Gold-plated mode reexamined: sin(2β) and $B^0 \to J/\Psi K_S$ in the standard model

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We study the corrections to the determination of sin(2β) from the time dependent CP asymmetry of $B^0 \to J/\Psi K_S$ which arise in the standard model. Although a precise prediction of these corrections is not possible we find that they are indeed extremely small, of the order of less than a per mil of the observed value. This means in turn that any deviation visible at the B factories will be a clear signal for new physics.

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I. INTRODUCTION

The measurement of the mixing-induced CP asymmetry in the so-called gold-plated mode $B^0 \to J/\Psi K_S$ is becoming a precision measurement [1]. Currently the relative uncertainty is at the level of five percent and is going to decrease further in the next few years as more data from the B factories are analyzed.

From the theoretical side the time dependent CP asymmetry in this channel is related in the standard model to sin(2β) in a very clean way [4], i.e. it is not plagued by hadronic uncertainties, at least at the level of current experimental precision. However, the precision of the experimental data will increase further, making this CP asymmetry an interesting probe for new physics.

Possible new physics effects have been discussed in some detail in a generic framework in [5] where both the charged and the neutral modes of $B \to J/\Psi K$ have to be taken into account. In [5] certain observables have been defined which are sensitive to different aspects of new physics.

However, in order to quantitatively pin down a possible new physics effect and to assess the reach to new physics, the small standard-model contributions have to be under control. Although estimates of these effects have been given some time ago [6], it is worthwhile to reconsider these estimates motivated by the experimental precision to be expected soon.

In this paper we shall try to get a quantitative estimate for the time-dependent CP asymmetry in this channel beyond the simple relations [in the convention used in Eq. (1) below]

$$C_{B\to J/\Psi K_S}=0 \qquad S_{B\to J/\Psi K_S}=\sin(2\beta).$$

The corrections to these relations originate from two sources. The first source is from the corrections to the $\Delta B=\pm 2$ part of the Hamiltonian. In the present paper we compute these contributions systematically in an effective field theory by subsequently integrating out heavy degrees of freedom down to a small hadronic scale. It is worthwhile to note that many elements of our calculation are similar to the calculation of the lifetime difference in the system of neutral B mesons which is performed in an effective field theory framework in [7].

The second source is the decay amplitude itself which has small contributions carrying a different CP phase compared to the leading piece. The latter is much harder to estimate and we shall refer to well known methods.

A full discussion of the standard-model effects would also require to include the effects from CP violation in the kaon system. We shall not discuss these effects in the present paper, we focus completely on the effects coming from the B system.

In the next sections we set up the calculation of these two contributions and discuss the methods of calculation. Finally we summarize our results and conclude.

II. BASIC RELATIONS

CP asymmetries are measured at the B factories by detecting the decay products of the coherent $B^0\overline{B^0}$ pair. One of them is identified as a flavor state using e.g. a leptonic tagging mode while the decay of the other into a CP eigenstate is observed. Usually, in the calculation of these rates a possible CP violation stemming from the mixing on the tagging side or in the tagging decay is neglected which results in the well-known formula for the decay $B^0 \to J/\Psi K_S$

$$[a_{CP}^{B\to J/\Psi K_S}]_0(t)=\frac{C_{B\to J/\Psi K_S}\cos(\Delta m t)-S_{B\to J/\Psi K_S}\sin(\Delta m t)}{\cosh(\Delta\Gamma t/2)+D_{B\to J/\Psi K_S}\sinh(\Delta\Gamma t/2)} \tag{1}$$

with

$$C_{B\to J/\Psi K_S}=\frac{1-|\lambda|^2}{1+|\lambda|^2}, \qquad S_{B\to J/\Psi K_S}=\frac{2\,\mathrm{Im}[\lambda]}{1+|\lambda|^2},$$

$$D_{B\to J/\Psi K_S}=\frac{2\,\mathrm{Re}[\lambda]}{1+|\lambda|^2} \tag{2}$$

and

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$$\lambda = \left(\frac{q}{p}\right)_B \cdot \left(\frac{p}{q}\right)_K \cdot \frac{A(\overline{B}^0 \to J/\Psi K_S)}{A(B^0 \to J/\Psi K_S)}. \tag{3}$$

Here p and q are the parameters characterizing the mixing in the K- and B-system, while $A(B^0 \to J/\Psi K_S)$ is the amplitude for the decay into the CP eigenstate $J/\Psi K_S$.

