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Brane cosmology as a dynamical system

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We investigate the qualitative dynamical properties of a brane model for the flat isotropic universe with a single matter component represented by a scalar field. We study the flat and quadratic potential. Three classes of behaviors of the scale factor are determined. In particular, in the case of the brane with dark negative radiation, via a fine tuning, the existence of oscillatory solutions is shown, which is not possible in the traditional flat FRW model.

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I. INTRODUCTION

Since the 1970s the search of chaos in cosmological models has been one of the permanent concerns of theoretical physics; essentially for two main reasons: one is purely cosmological, to find an explanation for the homogeneity and isotropy of the spatial universe, and the other purely theoretical, since chaos is ubiquitous in the universe, the universe itself, considered as a dynamical system, must be chaotic. The mixmaster model was the first step [1], and it was followed by many researchers where, curiously enough, the FRW models were almost absent. This models began to be studied in the 1990s (see e.g. [2]) and many, most interesting, papers followed (see e.g. [3]). From this last paper we learned that, at first order, we must find periodic solutions because second order perturbations may transform them in chaotic evolutions. So we began a systematic search of these solutions in papers [4], and we found some, but only in closed spatial FRW universes. Then superstrings and branes entered in the cosmological scenario and recently many papers were published on the evolution of the universe using a brane cosmology [5,7–9]. Good general references on branes are [10,11].

Therefore we are particularly interested in the study of brane cosmological models as dynamical systems [12,13], as a logical continuation of our studies of usual FRW models also as dynamical systems [4]. In both cases our aim is to find the qualitative properties of the universe evolutions (in particular the existence of periodic solutions) in a pure analytic way (i.e. not using numerical methods) as in papers [8,14].

Then we will try to describe the dynamic of a scalar field confined in a brane, which is assumed flat (and therefore a homogeneous and isotropic space), as in the spatially flat FRW model. But it is clear that in the new case we must add new brane terms in the Einstein equation of the traditional cosmology, namely: a term proportional to the square of the energy density corresponding to the scalar field and a second one due to dark radiation [7]. Since in this case the cosmological constant could be considered as extremely small we take it equal to zero.

It is well known that in FRW models with minimal coupling, when the scalar field potential is positive defined, the Hubble function has a monotonous evolution [3]. Then if initially the universe is in expansion it will expand forever. On the other hand, if initially the universe is in contraction it will inevitably collapse and there is no possibility of an oscillatory behavior and therefore of chaos. But in the brane case, with flat space, this monotonous behavior is lost, and under adequate conditions oscillatory solutions exist which are not possible in the traditional case, because they essentially depend on the dark radiation. As we just said the presence of these oscillatory solutions are most interesting since they can be the first step to find chaos in more complete models.

We will also study the evolution for small times to describe the radiation dominated era, and to find the matter dominated era in the expansive case. We will see that in the case of the expanding universe, we find the same traditional cosmology matter dominated era, since due to the expansion, the new terms introduced by the brane become irrelevant for large times.

The work is organized as follows. In Sec. II we present the general equations of the brane model. In Sec. III we study the case of constant potential. In Sec. IV we study the case of quadratic potential with no cosmological constant. In Sec. V we present the Lyapunov function of the just found dynamic. In Sec. VI we consider the oscillatory solutions. In Sec. VII we draw our conclusions.

II. GENERAL EQUATIONS

This paper is based on Refs. [5,6], and [7], so we use the same notation and system of unities.

