Nearly minimal magnetogenesis

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We propose a new mechanism for magnetic field generation from inflation, by which strong magnetic fields can be generated on cosmological scales. These fields may be observable by cosmic microwave background radiation measurements, and may have a dynamical impact on structure formation. The mechanism is based on the observation that a light nearly minimally coupled charged scalar may be responsible for the creation of a negative photon mass squared (provided the scalar field coupling to the curvature scalar is small but negative), which in turn results in abundant photon production—and thus in growing magnetic fields—during inflation.

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I. COSMOLOGICAL MAGNETIC FIELDS

There is growing evidence that magnetic fields permeate intergalactic medium [1–4]. The galactic α-Ω dynamo mechanism [5] represents the standard explanation, according to which small seed fields in protogalaxies are magnified to a microgauss strength correlated on kiloparsec scales observed today in many galaxies. Yet the dynamo does poorly as regards explaining the observed cluster fields, which are typically correlated over several kiloparsecs and can reach 10 microgauss strength. On larger scales the field strength drops quite dramatically, and it is characterized by a negative spectral index between -1.6 and -2 [4], consistent with the Kolmogorov spectrum of turbulence.

The seed field that fuels the dynamo, which amplifies the galactic (and possibly cluster) fields, has either primordial origin or it was created by the Biermann battery mechanism, operative at the time of structure and galaxy formation, when the Universe was about one billion years old [6–8]. The Biermann battery mechanism is operative in the presence of oblique shocks, which generate nonideal fluid that violates proportionality between the gradients of pressure and energy density. Such a fluid can produce the vorticity required for the generation of seed fields, whose strength is typically of the order 10^{-20} gauss. While in the protogalactic environments these seed fields may be sufficiently strong to fuel the galactic dynamo mechanism, whether they can be used to explain the observed cluster fields is a highly controversial question. Moreover, these fields may be insufficiently strong to explain the galactic fields observed in a couple of galaxies at higher redshifts [1,2].

The quest for the fields correlated on much larger scales has so far been unsuccessful. A notable exception is the result of Ref. [9], where a local field of about 10^{-8} gauss correlated on megaparsec scale is quoted. If established, the existence of extragalactic fields would point at a primordial origin. Perhaps the most promising method for detecting primordial fields correlated over supragalactic scales is through their impact on the cosmic microwave background radiation (CMBR) [10,11], and possibly through their dynamical influence on large scale structure and galaxy formation. The current upper bounds from the CMBR are of the order 10^{-9} gauss, while the bounds from structure formation are somewhat weaker. Fields stronger than about 10^{-5} gauss at the time of galaxy formation (which correspond to about 10^{-9} gauss comoving field strength) are dynamically relevant.

In this article we propose a novel mechanism for generation of large scale magnetic fields during cosmological inflation. Our mechanism is based on a model with a rather minimal set of assumptions: We study the dynamics of (one-loop) quantum scalar electrodynamics [12–16] in cosmological space-times, which can be naturally embedded into the standard model (where the role of the scalars is played by the charged components of the Higgs field), or into supersymmetric extensions of the standard model (the charged scalars are the sypersymmetric partners of the standard model fermions).

II. SCENARIO

We assume a standard model of the Universe, in which a period of inflation is followed by radiation and matter eras. The background space-time during inflation can be accurately approximated by the de Sitter metric, $g_{\mu\nu}=a^2\eta_{\mu\nu}$ ($\eta_{\mu\nu}=\mathrm{diag}[-1,1,1,1]$) with the scale factor $a=-1/(H\eta)$ (for $\eta<-\eta_H\equiv-1/H$), where η denotes conformal time, and H the inflationary Hubble parameter. After inflation the Universe undergoes a sudden transition to radiation domination, in which $a=H\eta$ (for $\eta>\eta_H$). When inflation lasts a sufficiently long time, such that a de Sitter invariant solution is established, the dynamics of photons that couple to *light* scalar particles, which in turn couple *nearly minimally* to gravity, can be described by the Proca Lagrangian [12]

$$\mathcal{L}_{\rm Proca}=-\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}-\frac{1}{2}a^2m_\gamma^2\eta^{\mu\nu}A_\mu A_\nu+O(\mathsf{s}^0), \tag{1}$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. The photon mass is given by

$$m_\gamma^2=\frac{\alpha H^2}{\pi\mathsf{s}},\qquad \mathsf{s}=\frac{m_\Phi^2+\xi\mathcal{R}}{3H^2}, \tag{2}$$

