Riemann tensor of the ambient universe, the dilaton, and Newton's constant

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We investigate a four-dimensional world, embedded into a five-dimensional spacetime, and find the five-dimensional Riemann tensor via generalization of the Gauss(-Codacci) equations. We then derive the generalized equations of the four-dimensional world and also show that the square of the dilaton field is equal to Newton's constant. We find plausible constant and nonconstant solutions for the dilaton.

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Since the pioneering work of Kaluza [1] and Klein [2], who unified gravitation with electromagnetism, the implications of possible extra dimensions to our world have been under intense investigation—see [3] for an extensive collection of papers on higher-dimensional unification. Jordan and Müller [4] and Thiry [5] used the equations of Kaluza-Klein theory to show that the gravitational constant can be expressed as a dynamical field. A constant solution for Newton's constant, however, is then allowed only if the square of the Maxwell electromagnetic tensor vanishes. Here we examine a *dual* setup in which this problem can be avoided. Based on the original Kaluza model, we are here considering a general embedding of a four-dimensional world into a five-dimensional ambient spacetime. We derive a generalization of the Gauss(-Codacci) equations by utilizing all degrees of freedom (the entire geometry) of the ambient spacetime, and we also show how they affect the physics of the four-dimensional world. As a result we find a system of equations for the electromagnetic field, gravitational field, and dilaton field. One of these equations is a plausible generalization of the gauge fixed Maxwell equations in the presence of a dilaton field. We also show that the square of the dilaton field is equal to (modulo numerical factors) the Newton's constant. The gauge freedom of the electromagnetic fields is transferred to a freedom in fixing the dilaton field. Apart from the constant solution for the dilaton, we give an example of a nonconstant solution describing a time-varying Newton constant in an expanding universe (see also [6–11] and others). We also give a general formula for generating different solutions for the dilaton field.

We consider a four-dimensional world M, embedded into a five-dimensional spacetime V (see [12–22] and references therein for a detailed discussion on embeddings). Let y^i $(i=1,2,3,4)$ denote the coordinates on M and x^μ $(\mu=1,2,3,4,5)$ denote the coordinates on V. Greek indices will be related to the five-dimensional spacetime V, while latin indices will be associated with the four-dimensional spacetime M. Let $\psi(x^\mu)=s=\text{const}$ be the equation of the hypersurface M. We assume that this equation is given *a priori*. The physical principles motivating the choice of type of hypersurface [specified by the function $\psi(x^\mu)$] and the choice of a particular four-dimensional spacetime slice (given by s) are open problems for multidimensional theories and not the subject of our analysis. We will retain s as a parameter of the model.

One can alternatively express the parametric equations of M as $x^\mu=x^\mu(y^j,s)$ and treat the parameter s as a coordinate; this then represents a coordinate transformation with inverse

$$y^j=y^j(x^\mu), \quad s=\psi(x^\mu). \tag{1}$$

We assume that this transformation is invertible at each point. This means that the Jacobi matrices of the transformation and its inverse have nonvanishing determinants everywhere. Thus, to globally parametrize the foliated five-dimensional spacetime V, it is sufficient to use the coordinates of the four-dimensional world M and the foliation parameter s.

The vector normal to the surface is

$$N_\mu=\frac{\partial\psi}{\partial x^\mu}. \tag{2}$$

Let us also define

$$e^\mu_j=\frac{\partial x^\mu}{\partial y^j}, \quad n^\mu=\frac{\partial x^\mu}{\partial s}, \quad E^i_\mu=\frac{\partial y^i}{\partial x^\mu}. \tag{3}$$

The derivatives are related as follows:

$$\partial_k\equiv\frac{\partial}{\partial y^k}=e^\mu_k\partial_\mu, \tag{4}$$

$$\partial_\nu\equiv\frac{\partial}{\partial x^\nu}=E^k_\nu\partial_k+N_\nu\partial_s. \tag{5}$$

Obviously, if we denote $e^\mu_5=n^\mu$ and $E^5_\mu=N_\mu$, then (e^μ_ν) and (E^μ_ν) will be the Jacobi matrices of the transformation $(x^\mu)\to(y^i,s)$ and its inverse. Therefore,

