

Observable primordial vector modes

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Primordial vector modes describe vortical fluid perturbations in the early Universe. A regular solution exists with constant nonzero radiation vorticities on superhorizon scales. Baryons are tightly coupled to the photons, and the baryon velocity only decays by an order unity factor by recombination, leading to an observable CMB anisotropy signature via the Doppler effect. There is also a large B -mode CMB polarization signal, with significant power on scales larger than $l \sim 2000$. This B -mode signature is distinct from that expected from tensor modes or gravitational lensing, and makes a primordial vector to scalar mode power ratio $\sim 10^{-6}$ detectable. Future observations aimed at detecting large scale B modes from gravitational waves will also be sensitive to regular vector modes at around this level.

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INTRODUCTION

Observations of the cosmic microwave background (CMB) show that the primordial perturbation was almost certainly dominated by adiabatic scalar (density) modes. However it is well known that there are several possible scalar isocurvature modes [1] that could be present at some level. In the presence of a primordial magnetic field, there is also an observable vector mode perturbation [2,3] sourced by the anisotropic stress of the magnetic field. Other sources such as topological defects can also source vector modes. Here we concentrate on the rarely considered regular primordial (un-sourced) vector modes, which are nondecaying solutions of the perturbation equations in the presence of free streaming neutrinos [4]. We show that a very small primordial regular vector mode amplitude could be observable.

In the absence of an initial large scale radiation vorticity the vector modes remain in a decaying mode and have essentially no observational signature. They are therefore not predicted to be present at any significant level in inflation or other simple models. However, there is a regular mode with a nonzero initial photon vorticity, having equal and opposite initial photon and neutrino angular momenta such that the total large scale angular momentum is zero. This is the vector analogue of the scalar neutrino isocurvature velocity mode discussed in Ref. [1], and constitutes a valid possible component of the general primordial perturbation. These velocity modes would have to be excited after neutrino decoupling and are hence difficult to produce and somewhat contrived. But they remain a logical possibility that can be constrained by observation, and if observed would be a powerful way to rule out most theoretical models (for constraints on the scalar mode see, e.g., Ref. [5] and references therein). The vector mode can be detected at very small amplitudes and distinguished from the various scalar modes because of its distinct nonzero B -mode (curl-like) CMB polarization signal that is absent with only linear scalar modes.

As we show, the vector B -mode signature is quite different from that expected from weak lensing or primordial ten-

sor modes. On large scales the spectrum is similar to that from tensors, so observations aimed at detecting the B modes from primordial tensors will also be sensitive to the large scale part of the vector power spectrum, but they can easily be distinguished by the vector mode power on smaller scales. The physical difference between the spectra is that tensor modes rapidly decay as soon as they come inside the horizon, whereas the vortical modes are nearly constant during radiation domination, decaying only on small scales though damping towards the end of tight coupling.

Even more contrived regular modes exist with nonzero primordial neutrino octopole (or higher) [4,6]; however, these have a much weaker observational signature and are not considered further here.

COVARIANT EQUATIONS

We consider linear perturbations in a flat Friedmann-Robertson-Walker (FRW) universe evolving according to general relativity with a cosmological constant, neglect any velocity dispersion of the dark matter and baryon components, and approximate the neutrinos as massless. Perturbations can be described covariantly in terms of a 3+1 decomposition with respect to some choice of observer velocity u_a (we use natural units, and the signature where $u_a u^a = 1$), following Refs. [7–9]. Projected spatial derivatives orthogonal to u_a can be used to quantify perturbations to scalar quantities, for example, the pressure perturbation can be described in terms of $D_a p$ where the spatial derivative is

$$D_a \equiv \nabla_a - u_a u^b \nabla_b. \quad (1)$$

Conservation of total stress-energy $\nabla^a T_{ab} = 0$ implies an evolution equation for the total heat flux q_a

$$\dot{q}_a + \frac{4}{3} \Theta q_a + (\rho + p) A_a - D_a p + D^b \pi_{ab} = 0, \quad (2)$$

where ρ is the energy density, $\dot{q}_a \equiv u^b \nabla_b q_a$, $\Theta \equiv \nabla^a u_a$ is three times the Hubble expansion, $A_a \equiv u_b \nabla^b u_a$ is the acceleration, and $\pi_{ab} \equiv T_{(ab)}$ is the total anisotropic stress. Angle

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brackets around indices denote the projected symmetric trace-tree part (orthogonal to u_a).

