gibbons [16] [see Eq. (3) of [16]], with $X_{\mu\nu}=R^\alpha_\mu R_{\alpha\nu}$.

IV. SELF-DUAL (ANTI-SELF-DUAL) ACTION IN FOUR DIMENSIONS

In four dimensions the action (9) can be generalized to the case of self-dual (anti-self-dual) gauge gravitational field. We define the self-dual (anti-self-dual) of $\mathcal{R}^{cd}_{\mu\nu}$ as

$${}^\pm\mathcal{R}^{ab}_{\mu\nu}=\frac{1}{2}{}^\pm M^{ab}_{cd}\mathcal{R}^{cd}_{\mu\nu}, \tag{22}$$

where

$${}^\pm M^{ab}_{cd}=\frac{1}{2}(\delta^{ab}_{cd}\mp i\epsilon^{ab}_{cd}). \tag{23}$$

Here, $\delta^{ab}_{cd}=\delta^a_c\delta^b_d-\delta^b_c\delta^a_d$. We also define

$${}^\pm\mathcal{R}^a_\mu\equiv e^\nu_b{}^\pm\mathcal{R}^{ab}_{\mu\nu}. \tag{24}$$

In four dimensions (9) becomes

$$S=-\frac{1}{4!}\int d^4x\varepsilon^{\mu_1\cdots\mu_4}\epsilon_{a_1\ldots a_4}\mathcal{R}^{a_1}_{\mu_1}\ldots\mathcal{R}^{a_4}_{\mu_4}. \tag{25}$$

Substituting Eq. (11) into this expression lead us to

$$S=-\frac{1}{4!}\int d^4x\varepsilon^{\mu_1\cdots\mu_4}\epsilon_{a_1\ldots a_4}(R^{a_1}_{\mu_1}+\lambda e^{a_1}_{\mu_1})\ldots(R^{a_4}_{\mu_4}+\lambda e^{a_4}_{\mu_4})=-\frac{\lambda^4}{4!}\int d^4x\varepsilon^{\mu_1\cdots\mu_4}\epsilon_{a_1\ldots a_4}e^{a_1}_{\mu_1}\ldots e^{a_4}_{\mu_4}$$
$$-\frac{4\lambda^3}{4!}\int d^4x\varepsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{a_1a_2a_3a_4}e^{a_1}_{\mu_1}e^{a_2}_{\mu_2}e^{a_3}_{\mu_3}R^{a_4}_{\mu_4}-\frac{6\lambda^2}{4!}\int d^4x\varepsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{a_1a_2a_3a_4}e^{a_1}_{\mu_1}e^{a_2}_{\mu_2}R^{a_3}_{\mu_3}R^{a_4}_{\mu_4}$$
$$-\frac{4\lambda}{4!}\int d^4x\varepsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{a_1a_2a_3a_4}e^{a_1}_{\mu_1}R^{a_2}_{\mu_2}R^{a_3}_{\mu_3}R^{a_4}_{\mu_4}-\frac{1}{4!}\int d^4x\varepsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{a_1a_2a_3a_4}R^{a_1}_{\mu_1}R^{a_2}_{\mu_2}R^{a_3}_{\mu_3}R^{a_4}_{\mu_4}. \tag{26}$$

Simplifying these expressions one finds

$$S=\lambda^4\int d^4xe+\lambda^3\int d^4xeR+\frac{\lambda^2}{2!}\int d^4xe(R^2-R^{\mu\nu}R_{\mu\nu})$$
$$+\frac{\lambda}{3!}\int d^4xe(R^3-3RR^{\mu\nu}R_{\mu\nu}+2R^{\mu\alpha}R_{\alpha\beta}R^\beta_\mu)$$
$$+\int d^4xe\det(R_{\mu\nu}). \tag{27}$$

Let us now generalize the action (25) in the form

$$S=-\frac{1}{4!}\int d^4x\varepsilon^{\mu_1\cdots\mu_4}\epsilon_{a_1\ldots a_4}{}^\pm\mathcal{R}^{a_1}_{\mu_1}\ldots{}^\pm\mathcal{R}^{a_4}_{\mu_4}. \tag{28}$$

It is not difficult to see that

$${}^\pm\mathcal{R}^a_\mu\equiv{}^\pm R^a_\mu+\lambda e^a_\mu. \tag{29}$$

Thus, we have

$$S=-\frac{1}{4!}\int d^4x\varepsilon^{\mu_1\cdots\mu_4}\epsilon_{a_1\ldots a_4}({}^\pm R^{a_1}_{\mu_1}+\lambda e^{a_1}_{\mu_1})\ldots({}^\pm R^{a_4}_{\mu_4}+\lambda e^{a_4}_{\mu_4}). \tag{30}$$

Therefore, we obtain

$$S=\lambda^4\int d^4xe+\lambda^3\int d^4xe^{\pm}R+\frac{\lambda^2}{2!}\int d^4xe({}^{\pm}R^2 - {}^{\pm}R^{\mu\nu\pm}R_{\mu\nu})+\frac{\lambda}{3!}\int d^4xe({}^{\pm}R^3-3{}^{\pm}R^{\pm}R^{\mu\nu\pm}R_{\mu\nu} + 2{}^{\pm}R^{\mu\alpha\pm}R_{\alpha\beta}{}^{\pm}R^{\beta}_{\mu})+\int d^4xe\,\det({}^{\pm}R_{\mu\nu}). \tag{31}$$

We recognize in the second term of this expression the Ashtekar action as proposed by Jacobson-Smolin-Samuel [20].

