Phantom cosmologies

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The dynamics of a minimally coupled scalar field in the expanding universe is discussed with special reference to phantom cosmology. The evolution of the universe with a phantom field *vis-à-vis* a quintessence field is compared. Phantom cosmologies are found to have two special features: (i) occurrence of a singularity where the scale factor, the energy density, and Ricci curvature scalar diverge to infinity (this singularity occurs at a finite time, depending on the value of w during cosmic evolution), and (ii) degeneracy in the determination of $w(z_m)$ for a given transition redshift z_m which seems to impart similar observational properties to corresponding phantom and quintessence models and makes both of them compatible with the cosmological observations. Although due to the uncertainties in the measurement of the Hubble constant H_0, the Hubble dependent observational parameters yield only loose constraints over the range of w, the duality in the determination of w with respect to transition redshift may be used to constrain w. An observational test, based upon the observations of low redshift galactic clusters, is suggested to discriminate between the quintessence and phantom dark energy.

I. INTRODUCTION

The combined analysis of SNe Ia observations [1,2], galaxy cluster measurements [3] and the latest cosmic microwave background (CMB) data [4] provides compelling evidence for the existence of dark energy which dominates the present day Universe and accelerates the cosmic expansion. The recent detection of integrated Sachs-Wolfe effect [5] also gives a strong and independent support to dark energy. In principle, any physical field with positive energy density and negative pressure, which violates the strong energy condition, may cause the dark energy effect of repulsive gravitation. Of late, phantom fields [6] have emerged as potential candidates for dark energy. Scalar fields with a supernegative equation of state ($p = w\rho, w < -1$) are called phantom fields as their energy density increases with the expansion of the universe in contrast to quintessence energy density ($w > -1$) which scales down with the cosmic expansion. The phantom models violate the dominant energy condition ($p + \rho) < 0$ as such they may not be physically stable models of dark energy; but, strangely enough, phantom energy is found to be compatible with most of the classical tests of cosmology [6] based on current data from SNe Ia observations, CMB anisotropy, and the mass power spectrum.

The peculiar nature of phantom energy, violation of the dominant energy condition and its strange consequences, possible ripoff of the large and small scale structures of matter, occurrence of future singularity, and probable decay of phantom energy have attracted many cosmologists [7–27], and made "phantom cosmologies" a hot topic of research.

In Sec. II, we discuss the dynamics of minimally coupled scalar fields with special reference to phantom fields. There is extensive literature in cosmology on rolling scalar fields [22,23], quintessence fields and tracker fields [24,28–45] the

cosmological constant Λ [28,46–50] and other forms of dark energy. The major problem in cosmology is to identify the form of dark energy that dominates the universe today whether it is phantom energy, quintessence, simply Λ, or something else. Maor, Brustein, Macmahon, and Steinhardt [51] have discussed the degeneracy in the measurement of the dark energy parameter w from SNe Ia data, its time variation, and pitfalls in taking w to be a constant.

In Sec. III, we have computed the present age t_0 of the Universe in two steps. First we calculate the expansion age up to the end of the matter-dominated era, denoted by t_m. In order to supplement it with the expansion age during the dark energy dominated era, we express t_0 in terms of t_m and thereby we calculate t_0. The advantage of this method is that the expansion age in the two segments can be expressed separately in terms of the redshift z_m at the end of the matter-dominated era, which is again a function of the parameter w.

In Sec. IV, assuming w to be constant, we have discussed a kind of degeneracy in the value of $w(z_m)$ which leads to duality in the behavior of phantom and quintessence models with respect to transition redshift from deceleration to the accelerating phase of expansion. In fact two distinct values of parameter w, usually one lying in the range of the quintessence field and another in the range of the phantom field, lead to the same transition redshift z_m.