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$$e^{\mu}_{\sigma}E^{\sigma}_{\nu}=\delta^{\mu}_{\nu}=E^{\mu}_{\sigma}e^{\sigma}_{\nu}. \tag{6}$$

This orthogonality condition is equivalent to

$$e^{\mu}_{i}E^{j}_{\mu}=\delta^{j}_{i}, \tag{7}$$

$$N_{\mu}n^{\mu}=1, \tag{8}$$

$$E^{i}_{\mu}e^{\nu}_{i}+N_{\mu}n^{\nu}=\delta^{\nu}_{\mu}, \tag{9}$$

$$N_{\mu}e^{\mu}_{j}=0, \tag{10}$$

$$n^{\mu}E^{j}_{\mu}=0. \tag{11}$$

(Note that $\delta^{\mu}_{\mu}=\dim V=5$ and $\delta^{i}_{i}=\dim M=4$.)

Thus the bases (e^{μ}_{j},n^{μ}) and (E^{j}_{μ},N_{μ}) are dual. They do not depend on the metric of either spacetime, but only on the particular embedding chosen.

Let us now introduce a scalar field $\phi(y^k,s)$, a vector field $A_i(y^k,s)$, and the metric tensor $g_{ij}(y^k,s)$, which are M-valued tensor functions on V. We further define the metric $G_{\mu\nu}$ of the five-dimensional spacetime V as an expansion over the basis vectors E^{i}_{μ} and N_{μ}:

$$G_{\mu\nu}=E^{i}_{\mu}E^{j}_{\nu}g_{ij}+(N_{\mu}E^{i}_{\nu}+N_{\nu}E^{i}_{\mu})A_i+N_{\mu}N_{\nu}\phi. \tag{12}$$

Taking $x^i=y^i$, $x^5=s=\text{const}$, i.e., $e^{\mu}_{i}=\delta^{\mu}_{i}$, $n^{\mu}=\delta^{\mu}_{5}$, $E^{i}_{\mu}=\delta^{i}_{\mu}$, $N_{\mu}=\delta^{5}_{\mu}$ in Eq. (12) corresponds to the original Kaluza model [1]. Klein's modification [2] $g_{ij}\to g_{ij}+A_iA_j$, together with the identification of ϕ as a dilaton is the model put forward by Jordan and Müller [4] and Thiry [5]. We note that the metric (12) has the same form as the inverse of the metric of Klein's model, and thus the two theories are dual: Kaluza's model corresponds to slicing, while Klein's model corresponds to threading of the five-dimensional spacetime [15]. The case with $A_i=0$ has also been considered (see, for example, [22,23] and references therein).

The lack of gauge invariance for the fields A_i, which we nevertheless will associate with the electromagnetic potentials, in view of the slicing-threading duality, is compensated by the freedom to fix ϕ. This, as will become clear later, is the freedom to fix the dilaton field.

Returning to Eq. (12), we define g^{ij} as the inverse of the metric g_{ij}. Thus $A^i=g^{ij}A_j$ and $A^2=g^{ij}A_iA_j$.

The inverse $G^{\mu\nu}$ of the metric $G_{\mu\nu}$ on V is then given by

$$G^{\mu\nu}=h^{ij}e^{\mu}_{i}e^{\nu}_{j}-\theta A^i(e^{\mu}_{i}n^{\nu}+e^{\nu}_{i}n^{\mu})+\theta n^{\mu}n^{\nu}, \tag{13}$$

where $\theta=(\phi-A^2)^{-1}$ and $h^{ij}=g^{ij}+\theta A^iA^j$. One can easily check that $G^{\mu\lambda}G_{\lambda\nu}=\delta^{\mu}_{\nu}$. Using the inverse $G^{\mu\nu}$ (13) of the metric $G_{\mu\nu}$, we can raise and lower the five-dimensional indices to get

$$N^{\mu}=G^{\mu\nu}N_{\nu}=\theta(n^{\mu}-A^ie^{\mu}_{i}), \tag{14}$$

$$N^2=N_{\mu}N^{\mu}=\theta, \tag{15}$$

$$n_{\mu}=G_{\mu\nu}n^{\nu}=A_iE^{i}_{\mu}+N_{\mu}\phi, \tag{16}$$

$$n^2=n_{\mu}n^{\mu}=\phi. \tag{17}$$

Note that when $A^i=0$ and $\phi=1$, then $N^{\mu}=n^{\mu}$ as in the Arnowitt-Deser-Misner approach [22].

