Active-sterile neutrino oscillations and pulsar kicks

M. Barkovich, J. C. D'Olivo, and R. Montemayor*

Departamento de Física de Altas Energías, Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apartado Postal 70-543, 04510 México, Distrito Federal, México

We develop a thorough description of neutrino oscillations in a magnetized protoneutron star, based on a resonance layer for neutrinos with different momentum directions. We apply our approach to the calculation of the asymmetry in the neutrino emission during the birth of a neutron star and the pulsar acceleration in the case of an active-sterile neutrino resonant conversion. The observed velocities can be obtained with the magnetic fields expected in the interior of a protoneutron star, for sterile neutrino masses of the order of KeV and small mixing angles.

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I. INTRODUCTION

The peculiar motion of pulsars is one of the challenging problems in modern astrophysics. Observations show that young pulsars move with velocities of 200–500 $\mathrm{km\,s^{-1}}$ on average, much larger than the mean velocity of ordinary stars in our Galaxy (~ 30 $\mathrm{km\,s^{-1}}$) [1]. This is strong evidence that pulsars receive a kick during their birth in supernovae explosions, but the physical origin of such an impulse remains unclear. A natal kick could be a result of global hydrodynamical perturbations in the pre-collapse core of the progenitor star or an anisotropic neutrino emission induced by the strong magnetic fields in the protoneutron star [2]. A significant fraction of the pulsar population has velocities over 1000 $\mathrm{km\,s^{-1}}$ which are difficult to account for assuming hydrodynamical kick mechanisms, as numerical simulations of supernova explosions indicate.

More than 99% of the gravitational energy released in the collapse is emitted in the form of neutrinos and antineutrinos. A small asymmetry ($\sim 1\%$) in the momentum taken away by neutrinos could produce an acceleration consistent with the measured pulsar velocities. A particle physics explanation of the origin of this asymmetry has been proposed by Kusenko and Segrè (KS) [3,4], based on adiabatic flavor ($\nu_e \leftrightarrow \nu_{\mu,\tau}$) oscillations in matter, in the presence of a magnetic field. In a magnetized stellar plasma there are anisotropic contributions to the neutrino refraction index [5,6], and thus the resonant transformation of neutrinos moving in different directions with respect to the magnetic field occurs at different depths within the protostar, producing a momentum flux asymmetry. With the standard (active) neutrinos, the mechanism works when the resonance region lies between the surface of the two neutrinospheres, but it requires an exceedingly high square mass difference ($\Delta m^2 \gtrsim 10^4$ eV^2), in conflict with the existing limits for standard neutrinos.

A variant of the idea of KS can be implemented in terms of matter neutrino oscillations between active and non interacting (sterile) states [7], in which case the above mentioned limitation can be avoided. More recently it has been shown that an asymmetric off-resonant emission of sterile neutrinos ν_s is also a feasible mechanism [8]. In both cases, the range of required oscillation parameters (mass and mixing angle) overlap with the allowed region for sterile neutrinos considered as the cosmological dark matter. Alternative neutrino oscillation mechanisms for the pulsar kick have been proposed in relation to the possible existence of transition magnetic moments [9], nonorthonormality of the flavor neutrinos [10], or violations of the equivalence principle [11]. The asymmetric neutrino emission could also produce a short gamma-ray burst [12] and gravity waves, whose detection can help to understand pulsar kicks [13].

In a previous article [14] we reexamined the viability of the KS mechanism for resonant active-active neutrino oscillations, and emphasized the relevance of the geometrical ($1/r^2$) dependence of the energy flux to produce an anisotropic neutrino emission. Following previous works on the subject, we use the concept of an induced deformation by the magnetic field of an averaged resonance surface, which acts as an effective $\nu_{\mu,\tau}$ neutrinosphere. In this work we analyze the active-sterile transformations incorporating a more realistic description in terms of a spherical layer where the resonant conversion of neutrinos with momentum k moving in all directions takes place. A geometrical expression for the neutrino flux anisotropy is derived in terms of the ratio of the thickness of the resonance shell, which depends on the intensity of the magnetic field and its average position. To quantify the effect we apply our results to two simple models for the protostar atmosphere.

The paper is organized as follows. In Sec. II we analyze the resonant conversion of active neutrinos into sterile neutrinos within the protostar atmosphere in the presence of a magnetic field, and we derive an expression for the thickness of the resonance region as a function of the magnetic field and the medium properties. In Sec. III, on the basis of the reasonable assumption that the neutrino flux is in the diffusive regime, we evaluate the asymmetry of the momentum flux taken out by neutrinos. In Sec. IV concrete results are obtained in the context of two specific models. Section V contains some comments and concluding remarks.

*Permanent address: Instituto Balseiro and CAB, Universidad Nacional de Cuyo and CNEA, 8400 Bariloche, Argentina.