In Sec. V, we have tried to constrain the range of the dark energy parameter w on the basis of data analysis of Kiselev [52], Freedman and Turner [53], Schubnell [54], and the precise observational data from the Wilkinson Microwave Anisotropy Probe (WMAP) [55] in combination with the Sloan Digital Sky Survey (SDSS) [56]. In Sec. VI, we conclude with some remarks on phantom energy.

II. DYNAMICS OF PHANTOM COSMOLOGY

Consider a 2-component cosmic fluid in a Friedmann universe comprising (i) pressure-free matter of energy density ρ_m and (ii) a minimally coupled scalar field of energy density ρ_x and equation of state $p_x = w\rho_x$ which contributes to dark

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energy in the Universe. The energy densities $\rho_m \sim a^{-3}$ and $\rho_x \sim a^{-3(1+w)}$ evolve independently in the expanding universe.

The dark energy might be due to (a) a quintessence field if $-1 < w < -\frac{1}{3}$, (b) the cosmological constant Λ if $w = -1$, or (c) a phantom field if $w < -1$. In the above classification, the equation of state parameter w plays the role of dark energy parameter.

The Friedmann equations are

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}[\rho_m + \rho_x] = H_0^2[\Omega_m^0(a_0/a)^3 + \Omega_x^0(a_0/a)^{3(1+w)}] \tag{1}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[\rho_m + \rho_x(1+3w)] = -\frac{4\pi G}{3}\rho_x[\Omega_x^{-1} + 3w]$$

$$= -\frac{4\pi G}{3}\rho_x\left[\frac{\Omega_m^0}{\Omega_x^0}\left(\frac{a_0}{a}\right)^{-3w} + 1 + 3w\right]. \tag{2}$$

The cosmic expansion decelerates as long as

$$\Omega_x^{-1} + 3w > 0. \tag{3}$$

With the growth of the phantom energy density parameter, Ω_x^{-1} goes on decreasing with time until the transition to accelerating phase takes place at cosmic time $t = t_m$. The transition epoch t_m corresponds to the redshift z_m given by

$$1 + z_m = \left[-(3w+1)\Omega_x^0/\Omega_m^0\right]^{-1/3w}. \tag{4}$$

It may be emphasized that the transition epoch t_m marks the end of the "effective matter dominated" era or the beginning of the accelerating phase in the cosmic expansion after which the large scale structure formation in the universe must cease although Ω_m may still be greater than Ω_x. It is evident from Eq. (4) that the transition epoch t_m depends upon the choice of w. Figure 3 shows the variation of z_m with w.

As Ω_x^{-1} decreases further after the transition epoch

$$\Omega_x^{-1} + 3w < 0 \tag{5}$$

and the Hubble expansion in the accelerating phase of the Universe is given by Eq. (1),

$$\frac{\dot{a}}{a} = H_0\sqrt{\Omega_x^0}\left(\frac{a_0}{a}\right)^{3(1+w)/2}\left[1 + \frac{\Omega_m^0}{\Omega_x^0}\left(\frac{a}{a_0}\right)^{3w}\right]^{1/2}. \tag{6}$$

Expanding binomially and integrating, we get

$$\chi t + c = \left(\frac{a}{a_0}\right)^{3(1+w)/2}\left[\frac{2}{3(1+w)}\right.$$

$$\left. - \frac{\Omega_m^0(a)^{3w}}{\Omega_x^0(a_0)^{3w}(9w+3)} + \cdots\right], \tag{7}$$
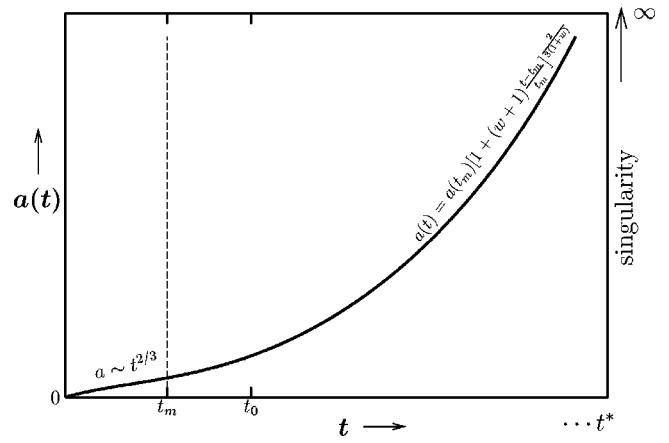