We define the vorticity vector $\Omega_a \equiv \text{curl } u_a$ where for a general tensor

$$\text{curl } X_{a_1 \dots a_l} \equiv \eta_{bcd(a_1} u^b D^c X^d_{a_2 \dots a_l)} \quad (3)$$

and round brackets denote symmetrization. It has the evolution equation

$$\dot{\Omega}_a + \frac{2}{3} \Theta \Omega_a = \text{curl } A_a \quad (4)$$

and is transverse $D^a \Omega_a = 0$. Remaining quantities we shall need are the ‘‘electric’’ E_{ab} and ‘‘magnetic’’ H_{ab} parts of the Weyl tensor C_{abcd} ,

$$E_{ab} \equiv C_{acbd} u^c u^d, \quad H_{ab} \equiv \frac{1}{2} \eta_{acdf} C_{be}{}^{cd} u^e u^f \quad (5)$$

(which are frame invariant) and the shear $\sigma_{ab} \equiv D_{\langle a} u_{b \rangle}$. The Einstein equation and the Bianchi identity give the constraint equations

$$\begin{aligned} D^a \sigma_{ab} - \frac{1}{2} \text{curl } \Omega_b - \frac{2}{3} D_b \Theta - \kappa q_b &= 0, \\ D^a E_{ab} - \kappa \left(\frac{\Theta}{3} q_b + \frac{1}{3} D_b \rho + \frac{1}{2} D^a \pi_{ab} \right) &= 0, \\ D^a H_{ab} - \frac{1}{2} \kappa [(\rho + p) \Omega_b + \text{curl } q_b] &= 0, \\ H_{ab} - \text{curl } \sigma_{ab} + \frac{1}{2} D_{\langle a} \Omega_{b \rangle} &= 0, \end{aligned} \quad (6)$$

and the evolution equations

$$\begin{aligned} \dot{\sigma}_{ab} + \frac{2}{3} \Theta \sigma_{ab} &= -E_{ab} - \frac{1}{2} \kappa \pi_{ab}, \\ \dot{E}_{ab} + \Theta E_{ab} &= \text{curl } H_{ab} + \frac{\kappa}{2} \left[\dot{\pi}_{ab} - (\rho + p) \sigma_{ab} \right. \\ &\quad \left. + \frac{\Theta}{3} \pi_{ab} \right], \\ \dot{H}_{ab} + \Theta H_{ab} &= -\text{curl } E_{ab} - \frac{\kappa}{2} \text{curl } \pi_{ab}. \end{aligned} \quad (7)$$

We use natural units where $c = 1$ and define $\kappa \equiv 8\pi G$.

A vector like A_a may be split into a scalar part $A_a^{(0)}$ and a vector part $A_a^{(1)}$ where $A_a = A_a^{(0)} + A_a^{(1)}$, $A_a^{(0)} = D_a A$ for some first order scalar A and the vector part is solenoidal $D^a A_a^{(1)} = 0$. This extends to a tensor where the vector part is given by $\sigma_{ab}^{(1)} = D_{\langle a} \Sigma_{b \rangle}$ for some first-order solenoidal vector Σ_b .

We can choose u_a to simplify the analysis. At linear order one can always write $u_a = u_a^\perp + v_a$, where u_a^\perp is hypersurface orthogonal and v_a is first order, so $\text{curl } u_a = \text{curl } v_a$. For a

zero-order scalar quantity X it follows that $D_a X = D_a^\perp X - v_a \dot{X}$. For vector modes $(D_a^\perp X)^{(1)} = 0$, and it is convenient to choose the frame u_a to be hypersurface orthogonal so that $\text{curl } u_a = 0$ and hence $(\bar{D}_a X)^{(1)} = 0$, where the bar denotes evaluation in the zero vorticity frame. From its propagation equation, vanishing of the vorticity also implies that $\bar{A}_a^{(1)} = 0$, so the zero vorticity frame coincides with the synchronous gauge. The cold dark matter (CDM) velocity is also zero in this frame modulo a mode that decays as $1/S$ where S is the scale factor.