The extrinsic curvature is

$$K_{jl}=e^{\mu}_{j}e^{\nu}_{l}Q_{\mu\nu}, \tag{18}$$

where $Q_{\mu\nu}=\nabla_{\mu}N_{\nu}=\nabla_{\nu}N_{\mu}$.

Multiplying Eq. (18) across by $E^{j}_{\alpha}E^{l}_{\beta}$ and applying the orthogonality conditions (7)–(11), one easily finds

$$Q_{\alpha\beta}=E^{i}_{\alpha}E^{j}_{\beta}K_{ij}+(N_{\alpha}E^{i}_{\beta}+N_{\beta}E^{i}_{\alpha})f_i+N_{\alpha}N_{\beta}\chi, \tag{19}$$

with $\chi=n^{\mu}n^{\nu}Q_{\mu\nu}$ and

$$f_j=n^{\mu}e^{\beta}_{j}Q_{\mu\beta}=A^iK_{ij}+\frac{1}{N}\partial_jN, \tag{20}$$

where $N=\sqrt{N_{\mu}N^{\mu}}$.

The four-dimensional Christoffel symbols

$$\gamma^{i}_{jk}=\frac{1}{2}g^{il}(\partial_kg_{lj}+\partial_jg_{lk}-\partial_lg_{jk}) \tag{21}$$

can then be expressed as

$$\gamma^{i}_{jl}=(\partial_je^{\mu}_{l}+e^{\alpha}_{j}e^{\beta}_{l}\Gamma^{\mu}_{\alpha\beta})E^{i}_{\mu}-K_{jl}A^i, \tag{22}$$

where $\Gamma^{\mu}_{\alpha\beta}$ are the five-dimensional Christoffel symbols. Further, the four-dimensional Riemann curvature tensor

$$r^{i}_{jkl}=\partial_k\gamma^{i}_{jl}-\partial_l\gamma^{i}_{jk}+\gamma^{m}_{jl}\gamma^{i}_{mk}-\gamma^{m}_{jk}\gamma^{i}_{ml}, \tag{23}$$

using Eqs. (22) and (7)–(10), becomes

$$\begin{aligned}r^{i}_{jkl}=&E^{i}_{\alpha}e^{\lambda}_{j}e^{\mu}_{k}e^{\nu}_{l}R^{\alpha}_{\lambda\mu\nu}+E^{i}_{\alpha}(\nabla_{\mu}n^{\alpha})(e^{\mu}_{k}K_{lj}-e^{\mu}_{l}K_{kj})\\&+\nabla_l(A^iK_{kj})-\nabla_k(A^iK_{lj})+A^iA^m(K_{ml}K_{kj}-K_{mk}K_{lj}),\end{aligned} \tag{24}$$

where $R^{\alpha}_{\lambda\mu\nu}$ is the five-dimensional Riemann curvature tensor. The above is a *generalization* of the Gauss equations. Taking $A^i=0$ and $\phi=1$, one simply recovers the well known Gauss equations (see, for example, [24])

$$r^{i}_{jkl}=E^{i}_{\alpha}e^{\lambda}_{j}e^{\mu}_{k}e^{\nu}_{l}R^{\alpha}{}_{\lambda\mu\nu}+K^{i}_{k}K_{lj}-K^{i}_{l}K_{kj}. \tag{25}$$

Let us now expand the five-dimensional Riemann curvature tensor over our basis. Using its symmetries we can write

$$\begin{aligned}R_{\lambda\mu\nu\sigma}=&E^{i}_{\lambda}E^{j}_{\mu}E^{k}_{\nu}E^{l}_{\sigma}U_{ijkl}+[(N_{\lambda}E^{j}_{\mu}-N_{\mu}E^{j}_{\lambda})E^{k}_{\nu}E^{l}_{\sigma}\\&+(N_{\nu}E^{j}_{\sigma}-N_{\sigma}E^{j}_{\nu})E^{k}_{\lambda}E^{l}_{\mu}]V_{jkl}\\&+(N_{\lambda}E^{j}_{\mu}-N_{\mu}E^{j}_{\lambda})(N_{\nu}E^{l}_{\sigma}-N_{\sigma}E^{l}_{\nu})W_{jl},\end{aligned} \tag{26}$$