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where $\alpha=e^2/4\pi$ is the fine structure constant, $\mathcal{R}=12H^2$ is the curvature scalar of de Sitter space, m_Φ is the scalar mass, and ξ describes the coupling of the scalar field to $\mathcal{R}$. When $0<m_\Phi^2\ll H^2$ and $-1/12\ll\xi<0$, both s and m_γ^2 can be negative. We emphasize that there is nothing pathological about this limit. Indeed, the de Sitter invariant (Feynman) scalar propagator $i\Delta_F(x,x')\equiv G_F(y)$ in $D=4$ reads [12]

$$G_F(y)=\frac{H^2}{4\pi^2}\left\{\frac{1}{y}-\frac{1}{2}\ln(y)+\frac{1}{2\mathbf{s}}-1+\ln(2)+O(\mathbf{s})\right\}, \tag{3}$$

where $y(x;x')\equiv aa'H^2\Delta x^2(x;x')$ denotes the de Sitter invariant length, and $\Delta x^2(x;x')=-(|\eta-\eta'|-i\epsilon)^2+\|\vec{x}-\vec{x}'\|^2$, $\epsilon\to0^+$, such that a negative small s introduces negative, but finite, correlations, which are at the heart of the mechanism discussed in this article.

The Lagrangean (1) was derived in Ref. [12] by expanding in powers of $|\mathsf{s}|\ll1$ the one-loop (order α) nonlocal effective action written in the Schwinger-Keldysh formalism. In Ref. [12] we proved that, when written in the generalized Lorentz gauge $\partial_\mu(a^2\eta^{\mu\nu}A_\nu)=0$ at the order $O(\alpha,\mathsf{s}^{-1})$, the effective action reduces to the Proca theory (1).

Due to spatial homogeneity, the general solution for the photon field of the Proca equation can be conveniently written as a superposition of spatial plane waves $A_\mu(x)=\varepsilon_\mu(\vec{k},\eta)e^{i\vec{k}\cdot\vec{x}}$, where $\varepsilon_\mu=(\varepsilon_0,\vec{\varepsilon})$. Then ε_0 is nondynamical and just traces the spatial components. The spatial components correspond to the three physical degrees of freedom, and can be decomposed into longitudinal $\vec{\varepsilon}_{\mathsf{L},\vec{k}}\equiv\vec{k}(\vec{k}\cdot\vec{\varepsilon})/\vec{k}^2$ and transverse part $\vec{\varepsilon}_{\mathsf{T},\vec{k}}\equiv\vec{\varepsilon}-\vec{\varepsilon}_{\mathsf{L},\vec{k}}$. From Eq. (1), one finds the following equation of motion for the transverse polarization $\vec{\varepsilon}_{\mathsf{T},\vec{k}}$ during inflation:

$$(\partial_\eta^2+k^2+m_\gamma^2a^2)\vec{\varepsilon}_{\mathsf{T},\vec{k}}(\eta)=0, \tag{4}$$

where $k\equiv\|\vec{k}\|$. Demanding that the solution of this equation corresponds to a (circularly polarized) vacuum state for $\eta\to-\infty$, one finds $\vec{\varepsilon}_{\mathsf{T},\vec{k}}(\eta)=A_{\vec{k}}(\eta)(\vec{\epsilon}^{\,1}_{\vec{k}}+i\vec{\epsilon}^{\,2}_{\vec{k}})/\sqrt{2}$, where $\vec{\epsilon}^{\,\mathcal{T}}_{\vec{k}}$ $(\mathcal{T}=1,2)$ are the transverse polarization vectors and