It is convenient to expand the vector components in terms of transverse eigenfunctions of the zero order Laplacian, Q_a^\pm where $S^2 D^2 Q_a^\pm = k^2 Q_a^\pm$ and \pm denotes the parity. A rank- ℓ tensor may be expanded in terms of rank- ℓ eigenfunctions $Q_{A_l}^\pm$ defined by

$$Q_{A_l}^\pm \equiv \left(\frac{S}{k} \right)^{l-1} D_{\langle a_1} \dots D_{a_{l-1}} Q_{a_l}^\pm, \quad (8)$$

which satisfy

$$D^{a_l} Q_{A_{l-1} a_l}^\pm = \frac{k}{S} \frac{(l^2 - 1)}{l(2l-1)} Q_{A_{l-1}}^\pm, \quad (9)$$

$$\text{curl } Q_{A_l}^\pm = \frac{1}{l} \frac{k}{S} Q_{A_l}^\mp. \quad (10)$$

Harmonic coefficients are defined by

$$\begin{aligned} \sigma_{ab}^{(1)} &= \sum \frac{k}{S} \sigma Q_{ab}^\pm, & H_{ab}^{(1)} &= \sum \frac{k^2}{S^2} H Q_{ab}^\pm, \\ q_a^{(1)} &= \sum q Q_a^\pm, & \Omega_a &= \sum \frac{k}{S} \Omega Q_a^\pm, \\ \pi_{ab}^{(1)} &= \sum \Pi Q_{ab}^\pm, \end{aligned} \quad (11)$$

where the k and \pm dependence of the harmonic coefficients is suppressed and $q_i = (\rho_i + p_i) v_i$ for each fluid component, where v_i is the velocity, and the total heat flux is given by $q = \sum_i q_i$. The sum is over k and the \pm parities. We write the baryon velocity simply as v .

The equations for the harmonic coefficients in the zero vorticity frame reduce to

$$\begin{aligned} k(\bar{\sigma}' + 2\mathcal{H}\bar{\sigma}) &= -\kappa S^2 \Pi, \\ H &= \frac{1}{2} \bar{\sigma}, & 2\kappa S^2 \bar{q} &= k^2 \bar{\sigma} \end{aligned} \quad (12)$$

where the prime denotes a derivative with respect to conformal time η , and $\mathcal{H} = S\Theta/3$ is the comoving Hubble parameter. The combination $v + \sigma$ (the Newtonian gauge velocity) is frame invariant, as are $\bar{\sigma} = \sigma + \Omega$ and $\bar{v} = v - \Omega$. By choosing to consider the zero vorticity frame we have simply

expedited the derivation of the above frame-invariant equations. Other papers use the Newtonian gauge [10], in which $\bar{\sigma}$ is the vorticity.

The evolution equation for the shear has the solution

$$\bar{\sigma} = \frac{-1}{S^2} \int d\eta \frac{\kappa S^4 \Pi}{k}. \quad (13)$$

In the absence of anisotropic stress it therefore decays as $1/S^2$. However, after neutrino decoupling the neutrinos will supply an anisotropic source, and solution of this equation requires a consistent solution for the neutrino evolution.

The baryon velocity is coupled to the photon velocity via Thomson scattering

$$\bar{v}' + \mathcal{H}\bar{v} = -\frac{\rho_\gamma}{\rho_b} S n_e \sigma_T \left(\frac{4}{3} v - I_1 \right) \quad (14)$$

where $I_1 = 4v\sqrt{3}$, n_e is the electron number density and σ_T is the Thomson scattering cross section.

The photon multipole equations [9] for vectors become

$$\begin{aligned} \bar{I}'_l + k \frac{l}{2l+1} \left[\frac{(l+2)}{(l+1)} \bar{I}_{l+1} - \bar{I}_{l-1} \right] \\ = -S n_e \sigma_T \left(\bar{I}_l - \frac{4}{3} \delta_{l1} \bar{v} - \frac{2}{15} \zeta \delta_{l2} \right) + \frac{8}{15} k \bar{\sigma} \delta_{l2} \end{aligned} \quad (15)$$

where $\zeta \equiv 3I_2/4 - 9E_2/2$ is a source from the photon anisotropic stress and E -mode polarization, and $\rho_\gamma I_2 = \Pi_\gamma$. In general I_l is an angular moment of the fractional photon density distribution, four times the fractional temperature anisotropy. The neutrino multipole equations are analogous but without the Thomson scattering terms. The solution is

$$\begin{aligned} I_l(\eta_0) = 4 \int^{\eta_0} d\eta e^{-\tau} \left[S n_e \sigma_T \bar{v} \Psi_l(\chi) \right. \\ \left. + \left(S n_e \sigma_T \frac{\zeta}{4} + k \bar{\sigma} \right) \frac{d\Psi_l(\chi)}{d\chi} \right] \end{aligned} \quad (16)$$

where $\Psi_l(x) \equiv l j_l(x)/x$, $j_l(x)$ is a spherical Bessel function, $\chi \equiv k(\eta_0 - \eta)$ and τ is the optical depth. In the approximation that the visibility $S n_e \sigma_T e^{-\tau}$ is a delta function at last scattering $\eta = \eta^*$ this becomes

$$\frac{I_l(\eta_0)}{4} \approx \left[(v + \sigma) \Psi_l + \frac{\zeta}{4} \frac{d\Psi_l}{d\chi} \right]_{\eta^*} + 2 \int_{\eta^*}^{\eta_0} d\eta H' \Psi_l. \quad (17)$$