FIG. 1. Expansion of the universe with matter and phantom fields: up to the end of the matter dominated phase t_m, the universe undergoes Einstein–de Sitter expansion with deceleration. For $t > t_m$ the cosmic expansion accelerates. In the case of phantom fields ($w < -1$), the scale factor diverges to infinity at finite time $t = t^*$.

where $\chi = H_0\sqrt{\Omega_x^0}$. The binomial expansion of the right hand side of Eq. (6) is valid under the condition $(\Omega_m^0/\Omega_x^0)(a/a_0)^{3w} < 1 < -(3w+1)$ which holds during the accelerating phase over the range (i) $a < a_0$, $w < 0$, and $\Omega_m^0 < \Omega_x^0$ and (ii) $a < a_0$, $w < 0$, and $(1+z)^{-3w} < -[(3w+1)\Omega_x^0]/\Omega_m^0$. This ensures that the accelerated expansion during the regime $\Omega_m > \Omega_x$ continues as long as $-(3w+1)\Omega_x^0 > \Omega_m^0$ (see Fig. 1).

Since $w < 0$, the successive terms on the right hand side of Eq. (7) decrease by $O(3w)$ of magnitude and the scale factor is effectively given by

$$a^{3(1+w)/2}(t) = 3(1+w)/2 \, \chi t + c. \tag{8}$$

It shows that the contribution of the matter density is almost negligible during the phantom dominated universe since the contribution of Ω_m^0 falls down steeply by 3 orders of magnitude or more with each successive term in Eq. (7). According to Eq. (1) also, the cosmic expansion in the accelerating phase is essentially driven by the phantom field since its energy density scales up as $\rho_x \sim a^{-3(1+w)}$ whereas ρ_m scales down as $\sim a^{-3}$.

During the deceleration phase, the Hubble expansion is given by

$$\frac{\dot{a}}{a} = H_0\sqrt{\Omega_m^0}\left(\frac{a_0}{a}\right)^{3/2}\left[1 + \frac{\Omega_x^0}{\Omega_m^0}\left(\frac{a_0}{a}\right)^{3w}\right]^{1/2}. \tag{9}$$

The deceleration condition (3) implies that $(\Omega_x^0/\Omega_m^0)(a_0/a)^{3w} < -1/(3w+1) < 1$ since $w < -1$. Therefore expanding Eq. (9) binomially and integrating, we get

$$\xi t = a^{3/2}\left[\frac{2}{3} - \frac{(1+z)^{3w}\Omega_x^0}{3(1-2w)\Omega_m^0} + \frac{(1+z)^{6w}}{4(1-4w)}\left(\frac{\Omega_x^0}{\Omega_m^0}\right)^2 + \cdots\right]. \tag{10}$$

Since $w < 0$, the first term on the righ- hand side of Eq. (10) dominates while the remaining terms decrease for high redshifts. Therefore during the matter dominated era, the scale factor is given by

$$a^{3/2}(t) = \tfrac{3}{2} \xi t. \tag{11}$$

Equation (11) holds at the epoch $t = t_m$, and as such

$$a^{3/2}(t_m) = \tfrac{3}{2} \xi t_m. \tag{12}$$

At the beginning of the dark energy dominated phase, Eq. (8) gives

$$a^{3(1+w)/2}(t_m) = \tfrac{3}{2} \chi t_m + c. \tag{13}$$

Matching the junction conditions at $t = t_m$, Eqs. (8), (12), and (13) yield

$$\left[\frac{a(t)}{a(t_m)} \right]^{3(1+w)/2} = 1 + \frac{3/2(w+1)\chi(t-t_m)}{a^{3(1+w)/2}(t_m)}$$

$$= 1 + (w+1)\frac{t-t_m}{t_m}. \tag{14}$$

Therefore the scale factor in the phantom (dark energy) dominated universe is given by

$$a(t) = \frac{a(t_m)}{[-w + (1+w)t/t_m]^{-2/[3(1+w)]}} \quad \text{for} \ \ t > t_m. \tag{15}$$