II. RESONANT ACTIVE-STERILE TRANSFORMATION IN A MAGNETIZED MEDIUM

Within the neutrinosphere neutrinos are trapped due to the opacity of the medium. This region can be considered as limited by a spherical surface with a radius R_{ν_l}, defined by the condition that above the surface neutrinos suffer less than one collision [15]

$$\int_{R_{\nu l}}^{\infty} \frac{dr}{\lambda_{\nu_l}} \simeq \frac{2}{3}, \qquad (1)$$

where λ_{ν_l} is the mean free path of the neutrino of flavor $l = e, \mu, \tau$. We consider the protostar atmosphere as constituted by a degenerate gas of relativistic electrons and a classical nonrelativistic gas of nucleons. The main contributions to the neutrino opacity in such a medium come from the neutral-current scattering reactions [16]

$$\nu_l + n \rightleftarrows \nu_l + n, \qquad (2)$$

$$\nu_l + p \rightleftarrows \nu_l + p, \qquad (3)$$

and the charged-current absorption reaction

$$\nu_e + n \rightleftarrows e^- + p. \qquad (4)$$

The cross sections for the reactions (2) and (3) are

$$\sigma_n = \frac{G_F^2}{4\pi}(1 + 3g_A^2)k^2, \qquad (5)$$

$$\sigma_p = \sigma_n \left[1 - \frac{8\sin^2\theta_W}{1 + 3g_A^2}(1 - 2\sin^2\theta_W) \right], \qquad (6)$$

where k is the neutrino momentum, $\sin^2\theta_W \simeq 0.23$, and $g_A \simeq 1.26$ is the renormalization of the axial-vector current of the nucleons. For the absorption reaction we have

$$\sigma_{abs} = 4\sigma_n \left(1 + \frac{Q}{k} \right) \left[\left(1 + \frac{Q}{k} \right)^2 - \left(\frac{m_e}{k} \right)^2 \right]^{1/2}, \qquad (7)$$

with $Q = m_n - m_p$ denoting the difference between the neutron and the proton masses.

In terms of thermally averaged cross sections, the mean free paths for the electron and muon (tau) neutrino are given by

$$\lambda_{\nu_e}^{-1} = N_n(\langle\sigma_{abs}\rangle + \langle\sigma_n\rangle) + N_p\langle\sigma_p\rangle \equiv \kappa_{\nu_e}k^2\rho, \qquad (8)$$

$$\lambda_{\nu_{\mu,\tau}}^{-1} = N_n\langle\sigma_n\rangle + N_p\langle\sigma_p\rangle \equiv \kappa_{\nu_{\mu,\tau}}k^2\rho, \qquad (9)$$

where $N_{n,p}$ are the neutron and proton number densities, ρ is the baryonic density, and κ_{ν_l} are the neutrino opacities. In a protoneutron star $N_n \simeq 0.9(N_n + N_p)$ and $k \gg Q$, which yields

$$\lambda_{\nu_e}^{-1} \simeq 5\lambda_{\nu_{\mu,\tau}}^{-1} \simeq 5N_n\langle\sigma_n\rangle, \qquad (10)$$

and

$$\kappa_{\nu_e} = 3 \times 10^{-25} \text{ MeV}^{-5}, \qquad (11)$$

$$\kappa_{\nu_{\mu,\tau}} = 0.7 \times 10^{-25} \text{ MeV}^{-5}. \qquad (12)$$

Equation (10) implies that $R_{\nu_e} > R_{\nu_{\mu,\tau}}$, as can immediately be seen from Eq. (1).

In the presence of a magnetic field, the effective potential for an active neutrino propagating in a plasma of electrons, nucleons, and neutrinos can be written as [5,17]:

$$V_{\nu_l} = b_{\nu_l} - ec_{\nu_l}\hat{\mathbf{k}} \cdot \mathbf{B}, \qquad (13)$$

with the coefficients b_{ν_l} and c_{ν_l} given by [6]

$$b_{\nu_e} = \sqrt{2}G_F\left[N_e + \left(\frac{1}{2} - 2\sin^2\theta_W \right)(N_p - N_e) - \frac{1}{2}N_n \right] + \tilde{b}_{\nu_e}, \qquad (14)$$

$$b_{\nu_{\mu,\tau}} = \sqrt{2}G_F\left[\left(\frac{1}{2} - 2\sin^2\theta_W \right)(N_p - N_e) - \frac{1}{2}N_n \right] + \tilde{b}_{\nu_{\mu,\tau}}, \qquad (15)$$

$$c_{\nu_e} = 2\sqrt{2}G_F\left[g_A\left(1 + 2m_p\frac{\kappa_p}{e} \right)C_p - g_A 2m_n\frac{\kappa_n}{e}C_n - C_e \right], \qquad (16)$$

$$c_{\nu_{\mu,\tau}} = 2\sqrt{2}G_F\left[g_A\left(1 + 2m_p\frac{\kappa_p}{e} \right)C_p - g_A 2m_n\frac{\kappa_n}{e}C_n + C_e \right], \qquad (17)$$

where $e > 0$ is the proton charge, $\kappa_{n,p}$ are the anomalous part of the nucleon magnetic moments

$$\kappa_n = -1.91\frac{e}{2m_n}, \quad \kappa_p = 1.79\frac{e}{2m_p}, \qquad (18)$$

and

$$\tilde{b}_{\nu_l} = \sqrt{2}G_F\left[N_{\nu_l} - N_{\bar{\nu}_l} + \sum_{l'}(N_{\nu_{l'}} - N_{\bar{\nu}_{l'}}) \right], \qquad (19)$$

$$C_{e,n,p} = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E}\frac{d}{dE}f_{e,n,p}. \qquad (20)$$

In these equations N_e and $N_{\nu_l}(N_{\bar{\nu}_l})$ are the number densities of electrons and neutrinos (antineutrinos) of flavor l, respectively and $f_{e,n,p}$ are the distribution functions of electrons, protons, and neutrons. Since sterile neutrinos do not interact with the medium

$$V_{\nu_s} = V_{\bar{\nu}_s} = 0,$$

which means that, unlike the oscillations among active neutrinos, the neutral current contributions to V_{ν_l} ($V_{\bar{\nu}_l}$) play a relevant role in the MSW equation governing the active-sterile oscillations in matter.