$$A_{\vec{k}}=\frac{1}{2}(-\pi\eta)^{1/2}\mathrm{H}_\nu^{(1)}(-k\eta),\qquad \nu=\sqrt{\frac{1}{4}-\frac{m_\gamma^2}{H^2}}, \tag{5}$$

where $\mathrm{H}_\nu^{(1)}$ is the Hankel function. The second solution is simply $\vec{\varepsilon}^{\,*}_{\mathsf{T},\vec{k}}(\eta)$, such that the Wronskian $W[\vec{\varepsilon}_{\mathsf{T},\vec{k}},\vec{\varepsilon}^{\,*}_{\mathsf{T},\vec{k}}]=i$.

III. VANISHING CONDUCTIVITY

When conductivity in the radiation era is negligibly small, a smooth matching of the inflationary epoch solution (5) to the radiation era modes

$$A^{\pm}_{\vec{k}}(\eta)=\frac{1}{\sqrt{2k}}e^{\mp ik\eta} \tag{6}$$

leads to the radiation era solution (cf. Ref. [17]). By matching Eq. (5) to a linear combination of these solutions, one finds

$$\mathcal{A}_0=\alpha_kA^+_{\vec{k}}|_{\eta=\eta_H}+\beta^*_kA^-_{\vec{k}}|_{\eta=\eta_H}, \tag{7}$$

$$\mathcal{A}'_0=\alpha_k(\partial_\eta A^+_{\vec{k}})|_{\eta=\eta_H}+\beta^*_k(\partial_\eta A^-_{\vec{k}})|_{\eta=\eta_H}, \tag{8}$$

where $\mathcal{A}_0\equiv A_{\vec{k}}|_{\eta=-\eta_H}$ and $\mathcal{A}'_0\equiv(\partial_\eta A_{\vec{k}})|_{\eta=-\eta_H}$. Solving for α_k,β^*_k gives

$$\alpha_k=\frac{\mathcal{A}_0+(i/k)\mathcal{A}'_0}{2A^+_{\vec{k}}|_{\eta=\eta_H}},\qquad \beta^*_k=\frac{\mathcal{A}_0-(i/k)\mathcal{A}'_0}{2A^-_{\vec{k}}|_{\eta=\eta_H}}, \tag{9}$$

such that the gauge field evolves in the radiation era as

$$A_{\vec{k}}(\eta)=\mathcal{A}_0\cos k(\eta-\eta_H)+\frac{\mathcal{A}'_0}{k}\sin k(\eta-\eta_H)(\eta>\eta_H). \tag{10}$$

For modes that are superhorizon at the end of inflation ($k\ll H$) one can use the small-argument expansion of the Hankel function $H_\nu^{(1)}(k/H)$ to get (for $\nu>0$)

$$\mathcal{A}_0=-i\frac{\Gamma(\nu)}{\sqrt{2\pi}}(2H)^{\nu-1/2}k^{-\nu}+O(k^\nu,k^{-\nu+1}), \tag{11}$$

$$\mathcal{A}'_0=\left(\nu-\frac{1}{2}\right)H\mathcal{A}_0, \tag{12}$$

This result is used below to calculate the magnetic field spectrum in the radiation era. Since matter era is a conformal space-time, the fields (as well as their spectra) in the matter era are inherited from the radiation era fields (10).

IV. LARGE CONDUCTIVITY

An opposite extreme is a sudden increase of conductivity in radiation era, which occurs, for example, in the case of a rapid thermalization after inflation, resulting in a large (thermal) conductivity. To leading logarithm (squared) in the coupling constant, the evolution of the photon field is then governed by the Bödeker-Langevin equation [19]

$$(a\sigma\partial_\eta+k^2)\varepsilon^{\mathcal{T}}_{\vec{k}}(\eta)=a^3\xi^{\mathcal{T}}(\vec{k},\eta), \tag{13}$$