Since $1 + w < 0$ for phantom fields, $a(t)$ diverges to infinity at $t^* = [w/(1+w)]t_m$. Prior to blow over time t^*, $H > 0$ and the deceleration parameter $q = -1 + 3(1+w)/2$ in contrast to $q_0 = 1/2 + 3/2w\Omega_x$ at the present epoch. On the contrary in the quintessence dominated universe $(1+w > 0)$, the cosmic expansion is singularity-free with the scale factor $a(t) = a(t_m)[1 + (1+w)(t-t_m)/t_m]^{2/[3(1+w)]}$.

Using Eq. (15), the energy density of the phantom universe $(t > t_m)$ is given by

$$\rho_x(t) = \frac{\rho(t_m)}{[-w + (1+w)t/t_m]^2}. \tag{16}$$

Accordingly ρ_x goes on increasing gradually, followed by a steep rise to an infinite value at $t^* = [w/(1+w)]t_m$. Therefore the phantom models have a finite lifetime ending in a singularity. On the other hand, ρ_x scales down with time in the quintessence universe whereas ρ_Λ remains stationary. For $t < t_m$, the Hubble expansion is dominated by matter density; accordingly $a \sim t^{2/3}$, $\rho_m \sim t^{-2}$ but ρ_x varies independently as $t^{-2(1+w)}$ as shown in Fig. 2.

The expansion and density singularity in the phantom universe correspond to the curvature singularity as the Ricci scalar R_i^i also tends to infinity at this epoch. The total age t^* of the phantom universe depends upon the choice of w and is larger for values of w closer to -1. For example, $t^* = 21 t_m$ for $w = -1.05$ whereas $t^* = 6 t_m$ for $w = -1.2$.
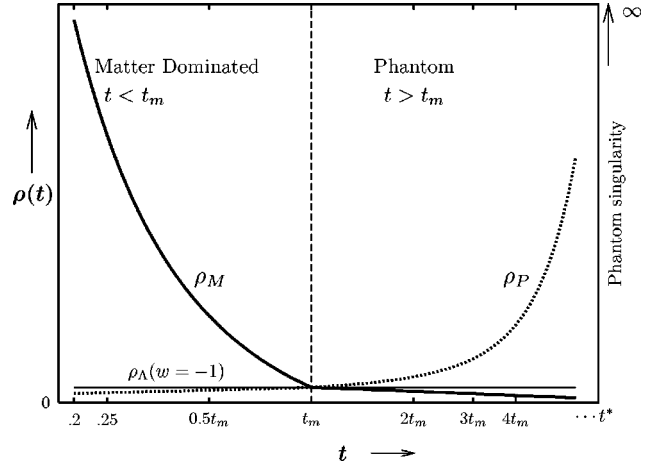

FIG. 2. Variation of the energy density in a phantom universe: variation of the matter density is shown by the thick solid line, variation of the vacuum energy density ρ_Λ is shown by the thin line, and the variation of the phantom energy density by the dotted line.

