Traversable Lorentzian wormholes in the vacuum low energy effective string theory in Einstein and Jordan frames

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Three *new* classes (II-IV) of solutions of the vacuum low energy effective string theory in four dimensions are derived. Wormhole solutions are investigated in those solutions including the class I case both in the Einstein and in the Jordan (string) frame. It turns out that, of the eight classes of solutions investigated (four in the Einstein frame and four in the corresponding string frame), massive Lorentzian traversable wormholes exist in five classes. Nontrivial massless limit exists only in class I Einstein frame solution while none at all exists in the string frame. An investigation of test scalar charge motion in the class I solution in the two frames is carried out by using the Plebański-Sawicki theorem. A curious consequence is that the motion around the extremal zero (Keplerian) mass configuration leads, as a result of scalar-scalar interaction, to a new hypothetical "mass" that confines test scalar charges in bound orbits, but does not interact with neutral test particles.

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I. INTRODUCTION

Currently, there exist an intense activity in the field of wormhole physics following particularly the seminal works of Morris, Thorne and Yurtsever [1]. Wormholes are topological handles that connect two distant otherwise disconnected regions of space. Theoretical importance of such geometrical objects is exemplified in several ways. For instance, they are invoked to interpret or solve many outstanding issues both in the local as well as cosmological scenarios [2–5]. Lorentzian wormholes could be threaded both by quantum and classical matter fields that violate certain energy conditions ("exotic matter") at least at the throat. In the quantum regime, several negative energy density fields are already known to exist. For instance, they occur in the Casimir effect, and in the context of Hawking evaporation of black holes, and also in the squeezed vacuum states [1]. Classical fields playing the role of exotic matter also exist. They are known to occur in the $R+R^2$ theory [6], scalar tensor theories [7–11], Visser's cut and paste thin shell geometries [12]. On general grounds, it has recently been shown that the amount of exotic matter needed at the wormhole throat can be made arbitrarily small thereby facilitating an easier construction of wormholes [13].

A commendable arena to look for classical exotic fields is the vacuum linear string theory which, in the low energy limit, reproduces a scalar tensor theory of gravity in four dimensions. The action can be written in the Jordan frame (JF), which is also called the string frame, and it is this form of action that appears in the original nonlinear σ-model and its solutions are what the string actually "sees." (We set the β-functions to zero for reasons of quantum conformal invariance.) The JF action is referred to here as string theory. The action can also be cast into the Brans-Dicke form with the coupling parameter $\omega=-1$ showing that the Machian philosophy is already imbedded into the string action. This Brans-Dicke action can be transferred to the conformally rescaled Einstein frame (EF) so that the Lagrangian assumes the form of Einstein-Hilbert action of (non-Machian) general relativity in which the scalar field (dilaton) couples to the gravitional sector minimally but with an arbitrary sign in the kinetic term. We choose to call the latter the Einstein massless scalar (EMS) field theory. Both the signs can be theoretically allowed as long as there does not appear any inconsistency. It should be noted that the positive sign before the kinetic term in the action represents conventional coupling while the negative sign corresponds to the unconventional one that leads to the violation of energy conditions.

The motivation for the present paper is provided by three key reasons: First, both the above frames exhibit certain symmetry properties, T-duality in the Jordan frame and S-duality in the Einstein frame. Second, there is as yet no consensus as to which frame is more physical although the Einstein frame is often advocated in view of energy considerations. As we are here concerned with only wormhole solutions, we need not be concerned with the violations of energy conditions in either of the frames. Overall, there is no canonical principle to rule out one frame in preference to the other and hence we shall examine the solutions in both of them. This is the third reason. In fact, recently, in the context of traversable Lorentzian wormholes in general relativity Armendáriz-Picón (AP) [14] has shown that the most simple form of Lagrangian that satisfies all the traversable wormhole conditions is that of EMS theory but with a *negative* sign before the kinetic term. The author has briefly discussed

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massive wormholes in a certain class (let us call it class I) of static, spherically symmetric solutions in the EMS theory and has also proved the existence and stability of zero mass wormholes. As remarked by the author, zero mass wormhole configuration is the simplest and it exemplifies Wheeler's concept of "charge without charge." However, there also exist other classes of EMS solutions although, unfortunately, they have not received as much attention in the literature as the class I solutions. Therefore, it is of interest to examine if wormholes exist in those other classes of static spherically symmetric solutions (II-IV) of EMS theory as well as in the string theory. In this paper, we start with the EMS solutions from our earlier paper and derive the corresponding new string frame solutions and then adopt a search for wormholes analyzing all solutions on a class by class basis in both the theories. The search result turns out to be quite encouraging in the sense that out of the eight, three and two classes represent massive wormhole solutions in the EMS and string theory respectively. However, no massless limit exists in the string theory. We shall also study the motion of test particles in the gravity-scalar field environment by adopting a different principle based on the Plebański-Sawicki theorem. We work mainly in the Morris-Thorne coordinate description for more transparency.

The paper is organized as follows: In Sec. II, we start from the linear string action and review the class I solution of the EMS theory. In Sec. III, we elucidate more pedagogical details of the EMS class I wormhole in coordinate description and revisit the zero mass limit. The contents will be useful in Sec. IV where we explore the wormhole nature of class I solution in the context of string theory. Investigation of other classes of solutions (II-IV) is contained in Sec. V. This section also includes the analyses of the corresponding string classes of solutions. In Sec. VI, we study test particle motion in the class I solution of the two theories while in Sec. VII, we summarize our results. An Appendix contains a comparison of notations for easy reference.