where $\varepsilon^{\mathcal{T}}_{\vec{k}}$ is defined by $\vec{\varepsilon}_{\mathsf{T},\vec{k}}=\Sigma_{\mathcal{T}}\varepsilon^{\mathcal{T}}_{\vec{k}}\vec{\epsilon}^{\,\mathcal{T}}_{\vec{k}}$. The stochastic force $\xi^{\mathcal{T}}$ satisfies the following Markowian fluctuation-dissipation relation:

$$\langle\xi^{\mathcal{T}}(\vec{k},\eta)\xi^{\mathcal{T}'*}(\vec{k}',\eta')\rangle=\frac{2\sigma T}{a^4}(2\pi)^3\delta^{\mathcal{T}\mathcal{T}'}\delta(\eta-\eta')\times\delta^{(3)}(\vec{k}-\vec{k}'), \tag{14}$$

where T denotes the equivalent temperature (average energy of the plasma excitations). The factor a^3, which multiplies $\xi^{\mathcal{T}}$ in Eq. (13), can be inferred from the fact that the transverse components of the current density are given by $j^{\mathcal{T}}=\sigma E^{\mathcal{T}}+\xi^{\mathcal{T}}$, and from the transformation properties of the conductivity and the electric field. Equation (13) can be easily integrated,

$$\varepsilon^{\mathcal{T}}_{\vec{k}}(\eta)=\int_{\eta_H}^{\eta}\exp\left(-\int_{\eta'}^{\eta}d\eta'' k^2/(a\sigma)\right)\frac{a^2\xi^{\mathcal{T}}}{\sigma}d\eta' + \exp\left(-\int_{\eta_H}^{\eta}\frac{k^2}{a\sigma}d\eta'\right)\varepsilon^{\mathcal{T}}_{\vec{k}}(\eta_H). \tag{15}$$

Using this and Eq. (14) one can derive the equal time correlator

$$\begin{aligned}\langle\varepsilon^{\mathcal{T}}_{\vec{k}}(\eta)\varepsilon^{\mathcal{T}'*}_{\vec{k}'}(\eta)\rangle &= \int_{\eta_H}^{\eta}d\eta'\ \exp\left(-2\int_{\eta'}^{\eta}d\eta''\frac{k^2}{a\sigma}\right)\frac{2T}{\sigma}(2\pi)^3 \\ &\quad\times\delta^{\mathcal{T}\mathcal{T}'}\delta^{(3)}(\vec{k}-\vec{k}') \\ &\quad+\exp\left(-\int_{\eta_H}^{\eta}d\eta'\frac{k^2+k'^2}{a\sigma}\right)\varepsilon^{\mathcal{T}}_{\vec{k}}(\eta_H)\varepsilon^{\mathcal{T}'*}_{\vec{k}'}(\eta_H).\end{aligned} \tag{16}$$

Assuming $\sigma, T\propto 1/a$ (which is justified in and close to thermal equilibrium), one can perform the integrations to obtain

$$\begin{aligned}\langle\varepsilon^{\mathcal{T}}_{\vec{k}}(\eta)\varepsilon^{\mathcal{T}'*}_{\vec{k}'}(\eta)\rangle &= \frac{aT}{k^2}[1-\mathrm{e}^{-(2k^2/a\sigma)\Delta\eta}](2\pi)^3\delta^{\mathcal{T}\mathcal{T}'}\delta^{(3)}(\vec{k}-\vec{k}') \\ &\quad+\exp\left(-\frac{k^2+k'^2}{a\sigma}\Delta\eta\right)\varepsilon^{\mathcal{T}}_{\vec{k}}(\eta_H)\varepsilon^{\mathcal{T}'*}_{\vec{k}'}(\eta_H),\end{aligned} \tag{17}$$

where $\Delta\eta=\eta-\eta_H$, such that the spectrum is neatly split into the thermal and primoridial contribution.

