Measurement of $K^+ \to \pi^0 \pi^0 e^+ \nu$ (K_{e4}^{00}) decay using stopped positive kaons

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The $K^+ \to \pi^0 \pi^0 e^+ \nu$ (K_{e4}^{00}) decay has been measured with stopped positive kaons for a data sample of 216 events. A comparison of the observed spectra with a Monte Carlo simulation determined the K_{e4}^{00} form factor. The results are compatible with the $K^+ \to \pi^+ \pi^- e^+ \nu$ data, as estimated from the $\Delta I = 1/2$ rule. We also established that the K_{e4}^{00} channel can be used to determine the π-π scattering lengths.

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$K \to \pi\pi e \nu$ (K_{e4}) decays are the subject of continuing interest for a variety of reasons [1–3]. The decay form factors serve as constraints on the parameters of the Lagrangians in the framework of chiral perturbation theory (ChPT). More importantly, they are clean sources of π-π pairs at low energy, from which one can deduce the scattering lengths of significance for models of hadron dynamics.

Recently, the BNL-E865 group reported on the precise measurement of the $K^+ \to \pi^+ \pi^- e^+ \nu$ (K_{e4}^{+-}) decay, yielding the decay form factors as well as the S-wave, isospin zero π-π scattering length a_0^0 [4]. In addition to the K_{e4}^{+-} decay, there are two other K_{e4} decays, $K^+ \to \pi^0 \pi^0 e^+ \nu$ (K_{e4}^{00}) and $K_L \to \pi^- \pi^0 e^+ \nu$ (K_{e4}^{-0}) [5]. Since these K_{e4} decays are theoretically related to each other by simple isospin arguments [6], comprehensive experimental studies for all channels provide deep insight into the understanding of K_{e4} physics. Among the three K_{e4} decays, the K_{e4}^{00} channel is the simplest because the decay kinematics can be described by only one form factor in view of the identity of the two π^0s in the final state. Thus far, only 37 K_{e4}^{00} events have been observed using an in-flight-K^+ technique [7,8]. A more accurate measurement of the K_{e4}^{00} channel was desirable as an additional independent check on the π-π scattering length and the $\Delta I = 1/2$ rule.

In this paper, we present a new measurement of K_{e4}^{00} decay. The experiment used a stopped K^+ beam in conjunction with a 12-sector iron-core superconducting spectrometer. We have been able to make a first attempt to deduce the q^2 dependent terms of the form factor and determine the π-π scattering length in a nearly model independent manner.

The experiment was performed at the KEK 12 GeV proton synchrotron. The experimental apparatus was based on the E246 experiment, a search for the T-violation in $K^+ \to \pi^0 \mu^+ \nu$ decay ($K_{\mu 3}$) [9,10]. Besides the T-violation search, spectroscopic studies for various decay modes have been successfully performed [11]. After the E246 data collection was completed in 2000, the E470 experiment to measure direct photon emission in the $K^+ \to \pi^+ \pi^0 \gamma$ ($K_{\pi 2\gamma}$) decay was carried out [12], and the K_{e4}^{00} events were simultaneously recorded.

A separated 660 MeV/c K^+ beam was stopped in an active target system located at the center of the spectrometer. The K_{e4}^{00} events were identified by analyzing the e^+ momentum with the spectrometer and detecting the four photons in the CsI(Tl) calorimeter. Charged particles from the target were momentum-analyzed using multiwire proportional chambers, as well as an array of ring scintillators. The e^+s were separated from μ^+s by determining the mass squared ($M_{\rm TOF}^2$) of the charged particles from the measured time-of-flight between TOF1 and TOF2 counters. Counter TOF1 surrounded the active target and counter TOF2 was located at the exit of the spectrometer. The photon energy and hit position were obtained, respectively, by summing the energy deposits and taking the energy-weighted centroid of the crystals sharing the shower.

K_{e4}^{00} decays at rest were extracted by the following procedure. The K^+ decay time, defined as the e^+ signal in the TOF1 counter, was required to be more than 1.4 ns later than the K^+ arrival time as measured by the Čerenkov counter. This reduced the fraction of K^+ decay in-flight contamination to the level of 10^{-3} of the stopped K^+s. Events with e^+ scattering on the magnet pole faces were eliminated by requiring the hit position in the ring counters to be consistent with the charged particle track. The selection of positrons required $-5000 < M_{\rm TOF}^2 < 5000$ MeV$^2/c^4$. Events with four photon clusters coming from two π^0s were selected, while