For antineutrinos the potentials change in sign, i.e. $V_{\bar{\nu}_l} = -V_{\nu_l}$, and in general the transition probability for $\nu_l \leftrightarrow \nu_s$ will be different for neutrinos and antineutrinos. As a consequence, their relative amounts will change, driving the potential locally to zero and the mixing angle in matter close to the one in vacuum throughout the protoneutron star [18]. In this work we will assume that this effect is not important because the resonance conversion takes place within a thin layer. Under such conditions the kick mechanism happens as described in Ref. [8].

For relativistic degenerate electrons $C_e = -\mu_e/8\pi^2$ and for classical nonrelativistic nucleons $C_{n,p} = -N_{n,p}/8Tm_{n,p}$ [19,6], which gives

$$c_{\nu_e} \simeq -4\sqrt{2}G_F\left(\frac{N_p}{5Tm_n} + \frac{3N_n}{20Tm_n} - \frac{\mu_e}{16\pi^2}\right), \quad (21)$$

$$c_{\nu_{\mu,\tau}} \simeq -4\sqrt{2}G_F\left(\frac{N_p}{5Tm_n} + \frac{3N_n}{20Tm_n} + \frac{\mu_e}{16\pi^2}\right), \quad (22)$$

where we have taken $m_p \simeq m_n$. Here T is the background temperature and $\mu_e = (3\pi^2 N_e)^{1/3}$ is the electron chemical potential. It should be noticed that in Ref. [7] c_{ν_e} vanishes because all the components of the stellar plasma are assumed to be degenerated, and the contributions due to the anomalous magnetic moment of the nucleons are not included. These expressions have been derive in the weak-field limit, i.e. $B \ll \mu_e^2/2e$ [19]. More general features of the neutrino propagation in magnetized media, incorporating the effect of strong magnetic fields, have been considered by several authors [17].

The neutrino fractions are much smaller than the electron fraction $Y_e = N_e/(N_n + N_p) \sim 0.1$ and, in the context of our analysis, the contributions to V_{ν_l} coming from the neutrino-neutrino interactions (denoted by $\tilde{b}_{\nu_l}$) can be neglected. With this approximation in mind, and taking into account that by electrical neutrality $N_e = N_p$, the coefficients in the effective potentials become:

$$b_{\nu_l} = -b_{\bar{\nu}_l} \simeq -\frac{G_F}{\sqrt{2}}(1 - \eta_l Y_e)\frac{\rho}{m_n}, \quad (23)$$

$$c_{\nu_l} = -c_{\bar{\nu}_l} \simeq -\sqrt{2}G_F\left(\frac{3 + Y_e}{5Tm_n^2}\rho - \xi_l\frac{\mu_e}{4\pi^2}\right) \quad (24)$$

where $\eta_e = 3$, $\xi_e = 1$ and $\eta_{\mu,\tau} = 1$, $\xi_{\mu,\tau} = -1$.

As is shown by Eq. (13), the electron fraction plays a very important role in the case of active-sterile oscillations. This quantity can be estimated as follows. From the equilibrium condition for the reaction $e^- + p \leftrightarrows n + \nu_e$, taking $\mu_{\nu_e} \simeq 0$ we have the relation $\mu_e + \mu_p \simeq \mu_n$ for the chemical potentials. Hence, $N_n = e^{-(Q-\mu_e)/T}N_p$ for a non-relativistic nucleon gas at a temperature T and

$$Y_e \simeq \frac{1}{1 + e^{\mu_e/T}}, \quad (25)$$

where we used the fact that $Q \ll \mu_e$ for typical densities in a protoneutron star. The above is an implicit equation for Y_e, which has to be solved numerically once ρ and T are known as a function of r.

Now we use the effective potentials with $b_{\bar{\nu}_l}$ and $c_{\bar{\nu}_l}$ given by Eqs. (23) and (24) to discuss the effect of a magnetic field on the active-sterile matter neutrino oscillations. To make our analysis simpler, let us consider the case of two states (ν_l, ν_s) that are a mixture of the mass eigenstates ν_1 and ν_2. For a momentum $\mathbf{k}$, a ν_l will undergo a resonant conversion into in a ν_s if the following condition is satisfied:

$$\left.\frac{\Delta m^2}{2k}\cos 2\theta = V_{\nu_l}\right|_{r=R(\mathbf{k})}, \quad (26)$$

where θ is the vacuum mixing angle and $\Delta m^2 = m_2^2 - m_1^2$ is the difference between the squares of the neutrino masses. Assuming $\Delta m^2 \cos 2\theta > 0$, if $B = 0$ the resonant condition for the electron neutrino (antineutrino) is satisfied when $Y_e > 1/3$ ($Y_e < 1/3$). In addition, since $Y_e < 1$ only the $\bar{\nu}_{\mu,\tau}$ (but not the $\nu_{\mu,\tau}$) can go through the resonance region. In what follows we assume that within the protoneutron star $Y_e < 1/3$, so that the resonant condition is verified only for antineutrinos.