V. MAGNETIC FIELD SPECTRUM

When suitably averaged over a physical scale $\ell_{\mathrm{ph}}=a\ell$, the magnetic field operator can be defined as

$$\vec{B}_{\ell}(\eta,\vec{x})\equiv\int d^3y W(\vec{x}-\vec{y},\ell)\vec{B}(\eta,\vec{y}), \tag{18}$$

where W denotes a coordinate space window function, which, e.g., can be chosen as $\mathtt{W}=[2\pi\ell^2]^{-3/2}\times\exp[-\|\vec{x}-\vec{y}\|^2/(2\ell^2)]$, and

$$\vec{B}(\eta,\vec{y})=\frac{1}{a^2}\int\frac{d^3k}{(2\pi)^3}i\vec{k}\times\sum_{\mathcal{T}}[A_{\vec{k}}(\eta)\vec{\epsilon}^{\mathcal{T}}_{\vec{k}}e^{i\vec{k}\cdot\vec{y}}\hat{a}^{\mathcal{T}}_{\vec{K}}-\mathrm{H.c.}]. \tag{19}$$

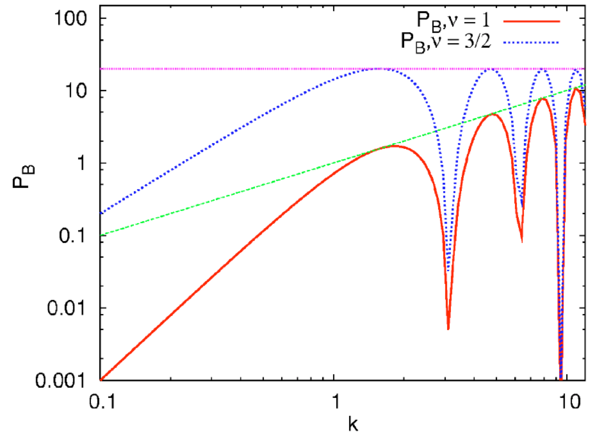

FIG. 1. Magnetic field spectrum in radiation era (21) for vanishing conductivity on a log-log scale for $\nu=1$ and $\nu=3/2$. (Normalization is arbitrary.)

The creation and annihilation operators $\hat{a}^{\mathcal{T}\dagger}_{\vec{K}}$ and $\hat{a}^{\mathcal{T}}_{\vec{K}}$ obey the commutation relation, $[\hat{a}^{\mathcal{T}}_{\vec{K}},\hat{a}^{\mathcal{T}'\dagger}_{\vec{k}'}]=(2\pi)^3\delta^{\mathcal{T}\mathcal{T}'}\delta^{(3)}(\vec{k}-\vec{k}')$, and $\hat{a}^{\mathcal{T}}_{\vec{K}}|0\rangle=0$. When (18) is squared and averaged over the vacuum state, one arrives at

$$\langle\vec{B}^2_{\ell}(\eta,\vec{x})\rangle\equiv\int\frac{dk}{k}|\mathcal{W}|^2\mathcal{P}_B,\quad \mathcal{P}_B=\frac{1}{a^4}\frac{k^5}{\pi^2}|A_{\vec{K}}(\eta)|^2, \tag{20}$$

where $\mathcal{P}_B=\mathcal{P}_B(\eta,k)$ defines the *magnetic field spectrum*, and $\mathcal{W}(\ell,k)=\int d^3z\mathtt{W}(\vec{z},\eta)\exp(i\vec{k}\cdot\vec{z})$ is the momentum space window function $[\mathcal{W}=\exp(-k^2\ell^2/2)$ for the coordinate space window function $\mathtt{W}$ mentioned above].

When conductivity is vanishingly small, the spectrum in radiation era can be calculated by inserting Eqs. (10)–(12) into (20). The result is (when $a\gg 1$)

$$\mathcal{P}_B\simeq\frac{1}{a^4}\frac{\Gamma^2(\nu)}{(2\pi)^3}\frac{(2H)^{2\nu+1}}{k^{2\nu-3}}\left(\nu-\frac{1}{2}\right)^2\sin^2[k(\eta-\eta_H)], \tag{21}$$

such that on superhorizon scales $k\eta\ll 1$ the spectrum (21) $\mathcal{P}_B\propto k^{5-2\nu}$, and on subhorizon scales the spectrum is oscillatory, with the envelope scaling as $\mathcal{P}_B\propto k^{3-2\nu}$. This spectrum is shown in Fig. 1 on a log-log plot. Note that scale invariance is reached for $\nu_{\mathrm{flat}}=5/2$ (superhorizon scales) $\nu_{\mathrm{flat}}=3/2$ (subhorizon scales).