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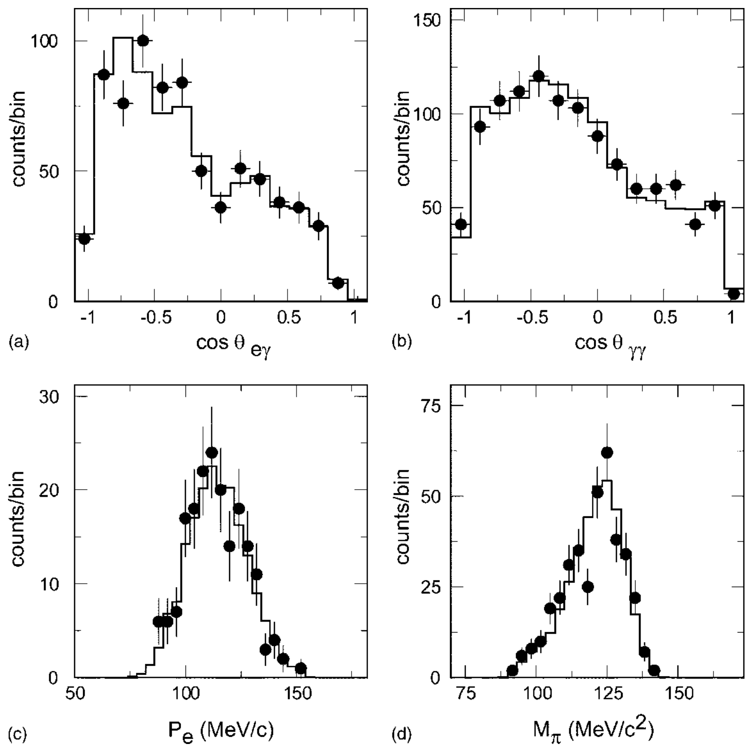

FIG. 1. K_{e4}^{00} spectra: (a) cos $\theta_{e^+\gamma}$, (b) cos $\theta_{\gamma\gamma}$, (c) e^+ momentum, (d) M_{π^0} for the experimental data (dots) and a Monte Carlo simulation of K_{e4}^{00} (histogram). Four and six combinations for cos $\theta_{e^+\gamma}$ and cos $\theta_{\gamma\gamma}$, respectively, were accumulated in the same figures. Both $M_{\pi_1^0}$ and $M_{\pi_2^0}$ values were also accumulated in (d).

events with other photon cluster number were rejected. Photon conversion backgrounds in the active target system could be removed by selecting only events in which there was one charged particle hit in TOF1 as well as in the ring counters.

Since there are three possible combinations to form two π^0s from the four photons, the K_{e4}^{00} events were reconstructed by introducing a quantity Q^2,

$$Q^2=(M_{\pi_1^0}-M_1)^2/\sigma_{M_1}^2+(M_{\pi_2^0}-M_2)^2/\sigma_{M_2}^2 +(\cos\theta_{\pi^0\pi^0}^{\text{meas}}-\cos\theta_{\pi^0\pi^0}^{\text{calc}}-\delta)^2/\sigma_\delta^2, \quad (1)$$

where M_{π^0} is the invariant mass of the selected pair (the first π^0 has higher energy) and $\theta_{\pi^0\pi^0}$ is the opening angle between the π^0s. The superscripts MEAS and CALC stand for the measured angle and the angle calculated from the measured photon energies, respectively. The photon pair with the minimum, Q_{min}^2, among the three combinations was adopted as the correct pairing. The σ and offset values in each term are $\sigma_{M_1}=12.74$ MeV/c^2, $\sigma_{M_2}=15.42$ MeV/c^2, $M_1=124.5$ MeV/c^2, $M_2=113.5$ MeV/c^2, $\delta=-0.019$, and $\sigma_\delta=0.336$. The choice of Eq. (1) and these parameters were determined to maximize the probability for the correct pairing using a GEANT based Monte Carlo code. Because of the shower leakage from the calorimeter holes, the σ and offset values differ from the ideal ones. The correct pairing probability was estimated to be 96% from the simulation.