where the coefficients in this expansion satisfy

$$U_{ijkl}=U_{klij}=-U_{ijlk}=-U_{jikl}, \tag{27}$$

$$V_{jkl}=-V_{jlk},\quad W_{jl}=W_{lj}. \tag{28}$$

Using Eq. (24), one can further find

$$U_{ijkl}=e_i^\lambda e_j^\mu e_k^\nu e_l^\sigma R_{\lambda\mu\nu\sigma}$$
$$=r_{ijkl}-(\pi_{ik}\pi_{lj}-\pi_{il}\pi_{kj}), \quad (29)$$
$$V_{jkl}=n^\lambda e_j^\mu e_k^\nu e_l^\sigma R_{\lambda\mu\nu\sigma}$$
$$=A^i U_{ijkl}-\frac{1}{N}(\nabla_k\pi_{lj}-\nabla_l\pi_{kj}), \quad (30)$$

where $\pi_{jl}=(1/N)K_{jl}$.

Finding the remaining tensor W_{jl} is more complicated. One can easily see that

$$W_{jl}=n^\mu n^\sigma e_j^\lambda e_l^\nu R_{\lambda\mu\nu\sigma}=S_{jl}+A^k V_{lkj}, \quad (31)$$

where

$$S_{jl}=\frac{1}{N^2}n^\sigma e_j^\lambda e_l^\nu(\nabla_\lambda Q_{\sigma\lambda}-\nabla_\sigma Q_{\nu\lambda}). \quad (32)$$

In the above we identify derivatives in the direction of n^σ. To handle this type of term, we will have to explicitly invoke the dependence on s.

First, by virtue of Eq. (5), we get the following expression for the extrinsic curvature (18):

$$K_{jl}=-\frac{N^2}{2}(\nabla_j A_l+\nabla_l A_j)+\frac{N^2}{2}\partial_s g_{jl}$$
$$-\frac{1}{2}N^\lambda(g_{kl}\partial_j E_\lambda^k+g_{kj}\partial_l E_\lambda^k+A_l\partial_j N_\lambda+A_j\partial_l N_\lambda)$$
$$+\frac{N^2}{2}G_{\mu\nu}[A^k\partial_k(e_j^\mu e_l^\nu)-\partial_s(e_j^\mu e_l^\nu)]. \quad (33)$$

Using Eqs. (20), (31), (32), and (33), it follows that

$$W_{jl}=\frac{1}{N^3}\nabla_j\nabla_l N-\frac{2}{N^4}(\partial_j N)(\partial_l N)+A^i A^k U_{ijkl}$$
$$+\frac{1}{N}[\nabla_j(A^k\pi_{kl})+\nabla_l(A^k\pi_{kj})]-\frac{1}{N}A^k\nabla_k\pi_{lj}$$
$$+\frac{1}{N^2}\pi_{kl}\pi_j^k-\frac{1}{N}\partial_s\pi_{jl}+\Omega_{jl}, \quad (34)$$

where Ω_{jl} contains only terms that are proportional to derivatives of the basis vectors and their duals with respect to (y^k,s).

The five-dimensional Ricci tensor can easily be calculated from Eq. (26):

$$R_{\mu\nu}=E_\mu^j E_\nu^l[h^{ik}U_{ijkl}-N^2A^k(V_{jkl}+V_{lkj})+N^2W_{jl}]+(N_\mu E_\nu^l$$
$$+N_\nu E_\mu^l)(-h^{jk}V_{jkl}+N^2A^jW_{jl})+N_\mu N_\nu h^{jl}W_{jl}. \quad (35)$$

Then the five-dimensional Einstein equations in vacuum

$$R_{\mu\nu}=0 \quad (36)$$

reduce to

$$h^{ik}U_{ijkl}-N^2A^k(V_{jkl}+V_{lkj})+N^2W_{jl}=0, \quad (37)$$
$$h^{jk}V_{jkl}-N^2A^jW_{jl}=0, \quad (38)$$
$$h^{jl}W_{jl}=0. \quad (39)$$

Multiplying Eq. (37) by A^j and adding it to Eq. (38) allows us to exclude W_{jl} from Eq. (38). Then, using the expressions (29) and (30) for U_{ijkl} and V_{jkl}, Eq. (38) becomes

$$\nabla_k\pi_l^k-\nabla_l\pi_k^k=0. \quad (40)$$

Equations (40) (as we will see below) are a *generalization* of Maxwell's equations in a fixed gauge.