In the limit of a large conductivity σ, the magnetic field spectrum can be split into the contribution from thermal excitations and the primordial contribution. A new scale arises in Eq. (17), associated with the momentum

$$k_{\sigma}=\sqrt{\sigma H/2}, \tag{22}$$

which we refer to as the *conductivity scale*. The thermal spectrum can be inferred from (17): In the ultraviolet $P^{\mathrm{th}}_B\propto k^3$ $(k\gg k_{\sigma})$, while in the infrared $P^{\mathrm{th}}_B\propto k^5$ $(k\ll k_{\sigma})$, imply-

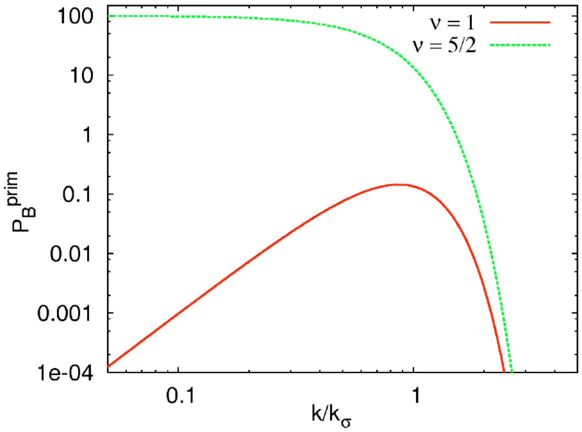

FIG. 2. Magnetic field spectrum in radiation (23) when a large conductivity sets in rapidly after inflation on a log-log scale for $\nu=1$ and $\nu=5/2$. (Normalization is arbitrary.)

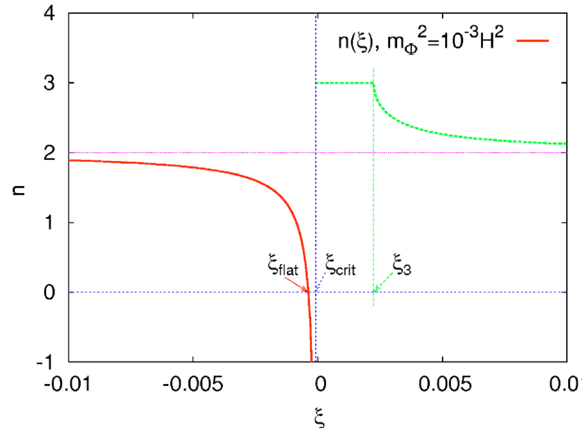

FIG. 3. Spectral index n for subhorizon modes in the low conductivity case [cf. Eq. (21)] as a function of the scalar coupling to gravity ξ (with $\alpha=1/137$ and the scalar mass $m_\Phi^2=10^{-3}H^2$).

ing that thermal excitations give rise to fields of negligible strength on cosmological scales (see Ref. [17]).

The primordial contribution to the spectrum can be obtained from Eqs. (11), (17), and (20),

$$\mathcal{P}_B^{\rm prim}\simeq\frac{1}{a^4}\frac{\Gamma^2(\nu)}{2\pi^3}\frac{(2H)^{2\nu-1}}{k^{2\nu-5}}\exp\left(-\frac{2k^2}{H\sigma}(1-a^{-1})\right), \tag{23}$$

such that the spectrum is exponentially cut off when $k>k_\sigma$. The flat spectrum is reached for $\nu_{\rm flat}=5/2$ for $k<k_\sigma$. This spectrum is shown in Fig. 2 on a log-log plot. Since typically in radiation era $\sigma\sim T\gg H/a$ $(a\gg1)$, $k_{\sigma,{\rm ph}}\equiv k_\sigma/a\gg H(t)=H/a^2$, implying that a rapid growth in conductivity after inflation freezes out large scales magnetic fields and destroys the small scales fields (electric field spectrum is completely destroyed), such that no oscillations are present on subhorizon scales. Thus, the absence or presence of subhorizon oscillations, and observation of a conductivity scale, can be testing grounds for the conductivity history during radiation era.