Since backgrounds due to $K^+\to\pi^0e^+\nu$ (K_{e3}) and $K^+\to\pi^0e^+\nu\gamma$ ($K_{e3\gamma}$) decays with accidental photons, photons split into multiple clusters, and bremsstrahlung photons do not satisfy the K_{e4}^{00} kinematics, the cut of $Q_{\text{min}}^2<3$ essentially removed these backgrounds. The fraction of the accidental background in the calorimeter was estimated to be 1.0% by changing the CsI(Tl) TDC gate widths. Using the Monte Carlo simulation, background fractions in the K_{e4} sample due to the bremsstrahlung photons and photons split into multiple clusters were obtained to be 0.02% and 0.06%, respectively. The total level of the backgrounds was thus derived to be 1.1% which was dominated by $K_{e3\gamma}$ with accidental photons. The opening angle distributions between e^+ and four γs ($\theta_{e^+\gamma}$) and between each photon ($\theta\gamma\gamma$) for the selected K_{e4} events were compared to the simulation to confirm the correctness of the simulation conditions, as shown in Figs. 1(a) and 1(b). No deviation of $\theta_{e^+\gamma}$ and $\theta_{\gamma\gamma}$ from the simulation at small angles was observed, supporting the correct estimation of these background fractions. After these selection conditions, the number of good K_{e4}^{00} events was found to be 216. The e^+ momentum and π^0 invariant mass spectra of the selected K_{e4}^{00} events are shown in Figs. 1(c),1(d).

In general, the K_{e4} kinematics can be written as a set of five configuration variables [1,3]: (I) $s_\pi=M_{\pi\pi}^2$, the effective mass squared of the dipion system, (II) $s_l=M_{e\nu}^2$, the effective mass squared of the dilepton system, (III) θ_π, the angle made by the π^0 in the dipion center of mass with respect to the kaon direction, (IV) θ_l, the angle made by the e^+ in the dilepton center of mass with respect to the kaon direction, and (V) ϕ, the angle between the decay planes formed by the dipion and dilepton systems. Because of the identity of the two π^0s, θ_π and ϕ can be defined in the region of $0<\theta_\pi<\pi/2$ and $0<\phi<\pi/2$. The most general form of the K_{e4} matrix element in terms of the hadronic vector and axial vector current contributions V^μ and A^μ is given by [1]

$$M=(G_F/\sqrt{2})V_{us}\overline{u}(p_\nu)\gamma_\mu(1-\gamma_5)v(p_e)(V^\mu-A^\mu), \quad (2)$$

$$A^\mu=FP^\mu+GQ^\mu+RL^\mu, \quad (3)$$

$$V^\mu=H\epsilon^{\mu\nu\rho\sigma}L_\nu P_\rho Q_\sigma, \quad (4)$$

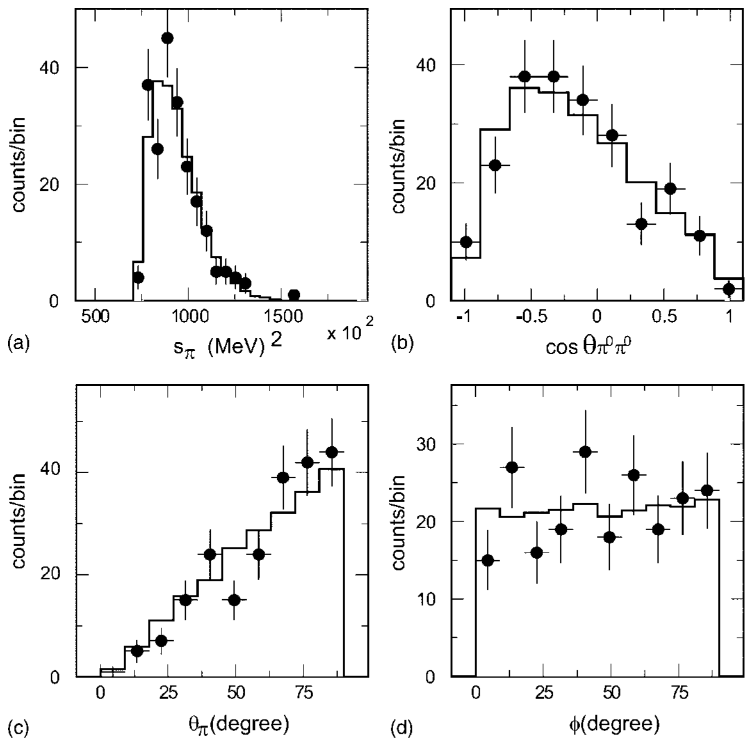

FIG. 2. K_{e4}^{00} spectra: (a) s_π, (b) cos $\theta_{\pi^0\pi^0}$, (c) θ_π, (d) ϕ, for the experimental data (dots) and the Monte Carlo simulation of K_{e4}^{00} with the best fitted f'/f_0 and f''/f_0 parameters (histograms).