II. THE ACTION AND CLASS I SOLUTION: A BRIEF REVIEW

Our starting point is the 4-dimensional, low energy effective action of heterotic string theory compactified on a 6-torus [15]. The tree level string effective action, keeping only linear terms in the string tension α' and in the curvature $\tilde{\mathbf{R}}$, takes the following form in the ordinary-matter free region ($S_{matter}=0$):

$$S_{eff}=\frac{1}{\alpha'}\int d^4x\sqrt{-\tilde{g}}e^{-2\tilde{\Phi}}[\tilde{\mathbf{R}}+4\tilde{g}^{\mu\nu}\tilde{\Phi}_{,\mu}\tilde{\Phi}_{,\nu}], \qquad (1)$$

where $\tilde{\Phi}$ is the dilaton field. Note that the zero values of other matter fields do not lead to any additional constraints either on the metric or on the dilaton [15]. One also avoids the complexity of abnormal scalar coupling with these fields in the EMS version. Such couplings are known to violate the principle of equivalence since the test particle rest mass depends on the scalar field. We shall comment on this principle later in Sec. VI. Under the substitution $e^{-2\tilde{\Phi}}=\phi$, the above action reduces to the JFBD action (we take the units $16\pi G=c=1$):

$$S_{JF}=\int d^4x\sqrt{-\tilde{g}}\left[\phi\tilde{\mathbf{R}}+\frac{1}{\phi}\tilde{g}^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right] \qquad (2)$$

in which the BD coupling parameter ω is set to the value $\omega=-1$. This particular value is actually a model independent prediction and it arises due to the fundamental symmetry of strings, viz. the target space duality [16]. It should be noted that the vacuum BD action has a conformal invariance characterized by a constant gauge parameter ξ. Arbitrary choice of ξ can lead to a shift from the value $\omega=-1$. This ambiguity can be removed either by allowing abnormal coupling to matter or simply by fixing the gauge [21]. We fix $\xi=0$. Under a further substitution

$$g'_{\mu\nu}=\phi\tilde{g}_{\mu\nu}, \qquad (3)$$

$$d\varphi'=\sqrt{\frac{2\omega+3}{2\alpha}}\frac{d\phi}{\phi}, \quad \alpha\neq 0, \qquad (4)$$

the action (2) goes into the EFBD action, for the string value $\omega=-1$,

$$S_{EF}=\int d^4x\sqrt{-g'}[\mathbf{R}'+\alpha g'^{\mu\nu}\varphi'_{,\mu}\varphi'_{,\nu}], \qquad (5)$$

where we have introduced a constant arbitrary parameter α that can have any sign. The action (5) is also called the string action in the Einstein frame but in this paper we distinguish it as the action of the EMS theory. If the kinetic term $\alpha g'^{\mu\nu}\varphi'_{,\mu}\varphi'_{,\nu}$ has an overall reverse (that is, negative) sign, we have what one calls unconventional coupling. However, no matter what the sign or value of α is, we can always proceed from EMS action (5) backwards up to the string action (1). We keep α unassigned until later. It seems remarkable that A-P [14] has ended up with action (5) as the simplest action arguing from a completely different angle, viz. by imposing wormhole constraints on the Lagrangian for a general class of microscopic scalar field. Obviously, the arguments have nothing to do with string theory yet the end action is quite the same. So we have here a picture in which the physics of dilatonic gravity meets that of wormholes. In what follows, we shall use slightly different notations that are in line with our earlier papers. These can be easily transcribed to those in Ref. [14], as shown in the Appendix.

To clearly demarcate the scope of what follows, we must state that we are not dealing here with time dependent cosmological wormholes, and/or wormholes with Euclidean signatures [17–19] which are qualitatively completely different from static Lorentzian wormholes. However, the role of Eq. (4) that connects JF and EF is the same. In this regard, note that we have imported a new parameter α in Eq. (4) and it is obvious that the ranges of ω and α can be chosen independently. In the context of cosmology, the choice of $\omega=0$ leaves the parameter α arbitrary in the EF [17,18]. It is also worth noting that Quiros, Bonal and Cardenas [19] have shown that the cosmological singularity occurring in the EF is removed in the JF in the range $-3/2<\omega\leq-4/3$. This result has significant impact on the question of which frame, JF or EF, is more physical and also on the status of quantum gravity [20]. However, in the context of string theory, we must use *only* the model independent, unique string value $\omega=-1$ in Eq. (4). In this case, we have $d\varphi' =(1/\sqrt{2\alpha})(d\phi/\phi)$, and the range of α is essentially left undetermined by the string theory field equations *per se* in the EF, viz. Eqs. (6) and (7). What actually determines α is the condition for the existence of wormholes *at the solution level* given, for instance, by $\beta^2>1$ [see Eq. (17) below], which in turn implies that φ' be imaginary for $\alpha>0$ [see Eqs. (11), (12)]. AP [14] has shown that the imaginary nature of φ' does not lead to any pathology or inconsistency in the physics of Lorentzian wormholes. A completely equivalent but alternative description, again at the solution level, is to regard φ' as a real function which then leads to $\alpha<0$. All these matters are developed in Secs. II and III. The important point is that *both* the cases [$\alpha>0$, φ' imaginary, or, $\alpha<0$, φ' real] lead to a negative sign before the kinetic term $\alpha g'^{\mu\nu}\varphi'_{,\mu}\varphi'_{,\nu}$ which is what we need for exotic matter. One is free to adopt any of the mutually exclusive theoretical alternatives without any loss of rigor in the wormhole analysis.

The field equations for the EMS theory, after dropping the primes in Eq. (5), are given by

$$R_{\mu\nu}=-\alpha\varphi_{,\mu}\varphi_{,\nu} \tag{6}$$

$$\Box^2\varphi=0. \tag{7}$$

In "isotropic" coordinates ($x^\mu,\mu=0,1,2,3$), the solution is given by [22].

$$\begin{aligned} ds^2&=g_{\mu\nu}dx^\mu dx^\nu \\ &=\left(1-\frac{m}{2r}\right)^{2\beta}\left(1+\frac{m}{2r}\right)^{-2\beta}dt^2-\left(1-\frac{m}{2r}\right)^{2(1-\beta)} \\ &\quad\times\left(1+\frac{m}{2r}\right)^{2(1+\beta)}[dr^2+r^2d\Omega_2^2], \end{aligned} \tag{8}$$

$$\varphi(r)=2\lambda\ \ln\left[\frac{1-\dfrac{m}{2r}}{1+\dfrac{m}{2r}}\right], \tag{9}$$

$$d\Omega_2^2=d\theta^2+\sin^2\theta d\phi^2, \tag{10}$$

where $\beta^2=1-2\alpha\lambda^2$. This solution can be directly obtained also by conformally rescaling the BD class I solution [9]. The two undetermined constants m and β are related to the source strengths of the gravitational and scalar parts of the configuration. To first order,

$$\varphi\approx\frac{\sigma}{r}, \tag{11}$$

where

$$\sigma=-2m\lambda=-2m[(1-\beta^2)/2\alpha]^{1/2} \tag{12}$$

is the strength of the scalar source. When $\beta=1$, we have $\varphi=0$, and the Schwarzschild metric is recovered in accordance with the no hair theorem. Using Einstein's energy momentum complex, we find that the total mass M of the configuration is given by

$$M=m\beta. \tag{13}$$

This is the conserved total mass of the configuration to be observed by asymptotic observers. Using this value, the metric (8) can be expanded in the weak field as

$$\begin{aligned} ds^2=&[1-2Mr^{-1}+2M^2r^{-2}+O(r^{-3})]dt^2-[1+2Mr^{-1} \\ &+O(r^{-2})][dr^2+r^2d\Omega_2^2]. \end{aligned} \tag{14}$$