The anisotropy therefore comes predominantly from the Newtonian gauge baryon velocity at last scattering, plus an integrated Sachs-Wolfe (ISW) term from the evolution of the magnetic Weyl tensor $H = \bar{\sigma}/2$ along the line of sight.

The vector polarization multipole equations [11] become

$$\begin{aligned} E_l^\pm{}' + \frac{(l+3)(l+2)l(l-1)}{(l+1)^3(2l+1)} k E_{l+1}^\pm - \frac{l}{2l+1} k E_{l-1}^\pm - \frac{2}{l(l+1)} k B_l^\mp = -S n_e \sigma_T \left(E_l^\pm - \frac{2}{15} \zeta^\pm \delta_{l2} \right), \\ B_l^\pm{}' + \frac{(l+3)(l+2)l(l-1)}{(l+1)^3(2l+1)} k B_{l+1}^\pm - \frac{l}{2l+1} k B_{l-1}^\pm + \frac{2}{l(l+1)} k E_l^\mp = 0 \end{aligned} \quad (18)$$

where E_l and B_l describe moments of the E (gradient-like) and B (curl-like) polarization. These equations have solutions

$$\begin{aligned} E_l(\eta_0) = \frac{l-1}{l+1} \int^{\eta_0} d\eta S n_e \sigma_T e^{-\tau} \left[\frac{d\Psi_l(\chi)}{d\chi} + \frac{2\Psi_l(\chi)}{\chi} \right] \zeta, \\ B_l(\eta_0) = -\frac{l-1}{l+1} \int^{\eta_0} d\eta S n_e \sigma_T e^{-\tau} \Psi_l(\chi) \zeta. \end{aligned} \quad (19)$$

Signs of E_l and B_l here follow the conventions of CMBFAST [12] and CAMB [13].

SOLUTIONS

At early times the baryons and photons are tightly coupled, and the opacity $\tau_c^{-1} \equiv S n_e \sigma_T$ is large. This means $v_\gamma \approx v$, and we can do an expansion in τ_c that is valid for $\epsilon \equiv \max(k\tau_c, \mathcal{H}\tau_c) \ll 1$. To lowest order

$$\bar{v}' = -\frac{R\mathcal{H}\bar{v}}{1+R} + \mathcal{O}(\tau_c) \quad (20)$$

where $R \equiv 3\rho_b/4\rho_\gamma$. The solution is

$$\bar{v} \approx \frac{\bar{v}_0}{1+R} \quad (21)$$

where \bar{v}_0 is the initial value. Hence if $\bar{v}_0 \neq 0$, by decoupling \bar{v} has only decayed an order unity factor depending on the matter and radiation density at the time. On smaller scales where $k\tau_c = \mathcal{O}(1)$ before decoupling the perturbations are damped by photon diffusion, giving a characteristic fall off in perturbation power on small scales.

We now perform a general series expansion in conformal time for the above equations in the early radiation dominated era to identify the regular primordial modes. We define ω

$\equiv \Omega_m \mathcal{H}_0 / \sqrt{\Omega_R}$, where $\Omega_R = \Omega_\gamma + \Omega_\nu$, and \mathcal{H}_0 and Ω_i are the Hubble parameter and densities (in units of the critical density) today. The Friedmann equation gives

$$S = \frac{\Omega_m \mathcal{H}_0^2}{\omega^2} \left(\omega \beta + \frac{1}{4} \omega^2 \tau^2 + \mathcal{O}(\tau^5) \right). \quad (22)$$