where $P=p_1+p_2$, $Q=p_1-p_2$, $L=p_e+p_\nu$, and p_1, p_2, p_e, p_ν are the four-momenta of π_1, π_2, e^+, ν in units of m_K, respectively. The form factors *F*, *G*, *R*, and *H* are dimensionless functions of s_π. The *R* term enters the decay distribution multiplied by m_e^2 and is therefore negligible. In general, *S* and *P* waves in the dipion system were considered to contribute to the K_{e4} decays [4]. However, odd angular momenta π-π states cannot be present in the K_{e4}^{00} channel due to symmetry arguments. Therefore, only a contribution from the *S*-wave component is possible and the decay kinematics are described only by the *F* form factor [6]. Here, a parametrization [13]

$$F=f_0+f'q^2+f''q^4 \tag{5}$$

is employed, where q^2 is the momentum transfer squared $q^2=s_\pi/(4m_\pi^2)-1$ to the lepton current. The observed K_{e4}^{00} spectra (s_π, cos $\theta_{\pi^0\pi^0}$, θ_π, ϕ) which are sensitive to the K_{e4}^{00} form factor are shown in Fig. 2.

The f'/f_0 and f''/f_0 values could be extracted by fitting the observed spectra to the Monte Carlo simulation. The simulation data were generated according to Pais and Treiman [2] and analyzed in the same manner as the experimental sample. The simulation spectra were then obtained as a function of the form factor by reweighting the reconstructed simulation events using the generated Monte Carlo kinematic variables. A program based on MINUIT minimized χ^2 and estimated the error of the parameters. The reproducibility of the experimental conditions in the simulation was carefully checked using $K_{\pi2}$ and $K_{\pi3}$ decays.

In the case of the K_{e4}^{+-} decay, the π-π scattering length can be determined from the phase-shift difference between the *S* and *P* waves [4]. However, a similar procedure would not be of use for the K_{e4}^{00} decay. Instead, one can assume that the s_π dependence of *F* is described by an *S*-wave phase-shift parameter δ_0^0 [1]:

$$F\propto(1/\beta)\sin[\delta_0^0(s_\pi)]e^{i\delta_0^0(s_\pi)}, \tag{6}$$

where $\beta=(1-4m_\pi^2/s_\pi)^{1/2}$ and the s_π dependence of the phase shift is given by the Chew-Mandelstam effective-range formula [14]:

$$\cot\delta_0^0=1/(\beta a_0^0)+(2/\pi)\ln[\sqrt{s_\pi}/(2m_\pi)(1+\beta)]. \tag{7}$$

To determine the a_0^0 value, this *F* form factor was used to generate the K_{e4}^{00} events in the simulation. Since the f'/f_0 and f''/f_0 parameters are numerically determined from a_0^0 by fitting Eq. (6) to Eq. (5), it is possible to check the consistency of the theories by comparing the results of the (f'/f_0, f''/f_0) and a_0^0 analyses.

The f'/f_0 and f''/f_0 values were determined from the density distribution of the two observables, s_π and $\theta_{\pi^0\pi^0}$. The experimental distribution $\rho(s_\pi,\theta_{\pi^0\pi^0})$ was fitted to a simulation spectrum with f'/f_0 and f''/f_0 being free parameters. They were obtained to be $f'/f_0=-0.45^{+0.75}_{-0.59}$ and $f''/f_0=0.32^{+0.72}_{-0.89}$. The reduced χ^2 was 0.91. Figure 3 shows the χ^2 contour plot in the (f'/f_0, f''/f_0) space. The histograms in Fig. 2 shows the simulation spectra with the best fit parameters, which are in good agreement with the experimental data. Using these form factors for the acceptance calculation, we obtained $\Gamma(K_{e4}^{00})=[(3.56\pm0.26(\text{stat.})^{+7.51}_{-2.92}(\text{syst.})]\times10^3$ s^{-1} by normalizing it to the theoretical value of internal bremsstrahlung in the $K_{\pi2\gamma}$ decay [15]. The large systematic error from the uncertainty of the form factors (i.e., uncertainty of the e^+ momentum distribution) was due to the fact that our experiment covered a very narrow region of phase space and the error of the de-

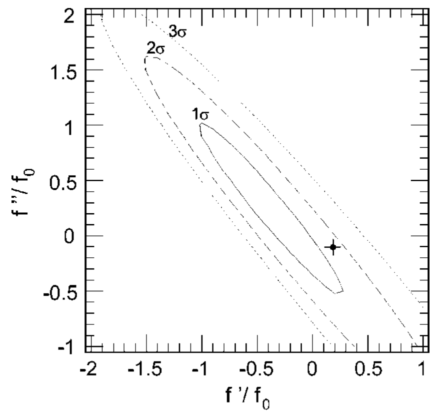

FIG. 3. χ^2 contour plot in the $(f'/f_0, f''/f_0)$ space. Contour levels of 1-, 2-, and 3σ constraints are drawn. The K_{e4}^{+-} experimental result by the BNL-E865 group is also plotted.