V. FINAL COMMENTS

In this work, we proposed the action (9) as an alternative to generalize to higher dimensions the Born-Infeld-gravity action of Deser and Gibbons. We proved that our action in four dimensions allows a generalization to self-dual (anti-self-dual) action. Moreover, we showed that such a self-dual action is reduced to the Jacobson-Smolin-Samuel action of the Ashtekar formulation.

It is remarkable that the only requirement for the proposed action (9) is the general covariance implied by the transition $SO(d,1)\rightarrow SO(d-1,1)$. It remains to check whether the action (9) satisfies the criteria of ghosts freedom and regularization of singularities used by Deser and Gibbons [16] in four dimensions and Wohlfarth [17] in higher dimensions. However, these requirements should be applied very carefully to the action (9) because under compactification (9) should lead not only to gravity but also to Yang-Mills and scalar fields. From the point of view of M-theory one may even think in a phase in which matter fields and ghosts are mixed implying a generalization of the action (9) to some kind of topological theory.

As we mentioned in the Introduction, supersymmetrizability provides to Wohlfarth [17] with one of the main motivation for considering BI gravity in higher dimensions. Wohlfarth's theory is constructed by using the symmetries of the Riemann tensor, considering tensors with pair of antisymmetrized indices. Specifically, the Wohlfarth's formulation is based on the action $\int\sqrt{-g}[(\det(\delta^A_B+\xi R^A_B)^{\varsigma}-1]$, where $R_{AB}=R_{[\mu\nu][\alpha\beta]}$, $\delta^A_B=\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu}-\delta^{\beta}_{\mu}\delta^{\alpha}_{\nu}$ and ξ, ς are constants (see Ref. [17] for details). Although interesting, it does not seem evident how to supersymmetrize this theory.

In contrast, our treatment admits a straightforward generalization to the supersymmetric version in four dimensions. In fact, in the action (9) we may replace the curvature $\mathcal{R}^{ab}_{\mu\nu}$ by its supersymmetric extension

$$\mathcal{R}^{ab}_{\mu\nu}=R^{ab}_{\mu\nu}+\Sigma^{ab}_{\mu\nu}+\Theta^{ab}_{\mu\nu},$$
$$\mathcal{R}^{i}_{\mu\nu}=R^{i}_{\mu\nu}+\Sigma^{i}_{\mu\nu}, \tag{32}$$
$$\mathcal{R}^{a}_{\mu\nu}=R^{a}_{\mu\nu}+\Sigma^{a}_{\mu\nu},$$

where

$$\Theta^{ab}_{\mu\nu}=\frac{1}{2}f^{ab}_{ij}\psi^i_{\mu}\psi^j_{\nu}, \tag{33}$$

$$\Sigma^{a}_{\mu\nu}=\frac{1}{2}f^{4a}_{ij}\psi^i_{\mu}\psi^j_{\nu}, \tag{34}$$

$$\Sigma^{i}_{\mu\nu}=f^{i}_{4aj}(e^a_{\mu}\psi^j_{\nu}-e^a_{\nu}\psi^j_{\mu}), \tag{35}$$

and

$$R^{i}_{\mu\nu}=\partial_{\mu}\psi^i_{\nu}-\partial_{\nu}\psi^i_{\mu}+\frac{1}{2}f^{i}_{cdj}(\omega^{cd}_{\mu}\psi^j_{\nu}-\omega^{cd}_{\nu}\psi^j_{\mu}). \tag{36}$$

Here, ψ^i_{μ} is a spinor field and the quantities f^{i}_{cdj} and so on are the structure constants of algebra associated to the Lie supergroup $Osp(1|4)$. It may be interesting for further research to see if the resulting supersymmetric Born-Infeld gravity theory is related to the $D=4$, $N=1$ Born-Infeld supergravity proposed by Gates and Ketov [21].

Since some authors [22,23] have found interesting connections between the non-Abelian Born-Infeld theory and D-branes it may also be interesting for further research to investigate the relation of the action (9) with string theory and with p-branes physics.

Recently, Liu and Li [24] have found an interesting application of Born-Infeld and brane worlds and García-Salcedo and Breton [25] have studied the possibility that Born-Infeld inflates the Bianchi cosmological models. It may be interesting to pursue an application of the action (9) in these directions.

Pilatnik [26] has found spherical static solutions of the Born-Infeld gravity theory which may also correspond to the theory described by the action (9). Feigenbaum [27] found that Born regulates gravity in four dimensions, so it appears attractive to see if the action (9) can be used to regulate gravity in higher dimensions. Furthermore, Vollick [28] used the Palatini formalism in Born-Infeld Einstein theory in four dimensions and identify the antisymmetric part of the Ricci tensor with the electromagnetic field. So, if we use similar Palatini's technics in connection with the action (9) one should expect to identify part of the Ricci tensor with generalized electromagnetic field, perhaps Yang-Mills field.

Finally, it has been shown (see [29]) that dualities in M-theory (see [30] and references therein) are deeply related to Born Infeld theory. Since eleven dimensional supergravity must be part of M-theory one should expect that Born-Infeld gravity theory in any dimensions may play an important role in this context. In this direction the derivations of Born-Infeld theory from supergravity [31] and the Nambu-Goto action from Born-Infeld theory [32] may be of particular interest to continue a further research in connection with the action (9).

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