Our aim is to investigate all effects correcting the leading contribution and therefore we take also into account the influence of the mixing in the B system on the tagging amplitude assuming that there is no CP violation in the tagging decay (cf. [2], the effect of the width difference and the K-$\overline{K}$ mixing on the CP asymmetry have been already considered some time ago in [3]). Defining

$$|\lambda_{\text{tag}}|^2 \equiv \left|\left(\frac{q}{p}\right)_B \cdot \frac{\overline{A}_{\text{tag}}}{A_{\text{tag}}}\right|^2 \approx \left|\left(\frac{q}{p}\right)_B\right|^2 =: 1+\epsilon, \tag{4}$$

the CP asymmetry becomes

$$a_{CP}^{B\to J/\Psi K_S}(t) = [a_{CP}^{B\to J/\Psi K_S}]_0(t) + \frac{\epsilon}{2} - \frac{\epsilon}{2}\sin^2(\Delta m t)\sin^2(2\beta). \tag{5}$$

These relations above are true in general and take into account a possible width difference, a possible direct CP asymmetry as well as the mixing effects on the tagging. The lifetime difference $\Delta\Gamma$ as well as $1-|\lambda|^2$ and ϵ are small and the leading term is obtained by neglecting these quantities. Furthermore, since the weak phase of the decay amplitude of the leading contribution vanishes in the standard convention, S measures the weak phase of the $\Delta B = 2$ contribution to the effective Hamiltonian which is—again to leading order—simply 2β.

We make use of the general relation

$$(C_{B\to J/\Psi K_S})^2 + (S_{B\to J/\Psi K_S})^2 + (D_{B\to J/\Psi K_S})^2 = 1, \tag{6}$$

which allows us to replace $D_{B\to J/\Psi K_S}$ by its leading order expression $D_{B\to J/\Psi K_S} = \cos(2\beta)$, since it is multiplied by the small quantity $\Delta\Gamma t$ in the expansion. Note that typically t is of the order of the B meson lifetime τ and we have $\Delta\Gamma\tau \ll 1$.

III. CORRECTIONS TO THE MIXING

The phenomenon of mixing between B and $\overline{B}$ is due to the box diagrams with a double W exchange in the full electroweak theory. These diagrams have been evaluated some time ago in the full standard model including the quark masses of all quarks in the loop [8]. After GIM cancellation, the leading term is due to the top quark, giving rise to a contribution of the order $(m_t/M_W)^2$ with the weak phase 2β. Subleading terms are either of the order $(m_b/M_W)^2$ which again carry the phase 2β and thus will not contribute to a modification of S, or of order $(m_c/M_W)^2$ which carry a different weak phase and hence will yield a correction to S.

Instead of calculating the box diagrams in the full theory one may also use an effective theory picture which we will do in this paper. The approach is similar to the one used for D-$\overline{D}$ mixing [9–11], and also for the $\Delta S = 2$ Hamiltonian in [12]. The advantage is that one may express the contributions to the mixing phase in terms of matrix elements of certain local operators, which can be estimated in factorization. Furthermore, one can resum systematically large logarithms using the renormalization group.