We will consider the five-dimension space metric

$$ds^2=\tilde{g}_{AB}dx^A dx^B=g_{\mu\nu}dx^\mu dx^\nu+b^2dy^2$$
$$=-n^2(\tau,y)d\tau^2+a^2(\tau,y)\gamma_{ij}dx^i dx^j+b^2(\tau,y)dy^2, \tag{1}$$

where the first term is the time term, the second one the 3D brane term and the last one the bulk term, $A,B,..=0,1,2,3,5$, $\mu,\nu,\ldots=0,1,2,3$, $i,j,\ldots=1,2,3$, γ_{ij} is a maximal symmetric 3-dimensional metric with $k=-1,0,1$. The five dimensional Einstein equation reads

$$\tilde{G}_{AB}=\tilde{R}_{AB}-\frac{1}{2}R\tilde{g}_{AB}=\kappa^2\tilde{T}_{AB}, \tag{2}$$

where

$$\tilde{T}^A_B=\tilde{T}^A_B|_{bulk}+T^A_B|_{brane},$$

$$\tilde{T}^A_B|_{bulk}=\mathrm{diag}(-\rho_B,P_B,P_B,P_B,P_T),$$

where $-\rho_B=P_B=P_T$, to mimic a cosmological constant, and

$$T^A_B|_{brane}=\frac{\delta(y)}{b}\mathrm{diag}(-\rho_b,p_b,p_b,p_b,0).$$

Then following the deduction of paper [6] and choosing $k=0$, and therefore $\gamma_{ij}=\delta_{ij}$, and the $T^\mu_\nu|_{brane}$ as the one produced by a scalar field ψ with potential $V(\psi)$ we obtain the following effective four-dimensional equation in the brane $(y=0)$ [7]:

$$\frac{\dot{a}^2}{a^2}=\frac{\tilde{\kappa}^2}{3}\left(\frac{1}{2}\dot{\psi}^2+V\right)+\frac{\kappa^4}{36}\left(\frac{1}{2}\dot{\psi}^2+V\right)^2+\frac{C}{a^4}, \tag{3}$$

$$\frac{\ddot{a}}{a}+\frac{2\dot{a}^2}{a^2}=\tilde{\kappa}^2V+\frac{\kappa^4}{48}(4V^2-\dot{\psi}^4)+\frac{C}{a^4}, \tag{4}$$

$$\ddot{\psi}=-3\frac{\dot{a}}{a}\dot{\psi}-V', \tag{5}$$

where the dots symbolize the time derivation, we have taken $n=1$, constants $\tilde{\kappa}$ y κ are defined in Refs. [5–7], and the constant C, associated to the dark radiation, can take positive, null, or negative values. These equations are independent of the metric outside the brane and the peculiar evolution of b [6].

To simplify the equations, we introduce a new variable (as in [4]):

$$u=\frac{a^2}{2}. \tag{6}$$

Then Klein-Gordon equation (5) reads

$$\ddot{\psi}+\frac{3}{2}\frac{\dot{u}}{u}\dot{\psi}+\frac{dV}{d\psi}=0. \tag{7}$$

We present the Friedmann equation (4) in a parametric oscillator form, namely

$$\ddot{u}=F(V,\dot{\psi})u, \tag{8}$$

where

$$F=\frac{1}{3}\tilde{\kappa}^2(4V-\dot{\psi}^2)+\frac{1}{18}\kappa^4(2V^2-\dot{\psi}^4-\dot{\psi}^2V). \tag{9}$$

Finally the Einstein condition (3) reads

$$\frac{1}{4}\frac{\dot{u}^2}{u^2}=\frac{1}{3}\tilde{\kappa}^2\left(\frac{1}{2}\dot{\psi}^2+V\right)+\frac{1}{36}\kappa^4\left(\frac{1}{2}\dot{\psi}^2+V\right)^2+\frac{1}{4}\frac{C}{u^2}. \tag{10}$$

III. THE CONSTANT POTENTIAL CASE

Let us consider the case of a constant potential V_0. The first integral of Eq. (7) is

$$\dot{\psi}=\alpha u^{-3/2}, \tag{11}$$

where α is constant. Then since the system defined by Eqs. (7), (8), and (9) has dimension four, the two first integrals Eqs. (10) and (11) make the system a two dimensional one, therefore chaotic solutions are not possible.