This metric exactly coincides with the weak field Robertson expansion [23] of a centrally symmetric gravitational field. Assuming that the neutral test particles follow the geodesics determined by the metric (8), that is no abnormal coupling of ordinary matter with gravity, we see that all the well known solar tests of gravity are described just as precisely as does the exterior Schwarzschild metric. The parameter β does not appear separately in the expansion (14) and hence its effect cannot be measured by any metric test of gravity. The parameter α does not appear here either but it does appear in the expression for the scalar field in Eq. (9) or (11) and we can use its sign to fix the nature of φ. Let us return to Eq. (13) which can be immediately rewritten as

$$M^2=m^2-\frac{1}{2}\alpha\sigma^2. \tag{15}$$

It is quite apparent from Eq. (12) that σ can assume real or imaginary values depending on the values assigned to β and α. We shall now see what kind of values could be assigned to these parameters if the metric (8) is to represent a traversable wormhole.

III. CLASS I WORMHOLE IN THE EMS THEORY

For a coordinate description of wormholes that encapsules all the essential details, the Morris-Thorne [1] form is most useful, which is given by

$$ds^2=e^{2\Phi(R)}dt^2-\frac{1}{1-\dfrac{b(R)}{R}}dR^2-R^2d\Omega_2^2, \tag{16}$$

where $\Phi(R)$ is the redshift function and $b(R)$ is the shape function. Casting metric (8) in that form and manipulating a little, the wormhole throat is found to occur at the isotropic r coordinate radii

$$r_0^\pm=\frac{m}{2}[\beta\pm(\beta^2-1)^{1/2}]. \tag{17}$$

The value $\beta^2=1$ corresponds to a massive nontraversable wormhole since $r_0^{\pm}$ coincides with the horizon radius $r_s = m/2$ and we are not interested in this case. In order to build a traversable wormhole, one needs to avoid this radius and therefore, one must have real $r_0^{\pm}>m/2$. This requires that $\beta^2>1$. Now consider scalar field energy density ρ and the Ricci scalar $\mathbf{R}$ which work out to be

$$\rho=\frac{1}{2}\times\frac{m^2(1-\beta^2)}{\left(1-\frac{m^2}{4r^2}\right)^2}\times\left(r+\frac{m}{2}\right)^{-2(1+\beta)}\times\left(r-\frac{m}{2}\right)^{-2(1-\beta)}, \tag{18}$$

$$\mathbf{R}=2m^2r^4(1-\beta^2)\times\left(r-\frac{m}{2}\right)^{-2(2-\beta)}\times\left(r+\frac{m}{2}\right)^{-2(2+\beta)}. \tag{19}$$

They become finite at $r=m/2$ if $\beta\geqslant 2$, which accords well with the wormhole condition. In fact, it can be verified that all curvature invariants are also finite under the condition $\beta \geqslant 2$. So, one indeed has a regular spacetime, but the problem is that the surface area becomes infinite at $r=m/2$. But this could be due to a wrong choice of coordinates. Bronnikov *et al.* [24] called such spacetimes as representing cold black holes (CBH) because of zero Hawking temperature. Some of their interesting properties have also been discussed in the literature [25]. In any case, the wormhole flares out to two asymptotically flat regions connected by the throat and is traversable because the tidal forces can be shown to be finite at the throat and elsewhere.

For the wormhole value of β^2, viz. $\beta^2>1$, then, we have two equivalent situations: (i) Take $\alpha<0$, say $\alpha=-2$. This means breaking the energy conditions "by hand" (we shall provide an example later) in the source term in Eq. (6) so that we can have, from Eq. (12), a real scalar charge σ, that is $\sigma^2>0$, or (ii) take $\alpha>0$, say $\alpha=2$, then we have an imaginary scalar charge from Eq. (12) so that $\sigma^2=-\sigma'^2 <0$. In either case, of course, we have a reversed sign kinetic term in the action. Also in Eq. (6), we have a stress tensor that violates all energy conditions giving the kind of classical exotic matter necessary for the threading of traversable wormholes. Then, from Eq. (15), we have

$$M^2=m^2+\sigma'^2. \tag{20}$$

A wormhole with zero total mass, that is, $M=0$, immediately implies $m=0$ and $\sigma'=0$. In other words, we have the trivial case of a flat metric and zero scalar field. However, it is possible to avoid this uninteresting case by making m also imaginary and noting from Eq. (13) that we can also have $M=0$ if we set $\beta=0$. This is actually the case considered by AP [14]. In fact, taking $\alpha=2,\beta=0$ we have from Eq. (15),

$$\sigma^2=m^2. \tag{21}$$

Clearly, if σ^2 is negative, then so is m^2 and vice versa. It is thus enough in this particular case to assume the imaginary nature of any *one* of them. Defining the proper distance l as

$$l=r-\frac{\sigma'^2}{4r}, \tag{22}$$

the metric (8) can be rewritten in the form

$$ds^2=dt^2-dl^2-(l^2+\sigma'^2)d\Omega_2^2, \tag{23}$$

$$\varphi=\ln\left[\frac{1-\frac{i\sigma'}{2r(l)}}{1+\frac{i\sigma'}{2r(l)}}\right], \tag{24}$$

$$r(l)=\frac{l\pm\sqrt{l^2+\sigma'^2}}{2}. \tag{25}$$

Since m^2 is also negative, i.e., $m^2=-m'^2<0$, the wormhole throat at $l=0$ implies the real coordinate values $r_0^{\pm} =\pm\sigma'/2=\mp m'/2$ and the scalar field φ becomes imaginary but does not blow up at this value. Also, in the units considered, we have at $r=r_0^{\pm}$, $\rho=-1/2\sigma'^2$, $\mathbf{R}=-2/\sigma'^2$. Thus, we indeed have the simplest well behaved wormhole. Under the considerations above, we now have a real equation that Eq. (20) translates into, viz. $M^2=-m'^2+\sigma'^2$ obviously implying that $M=0$ wormholes are *extremal* in nature. This amounts to saying that we have a configuration in which the stresses of the φ field contribute an amount of energy just sufficient to nullify the effect of gravitational potential making the total energy zero. In other words, we have nontrivial energy sources residing at the origin of central symmetry in such a way as to make a configuration that is gravitationally indifferent to neutral test particles. Note that the extremal configuration can arise even when no exotic matter is involved, that is, $\beta^2<1$. In this case also, we can have $M =0\Rightarrow m=\sigma$ from Eq. (15) simply by choosing $\alpha=2$. The foregoing analyses will be helpful in what follows.