The first step is the usual one, in which the W boson and the top quark are integrated out at a common scale $\mu \sim m_t \sim M_W$. The result is the well known $\Delta B = 2$ operator of dimension six, which is

$$H_{\text{eff}}^{\Delta B=2} = \frac{G_F^2}{4\pi^2}\lambda_t^2 m_t^2 C_0(x_t) Q_0$$

with

$$Q_0 = (\overline{b}_L \gamma_\mu d_L)(\overline{b}_L \gamma^\mu d_L) \tag{7}$$

where

$$\lambda_q = V_{qb}^* V_{qd}, \quad x_t = m_t^2/M_W^2$$

and

$$C_0(x_t) = \frac{4 - 11x_t + x_t^2}{4(1-x_t)^2} - \frac{3x_t^2 \ln x_t}{2(1-x_t)^3} \tag{8}$$

is obtained from the usual Inami-Lim function [13]. Note that we extracted explicitly a factor m_t^2 which is reminiscent of the GIM mechanism.

However, Eq. (7) is not the only contribution to $\Delta B = 2$. Other contributions originate from two insertions of $\Delta B = 1$ operators

$$T^{\Delta B=2} = -\frac{i}{2}\int d^4x\, T[H_{\text{eff}}^{\Delta B=1}(x) H_{\text{eff}}^{\Delta B=1}(0)], \tag{9}$$

where the relevant operators are

$$\begin{aligned} H_{\text{eff}}^{\Delta B=1} = \frac{4G_F}{\sqrt{2}} [& V_{cb}^* V_{cd} (\overline{b}_L \gamma_\mu c_L)(\overline{c}_L \gamma^\mu d_L) \\ & + V_{cb}^* V_{ud} (\overline{b}_L \gamma_\mu c_L)(\overline{u}_L \gamma^\mu d_L) + V_{ub}^* V_{cd} (\overline{b}_L \gamma_\mu u_L) \\ & \times (\overline{c}_L \gamma^\mu d_L) + V_{ub}^* V_{ud} (\overline{b}_L \gamma_\mu u_L)(\overline{u}_L \gamma^\mu d_L)] \end{aligned} \tag{10}$$

which yield non-local contributions at the scale $\mu \sim M_W$.

Furthermore, the matching at the scale M_W yields—aside from the above dimension-6 operator appearing in Eq. (7)—dimension-8 contributions involving two covariant derivatives. These have been partially calculated in [14]; however, we do not need these contributions for our purposes, since these pieces are again dominated by the top quark and thus have the same weak phase as the leading part.

In addition, keeping a non-zero charm mass, the matching calculation yields an operator of the form $m_c^2(\overline{b}_L \gamma_\mu d_L)(\overline{b}_L \gamma^\mu d_L)$ which we shall treat as a dimension-8 operator as well. The matching of this operator involves the calculation of the box diagrams keeping the charm mass nonzero.

Lowering the scale turns the high momentum part of the non-local contributions into local operators. This effect is described by a renormalization-group mixing of the non-local operators into local ones. We define the non-local operators as

$$T_1 = -\frac{i}{2}\int d^4x T[(\overline{b}_L\gamma_\mu c_L)(x)(\overline{c}_L\gamma^\mu d_L)(x)(\overline{b}_L\gamma^\nu c_L)(0) \times(\overline{c}_L\gamma_\nu d_L)(0)] \tag{11a}$$

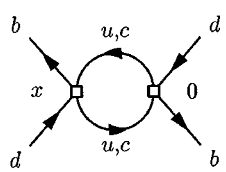
$$T_2 = -\frac{i}{2}\int d^4x T[(\overline{b}_L\gamma_\mu c_L)(x)(\overline{u}_L\gamma^\mu d_L)(x)(\overline{b}_L\gamma^\nu u_L)(0) \times(\overline{c}_L\gamma_\nu d_L)(0)] \tag{11b}$$

$$T_3 = -\frac{i}{2}\int d^4x T[(\overline{b}_L\gamma_\mu u_L)(x)(\overline{u}_L\gamma^\mu d_L)(x)(\overline{b}_L\gamma^\nu u_L)(0) \times(\overline{u}_L\gamma_\nu d_L)(0)] \tag{11c}$$

and find that these operators mix already at order $(\alpha_s)^0$ into local $\Delta B = 2$ operators of dimension 8 [30]

$$\begin{aligned} Q_1 &= \Box(\overline{b}_L\gamma_\mu d_L)(\overline{b}_L\gamma^\mu d_L) \\ Q_2 &= \partial^\mu\partial^\nu(\overline{b}_L\gamma_\mu d_L)(\overline{b}_L\gamma_\nu d_L) \\ Q_3 &= m_c^2(\overline{b}_L\gamma_\mu d_L)(\overline{b}_L\gamma^\mu d_L). \end{aligned} \tag{12}$$