One has to make a very important point here. Klein's theory corresponds to a threading decomposition of the five-dimensional spacetime [15]. Rigorous analysis [16] shows that the curvature tensor of the hypersurface formed is given by Zelmanov's curvature tensor, which differs from the ordinary Riemann curvature tensor by additional terms containing s derivatives of the four-dimensional metric [17–21]. Independence of the parameter s forces the two curvature tensors to be equal and thus represents a surface forming condition. In Kaluza's theory, the foliation of the five-dimensional spacetime corresponds to slicing [15]. Then the four-dimensional metric g_{ij} naturally appears as the slicing metric, and imposing independence on the parameter s is not at all necessary.

To simplify the analysis of the physics described by the fields A_i, ϕ, and N, we will, however, put aside the s-dependent terms. Also, for simplicity, we will assume that the basis elements and their duals are constant (thus recovering the original Kaluza theory). The tensor Ω_{jl} will then vanish from Eq. (34).

Equation (40) becomes

$$\nabla_k F^{kl}=-2A_k r^{kl}+\frac{2}{N^2}(\pi^{kl}-\pi_j^j g^{kl})\partial_k N. \quad (41)$$

Here $F_{kl}=\nabla_k A_l-\nabla_l A_k$ is the Maxwell electromagnetic tensor with A_k being the electromagnetic potential.

The first term on the right-hand side of Eq. (41) describes an interaction between electromagnetic and gravitational fields. We assume that it is much smaller than the remaining terms, so that we can neglect it. Note that A_k cannot be "gauged up" to increase the scale of $A_k r^{kl}$. Furthermore, if N is a constant, then Eq. (41) becomes the usual Maxwell equations

$$\nabla_k F^{kl}=0. \quad (42)$$

The remaining two equations are

$$\nabla_k\left(\frac{f^k}{N}\right)=0, \quad (43)$$
$$r_{jl}-\frac{1}{2}g_{jl}r=\frac{N^2}{2}T_{jl}, \quad (44)$$

where $r=g^{ik}r_{ik}$ and r_{jl} are the four-dimensional scalar curvature and four-dimensional Ricci tensor, respectively.

The energy-momentum tensor T_{jl} is therefore given by

$$T_{jl}=T_{jl}^{\text{Maxwell}}+g^{ik}\nabla_i B_{jlk}+C_{jl}+D_{jl}, \tag{45}$$

where

$$T_{jl}^{\text{Maxwell}}=g^{ik}F_{ij}F_{kl}-\frac{1}{4}g_{jl}F_{ik}F^{ik},$$

$$B_{jlk}=A_k\nabla_l A_j-A_l\nabla_k A_j-A_jF_{kl}+\nabla_j(A_kA_l)+g_{jl}(A^i\nabla_k A_i-A_k\nabla_i A^i), \tag{46}$$

$$C_{jl}=g_{jl}A^iA^kr_{ik}-2A^iA_lr_{ij}-2A^iA_jr_{il}, \tag{47}$$

$$D_{jl}=\frac{4}{N^4}(\partial_jN)(\partial_lN)-\frac{2}{N^3}\nabla_j\nabla_lN-\frac{2}{N^2}\pi_k^k(A_l\partial_jN+A_j\partial_lN)+\frac{2}{N^2}[-A^k\pi_{jl}+A_l\pi_j^k+A_j\pi_l^k-g_{jl}(A^i\pi_i^k-A^k\pi_i^i)]\partial_kN. \tag{48}$$

We will analyze each of these terms separately. The first one T_{jl}^{Maxwell} is the Maxwell energy-momentum tensor. The tensor C_{jl} describes the interaction between the electromagnetic and gravitational fields. From Eq. (44) we see that, if N^2 is very small (as we will confirm later), then r_{jl} will be of the order of N^2, which justifies the neglect of the interaction terms in Eq. (41) and the tensor C_{jl}.