Substituting Eq. (11) in Eq. (8), we obtain the first integral

$$\frac{1}{2}\dot{u}^2=f_C(u), \tag{12}$$

where

$$f_C=\frac{1}{72}\kappa^4\alpha^4\frac{1}{u^4}+\alpha^2\left(\frac{1}{3}\tilde{\kappa}^2+\frac{1}{18}\kappa^4V_0\right)\frac{1}{u}+\frac{C}{2}+\frac{1}{2}V_0\left(\frac{4}{3}\tilde{\kappa}^2+\frac{1}{9}\kappa^4V_0\right)u^2, \tag{13}$$

where the integration constant is consistent with Eq. (10).

Equation (12), defines a one dimension dynamic, thus also in this case there are not oscillatory solutions $a(t)$ has a monotonous behavior as we will see. In fact, since u is nonnegative, if f_C has no zeros then u is monotonous, thus it diverges or vanishes. If f_C has one zero then it is a fixed point of Eq. (12), and then $u\to$const. These are the only possible solutions since $u\geqslant 0$.

A. Behavior for $u\ll 1$

For $u\ll 1$, Eq. (12) can be approximated as

$$\dot{u}^2\simeq\frac{1}{36}\kappa^4\alpha^4\frac{1}{u^4},$$

then, in the first instants of the universe evolution, the space curvature and the dark radiation have not a dominant contribution and every dependence of C is lost. Then we have

$$u^3\simeq\frac{1}{6}\kappa^2\alpha^2t.$$

From Eq. (6), the scale factor a reads

$$a\simeq\beta t^{1/6},$$

where

$$\beta=\left(\frac{4}{3}\kappa^2\alpha^2\right)^{1/6}.$$

Let us remember that in Ref. [7] this result is presented just like an ansatz.

B. Asymptotic behavior of the scale factor

Let us now consider the case where the limit $u\to\infty$ is possible.

The asymptotic expression of Eq. (12) is

$$\dot{u}\simeq\sqrt{V_0\left(\frac{4}{3}\tilde{\kappa}^2+\frac{1}{9}\kappa^4V_0\right)}u,$$

that corresponds to positive constant Hubble function.

Integrating we can find the scalar factor a:

$$a\simeq\exp\frac{1}{2}\sqrt{V_0\left(\frac{4}{3}\tilde{\kappa}^2+\frac{1}{9}\kappa^4V_0\right)}t.$$

In the case when $V_0(\frac{4}{3}\tilde{\kappa}^2+\frac{1}{9}\kappa^4V_0)=0$, the asymptotic expression of Eq. (12) is

$$\dot{u}\simeq\sqrt{\frac{C}{2}}$$

that only allows solutions for $C>0$, then $a\simeq t^{1/2}$, which corresponds to the radiative era in the Friedmann model [15].

IV. THE QUADRATIC POTENTIAL CASE

In the previous section we have demonstrated that when the potential is constant, the scale factor a is monotonous. We will now study the potential

$$V(\psi)=\frac{1}{2}m^2\psi^2, \tag{14}$$

where we have considered that the cosmological constant is zero and m is the mass.

A. Properties of the model

Then we have the following properties.

1. Property 1

The dynamics of Eqs. (7) and (8) are invariant under the scale transformation

$$t\to\alpha t, m\to m/\alpha,\quad u\to\alpha^2u, \tag{15}$$

$$\tilde{\kappa}\to\tilde{\kappa},\quad \kappa\to\alpha^{1/2}\kappa, C\to\alpha^2C, \psi\to\psi, \tag{16}$$

where α is an arbitrary positive constant. Then we are allowed to choose $m=1$ with no lost of generality.

2. Property 2

There are fixed points only when $C=0$, in which case the fixed points of Eqs. (7) and (8), that satisfy the constraint (10), are $\psi=0,\dot{\psi}=0,u>0,\dot{u}=0$.