This order $(\alpha_s)^0$ mixing happens via the diagrams



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The anomalous dimension matrix at order $(\alpha_s)^0$ is

$$\gamma = \frac{1}{48\pi^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 6 & 0 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{13}$$

and the operators are gathered into $\vec{O} = (Q_1, Q_2, Q_3, T_1, T_2, T_3)^T$.

The initial conditions for the renormalization-group running are from matching. As discussed above, we do not explicitly need the matching conditions for Q_1 and Q_2, so we get

$$\begin{aligned} \hat{C}_1(M_W, x_t) &= -\frac{\lambda_t^2}{96\pi^2}C_{12}(x_t), \\ \hat{C}_2(M_W, x_t) &= -\frac{\lambda_t^2}{48\pi^2}C'_{12}(x_t), \\ \hat{C}_3(M_W, x_t) &= \frac{1}{32\pi^2}(2\lambda_c\lambda_t C_3(x_t) + \lambda_c^2), \\ \hat{C}_4(M_W) &= \lambda_c^2, \quad \hat{C}_5(M_W) = 2\lambda_c\lambda_u, \\ \hat{C}_6(M_W) &= \lambda_u^2, \end{aligned} \tag{14}$$

where we have used the tree-level matching of the $\Delta B = 1$ operators for the non-local terms. The function

$$C_3(x_t) = \ln x_t - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \ln x_t}{4(1-x_t)^2} \tag{15}$$

is derived from the Inami-Lim function; C_{12} and C'_{12} are functions of x_t which could be obtained from the matching of the dim-8 operators with two derivatives. We do not give these functions here, since they are not needed in the following.

The next step is to perform the renormalization-group running using the renormalization-group functions at order $(\alpha_s)^0$

$$\left(\mu\frac{\partial}{\partial\mu} - \gamma^T\right)\vec{C}(\mu) = 0 \tag{16}$$

where $\vec{C} = (\hat{C}_1, \hat{C}_2, \hat{C}_3, \hat{C}_4, \hat{C}_5, \hat{C}_6)$ are the coefficients of the operators.

Running down to the scale of the b quark mass we switch at $\mu \sim m_b$ again to another effective field theory in which the b quark becomes static. At this scale one has to replace the derivatives acting on the b quark field by [15]

$$i\partial_\mu b \to (m_b v_\mu + i\partial_\mu)h_v \tag{17}$$

where h_v is the static b quark field moving with velocity v. Keeping only the leading term in the $1/m_b$ expansion we have to match onto the operators

$$\begin{aligned} P_0 &= (\overline{h}_{v,L}^{(+)}\gamma_\mu d_L)(\overline{h}_{v,L}^{(-)}\gamma^\mu d_L) \\ P_1 &= m_b^2(\overline{h}_{v,L}^{(+)}\gamma_\mu d_L)(\overline{h}_{v,L}^{(-)}\gamma^\mu d_L) \\ P_2 &= m_b^2(\overline{h}_{v,R}^{(+)}d_L)(\overline{h}_{v,R}^{(-)}d_L) \\ P_3 &= m_c^2(\overline{h}_{v,L}^{(+)}\gamma_\mu d_L)(\overline{h}_{v,L}^{(-)}\gamma^\mu d_L), \end{aligned} \tag{18}$$

where $h_{v,L}^{(+/-)}$ denotes the static quark/antiquark field, which have become completely different fields in the static limit.