Using Eq. (33) in Eq. (20) and then Eq. (20) in Eq. (43), we see that a constant solution for N is allowed by Eq. (43) if ϕ satisfies

$$\nabla^k\partial_k\phi=\frac{1}{2}F^{ik}F_{ik}, \tag{49}$$

where F^{ik} is a solution of Eq. (42). For the constant solution for N, the tensor D_{jl} vanishes. Moreover,

$$0\equiv g^{ij}\nabla_iT_{jl}=g^{ij}\nabla_iT_{jl}^{\text{Maxwell}}, \tag{50}$$

since $g^{ml}g^{nk}\nabla_m\nabla_nB_{jlk}=-(2/N)\nabla_j\nabla_k(f^k/N)=0$ in view of Eq. (43).

In other words, the conservation law (50) is given by the usual Maxwell energy-momentum tensor T_{jl}^{Maxwell} and N^2 plays the role of the Newton constant G_N:

$$\frac{N^2}{2}=\frac{8\pi G_N}{c^4}. \tag{51}$$

The generalized Einstein and Maxwell equations (41)–(48) will be modified further upon inclusion of s-derivative terms. Equation (51), however, holds regardless of whether the s-derivative terms are included or not. One has to point out here that in the setup of Thiry [5], and in [14], $G_N\simeq\phi^2$, where ϕ satisfies Eq. (49). This implies that a constant solution for ϕ and, consequently, G_N is possible only when the unphysical constraint $F^{ik}F_{ik}=0$ is satisfied.

In contrast, in the dual setup, a constant solution is possible. However, N (together with A_i and g_{ij}) is a solution to the system of equations (41), (43), and (44) and, in general, does not need to be a constant. Then it plays the role of a dilaton field.

To illustrate this, consider the standard cosmological metric [11] with $E_\mu^\nu=\delta_\mu^\nu$:

$$ds_{(5)}^2=-s^2dt^2+t^{2/\alpha}s^{2/(1-\alpha)}(dr^2+r^2d\Omega^2)+\alpha^2(1-\alpha)^{-2}t^2ds^2. \tag{52}$$

Changing variables by $r\to s^\gamma e^{\beta r}$ with $\gamma=-(1/2)(1+\alpha)/(1-\alpha)$ and $t^{1/\alpha}\to a(t)$, we get

$$ds_{(5)}^2=-s^2\alpha^2[a(t)]^{2\alpha-2}\dot{a}^2(t)dt^2+2A_rdrds+sa^2(t)e^{2\beta r}(\beta^2dr^2+d\Omega^2)+\phi ds^2, \tag{53}$$

where $A_r=\gamma\beta a^2(t)e^{2\beta r}$, $A_t=A_\varphi=A_\theta=0$, and $\phi=(\gamma^2/s)a^2(t)e^{2\beta r}+\alpha^2(1-\alpha)^{-2}a^{2\alpha}(t)$.

We take β to be a negative constant, so that the field A_r will fall off toward infinity. Since $a(t)$ describes the inflation, we note that the field A_r expands as $a^2(t)$. The dilaton (which models Newton's constant) varies as

$$N^2=(1-\alpha)^2\alpha^{-2}[a(t)]^{-2\alpha}. \tag{54}$$

Thus,

$$\frac{\dot{G}}{G}=-2\alpha\frac{\dot{a}}{a}=-2\alpha H, \tag{55}$$

where H is Hubble's constant. Observational limits [7] put $\alpha<10^{-3}$.

One should note that the four-dimensional metric is now s dependent, but this does not pose a problem in the slicing formulation. Only the term ∂_sg_{jl} from the extrinsic curvature (33) should be recovered.

Finally, we note that the general solution for the dilaton field can be written as

$$N^2=\frac{(\det g_{ik})(\det E_\nu^\mu)^2}{\det G_{\mu\nu}}, \tag{56}$$

for a solution $G_{\mu\nu}$ of Eq. (36) and embedding specified with E_ν^μ.

To recapitulate, we have found plausible generalizations of the Einstein-Maxwell equations and explained the origin of the constant solution for the dilaton (representing Newton's constant G_N) as well as the possibilities for modeling nonconstant solutions for different cosmologies (representing time-varying G_N) in relation to the gauge freedom of our model.

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