3. Property 3

The divergence of the vectorial field of the Eqs. (7) and (8) is $-6H$. Then, when the universe is expanding $H>0$, we have a dissipative behavior.

4. Property 4

After the change of variables of Eq. (6) Hubble function reads

$$H=\frac{1}{2}\frac{\dot{u}}{u}, \tag{17}$$

then, using Eqs. (9) and (10), Eq. (8) becomes

$$\dot{H}=-\tilde{\kappa}^2\frac{1}{2}\dot{\psi}^2-\frac{1}{12}\kappa^4\dot{\psi}^2\left(\frac{1}{2}\dot{\psi}^2+\frac{1}{2}m^2\psi^2\right)-\frac{1}{2}\frac{C}{u^2}. \tag{18}$$

From this equation we conclude that if the dark radiation constant C is positive or null we have a simple dynamics since Hubble function is monotonous.

Thus, only in the case $C<0$ we can have an oscillatory behavior of the scale factor.

5. Property 5

Let us qualitatively describe the solutions as curves in phase space. The only solutions with physical interest are those that $\dot{a}(t=0)>0$ namely those with an expansive initial behavior. Then we define the tridimensional space of variables H, V and $Z=\frac{1}{2}\dot{\psi}^2$and we rewrite Eq. (10) as

$$H^2=H_0^2+\frac{1}{4}\frac{C}{u^2}, \tag{19}$$

where

$$H_0^2=\frac{1}{3}\tilde{\kappa}^2(Z+V)+\frac{1}{36}\kappa^4(Z+V)^2. \tag{20}$$

Then, using these variables $H,V,Z,$ from property 2 we can deduce that, when $C=0$, the only fixed point is $H=0$, $Z=0$, $V=0$.

On the other hand, Eq. (20) defines two surfaces $\Sigma_{\pm}$ in space $H,Z,V,$ namely

$$H_0=\pm\sqrt{\frac{1}{3}\tilde{\kappa}^2(Z+V)+\frac{1}{36}\kappa^4(Z+V)^2}.$$

Then, according to the values of C, we have the following cases:

(i) When $C=0$, the solutions are contained either in Σ_+ or in Σ_- because they cannot go from Σ_+ to Σ_- or vice versa since $H=0,Z=0$, $V=0$ is a fix point. From Eq. (18),

$\dot{H}<0$, then the trajectories contained in Σ_+ asymptotically converge to the origin, but the universe can also be expansive, since we may have decreasing $H>0$, then the scale factor has only two possibilities: either $u\to N>0$ (a finite positive value of u) or $u\to\infty$. The solutions in Σ_-, correspond to contracting universes that will collapse in a finite time t_1, namely $\lim_{t\to t_1}a=0$.

(ii) When $C>0$, from Eq. (19), we see that the expansive solutions are contained in a volume v_+ with the lower bound Σ_+, while the contracting trajectories belongs to a volume v_- with the upper bound Σ_-. In both cases the trajectories are confined either in the upper region v_+ $(H>0)$ or in the lower region v_- $(H<0)$. Since the origin is not a fixed point it belongs to a trajectory just in the case $u\to\infty$. This fact shows that, also in this case, a trajectory cannot go from v_+ to v_- or vice versa. The solutions contained in v_+, asymptotically converge to surface Σ_+ since $\dot{H}<0$. Then, since we are considering the case $C>0$, we have $u\to\infty$. Instead, the trajectories of v_- behave as $\lim_{t\to t_1}a=0$.

In both cases, (i) or (ii), from the monotonous behavior of H, we can say that the scalar field, and therefore the energy density $\rho=Z+V$, is decreasing in v_+ and growing in v_-, since the time variation of ρ is

$$\frac{d\rho}{dt}=-6HZ.$$

(iii) When $C<0$, the sign of H is not fix and the trajectories are confined in a bounded region by surfaces $\Sigma_\pm$ (either upper or lower). In this case oscillatory solutions are possible for the Hubble function.