Finally, it must be noted that class I EMS solutions have received good attention in the literature [26,27]. For instance, using Eqs. (23)–(25), particle models in general relativity have been constructed by Ellis [26] by way of an ether flow through a drainhole. Geometrical optics, classical and quantum scattering problems have been studied in the Ellis geometry by Chetouani and Clement [28] and by Clement [29].

IV. CLASS I WORMHOLE IN THE STRING THEORY

Starting with the solutions (8) and (9) and working backwards up to action (1), we can straightaway write down the corresponding string solution as

$$\begin{aligned} d\tilde{s}^2&=\tilde{g}_{\mu\nu}dx^\mu dx^\nu\\ &=\left(1-\frac{m}{2r}\right)^{2(\beta-\lambda\sqrt{2\alpha})}\left(1+\frac{m}{2r}\right)^{-2(\beta-\lambda\sqrt{2\alpha})}\\ &\quad\times dt^2-\left(1-\frac{m}{2r}\right)^{2(1-\beta-\lambda\sqrt{2\alpha})}\end{aligned}$$

$$\times\left(1+\frac{m}{2r}\right)^{2(1+\beta+\lambda\sqrt{2\alpha})}[dr^2+r^2d\Omega_2^2], \quad (26)$$

$$\tilde{\Phi}=-\lambda\sqrt{2\alpha}\ln\left[\frac{1-\frac{m}{2r}}{1+\frac{m}{2r}}\right]. \quad (27)$$

Under a suitable reidentification of constants and coordinates, the above solution reduces to that discussed by Kar [15]. To first order in $(1/r)$, we have the strength of the dilatonic source $\tilde{\sigma}$ given by

$$\tilde{\Phi}\approx\frac{\tilde{\sigma}}{r}, \quad \tilde{\sigma}=m\lambda\sqrt{2\alpha}. \quad (28)$$

One recovers the Schwarzschild metric in the limit $\beta=1$. However, it does not seem possible to recover the seed solutions (8) and (9) which shows that we are dealing here with a class of solutions that is essentially distinct from its counterpart either in BD or in the EMS theory.

Expanding the metric (26) as in Eq. (14), and identifying the Keplerian mass M^*, the tensor mass $M_T=m\beta$ and the scalar mass $M_S=-m\sqrt{1-\beta^2}$ (cf. [30] for these definitions), we have

$$d\tilde{s}^2=[1-2M^*r^{-1}+2M^{*2}r^{-2}+O(r^{-3})]dt^2$$
$$-\left[1+2M^*r^{-1}\left(\frac{\beta+\sqrt{1-\beta^2}}{\beta-\sqrt{1-\beta^2}}\right)+O(r^{-2})\right]$$
$$\times[dr^2+r^2d\Omega_2^2], \quad (29)$$

where $M^*=m(\beta-\sqrt{1-\beta^2})\equiv M_T+M_S$. Solar observations can put a limit to β, which obviously is expected to be $\beta\approx1$. Note that the motion of ordinary test particle measures the Keplerian mass and it is assumed that the particle has negligible self energy so that the Nordvedt effect can be ignored. The tensor mass is measured by the motion of a Schwarzschild black hole in the metric (8) [30].

We shall now investigate if the solutions (26) and (27) represent traversable wormholes. To this end, we cast the metric (26) in the Morris-Thorne form by redefining the radial variable $r\rightarrow R$ as

$$R=r\left(1-\frac{m}{2r}\right)^{(1-\beta-\lambda\sqrt{2\alpha})}\left(1+\frac{m}{2r}\right)^{(1+\beta+\lambda\sqrt{2\alpha})}. \quad (30)$$

The redshift function $\Phi(R)$ and the shape function $b(R)$ turn out to be

$$\Phi(R)=[\beta-\sqrt{1-\beta^2}]\times\left[\ln\left(1-\frac{m}{2r}\right)-\ln\left(1+\frac{m}{2r}\right)\right], \quad (31)$$

$$b(R)=R\left[1-\left(\frac{r^2+\frac{m^2}{4}-m\tilde{\beta}r}{r^2-\frac{m^2}{4}}\right)^2\right], \quad (32)$$

$$\tilde{\beta}\equiv\beta+\sqrt{1-\beta^2}. \quad (33)$$

Clearly, $\beta=1\Rightarrow\tilde{\beta}=1$. The throat occurs at the radii

$$\tilde{r}_0^{\pm}=\frac{m}{2}[\tilde{\beta}\pm(\tilde{\beta}^2-1)^{1/2}] \quad (34)$$

and as before the wormhole requirement is that $\tilde{\beta}^2>1$. The energy density $\tilde{\rho}$ is given by

$$\tilde{\rho}=\frac{2}{R^2}\times\frac{m^2r^2}{(r^2-m^2/4)^2}\times(1-\tilde{\beta}^2), \quad (35)$$

which is negative for $\tilde{\beta}^2>1$, as expected. The tidal forces are finite and so the solution represents a traversable wormhole. The total conserved mass of the configuration as observed by asymptotic observers can be identified as

$$\tilde{M}=m\tilde{\beta}. \quad (36)$$

In the zero mass limit: $\tilde{\beta}=0\Rightarrow\tilde{M}=0$. This implies $\beta=-1/\sqrt{2}$ which in turn implies a dilatonic field strength $\tilde{\sigma}=m/\sqrt{2}$. It is now useful to recall the discussions surrounding Eqs. (21)–(25). The situation here is that $\widetilde{r_0}^{\pm}$ is imaginary as before but to make it real one can assume m to be imaginary which automatically implies that $\tilde{\sigma}$ is also imaginary. Under the transformation $l=r(1+m^2/4r^2)$, the metric (26) becomes

$$d\tilde{s}^2=\left(\frac{l+m}{l-m}\right)^{\sqrt{2}}dt^2-dl^2-(l^2-m^2)d\Omega_2^2,$$

$$\tilde{\Phi}=-\frac{1}{2\sqrt{2}}\ln\left(\frac{l-m}{l+m}\right). \quad (37)$$

With m imaginary, we have a positive definite minimum surface area $-4\pi m^2$ but at $l=0$, we have $\tilde{g}_{00}=(-1)^{\sqrt{2}}$ which is a many valued function. Also for $l\neq0$, $\tilde{g}_{00}$ becomes imaginary which requires us to go beyond the real manifold that we have been considering. Hence, although massive wormholes exist, zero mass wormholes seem untenable, at least in the simplest form of the string theory that is under present investigation. We shall also state a physical reason in Sec. VI as to why they are untenable. Nonetheless, Eq. (37) is still a *formal* zero mass solution of the string action (1). We shall encounter solutions of similar nature in the next section.