In a linear approximation, we can write

$$R(\mathbf{k}) = R_r(k) + \delta(k)\hat{\mathbf{k}}\cdot\hat{\mathbf{B}}, \quad (27)$$

where $R_r(k)$ is the radius of the resonance sphere in the absence of the magnetic field. For a certain value of the magnitude of the momentum k, Eq. (27) determines a spherical shell limited by the spheres of radii $R_r(k) \pm \delta(k)B$, where the resonance condition is verified for antineutrinos moving in different directions with respect to $\mathbf{B}$. Using the expression (13) for $V_{\bar{\nu}_l}$ in Eq. (26), we get

$$\delta_{\bar{\nu}_l}(k) = \mathcal{D}_{\bar{\nu}_l}eB, \quad (28)$$

where

$$\mathcal{D}_{\bar{\nu}_l} = \left.\frac{1}{h_{b_{\bar{\nu}_l}}^{-1}}\frac{c_{\bar{\nu}_l}}{b_{\bar{\nu}_l}}\right|_{r=R_r(k)}, \quad (29)$$

and we have defined $h_g^{-1} \equiv (d/dr)\ln g$ for any function $g(r)$. From Eqs. (23) and (24) we find explicitly

$$\mathcal{D}_{\bar{\nu}_l} = \frac{2}{(1-\eta_l Y_e)h_\rho^{-1} - \eta_l Y_e h_{Y_e}^{-1}}$$
$$\times \left.\left(\frac{3+Y_e}{5Tm_n} - \xi_l\frac{m_n\mu_e}{4\pi^2\rho}\right)\right|_{r=R_r(k)}. \quad (30)$$

By means of Eq. (25), $h_{Y_e}^{-1}$ can be expressed in terms of h_ρ^{-1} and h_T^{-1} as follows

$$h_{Y_e}^{-1} = -(h_\rho^{-1} - 3h_T^{-1})\frac{1-Y_e}{1-Y_e+\dfrac{3T}{\mu_e}}. \qquad (31)$$

The efficiency of the flavor transformation depends on the adiabaticity of the process. The conversion will be adiabatic when the oscillation length at the resonance is smaller than the width of the enhancement region. This imposes the condition

$$\left|\frac{dV_{\bar{\nu}_l}}{dr}\right|_{r=R_r(k)} < \left(\frac{\Delta m^2}{2k}\sin 2\theta\right)^2\bigg|_{r=R_r(k)}. \qquad (32)$$

From Eq. (13) and neglecting the contribution of the magnetic field we get

$$\tan^2 2\theta > \left|\frac{1}{b_{\bar{\nu}_l}}\left(h_\rho^{-1} - \frac{\eta_l Y_e}{1-\eta_l Y_e}h_{Y_e}^{-1}\right)\right|_{r=R_r(k)}, \qquad (33)$$

with $b_{\bar{\nu}_l}$ given by Eq. (23).

III. NEUTRINO MOMENTUM ASYMMETRY

The energy flux in the atmosphere is radial and determined by the diffusive part of the neutrino distribution function. Consequently, in previous works on the subject the resonance condition has been evaluated in terms of an average neutrino momentum $\langle k \rangle = [7\pi^4/180\zeta(3)]T \simeq 3.15T$ pointing in the radial direction. This procedure leads to the concept of a deformed average surface of resonance, which acts as the boundary of an effective neutrinosphere. In a more detailed description, at each point of the neutrinosphere we have neutrinos with momentum **k** pointing in all directions. Hence, the resonance condition as given by Eq. (26) actually defines a spherical surface for each value of $\hat{\mathbf{k}}\cdot\hat{\mathbf{B}}$, which acts as a source of sterile neutrinos moving in the $\hat{\mathbf{k}}$ direction. Accordingly, as we mentioned above, for a given k the resonant transition takes place within a spherical shell instead of a deformed spherical surface. We will discuss the situation where the resonance shell lies totally within the $\bar{\nu}_l$-neutrinosphere. If the mixing angle is small and the adiabatic condition of Eq. (33) is satisfied, then the $\bar{\nu}_l$ are almost completely converted into $\bar{\nu}_s$. The outgoing sterile antineutrinos freely escape from the protostar, while the ingoing ones propagate through the medium until they cross again the resonance surface and are reconverted into $\bar{\nu}_l$, which are thermalized. In consequence, only those $\bar{\nu}_s$ going outward leave the star and the resonance surfaces for each momentum direction act as effective emission semispheres. For **k** pointing in opposite directions the respective semispheres are at $R_r(k) \pm \delta(k)kB$ and therefore have different areas, which generates a difference in the momentum carried away by the sterile neutrinos leaving the star in opposite

directions. This difference adds up to building a nonvanishing asymmetry in the total momentum $\mathcal{K}$ emitted by the cooling neutron star, which we now compute explicitly in terms of the whole neutrino distribution function.