B. Asymptotic behavior of the scale factor

We would like to find the divergent behavior of u for $t \gg 1$, which is only possible if $C\geqq 0$ as we saw in Sec. III D. Then we can make the following *Ansätze*:

$$u\sim t^{\alpha},\alpha>0. \tag{21}$$

Then Eq. (17) reads

$$\frac{\dot{u}}{u}=\frac{\alpha}{t}, \tag{22}$$

and the asymptotic expression of Eq. (7) is

$$\ddot{\psi}+\frac{3}{2}\frac{\alpha}{t}\dot{\psi}+\psi=0. \tag{23}$$

As $\alpha>0$ Eq. (23) is dissipative, then $\lim_{t\to\infty}\psi=0$. The solution of the last equation are linear combinations of Bessel functions like $c_J J_{1/2-3/4\alpha}+c_Y Y_{1/2-3/4\alpha}$. Then for $t\gg 1$, we have

$$\psi_\infty\sim t^{-3/4\alpha}\sin t.$$

This asymptotic solution of Eq. (7) must satisfy the Einstein condition Eq. (10) and Eq. (8); then we can see that $\alpha=4/3$. So finally the scale factor is $a\sim t^{2/3}$ which corresponds to a matter dominated universe, as in a flat FRW model. This result is independent of the value of C since in Eq. (10) the radiative term and the brane term (the square of the energy density) are negligible with respect to the other ones.

V. LYAPUNOV FUNCTION

We propose as the Lyapunov function, relevant for our problem, the function u defined in Eq. (6), since, by its own definition, this function has a necessary condition, it has a constant sign. On the other hand, its derivative is given by Eq. (10). Then we have the following cases:

(i) If $C>0$, $\dot{u}$ never vanishes so u is monotonous.

(ii) If $C=0$, according to property 2, $\dot{u}$ can only vanish asymptotically, namely $\dot{u}$ do not vanish for finite times and therefore u is monotonous.

(iii) If $C<0$, from Eq. (10), $\dot{u}$ can vanish and therefore we cannot say if it has a monotonous behavior. So in this case u can be an oscillatory solution.

VI. THE OSCILLATORY SOLUTIONS

From property 5 we know that when $C<0$ oscillatory solutions may exist.[1] In this section we will show under what conditions these oscillatory solutions are possible. We will use the following *Ansätze*:

(a) We decompose the square of the scale factor by

$$u=U+f(t),$$

where $U>0$ is a constant to be found and f a function also to be found.

(b) Let us suppose that f satisfies

$$\left|\frac{\dot{u}}{u}\right|\approx\left|\frac{f}{U}\right|\ll 1,$$

for every time and also that

$$\left|\frac{3}{2}\frac{\dot{u}}{u}\dot{\psi}\right|\ll\max(\dot{\psi},\ddot{\psi}). \tag{24}$$

From *Ansätze* (a) and (b), a solution of Eq. (7) is

$$\psi=c_s\sin(t), \tag{25}$$

where c_s is a integration constant. Using Eqs. (8) and (9) and taking into account Eq. (24) we obtain

[1]As the whole treatment is independent of the bulk [see under Eq. (5)] for simplicity we may take a static one. In this case as $C<0$ we can use Birkhoff theorem [16] and demonstrate that the bulk has a AdS-Schwarzschild metric, where C in the mass of the corresponding black hole. Then we would have a naked singularity in the bulk that would be a major drawback to our formalism. Fortunately, according to string theory, it is natural the presence of at least another brane. Then an extra brane can shield the singularity avoiding the problem (see [10] for details).

$$\ddot{f} \cong F(V,\dot{\psi})U,$$

where F can be evaluated from Eq. (25). Integrating we have

$$f = U(\alpha t^2 + \epsilon\cos(2t) + c_1 + c_2 t),$$

where c_1 and c_2 are constant of integration and

$$\alpha = \frac{1}{144}(12\tilde{\kappa}^2 c_s^2 - \kappa^4 c_s^4),$$

$$\epsilon = \frac{1}{96}c_s^2 \sin(\kappa^4 c_s^2 + 12\tilde{\kappa}^2). \tag{26}$$