The developments in Secs. III and IV immediately reveal certain interesting features about the images of EMS class I wormholes in the string theory. It follows that both $\beta=0$ (zero mass) and $\beta=1$ EMS wormholes have the image of only a nontraversable wormhole in the string theory with the throat occurring at $\tilde{r}_0^{\pm}=m/2$. For $\beta=0$ we have $\tilde{g}_{00}\to\infty$ at $\tilde{r}_0^{\pm}=m/2$, which is undesirable. For the range of values $\beta^2>1$, traversable Lorentzian wormholes do exist in the EMS theory but they have no counterpart in the string theory since $\tilde{\beta}$ and $\tilde{r}_0^{\pm}$ become imaginary. However, ordinary EMS solutions for $\beta^2<1$ have *only* wormhole images in the string regime due to the fact that $\beta^2<1\Rightarrow\tilde{\beta}^2>1$ which was shown to be a necessary condition for string wormholes.

V. CLASS II-IV SOLUTIONS IN THE EMS AND STRING THEORY

By conformal rescaling of the BD class II-IV solutions, it is possible to obtain the corresponding solutions in the EMS theory [31]. Alternatively, they can be obtained by solving the EMS equations (6) and (7) by standard procedures. We take the general form of the metric as

$$ds^2=P(r)dt^2-Q(r)[dr^2+r^2d\Omega_2^2]. \tag{38}$$

(a) Class II EMS solutions are

$$P(r)=e^{2\alpha_0+4\gamma\arctan(r/b)},$$

$$Q(r)=\left[1+\frac{b^2}{r^2}\right]^2 e^{2\beta_0-4\gamma\arctan(r/b)}, \tag{39}$$

$$\varphi(r)=2\lambda\arctan\left(\frac{r}{b}\right), \tag{40}$$

where $\lambda\equiv[8(1+\gamma^2)/\alpha]^{1/2}$ and $\alpha_0,\beta_0,\gamma,b$ are arbitrary constants. Asymptotic flatness requires that $\alpha_0=-\pi\gamma,\beta_0=\pi\gamma$. The solution (39) has a conserved total energy $M=2b\gamma$ as can be verified by computing the Einstein complex of stress energy. With this value of M, the metric expands exactly like Eq. (14) for $r^2\geqslant b^2$, and thereby explains all the solar system tests of gravity. To see if the solution set (38)–(40) represent wormholes, we cast it in the Morris-Thorne form to find that the coordinate throat radii occur at

$$r_0^{\pm}=b[\gamma\pm\sqrt{1+\gamma^2}]. \tag{41}$$

Note that, in the solution (40), one has $1+\gamma^2<0$, and this inequality is a result of the field equations (8) and (9), so that one has an imaginary φ. Alternatively, we can choose $\alpha<0$, and have a real φ. But this is no real problem as the two situations are equivalent, as explained earlier. Although $r_0^{\pm}$ is imaginary, one might choose b to be imaginary to make $r_0^{\pm}$ real. In this case, M also becomes real. As is obvious, we are employing the same arguments as we did in Sec. III for the zero mass case. Note that, although γ and b are imaginary, the metric functions $P(r)$ and $Q(r)$ are real. All the curvature invariants are finite everywhere [31] and the tidal forces experienced by a geodesic traveler can be shown to be finite at the wormhole throat and these tend to vanish at the asymptotic region. Most importantly, the exponential function $P(r)$ does not vanish anywhere so that the solution has no horizon. In this sense, it shares the features of Morris-Thorne "$\Phi=0$" (no horizon) wormholes [1]. Thus, the EMS class II solutions also represent traversable wormholes. Although in the zero mass limit, viz. $\gamma=0$, the metric (38) in proper distance language looks promising, that is,

$$ds^2=dt^2-dl^2-(l^2+4b^2)d\Omega_2^2, \tag{42}$$

such wormholes unfortunately do not exist as the limit itself $(\gamma=0)$ conflicts with the inequality $1+\gamma^2<0$.

The string version of the class II solution is given by

$$d\tilde{s}^2=\tilde{P}(r)dt^2-\tilde{Q}(r)[dr^2+r^2d\Omega_2^2], \tag{43}$$

$$\tilde{P}(r)=e^{2\alpha_0+4[\gamma-\sqrt{1+\gamma^2}]\arctan(r/b)},$$

$$\tilde{Q}(r)=\left[1+\frac{b^2}{r^2}\right]^2 e^{2\beta_0-4[\gamma+\sqrt{1+\gamma^2}]\arctan(r/b)}, \tag{44}$$

$$\tilde{\Phi}(r)=-(\gamma-\sqrt{1+\gamma^2})\arctan\left(\frac{r}{b}\right). \tag{45}$$

Other relevant quantities, e.g., the mass and throat radii, are respectively given by

$$\tilde{M}=2b\tilde{\gamma}=2b[\gamma+\sqrt{1+\gamma^2}],\quad \tilde{r}_0^{\pm}=b[\tilde{\gamma}\pm(1+\tilde{\gamma}^2)^{1/2}]. \tag{46}$$

All these quantities are real if b is imaginary, as we assumed. Here again, for the same reasons described in Sec. IV, massive, i.e., $\tilde{M}\neq0$ traversable wormholes do exist but the massless limit does not, as no value of γ can make $\gamma+\sqrt{1+\gamma^2}=0$, a condition that is required to make $\tilde{M}=0$.