III. PRESENT AGE OF THE UNIVERSE

The present age t_0 of the Universe depends on the cosmological density parameters and the equation of state parameter w which determines the redshift z_m at the transition epoch t_m (at the end of matter-dominated phase). We calculate t_0 in two steps. During the matter-dominated phase, the Hubble expansion is given by Eq. (9). Integrating the first term in the binomial expansion of Eq. (9) over the redshift range from infinity to z_m, we calculate the age of the universe from the beginning to the end of matter-dominated era as given by the expression

$$t_m = (H_0 \sqrt{\Omega_m^0})^{-1} \left[\tfrac{2}{3} (1+z_m)^{-3/2} \right], \tag{17}$$

where $\Omega_m^0 = 0.27, \Omega_x^0 = 0.73$. Using Eq. (15), the present age t_0 is given by the equation

$$t_0 = \left[1 + \frac{(1+z_m)^{3(1+w)/2} - 1}{1+w} \right] \times t_m \tag{18}$$

Combining Eqs. (17) and (18), the present age of the Universe is calculated for a wide range of w as shown in Table I.

IV. DUALITY IN QUINTESSENCE AND PHANTOM BEHAVIOR

We have investigated the correlation between the transition redshift z_m (corresponding to the end of matter-dominated era) and the dark energy parameter w and found a sort of duality in the behavior of quintessence fields (Q) and phantom fields (P) (see Fig. 3). For every value of z_m in Fig. 3, the corresponding parameter w has, in general, two values, one lying in the range of Q fields and the other in the range of P fields, both leading to cosmological parameters (like the present age t_0 and the deceleration parameter q_0), which seem to be compatible with the observational data. For example, if the transition from the decelerating phase to accelerating phase occurs at $z_m = 0.739$, the corresponding dark

TABLE I. Age of the universe versus dark energy parameter w. In the table $H_0^{-1} = 13.65$ Gyr.

w	z_m	$t_m(H_0^{-1})$	$t_0 \ (H_0^{-1})$	t_0 (Gyr)
-0.66	0.644	0.602	1.112	15.18
-0.70	0.678	0.583	1.090	14.87
-0.75	0.707	0.568	1.060	14.46
-0.80	0.739	0.554	1.053	14.37
-0.85	0.752	0.547	1.037	14.15
-0.90	0.757	0.545	1.024	13.97
-0.93	0.758	0.545	1.020	13.92
-0.95	0.756	0.546	1.016	13.88
-1.00	0.755	0.549	1.012	13.81
-1.02	0.749	0.548	1.004	13.70
-1.05	0.745	0.550	1.001	13.66
-1.10	0.739	0.554	0.995	13.58
-1.15	0.726	0.559	0.991	13.53
-1.18	0.721	0.562	0.987	13.48
-1.20	0.719	0.563	0.985	13.45
-1.35	0.683	0.583	0.979	13.36
-1.50	0.647	0.601	0.976	13.32

energy parameter may be either $w = -0.8$ (quintessence field) or $w = -1.1$ (phantom field), which yield 14.37 Gyr and 13.58 Gyr, respectively, for the present age of the Universe. Hence both the Q model and the P model seem to be compatible with the observational value [43] of t_0. This duality poses a question whether the phantom fields really exist or they are merely ghost fields which replicate the quintessencelike behavior for a supernegative equation of state, violating the dominant energy condition. It might explain the concordance of SNIa and galaxy cluster abundance observations in the extended w-Ω_m parameter space for $w < -1$ (phantom models) as shown by Caldwell et al. [7].

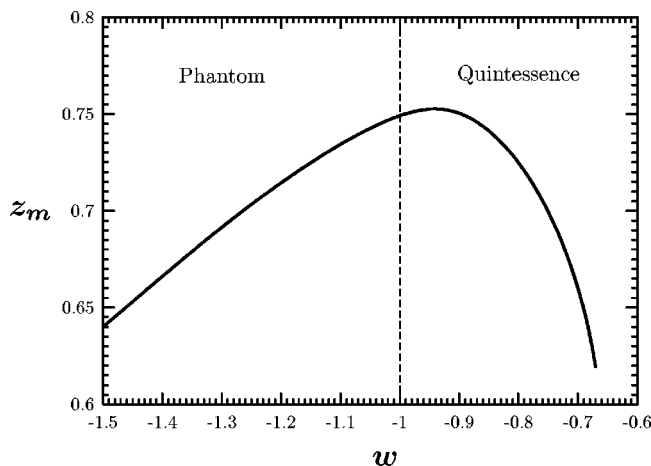The above-mentioned duality is essentially different from the form-invariance symmetry between standard cosmology