tector acceptance became large. As shown in Fig. 2, the experimental θ_π and ϕ distributions were well accounted for by the simulations without any *D*-wave component. Thus, the data are compatible with an *S*-wave only description. The reduced χ^2 was 0.92 for θ_π and 1.19 for ϕ. This feature also supports the previous K_{e4}^{+-} measurement in which the *D*-wave contribution was assumed to be negligible [4].

Then, using Eq. (6) as the F form factor, the a_0^0 value which gave the best fit to the experimental $\rho(s_\pi, \cos\theta_{\pi^0\pi^0})$ distribution was determined to be $a_0^0 = 0.45 \pm 0.43$ in units of m_π. The reduced χ^2 was 0.91. While the large error precludes a meaningful comparison with results from other channels, a well designed experiment at the future high intensity facilities such as J-PARC will be able to make a significant contribution to the determination of the a_0^0 parameter. The f'/f_0 and f''/f_0 parameters were determined from this a_0^0 value to be $f'/f_0 = -0.30^{+0.29}_{-0.33}$ and $f''/f_0 = 0.12^{+0.18}_{-0.12}$, which are in agreement with the $(f'/f_0,\ f''/f_0)$ fitting.

The $\Delta I = 1/2$ rule predicts that the q^2 dependence of K_{e4}^{00} and K_{e4}^{+-} form factors are identical. As seen in Fig. 3, our K_{e4}^{00} result overlaps with the K_{e4}^{+-} form factor reported by the BNL-E865 group within two standard deviations [4]. The f'/f_0 values were obtained to be $-0.19^{+0.19}_{-0.17}$ and $-0.11^{+0.19}_{-0.17}$ for $f''/f_0 = 0$ and -0.10 (K_{e4}^{+-} result), respectively, which also overlaps with the K_{e4}^{+-} result within the error [4]. The $\Delta I = 1/2$ rule also predicts a simple relation for the decay widths $\Gamma(K_{e4}^{00}) = 1/2\Gamma(K_{e4}^{+-}) - 1/4\Gamma(K_{e4}^{-0})$ [6]. The world averages for the K_{e4}^{+-} and K_{e4}^{-0} channels [16] give $\Gamma(K_{e4}^{00}) = (1.40 \pm 0.04) \times 10^3\ \mathrm{s}^{-1}$. Using the K_{e4}^{+-} form factor [4] for the acceptance calculation of the present experiment to reduce the systematic error, we deduced $\Gamma(K_{e4}^{00}) = [1.85 \pm 0.13(\mathrm{stat.}) \pm 0.24(\mathrm{syst.})] \times 10^3\ \mathrm{s}^{-1}$. The average of the previous K_{e4}^{00} experiments, under the assumption of constant form factor, is quoted as $\Gamma(K_{e4}^{00}) = (1.70 \pm 0.32) \times 10^3\ \mathrm{s}^{-1}$ [16]. While these numbers are all consistent with each other, the large systematic error of the $\Gamma(K_{e4}^{00})$ value determined in the present experiment precludes a meaningful comparison with other results. Further measurements will be essential before we can draw firm conclusions on the $\Delta I = 3/2$ contributions.

In conclusion, we have performed a new measurement of one of the K_{e4} channels $K^+ \to \pi^0 \pi^0 e^+ \nu$ (K_{e4}^{00}) using stopped positive kaons. The data sample of 216 events is, while a factor of six improvement over the world data set, still very limited. The detector response and acceptance functions were evaluated by the Monte Carlo simulation. The backgrounds due to K_{e3} and $K_{e3\gamma}$ decays were estimated to be 1.1%. The F form factor of the K_{e4}^{00} decay was determined by fitting the observed $\rho(s_\pi, \theta_{\pi^0\pi^0})$ distribution to the simulation. A first attempt to deduce the *S*-wave π-π scattering length from the $\rho(s_\pi, \theta_{\pi^0\pi^0})$ distribution was made. The good reproducibility of the experimental θ_π and ϕ spectra by the simulation indicates that the data are compatible with an *S*-wave only description. These results are consistent with the previous $K^+ \to \pi^+ \pi^- e^+ \nu$ measurement within errors. By this measurement, despite the limited statistics, we established the physics potential of the K_{e4}^{00} channel.

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