Performing the matching and the renormalization-group running we may evolve down to scales around the charm-quark mass. At such a low scale all contributions become local, once the up-quark mass is neglected. In fact the dim-8 operators of the generic structure

$$(\overline{h}_{v,R}^{(+)} D d_L)(\overline{h}_{v,R}^{(-)} D d_L),$$

with D being a derivative inserted in any possible way, will yield contributions proportional to $\Lambda_{\rm QCD}^2$. Compared to the pieces considered here, these are suppressed by a factor $\Lambda_{\rm QCD}^2/m_c^2$ and thus may be neglected.

Inserting back all the CKM factors, we make use of CKM unitarity, i.e. the fact that $\lambda_u+\lambda_c+\lambda_t=0$. This separates the running from CKM factors, which means that each weak phase is multiplied by a renormalization-group invariant. Thus the $\Delta B=2$ contributions at the scale $\mu\sim m_c$ can be expressed in terms of the local effective Hamiltonian

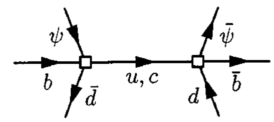
$$\mathcal{H}_{\rm eff}^{\Delta B=2}=\frac{G_F^2}{4\pi^2}\left\{\lambda_t^2 m_t^2 C_0(x_t)P_0+\frac{\lambda_t^2}{3}\left[C_{12}(x_t)-\ln\left(\frac{m_c^2}{M_W^2}\right)\right]P_1-\frac{2\lambda_t^2}{3}\left[C_{12}'(x_t)-\ln\left(\frac{m_c^2}{M_W^2}\right)\right]P_2+\left(2\lambda_c\lambda_t\left[C_3(x_t)-\ln\left(\frac{m_c^2}{M_W^2}\right)\right]+\lambda_c^2\right)P_3\right\}. \quad (19)$$

The first term in this expression is the well known leading term, while two of the subleading terms are proportional to m_b^2 and carry the same weak phase as the leading term. In these contributions we have the two unknown matching functions C_{12} and C_{12}'; however, the running gives the large logarithm which in this framework is assumed to dominate. Furthermore, the renormalization group reproduces the well known result for the term proportional to m_c^2, which will modify the mixing phase. Note that this effective Hamiltonian is given in terms of local operators P_i. We shall estimate the matrix elements of this effective Hamiltonian using naive factorization at the scale $\mu\sim m_c$.

From this effective Hamiltonian we can obtain the correction to the mixing phase. Numerically we find

$$\Delta\,\mathrm{Im}\left[\frac{M_{12}}{|M_{12}|}\right]=\frac{1}{C_0(x_t)}\frac{m_c^2}{m_t^2}\frac{|V_{cd}||V_{cb}|}{|V_{td}||V_{tb}|}\left[\frac{|V_{cd}||V_{cb}|}{|V_{td}||V_{tb}|}\times\sin(2\beta)\cos(2\beta)+2\left(C_3(x_t)-\ln\left(\frac{m_c^2}{M_W^2}\right)\right)\times(\sin\beta-\sin(2\beta)\cos\beta)\right]=-(4.48\pm2.55)\times10^{-4}. \quad (20)$$

At the tree level, the next order in the $1/m_b$ expansion consists of operators of dimension-9, which are six-quark operators (cf. e.g. [10]). They originate from diagrams like ($\psi\equiv u,c$)



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The contributions from these operators can be brought into the form

$$\langle B^0|\mathcal{O}_{6\ \rm quarks}|\overline{B^0}\rangle\sim\langle B^0|(\overline{h}_v d)(\overline{h}_v d)(\overline{u}u)|\overline{B^0}\rangle\times\left[\lambda_t^2-\lambda_c\lambda_t\left(\frac{m_c^2}{m_b^2}+\xi\right)+\mathcal{O}\left(\frac{m_c^4}{m_b^4}\right)\right], \quad (21)$$

where ξ is defined as

$$\frac{\langle B^0|(\overline{h}_v d)(\overline{h}_v d)(\overline{u}u)|\overline{B^0}\rangle}{\langle B^0|(\overline{h}_v d)(\overline{h}_v d)(\overline{c}c)|\overline{B^0}\rangle}=1+\xi. \quad (22)$$

The GIM mechanism as well as the OPE guarantee that ξ is of the same order as the m_c^2/m_b^2 terms. Altogether, the dim-9 operators are suppressed with respect to the dim-6 operators by a factor of $\Lambda_{\rm QCD}^2/m_b^2$, as could have been guessed from the OPE. Hence they are a negligible contribution, at most of the absolute order of 10^{-6}.