Within the neutrinosphere the $\bar{\nu}_l$ satisfy the diffusion regime [14,20] with a distribution function

$$f_{\bar{\nu}_l} = f_{\bar{\nu}_l}^{eq} - \frac{1}{\kappa_{\bar{\nu}_l}\rho k^3}\mathbf{k}\cdot\boldsymbol{\nabla}f_{\bar{\nu}_l}^{eq}, \qquad (34)$$

where $f_{\bar{\nu}_l}^{eq} = (1+e^{k/T})^{-1}$ is the distribution function at equilibrium. We have again assumed that the neutrino chemical potential is negligible. The momentum flux emitted in the radial direction is

$$dF_r = \frac{\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}}{(2\pi)^3}f_{\bar{\nu}_l}kd^3k. \qquad (35)$$

Let us now adopt a reference frame (x,y,z) fixed to the star, where the magnetic field coincides with the z-axis, $\mathbf{B}=B\hat{\mathbf{z}}$. In addition, at each point within the spherical shell determined by Eq. (27) we use a local frame (x',y',z') with $\hat{\mathbf{r}}=\hat{\mathbf{z}}'$, such that $\hat{\mathbf{k}}=\cos\theta'\hat{\mathbf{z}}' + \sin\theta'\cos\phi'\hat{\mathbf{x}}' + \sin\theta'\sin\phi'\hat{\mathbf{y}}'$. In this way the total momentum carried away by the $\bar{\nu}_l$ can be expressed as

$$k_r = \frac{1}{(2\pi)^2}\int_0^\infty k^3 dk \int_0^\pi \sin\theta d\theta \int_0^{\pi/2}\sin\theta' d\theta'$$
$$\times \int_0^{2\pi}d\phi' R^2(\mathbf{k})f_{\bar{\nu}_l}\hat{\mathbf{k}}\cdot\hat{\mathbf{r}} \qquad (36)$$

$$= \frac{1}{2\pi}\int_0^\infty dk k^3 R_r^2(k)\left(f_{\bar{\nu}_l}^{eq} - \frac{2}{3\kappa_{\bar{\nu}_l}\rho k^2}\frac{df_{\bar{\nu}_l}^{eq}}{dr}\right). \qquad (37)$$

The momentum emitted in direction of the magnetic field k_B is calculated in a similar manner, with $\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}$ replaced by $\hat{\mathbf{k}}\cdot\hat{\mathbf{B}} = \cos\theta\cos\theta' - \sin\theta\sin\theta'\sin\phi'$:

$$k_B = \frac{2}{3\pi}\int_0^\infty dk k^3 R_r(k)\delta(k)\left(f_{\bar{\nu}_l}^{eq} - \frac{1}{2\kappa_{\bar{\nu}_l}\rho k^2}\frac{df_{\bar{\nu}_l}^{eq}}{dr}\right), \qquad (38)$$

where we have used Eq. (27) keeping at most terms that are linear in $\delta(k)$.

In terms of the above quantities the $\bar{\nu}_l$ contribution to the fractional asymmetry in the total momentum becomes

$$\frac{\Delta\mathcal{K}}{\mathcal{K}} = \frac{1}{6}\frac{k_B}{k_r}, \qquad (39)$$

where the factor 1/6 comes from the fact that we are assuming that $\mathcal{K}$ is equipartitioned among all the neutrino and an-

tineutrino flavors. In order to evaluate the remaining integrals in Eqs. (37) and (38) we need to know the explicit dependence on k of the functions R_r and δ. Simple analytical results can be derived by replacing these functions by their values at the average momentum $\langle k \rangle$. Proceeding in this manner we get

$$k_r = \frac{\pi R_r^2}{48} \left(\frac{7\pi^2 T^4}{5} - \frac{4}{3\kappa_{\bar{\nu}_l}\rho} \frac{dT^2}{dr} \right), \qquad (40)$$

$$k_B = \frac{\pi R_r \delta}{36} \left(\frac{7\pi^2 T^4}{5} - \frac{1}{\kappa_{\bar{\nu}_l}\rho} \frac{dT^2}{dr} \right). \qquad (41)$$

In these expressions the terms that depend on dT^2/dr come from the diffusive part of the distribution function. In the regime considered here they happen to be smaller than the first terms and we will discard them. This can be verified in the context of specific models like the one considered in the next section. Using this approximation, we finally obtain

$$\frac{\Delta \mathcal{K}}{\mathcal{K}} \simeq \frac{2}{9} \frac{\delta}{R_r}, \qquad (42)$$

which is four times bigger than the asymmetry we derived by assuming a single (deformed) resonance surface [14]. This means that if Eq. (42) were applied to the case of active-active oscillations, when the resonance shell is at the border of the electron neutrinosphere, we would obtain $B \simeq 7.5 \times 10^{15}$ G for the intensity of the magnetic field required to produce the observed pulsar velocities.

Up to this point we have considered the resonant oscillation between a sterile antineutrino and an active antineutrino of a given flavor. The observational evidence indicates that active neutrinos mix. Furthermore, it is reasonable to assume that the three active antineutrinos could be mixed with the sterile ones. Consequently, the situation can be more complicated than the simple pairwise neutrino oscillation already discussed. In general different mixing schemes will render gives different results for the magnetic field, but these results will differ in factors of the order of the unity. Therefore, to make an estimation of the magnetic field we will consider the simplest possible scheme, in which the mixing between active neutrinos is neglected and pairwise oscillation with the sterile ones are taken. In this context the total momentum asymmetry can be written

$$\frac{\Delta \mathcal{K}}{\mathcal{K}} \simeq \frac{2}{9} \left(\frac{\delta_{\bar{\nu}_e}}{R_{r_{\bar{\nu}_e}}} + 2 \frac{\delta_{\bar{\nu}_{\mu,\tau}}}{R_{r_{\bar{\nu}_{\mu,\tau}}}} \right). \qquad (43)$$

In what follows we will make the calculation for the active-sterile conversion in the framework of this approximation by assuming that all the resonance layers are entirely within the $\nu_{\mu,\tau}$-neutrinosphere.