FIG. 3. Duality in the behavior of the phantom and the quintessence field is shown with respect to any chosen value of the transition redshift z_m. The peak value of z_m lies in the quintessence field; the nearby w in this field also show degeneracy with respect to z_m.

and phantom cosmology pointed out independently by Dabrowski et al. [24] and Chimento and Lazkoz [57]. They have shown that in the case of a single component cosmological model, the scale factors of the standard and phantom models for a given energy density have a reciprocal relationship with the equation of state parameter $w_{ph} = -w - 2 (w > -1)$. This formalism seems inadequate to describe the evolution of the scale factor in dark energy models in conjunction with pressure-free matter.

V. CONSTRAINTS OVER PHANTOM COSMOLOGIES

One of the greatest challenges in cosmology is to understand the nature of the dark energy. Dark energy models are characterized by two parameters Ω_x and w. From the analysis of WMAP data [55], $\Omega_x = 0.73 \pm 0.04$ is known up to high precision but $w < -0.8$ [95% confidence limit (CL)] leaves the field open to speculation whether the dark energy is phantom energy or quintessence energy. According to the analysis of Melchiorri et al. [13], $-1.38 < w < -0.82$. We have examined the possibility of constraining the range of w by comparison of the theoretically calculated age of the Universe in Table I with the age derived from the observational data from WMAP, SDSS, and data analysis of Tegmark et al. [56]. By assuming $H_0 = 71^{+4.0}_{-3.0}$ and $t_0 = 13.7 \pm 0.2$ Gyr from WMAP data, we can find a narrow range $-1.18 < w < -0.93$ for the dark energy parameter and the corresponding range $-0.8 < q_0 < -0.52$ for the deceleration parameter (consistent with Kisilev's analysis [52]), but it would be more realistic to allow for errors in the measurement of Hubble constant and take $H_0 = 72 \pm 7.0$ and $t_0 = 13.0 \pm 1.5$ Gyr [53] which yields a wider range $-1.5 < w < -0.75$ for the variation of dark energy parameter. It seems to be reasonably compatible with the recent estimates for w.

However, the degeneracy in the determination of $w(z_m)$ for a chosen transition redshift may be used to constrain w and discriminate between the quintessence and phantom dark energy. Since the formation of the galactic clusters ceases with the end of the matter-dominated era at redshift z_m, the lowest redshift observations of the galactic clusters can indicate the most probable value of the redshift at which large scale structure formation in the Universe would stop. In general there might be two values of w corresponding to this particular value of z_m of which the one satisfying the WMAP range for the present age of the Universe ($t_0 = 13.7 \pm 0.2$) may be taken as the correct value of w to determine the genuine candidate for dark energy filling the Universe.

VI. CONCLUDING REMARKS

The combined analysis of the latest cosmological observations provides a definite clue of the existence of dark energy in the Universe but it is difficult to distinguish between the various forms of dark energy at present. As far as phantom energy is concerned, it is found to be compatible with SNe Ia observations and CMB anisotropy measurements, but the violation of the "dominant energy condition" makes phantom models physically unstable. However, phantom models may be considered to be phenomenologically viable

provided their age happens to be less than the time scale of the singularity. In case the instability occurs earlier and the dark energy decays into gravitons, as discussed by Carroll *et al.* [14], the Universe might escape the ordeal of "rip-off" [7] and phantom singularity. With the large number of SNe Ia observations expected from SNAP, LOSS, and other surveys in the coming years, a clear picture of the dark energy profile is likely to emerge that would reveal whether ΛCDM, QCDM, or PCDM is the correct cosmology of the Universe.

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