(b) Class III EMS solutions are

$$P(r)=\alpha_0e^{-(\gamma r/b)},\quad Q(r)=\beta_0\left(\frac{r}{b}\right)^{-4}e^{(\gamma r/b)},\quad \varphi(r)=\frac{\gamma r}{2b}. \tag{47}$$

This solution is not asymptotically flat and hence does not meet the requirement of asymptotic flaring out of the wormholes. However, it is flat in the limit $r\to0$. If we still formally impose the zero mass condition $\gamma=0$ and define $l=-b^2/r$, we have the metric

$$ds^2=\alpha_0dt^2-\beta_0dl^2-\beta_0l^2d\Omega_2^2. \tag{48}$$

Under a further rescaling $\sqrt{\alpha_0}t\to t'$, $\sqrt{\beta_0}l\to l'$, we end up with a trivial metric. The string class III metric is

$$\tilde{P}(r)=\alpha_0e^{-(3\gamma r/2b)},\quad \tilde{Q}(r)=\beta_0\left(\frac{r}{b}\right)^{-4}e^{(\gamma r/2b)},$$

$$\tilde{\Phi}(r)=-\frac{\gamma r}{2b}, \tag{49}$$

and we again have a flat Minkowski metric like Eq. (48) for the case $\gamma=0$.

(c) Class IV EMS solutions exhibit some good properties. They are given by

$$P(r)=\alpha_0 e^{-[\gamma/(br)]},\quad Q(r)=\beta_0 e^{[\gamma/(br)]},\quad \varphi(r)=-\frac{\gamma}{2br}. \tag{50}$$

Asymptotic flatness fixes $\alpha_0=\beta_0=1$. The horizon appears at $r=0$. First of all, under the transformation $r\rightarrow 1/r$, $\beta_0 \rightarrow b^4\beta_0$, the solution goes over to the class III EMS solution (47) and hence the two classes are not essentially distinct, although in the original JF version, they are cited as different classes of solutions [32]. Secondly, all the curvature invariants are finite everywhere including $r=0$. In fact, the solution could be interpreted as a CBH [25] since the area at $r=0$ is infinite. But the geodesic congruences can not reach the origin, but reach a minimum distance $r_0=M$ (see below) away from it, corresponding to a finite surface area. Thereafter, they diverge so that it is more likely that it represents a pure wormhole. Thirdly, the total conserved mass for the solution is given by a real $M=\gamma/2b$ and the metric exactly coincides with the expansion (14) up to the orders considered. Hence, it describes all the weak field tests of general relativity just as good as the Schwarzschild metric does. However, for $\gamma=0$ for which $M=0$, we again have only a flat spacetime and a vanishing scalar field and consequently no zero mass wormholes. In the scalar field theory, there is a black hole counterpart which usually occurs when the scalar field is set to zero in accordance with the "no hair" theorem. This situation obviously does not arise here and that is another reason why we prefer to call class IV solutions as pure wormholes.

To see if class IV EMS solutions represent massive wormholes, we cast the metric (50) in the Morris-Thorne form to obtain the shape and redshift functions, respectively, as

$$b(R)=re^{M/r}\left[1-\left(1-\frac{M}{r}\right)^2\right],\quad \Phi(R)=-\frac{M}{r}, R=re^{M/r}. \tag{51}$$

The wormhole throat appears at $r_0=M\Rightarrow R=Me$ which is greater than the horizon radius. The density ρ and the radial tension τ are

$$\rho=-\frac{M^2}{R^2r^2}<0,\qquad \tau=\frac{M^2}{R^2r^2}, \tag{52}$$

such that $\tau-\rho>0$ not only at the throat but everywhere. Hence the flaring out condition is satisfied. The tidal forces are finite for static as well as for freely falling observers [21]. The forces, however, could be large for small values of M. So, everything put together, the solution indeed represents a massive Lorentzian wormhole that is traversable at least in principle.

Note that the solution (50) solves the field equations (8) and (9) for $\alpha=-2$ so that here we have an example where all energy conditions are broken by hand, since φ is real. In order to go to the string metric, we need ϕ which becomes, with this value of α, $\phi^{-1}=e^{-2i\varphi}$. This imports an imaginary factor to the string metric. Therefore, the string version of class IV solution has the form

$$\tilde{P}(r)=e^{-(2M/r)(1-i)},\quad \tilde{Q}(r)=e^{(2M/r)(1+i)},\quad \tilde{\Phi}=\frac{iM}{r}. \tag{53}$$

Clearly, Eqs. (53) do not represent wormholes in real spacetimes although it is an interesting formal solution of string field equations of the action (1) in the same way as the zero mass solution, Eq. (37) is. Here again, $\tilde{M}=M(1+i)=0\Rightarrow M=0$, and this leads to a trivial Minkowski spacetime so that there are no zero mass wormholes.

VI. CHARGED TEST PARTICLE MOTION

In this section, we consider motion only in the class I solution of EMS and string theory. As mentioned in Sec. II, from the expansion (14) the effect of β (or the source scalar charge σ) can not be separately explored by the motion of neutral test particles at least in the power of $1/r^2$ since the total conserved gravitating mass M appears only as a product of m and β. The expansion (29) in the string version does separate the effect of β and neutral test particle probes are able to put a limit on its value. However, here we wish to consider not neutral but charged particle motions. To this end, following an interesting approach by Buchdahl [22], we regard φ (or in the string context, $\tilde{\Phi}$) as representing some medium or long range force field existing in spacetime $g_{\mu\nu}$ (or $\tilde{g}_{\mu\nu}$) and imagine a test scalar charge responding to this field directly in addition to interacting indirectly via the metric. The situation is analogous to the motion of an electrically charged particle in the Reissner-Nordström spacetime. In order to have bound orbits, we shall assume that the source and test charges have opposite signs. With this understanding, let us consider the equation of motion of a test particle with infinitesimally small mass δ and scalar charge ε. In virtue of the Plebański-Sawicki theorem [33], the geodesic equations are given by

$$u^{\mu}[(\delta-\varepsilon\varphi)u^{\nu}]_{;\mu}=-\varepsilon\varphi_{;}^{\nu}, \tag{54}$$

where u^{μ} is the four velocity of the particle and ; denotes covariant derivative with respect to $g_{\mu\nu}$ defined in Eq. (8). These equations have the first integral $u_{\mu}u^{\mu}=1$ and the particle trajectories correspond to those defined by the metric [22]

$$ds'^2=(\delta-\varepsilon\varphi)^2 g_{\mu\nu}dx^{\mu}dx^{\nu}. \tag{55}$$

By carrying out the expansion plugging in the expressions of $g_{\mu\nu}$ and φ from Eqs. (8) and (9), we have the metric