IV. NUMERICAL RESULTS

Taking into account the total momentum emitted by the protostar, to have the required kick $\Delta \mathcal{K}/\mathcal{K}$ must be of the order of 0.01. Inserting this value for the momentum asymmetry on the left hand side of Eq. (43) we see that $\delta_{\bar{\nu}_l} \sim 0.015 R_{r_{\bar{\nu}_l}}$, which means that the thickness of the resonance shell is of the order of one hundred of meters. From Eqs. (28) and (43), using $R_{r_{\bar{\nu}_e}} \simeq R_{r_{\bar{\nu}_{\mu,\tau}}} \simeq R_r$, we find

$$eB = 0.045 \frac{R_r}{\mathcal{D}_{\bar{\nu}_e} + 2\mathcal{D}_{\bar{\nu}_{\mu,\tau}}}. \qquad (44)$$

The magnitude of the magnetic field needed to produce the kick can be evaluated from this result and Eq. (30), once the temperature and density profiles are known. In this section we will perform this calculation for two simple models of a protoneutron star atmosphere. One of them is the spherical Eddington model [14] and the other is the one introduced in Ref. [21], on the basis of a fitting of numerical results.

The Eddington model [20] gives a satisfactory description of a neutrinosphere locally homogeneous and isotropic, with a conserved energy flux. In the case of a protoneutron star the results are in reasonable agreement with those obtained by numerical analysis. For a spherical geometry the model was developed in Ref. [14], where the reader is referred for details. Here we summarize those features that are relevant for our analysis. The atmosphere of the protostar is described as an ideal nucleon gas in hydrostatic equilibrium

$$P = \frac{\rho T}{m_n}, \qquad (45)$$

$$\frac{dP}{dr} = -\rho \frac{GM(r)}{r^2}, \qquad (46)$$

with $M(r) = 4\pi \int_0^r dr' r'^2 \hat{\rho}(r')$. The total neutrino momentum flux satisfies the diffusion equation

$$F(r) = -\frac{1}{36} \frac{1}{\bar{\kappa}\rho(r)} \frac{d}{dr} T^2(r), \qquad (47)$$

where $\bar{\kappa} = (\kappa_{\bar{\nu}_e}^{-1} + 2\kappa_{\bar{\nu}_{\mu,\tau}}^{-1})^{-1} = 3 \times 10^{-26}$ MeV^{-5}, and the flux conservation

$$F(r) = \frac{L_c}{4\pi r^2}, \qquad (48)$$

where L_c is the luminosity of the protostar. Equation (47) is obtained from Eq. (35) by integrating over the momentum.

The above system of equations has no exact analytical solution. However, a good approximate solution has been found in Ref. [14]. It allows us a consistent treatment, from which we obtain the following expressions:

$$h_T^{-1} = -\frac{\lambda_c}{1-a} \frac{R_c}{r^2} \frac{T_c}{T} \left(1 - \frac{T_c^2}{T^2} a \right), \qquad (49)$$

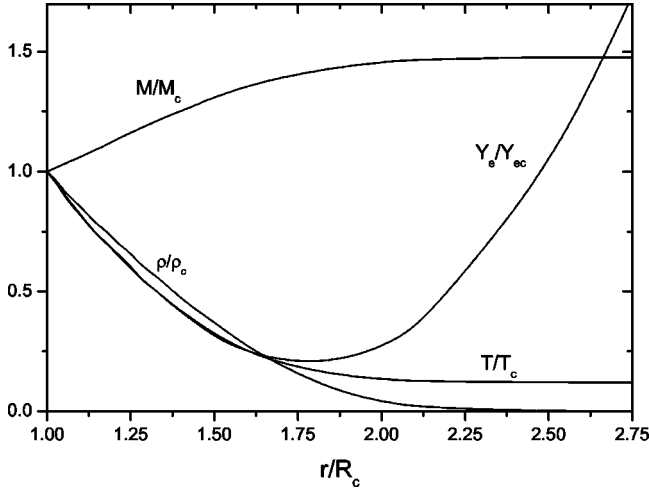$$h_\rho^{-1} + h_T^{-1} = -\frac{Gm_n M(r)}{r^2 T}, \qquad (50)$$

FIG. 1. Characteristic profiles for the Eddington model. The different functions are normalized to the corresponding value at the core: $R_c = 10$ km, $T_c = 40$ MeV, $\rho_c = 10^{14}$ g cm^3, $M_c = 1\ M_\odot$, and $Y_{ec} = 0.08$.

with $\lambda_c = (9\kappa L_c \rho_c)/(2\pi T_c^2 R_c)$. Here ρ_c and T_c are the density and the temperature at the radius of the core R_c, and a is a function of the temperature

$$a = 1 - \frac{1}{\alpha_c} - \frac{T_c - T}{T_c - T_s}\left(1 - \frac{1}{\alpha_c} - \frac{T_s^2}{T_c^2}\right), \qquad (51)$$

where $\alpha_c = (GM_c m_n)/(2\lambda_c T_c R_c)$ and T_s is the asymptotic value of the temperature. In Fig. 1 we show the resulting profiles for $\rho(r)$, $T(r)$, and $M(r)$, corresponding to a configuration like the one considered in Ref. [14]: $M_c = M_\odot$, $R_c = 10$ km, $L_c = 2 \times 10^{52}$ erg s^{-1}, $\rho_c = 10^{14}$ g cm^{-3}, $T_c = 40$ MeV, and $T_s = 4.8$ MeV. In the same figure, we plot $Y_e(r)$ as obtained by solving Eq. (25) at every point in the atmosphere.