VI. DISCUSSION

The spectrum (21) can be thought of as a function of the scalar coupling to gravity ξ and its mass m_Φ, which are fundamental parameters characterizing a scalar field. To illustrate this point more precisely, we show in Fig. 3 the spectral index $n=3-2\nu$ characterizing the spectrum envelope of $\mathcal{P}_B\propto k^n$ in Eq. (21) on subhorizon scales, $k/a>H(t)$ [or $B_\ell\equiv\langle\vec{B}_\ell^2\rangle^{1/2}\propto\ell^{-n/2}$ $(n\neq0)$, $B_\ell\propto-\ln(\ell)$ $(n=0)$ in Eq. (20)]. Between $\xi=\xi_{\rm crit}\equiv-m_\Phi^2/(12H^2)$ and $\xi=\xi_3\equiv(\alpha/\pi)-m_\Phi^2/(12H^2)$, $n=3$, equal to that of thermal spectrum. Above $\xi>\xi_3$, n drops, reaching an asymptotic value $n\to n_\infty=2$ when $\xi\to\infty$. The spectral index on superhorizon scales is obtained simply by replacing $n+2\to n=5-2\nu$. When compared with the vacuum spectrum, $\mathcal{P}_B^{\rm vac}\propto k^4$ $(k<H)$, a spectrum with $n\in(2,3)$ exhibits an enhancement of magnetic fields on subhorizon scales, which is due to a conversion of the electric energy (which is enhanced during inflation) into the magnetic energy during radiation era. The conversion is efficient provided the radiation era is characterized by a low conductivity. This mechanism was used in Refs. [12–18] to argue that a spectrum $B_\ell\propto\ell^{-1}$ can be obtained from inflation, which could be sufficient to seed the galactic dynamo mechanism.

When $\xi\in(-\infty,\xi_{\rm crit})$, however, gauge fields exhibit instability and are enhanced during inflation, such that the spectral index of subhorizon modes drops from $n=2$ when $\xi\to-\infty$, to flat spectrum $(n=0)$, when

$$\xi_{\rm flat}=-\frac{\alpha}{8\pi}-\frac{m_\Phi^2}{12H^2}, \tag{24}$$

to $n<0$, when $\xi\in(\xi_{\rm flat},\xi_{\rm crit})$. A negative spectral index $n\leqslant-2$ would imply a growth in magnetic field energy during inflation, resulting in a divergent magnetic (and electric) field energy, as it can be inferred from Eqs. (21), (23), and (20), with $\mathcal{W}=1$. A proper study of this case would require an inclusion of backreaction of the electromagnetic field on the background space-time, which is beyond the scope of this article. A spectral index $n>-2$ on subhorizon scales (or $n>0$ on superhorizon scales), results in an acceptable magnetic field spectrum on cosmological scales, whose dynamical impact on CMBR and large scale structure formation should be considered.

Spectrum normalization today can be determined from magnetic field strength at a co-moving scale corresponding to the Hubble scale at the end of inflation $(k=H)$. At the co-moving inflation scale $\ell_H\sim3$ m$\sim10^{-16}$ parsec (corresponding to $H\sim10^{37}$ Hz$\sim10^{13}$ GeV) the field strength is about $B_H\sim10^{-12}-10^{-11}$ gauss [20]. Therefore, if the spectrum is (almost) flat, the magnetic field is potentially observable by the next generation of CMBR experiments (PLANCK satellite), as well as of importance for the dynamics of large scale structures of the Universe.

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