Defining the ratios $R_\nu \equiv \Omega_\nu / \Omega_R$, $R_\gamma \equiv \Omega_\gamma / \Omega_R$, $R_b \equiv \Omega_b / \Omega_m$, and keeping lowest-order terms the regular solution (with zero initial anisotropies for $l > 2$) is

$$\bar{\sigma} = \bar{\sigma}_0 \left(1 - \frac{15}{2} \frac{\omega \eta}{4R_\nu + 15} \right), \quad (23)$$

$$\bar{v}_\gamma = \bar{\sigma}_0 \frac{4R_\nu + 5}{R_\gamma} \left(\frac{1}{4} - \frac{3R_b}{16R_\gamma} \omega \eta \right), \quad (24)$$

$$\bar{v}_\nu = -\frac{\bar{\sigma}_0}{4} \frac{4R_\nu + 5}{R_\nu}, \quad (25)$$

$$I_2 \equiv \frac{\Pi_\nu}{\rho_\nu} = -\frac{2}{3} \frac{k \eta_-}{R_\nu} \bar{\sigma}_0, \quad (26)$$

where we have neglected small contributions from the scattering-suppressed photon anisotropic stress. This regular mode is the vector analogue of the neutrino velocity isocurvature mode discussed in Ref. [1]. The shear $\bar{\sigma}$ is initially constant on superhorizon scales, supported by the growing anisotropic stress of the neutrinos. On subhorizon scales in radiation domination it decays as the neutrino anisotropic stress starts to oscillate rather than grow.

The photon and neutrino vorticities are constant on superhorizon scales during radiation domination. This is consistent with angular momentum conservation because of the energy redshift. The photon vorticity is tightly coupled to the baryons, so both are initially nearly constant, with some decay due to drag from the baryons through matter radiation equality. On superhorizon scales there is only an order unity decay, so a significant large scale photon quadrupole will be present at low redshift to source a significant additional large scale polarization signal from reionization. The evolution is illustrated in Fig. 1.

On large scales the early ISW contribution is about 20% as $\bar{\sigma}$ decays as the matter becomes more dominant. On subhorizon scales at recombination there is no ISW contribution as $\bar{\sigma}$ has already decayed. We neglect the effect of magnetic field generation by the photon-baryon vorticity [4].

OBSERVATIONS

We now compute the observable CMB anisotropy signal. We define the dimensionless first order transverse vector σ_a such that $\sigma_{ab}^{(1)} = D_{(a} \sigma_{b)}$, and quantify the primordial vector modes by their power spectrum $P_{\bar{\sigma}}$ defined so that

$$\langle |\bar{\sigma}_a|^2 \rangle = \int d \ln k P_{\bar{\sigma}}. \quad (27)$$

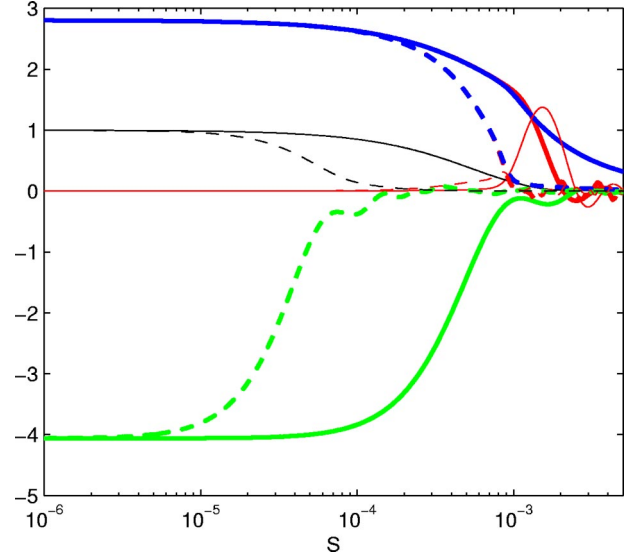


FIG. 1. Evolution of the vector perturbations with wave number $k = 0.02 \text{ Mpc}^{-1}$ (solid lines) and $k = 0.2 \text{ Mpc}^{-1}$ (dashed lines). Thick lines are the velocities of the baryons (top), photons (until decoupling same as the baryon velocity), and neutrinos (bottom). Thin lines are $\bar{\sigma}$ (with $\bar{\sigma}_0 = 1$) and the photon anisotropic stress. The baryon velocity evolves independently of wave number on large scales, but is damped on small scales.

The corresponding expressions for the CMB temperature and polarization power spectra are derived in Ref. [3].

To account for the small scale damping effect accurately, as well as a detailed treatment of recombination and reionization, we compute sample CMB power spectra numerically by a straightforward modification of CAMB.¹ The CMB power spectra (C_l) depend on $P_{\bar{\sigma}}$. For a scale-invariant spectrum, the temperature C_l has a broad peak around $\ell \sim 50$, as shown in Fig. 2. The polarization power spectra peak at around $\ell \sim 500$, with the B mode dominating in accordance with Ref. [10].