According to Eq. (10) the electron neutrino mean free path is

$$\lambda_{\bar{\nu}_e} \simeq 8\left(\frac{\rho_c}{\rho}\right)\left(\frac{T_c}{T}\right)^2 \text{ cm}. \qquad (52)$$

The radius of the neutrinosphere is estimated by inserting the above expression into Eq. (1) and evaluating the resulting integral numerically, the result is $R_{\bar{\nu}_e} \simeq 2.6\ R_c$. This yields $\rho(R_{\bar{\nu}_e}) \simeq 1.3 \times 10^{11}$ g cm^{-3}, $T(R_{\bar{\nu}_e}) \simeq T_s$, and $Y_e(R_{\bar{\nu}_e}) \simeq 0.1$ which are in good agreement with typical values for the electron neutrinosphere in a protoneutron star. In the same way, for the muon (tau) neutrino we get $R_{\bar{\nu}_{\mu,\tau}} \simeq 2.2\ R_c$.

Using Eqs. (48) and (47), the magnitude of the ratio of the second and the first terms in Eq. (40) is $(L_c \kappa_{\nu_l})/(3\bar{\kappa} T^4 r^2)$, which varies between 10^{-4} and 10^{-1} for $R_c \lesssim r \lesssim R_{\bar{\nu}_{\mu,\tau}}$. Therefore, the approximation done in writing Eq. (42) is justified and the intensity of the magnetic field required to produce the pulsar kick can be evaluated from Eq. (44). As shown in Fig. 2, at the surface of the core $B_c \simeq 8 \times 10^{16}$ G and B remains approximately constant when the position of
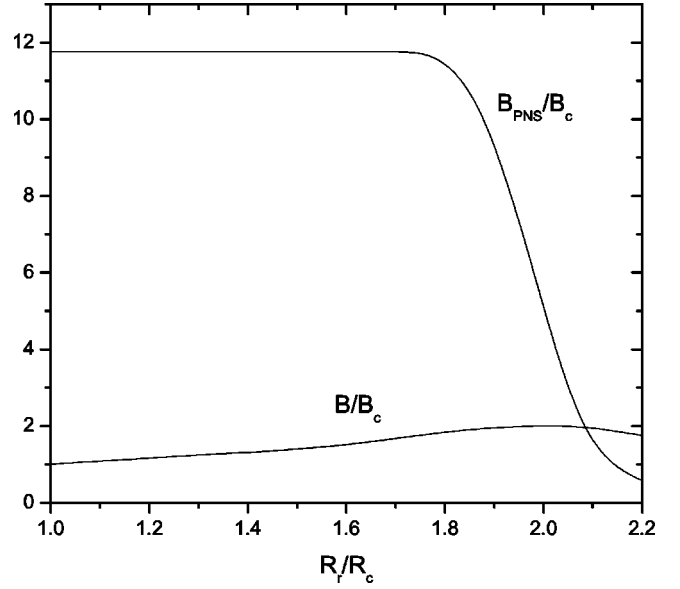

FIG. 2. Magnetic field required to produce the kick, B, for the Eddington model and the expected maximum magnetic field, B_{PNS}, as functions of the resonance radius and normalized to the field required at the core, $B_c = 8 \times 10^{16}$ G.

the resonance moves through the protostar atmosphere. According to Eq. (26), in the interval $R_c \leqslant R_r \leqslant R_{\bar{\nu}_{\mu,\tau}}$ the allowed values of the oscillations parameters are 10^9 eV$^2 \gtrsim \Delta m^2 \cos 2\theta \gtrsim 10^6$ eV^2, that for small mixing leads to 30 KeV $\gtrsim m_s \gtrsim 1$ KeV. From Eq. (33) it can be seen that transitions will be adiabatic for $\tan^2 2\theta > 7 \times 10^{-12}$ at $R_r = R_c$ and $\tan^2 2\theta > 3 \times 10^{-9}$ at $R_r = R_{\bar{\nu}_{\mu,\tau}}$.

In the second model, both the baryon density and the temperature profiles follows simple potential laws and are related according to [21]

$$\rho = \rho_c \left(\frac{T}{T_c}\right)^3. \qquad (53)$$

From this expression we see that $h_\rho^{-1} = 3h_T^{-1}$, which in turn implies that $h_{Y_e}^{-1} = 0$ [see Eq. (31)]. As a consequence, the electron fraction Y_e and also the ratio μ_e/T remain constants in the protostar atmosphere. Employing the same set of core parameters than in the preceding model we have $Y_e = 0.08$ and $\mu_e/T = 2.5$. In what follows, for the density profile we use $\rho = \rho_c(R_c/r)^4$, which gives $h_\rho^{-1} = -4/r$, and the border of the $\bar{\nu}_{\mu,\tau}$-neutrinosphere is located at $\rho(R_{\bar{\nu}_{\mu,\tau}}) \simeq 10^{12}$ g cm^{-3}, that corresponds to $R_{\bar{\nu}_{\mu,\tau}} \simeq 3\ R_c$ and $T(R_{\bar{\nu}_{\mu,\tau}}) \simeq 9$ MeV. The ratio of the diffusive term to the isotropic term in Eq. (40) can be expressed as $10^{-4}(r/R_c)^{17/3}$ and the approximation (42) for $\Delta\mathcal{K}/\mathcal{K}$ is also valid in this model. Taking these results into account and, from Eqs. (29) and (44) the following simple expression can be derived for the required magnetic field:

$$B = 3 \times 10^{17}\left(1 + 0.09\frac{T_c}{T}\right)^{-1}\frac{T}{T_c}\ G, \qquad (54)$$

with $T = T_c (R_c/r)^{4/3}$. Now the intensity of the magnetic field decreases monotonically from 3×10^{17} G at $R_r = R_c$ to 5×10^{16} G at $R_r = R_{\bar{\nu}_{\mu,\tau}}$. The allowed range for the sterile neutrino mass is similar to the one derived in the case of the Eddington model, while the requirement that the resonant conversion be adiabatic imposes the condition $\tan^2 2\theta \gtrsim 3 \times 10^{-11} (R_r/R_c)^3$ on the mixing angle.

A comment is in order. Our results have been derived on the basis of the weak field limit for the effective neutrino potentials in a magnetized medium. As it has been already mentioned in Sec. II, this approximation is valid whenever $B \ll \mu_e^2/2e$. In one of the examples (the Eddington model) this constraint is verified in the interior but not in the external region of the protostar. A more careful treatment would require to incorporate the effect of strong magnetic fields along the lines examined in Ref. [19].

V. FINAL REMARKS

We have reexamined the anisotropic momentum emission by a nascent neutron star driven by the resonant neutrino conversion in presence of strong magnetic fields. The protostar atmosphere was modeled by a nonrelativistic nucleon gas and a degenerated electron gas. We elaborate the concept of a resonance spherical shell for neutrinos with a given momentum k moving in different directions relative to **B**. This concept provides a more precise description of the problem than a deformed spherical surface for neutrinos with an average radial momentum. An important ingredient for the existence of the effect is that the ingoing neutrinos become thermalized. For active-active neutrino oscillations ($\nu_e \leftrightarrow \nu_{\mu,\tau}$) the thermalization proceeds by any of the following reasons: either because the ingoing $\nu_{\mu,\tau}$ moving toward the interior of the protoneutron star enter their own neutrinosphere, or because they cross again the resonance region and are reconverted into ν_e, which are now within their neutrinosphere. For sterile neutrinos solely the second process is effective. In any case only the outgoing neutrinos contribute to the total momentum flux. The difference in the areas of the semispherical emission surface for neutrinos moving in opposite directions with the same momentum magnitude originates a non null fractional asymmetry in the total momentum. We apply our approach to the study of the generation of a natal pulsar kick through the $\bar{\nu}_l \leftrightarrow \bar{\nu}_s$ oscillations. The kick was calculated in a two simple model for the stellar atmosphere and in both cases a magnetic field of the order of 10^{17} G is required to produce the observed velocities. The ranges of the mass and the mixing angle are compatible with
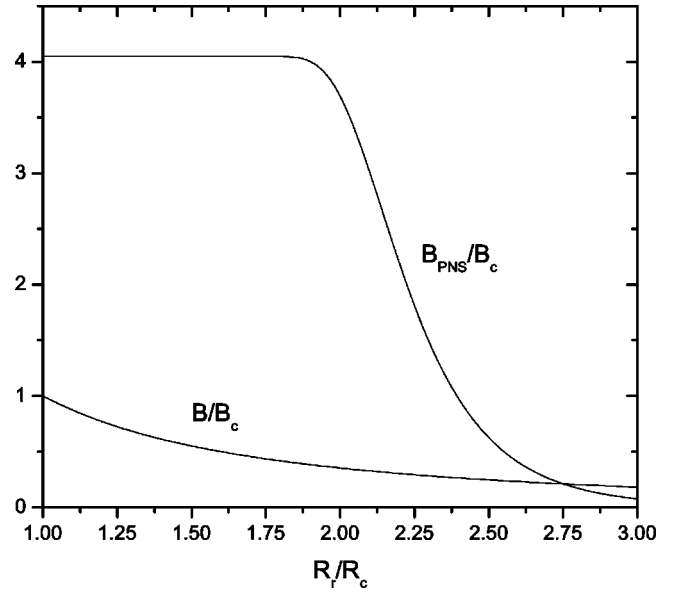

FIG. 3. Magnetic field required to produce the kick, B, for the potential model and the expected maximum magnetic field, B_{PNS}, as functions of the resonance radius and normalized to the field required at the core, $B_c = 3 \times 10^{17}$ G.

a massive sterile neutrino as a warm dark matter candidate [22]. Fields of this magnitude are possible in protoneutron stars. For example, magnetic fields of the order of 10^{18} G have been considered in the literature for the central regions [23], with a dependence in the baryon density parametrized as [24]

$$B_{PNS} = B_s + B_c [1 - e^{-\beta(\rho/\rho_s)^\gamma}], \qquad (55)$$

with $\beta = 10^{-5}$ and $\gamma = 3$. In Eq. (55) $\rho_s = 10^{11}$ g cm^{-3}, $B_s = 10^{14}$ G, and $B_c = 10^{18}$ G are the strengths of the magnetic field at the surface and the core, respectively. For comparison, in Figs. 2 and 3 we have also shown the magnetic field as given by Eq. (55). Taking this curve as an upper value of the magnetic field in the protostar, we see that the resonant neutrino transformation could be an effective mechanism for the pulsar kick in most of the neutrinosphere.

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