$$ds'^2=h'(r)dt^2-p'(r)[dr^2+r^2d\Omega_2^2], \tag{56}$$

where

$$h'(r)=1-2(1-\varpi)\frac{M}{r}+(2-4\varpi+\varpi^2)\frac{M^2}{r^2}+O\left(\frac{1}{r^3}\right), \tag{57}$$

$$p'(r)=1+2(1+\varpi)\frac{M}{r}+O\left(\frac{1}{r^2}\right), \quad \varpi=\frac{\sigma\varepsilon}{\delta M}, \quad (58)$$

in which ϖ can take on only negative values as ϵ and σ are assumed to have opposite signs. All the observable quantities relating to the trajectory of the test scalar charge can be calculated in the usual way. For instance, if κ is the precession of the pericenter per revolution for a given ϖ and κ_0 is the precession for $\varpi=0$, then

$$\frac{\kappa}{\kappa_0}=1-\frac{2\varpi}{3}-\frac{\varpi^2}{6}. \quad (59)$$

Note that $\kappa=\kappa_0\Rightarrow\varpi=-4$. Now, in the environment of a $M=0$ extremal configuration, we immediately find that the metric functions reduce to

$$h_0'(r)=1-\frac{2M'}{r}+\frac{M'^2}{r^2}+O\left(\frac{1}{r^3}\right), \quad (60)$$

$$p_0'(r)=1+\frac{2M'}{r}+O\left(\frac{1}{r^2}\right), \quad (61)$$

that can be thought of as generated by a hypothetical scalar "mass"

$$M'=\frac{\sigma\varepsilon}{\delta}>0. \quad (62)$$

Although neutral test particles follow straight paths due to the fact that the Keplerian source mass $M=0$, the test scalar charge executes a motion that closely resembles that of a neutral test particle in the Schwarzschild spacetime generated by the mass $M'=\sigma\varepsilon/\delta$. There will of course be a slight difference in the numerical value of the precession of orbits due to the lack of factor 2 in the $(1/r^2)$ term in Eq. (60). Nevertheless, what we have here is a purely scalar-scalar interaction leading to a scalar mass M' that restrains the test charges in their geodesics but does not respond gravitationally. In the limit $\delta\to 0, M'$ could be very large. This is an interesting feature of string theory if we believe that physics is described by the EMS action (5). One finds that the scalar-scalar interaction in an otherwise flat space (as noticed by neutral particles) describes a kind of confinement of the test scalar charge in bound orbits.

In the string environment, φ should be replaced by $\tilde{\Phi}$ and $g_{\mu\nu}\to\tilde{g}_{\mu\nu}$, and then the counterpart of the metric (56) becomes

$$d\tilde{s}'^2=(\delta-\varepsilon\tilde{\Phi})^2 d\tilde{s}^2=\tilde{h}(r)dt^2-\tilde{p}(r)[dr^2+r^2 d\Omega_2^2]. \quad (63)$$

Using the metric functions given by Eq. (29), we have

$$\tilde{h}(r)=1-2(1-\varpi_1)\left(\frac{M^*}{r}\right)+(2-4\varpi_1+\varpi_1^2)\left(\frac{M^*}{r}\right)^2+O\left(\frac{1}{r^3}\right), \quad (64)$$

$$\tilde{p}(r)=1+2\left[(1+\varpi_1)\left(\frac{\beta+\sqrt{1-\beta^2}}{\beta-\sqrt{1-\beta^2}}\right)\right]\frac{M^*}{r}+O\left(\frac{1}{r^2}\right),$$

$$\varpi_1=\frac{\varepsilon\tilde{\sigma}}{\delta M^*}. \quad (65)$$

For $\tilde{M}=0$ ordinary string configuration (not wormhole, they do not exist as shown in Sec. IV) corresponding to $\tilde{\beta}=0$, we find that the term in the square bracket vanishes identically. Therefore, the motion in this situation would be somewhat different from the one described by Eqs. (56)–(58). If in addition, we assume that the Keplerian mass $M^*=0$ in Eq. (64), we have the metric function $\tilde{h}(r)$ analogous to Eq. (62) with M' replaced by $\varepsilon\tilde{\sigma}/\delta$, but a flat $\tilde{p}(r)$ up to first order. Thus, in a flat metric with only the dilatonic field coupling to it, there is still some sort of a dilaton-dilaton interaction generating a certain mass if the physics is believed to be described by the action (1).

The developments of this section have a direct bearing on the principle of equivalence that is usually discussed in terms of motion of the test particles in a given metric. If the motion follows the geodesics of the metric, one says that the principle is obeyed. First consider the BD theory. The weak principle of equivalence (WEP) is satisfied as small (negligible binding energy) neutral test particles do move along the geodesics determined by the BD metric. But the strong equivalence principle (SEP), which states that WEP must be satisfied even for bodies with large gravitational self-energy, is violated in the BD theory due to the appearance of two types of masses [30]. In the string theory, too, WEP is satisfied to the same extent as in BD theory, but we see that SEP could be violated because of the appearance of two masses M_T and M_S in the metric (26). The ratio of the two masses, viz. $M_S/M_T=(\sqrt{1-\beta^2})/\beta$ depends on the gravitational binding energy of the source. Indeed, in the limit $\tilde{\beta}\to 0$, one has $|M_S/M_T|\to 1$, in contrast to the EMS case in which one has $M_S=0$. Thus, in the string zero mass configuration, the self energy becomes maximum. This is probably an indicator as to why zero mass wormholes in the string theory do not exist.

In this context, recall that the metric $g_{\mu\nu}$ (it is also called the Pauli metric) couples to dilaton in a "normal" way, that is, by way of EMS action (5) and it has been argued that the dilatonic test particle (and not the ordinary neutral particle) should follow the geodesics of $g_{\mu\nu}$ and satisfy the WEP [21]. If we endow the dilaton with an infinitesimal mass δ (e.g., pseudo Goldstone bosons) and charge ε, then the argument seems to be at variance with the Plebański-Sawicki theorem due to the fact that the dilaton follows the geodesics of ds'^2 and *not* of ds^2. Indeed, a value of κ/κ_0 away from unity indicates a violation of WEP by charged particle motion. However, this need not necessarily be the case. One may always take the usual viewpoint that the effect of the source scalar field is already subsumed in the metric and the motion of test dilaton, by definition, does not alter the background geometry. It is only if one allows a departure from this viewpoint by introducing an extra scalar-scalar interaction *a la* Buchdahl that one comes up with what is embodied in the Eqs. (60)–(65).