In order that the ansatz of Eq. (24) would be verified it is necessary that $c_1 = 0$, $c_2 = 0$, and $\alpha = 0$. Then

$$u \cong U(1 + \epsilon\cos(2t)). \tag{27}$$

Using Eqs. (25), (27) and (10) we obtain

$$0 = \frac{1}{144}\kappa^4 c_s^4 + \frac{1}{6}\tilde{\kappa}^2 c_s^2 + \frac{C}{4U^2(1+\epsilon\cos(2t))^2} - \frac{\epsilon^2 \sin^2(2t)}{(1+\epsilon\cos(2t))^2}. \tag{28}$$

When $t = 0$ we have

$$0 = \frac{1}{144}\kappa^4 c_s^4 + \frac{1}{6}\tilde{\kappa}^2 c_s^2 + \frac{C}{4U^2}, \tag{29}$$

then this equation has a solution if $C < 0$.

Using Eq. (26) and condition $\alpha = 0$, we obtain

$$c_s^2 = 48\frac{\tilde{\kappa}^2}{\kappa^4}, \tag{30}$$

$$\epsilon = 30\frac{\tilde{\kappa}^4}{\kappa^4}.$$

From Eqs. (29) and (30) we can find the relation between C and U

$$C = -96\frac{\tilde{\kappa}^4}{\kappa^4}U^2. \tag{31}$$

Therefore, for each value of C there exists a unique oscillatory solution. Then, in phase space, we have a vanishing measure set of initial conditions for oscillatory solution which are, therefore, less abundant than nonoscillatory ones. This fine-tuning explains why it is so difficult to find these oscillatory solutions numerically.[2]

As a function of Planck mass $M_{(5)}$, the brane tension λ is given in the papers [5,7] by the equation

$$\frac{\tilde{\kappa}^4}{\kappa^4} = \frac{16}{9}\pi^2 M_{(5)}^{-6}\lambda.$$

Once the parameters c_s, ϵ are determined it is necessary to verify the consistency with the proposed ansatzs. From Eqs. (24) it is necessary that $\epsilon \ll 1$, and this condition must be imposed on the brane tension. To have an oscillatory solution it is necessary that the terms associated to the brane would be dominant, as we can see from Eqs. (9) and (10). Moreover, to satisfy Eqs. (28) for all t we must have that $\epsilon \ll U^2$.

In this way we have shown the existence of periodic solutions for the case $C < 0$ as we promise in property 5.

Remark. These solutions appear in our flat model as a consequence of the brane equations and the negative C constant. This is also the case of the FRW models with no branes, but with a spherically spatial geometry, where, as we explained in the Introduction, we have also found oscillatory solutions [4]. So in flat space model branes some how mimics the effect of a spatial curvature. Thus we conclude that, at least based on the qualitative behavior of the solution, we cannot distinguish between these two effects.

Further research on the subject will lead us to evaluate if there is a fractal frontier between contracting and expanding zones of phase space (as in [3]) and, most interesting, if in more complete brane models new terms might transform periodic solutions into chaotic ones.

VII. CONCLUSION

In flat FRW cosmological model with quadratic potential there are no oscillatory solutions for the scale factor and therefore chaos is impossible. In this paper we have shown that for the same model but in the brane world, we have solutions with collapsing scalar factor and oscillatory behavior when the dark radiation is negative. This behavior corresponds to a fine-tuning in phase space and therefore it is difficult to find. On the other hand, where the potential is constant the scale factor is monotonous.

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[2]We have tried to do it but, for the explained reasons, the result are not good enough to be used as a further illustration (see similar numerical calculations in [3] and [7]).

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