The large scale reionization signal is rather similar to that expected from tensor modes, and thus experiments aimed at detecting this tensor signal will also be sensitive to vector modes. Incomplete sky coverage only decreases the sensitivity by an order unity factor due to E - B mode mixing [14,15] even on the largest scales. From Fig. 2 we see that the large scale B modes are more sensitive to vector power by a factor of about 100; thus sensitive observations of tensor modes will also be good probes of regular vector modes. To distinguish the two, one just needs to measure the spectrum at $\ell \gtrsim 100$ where the tensor power falls but the vector power continues to grow.

The dominant confusion on small scales is likely to be from weak lensing of the scalar modes, which peaks on similar scales. There are about 10^6 observable modes, so one can ideally expect to detect a vector contribution $\sim 1/1000$ of the power of the lensing signal. Since they are of comparable power for a scale-invariant primordial power spectrum ratio

¹<http://camb.info/>

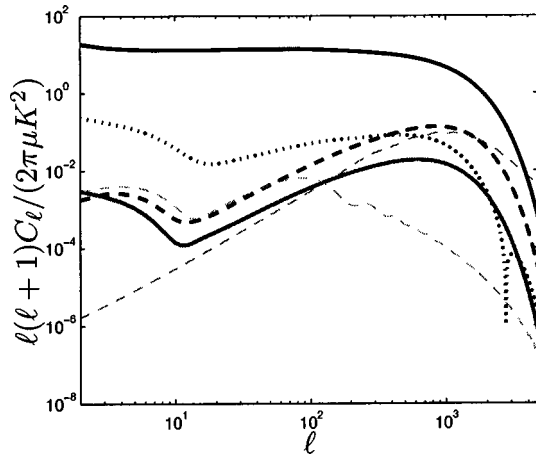


FIG. 2. Typical CMB temperature (top solid), polarization EE (bottom solid), BB (dashed thick), and cross-correlation TE (dotted; absolute value) power spectra for regular vector modes assuming a primordial vector to scalar power ratio $\sim 10^{-3}$ and scale-invariant vector mode spectrum $P_{\vec{\sigma}}$. The other dashed lines show the B -mode spectrum from weak lensing (peaking at $\ell \sim 1000$), and primordial tensors with initial power ratio $\sim 10^{-1}$ (peaking at $\ell \sim 100$).

$P_{\chi}/P_{\vec{\sigma}}$ of $\sim 10^{-3}$ (P_{χ} is the power in the comoving curvature perturbation), this implies that vector modes with only 10^{-6} of the scalar power may be detectable irrespective of the tensor mode amplitude. Since the lensing signal is non-Gaussian, and in the absence of vector modes is partially subtractable [16–18], the in-principle limit is probably much lower, though this depends on the spectrum of the vector modes. The ultimate limit may be around the level where there should be a sourced vector mode signal from second-order effects [19].

Primordial magnetic fields source a B -mode spectrum similar to that from primordial vector modes [2]. However, the perturbations are expected to be highly non-Gaussian for magnetic fields and hence easily distinguishable from pri-

mordial vector modes if they are approximately Gaussian, at least until the lensing confusion limit. Magnetic fields also provide a constant source that partly compensate the damping, so there is more magnetic field vector mode power on very small scales. The detailed signature of magnetic fields in the CMB is discussed in Ref. [3], including the additional large scale signature from tensor modes.

Topological defects can also source similar B -mode spectra [20], though again the spectrum is expected to be non-Gaussian, and (at least for strings) there is more power on very small scales due to the continuous sourcing of the vector modes.

CONCLUSION

We have shown that regular primordial vector modes have a strong observational signature, allowing the possibility that tiny primordial amplitudes can be constrained from future high-sensitivity CMB polarization B -mode observations. Any signature of vector modes would be powerful evidence against simple inflationary models. The Planck² satellite should be able to detect the B -mode signature from primordial vector modes at the 10^{-3} level, and distinguish them from tensor modes by the presence of small scale power. A full Bayesian joint analysis of all the CMB power spectra should be straightforward using Markov-Chain-Monte Carlo (MCMC) techniques, and may give better constraints than suggested here. Separating a vector mode signal at the 10^{-6} level from that generated by lensing of scalar modes would be a serious challenge for the future.

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²<http://astro.estec.esa.nl/Planck>

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