VII. SUMMARY

The contents of the paper may be summarized as under:

(1) Using the four classes (I-IV) of EMS solutions [action (5)] as seed solutions derived earlier [31], corresponding classes of solutions in the low energy effective string theory [action (1)] have been obtained and analyzed. They can be grouped as string class I [Eqs. (26), (27)], class II [Eqs. (44), (45)], class III [Eq. (49) and class IV [Eq. (53)] static spherically symmetric solutions. The string class I solutions can be identified with those discussed recently by Kar [15], but the rest of the three sets of solutions given here are essentially *new* to our knowledge. Note that it is not possible to recover the seed EMS solutions by any choice of the free parameters. The metric parts of the string solutions resemble those in the BD theory with $\omega=-1$, while the scalar parts correspond to those of EMS theory up to a redefinition of constants. In other words, the solutions have one leg in the BD and other in the EMS theory. These solutions could be derived also by solving the whole plethora of string field equations coming from the action (1). The last solution set, Eqs. (53), is complex but that is not unexpected as the seed BD class IV solution is also complex at the string value $\omega=-1$. It should be emphasized that the solutions discussed here do not exhaust all the possible spherically symmetric solutions of the string theory that might exist.

(2) Wormhole solutions have been explored in all the above solutions of EMS and string theories. The following results are obtained: Massive Lorentzian traversable wormholes exist in classes I, II and IV in the EMS theory, but not in class III. However, it has been pointed out that class III solution is related to class IV through a coordinate inversion and are not really distinct solutions. However, for the wormhole flaring out condition, the latter form (IV) is more suitable due to its asymptotic flatness at $r=\infty$. The zero mass limit is mathematically forbidden in class II while the class IV solution leads to a trivially flat spacetime in this limit. In the string theory, massive wormholes exist in the three corresponding classes of solutions (I,II,IV). A remarkable result is that zero mass wormholes do *not* exist in the string theory at all, at least within the solution sets considered. A physical reason for this could be that the gravitational self-energy becomes very large. In this sense, the nonexistence of zero mass wormholes is a result of the violation of SEP in the string frame. Stable zero mass wormholes that exemplify Wheeler's "charge without charge" discussed by Armendáriz-Picón [14] can exist only in EMS class I, and not in other EMS classes, as shown in this paper. From the analysis in Sec. III, one can now discern the underlying physical reason: The scalar mass M_S does not appear in the metric (8) so that the gravitational self energy is negligible. Only the EMS class I solution has this very desirable property.

(3) The motion of a test particle endowed with an infinitesimal mass and scalar charge has been investigated in both EMS and string theory following an approach by Buchdahl [22] who used the Plebański-Sawicki theorem. The approach is based on the idea that the test charge responds directly to the scalar field and not only indirectly via the metric. The metric expansions in the case of only indirect response have been demonstrated in Eqs. (14) and (29). If both direct and indirect responses are taken into account, then the metrics expand like Eqs. (56) and (63). An interesting result in the case of extremal $M=0$ EMS environment [action (5)] is that the scalar-scalar interactions produce a hypothetical scalar "mass" that can confine test charges. Similar, but not quite the same, effects occur also in the string theory described by action (1).

As a mere curiosity, one can speculate a possible cosmological implication of this phenomenon. Will and Steinhardt [35] conjectured that an inflation induced oscillation of a massive gravitational scalar field could account for the "missing mass" required to close the universe. The scalar-scalar interaction at a classical level as considered here could provide a possible mechanism for the production of the missing mass in the universe, if one is prepared to allow a violation of WEP. Ordinary neutral particles does not respond to the scalar mass M' (since the Keplerian mass of the configuration $M=m\beta=0$) but M' does curve the local spacetime by way of the metric (60), (61). One could think of zero Kepler mass (but $M'\neq 0$) microscopic wormholes populating the universe and the contributions from M' leading to the closure of the visible universe. However, it is stressed that resolving the missing mass issue is not the main purpose of the present paper as the problem involves several other different considerations.

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APPENDIX

The equivalence of notations of AP with those in the present paper can be readily achieved by the identifications: $M_{our}\to m_{A-P}, m_{our}\to \eta_{A-P}, \sigma_{our}\to q_{A-P}$. The expression $m^2=\eta^2-q^2$ used in Ref. [14] with the assumption $q^2<0$ is identical to our Eq. (20) above. It should also be pointed out that the solutions (8) and (9) reduce, under the radial transformation $r=\rho(1+m/2\rho)^2$ and in the AP notation, to the Janis-Newman-Winnicour (JNW) form:

$$ds^2 = \left(1-\frac{2\eta}{\rho}\right)^{m/\eta} dt^2 - \left(1-\frac{2\eta}{\rho}\right)^{-m/\eta} d\rho^2 - \left(1-\frac{2\eta}{\rho}\right)^{1-m/\eta} \rho^2 d\Omega_2^2, \tag{A1}$$

$$\varphi(r) = \sqrt{\frac{1-\frac{m^2}{\eta^2}}{2\alpha}} \ln\left(1-\frac{2\eta}{r}\right). \tag{A2}$$

For more details, see Ref. [34]. The metric (A1) above is precisely the Eq. (21) in AP [14]. Now, impose the condition of zero total mass, viz. $M_{our} \to m_{A-P} = 0$ on Eq. (A1). Redefining $\rho = l$, one gets, from Eq. (A1)

$$ds^2 = dt^2 - dl^2 - (l^2 - 2\eta l) d\Omega_2^2, \tag{A3}$$

$$\varphi = \frac{1}{2} \ln\left(1-\frac{2\eta}{l}\right). \tag{A4}$$

Whatever be the nature or sign of η the minimum surface area is zero that occurs either at $l=0$ or at $l=2\eta$ and the scalar field either blows up or becomes undefined at those values. Thus this form of metric is not suitable since it represents a naked singularity at $l=0$ or $l=2\eta$. It should be noted that the solution (8) and (9), which represents the *same* solution as the one in Eqs. (A1) and (A2) but only rewritten in isotropic form, also exhibits a globally strong naked singularity at $r=m/2$. (Visser [12] called such wormholes "diseased.") What is interesting is that, only in the zero mass limit, the disease disappears in one but persists in the other coordinate system.

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