Another class of corrections are the $\mathcal{O}(\alpha_s)$ QCD corrections which are known for most of the processes at the two-loop level [16]. For the box diagrams they have been already calculated some time ago (see the references in [16]). However, one may use the effective field theory to resum large logarithms of the form $\alpha_s\ln(M_W/\mu)$, where μ is a hadronic scale. Since there are large logarithms already at order $(\alpha_s)^0$ the situation is similar to the one in the transition $s\to d\ell\ell$, which has been discussed in [17].

The result given in Eq. (19) does not resum the logarithms of the form $\alpha_s\ln(M_W/\mu)$. For the case of the $\Delta S=2$ effective Hamiltonian, the next-to-leading result has been given in [12], however, in our case the situation is slightly different due to the fact that the mass of the bottom quark sets a large scale and thus a matching to an effective theory with a static b quark at the scale $\mu\sim m_b$ is possible, which is still perturbative. The running below the scale m_b down to the scale m_c resumes logarithms of the form $\ln m_b^2/m_c^2$; however, these logarithms are not large and hence the resummation is not really needed. In principle one could run perturbatively even below the charm mass, yielding a result like Eq. (19) in terms of local operators, but the running below m_b is a very small effect which we shall neglect in the following.

Working at one loop requires to include the one-loop correction to the $\Delta B = 1$ effective Hamiltonian as well as the mixing among the local $\Delta B = 2$ operators of dimension 8. However, since we are only interested in the contributions that modify the relation between sin(2β) and $S_{B\to J/\Psi K_s}$ we shall simplify the discussion by neglecting the mixing among the $\Delta B = 2$ operators of dimension 8, since only the single dim-8 operator proportional to m_c^2 will contribute to this effect.

The running of the $\Delta B = 1$ effective Hamiltonian forces us to introduce operators T_i with different color combinations. It is well known that for the $\Delta B = 1$ operators the renormalization-group evolution is diagonalized by the combinations [18]

$$[(\overline{b_L}\gamma_\mu q_L)(\overline{q'_L}\gamma^\mu d_L)]_\pm = \frac{1}{2}[(\overline{b_{i,L}}\gamma_\mu q_{i,L})(\overline{q'_{j,L}}\gamma^\mu d_{j,L}) \pm (\overline{b_{i,L}}\gamma_\mu q_{j,L})(\overline{q'_{j,L}}\gamma^\mu d_{i,L})]. \tag{23}$$

Thus it is convenient to introduce the non-local operators in the form

$$T^{\sigma\sigma'}_{qq'} = -\frac{i}{2}\int d^4x T\{[(\overline{b_L}\gamma_\mu q_L)(x)(\overline{q'_L}\gamma^\mu d_L)(x)]_\sigma \times [(\overline{b_L}\gamma^\nu q'_L)(0)(\overline{q_L}\gamma_\nu d_L)(0)]_{\sigma'}\}, \tag{24}$$

where σ and σ' may take the values $+,-$. Ignoring the mixing among the local dim-8 operators we choose as the basis

$$\vec{O} = (Q_3, T^{++}_{cc}, T^{+-}_{cc}, T^{--}_{cc}, T^{++}_{cu}, T^{+-}_{cu}, T^{--}_{cu}, T^{++}_{uu}, T^{+-}_{uu}, T^{--}_{uu})^T.$$

For the case at hand the relevant contribution is the mixing of the operators from the time-ordered products into the local operator Q_3, for which we also keep the mixing with itself. The anomalous dimension matrix, including tree level and this restricted set of α_s corrections, becomes

$$\gamma = \left(\begin{array}{c||ccc|ccc|ccc}
-\frac{\alpha_s}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline\hline
\frac{3}{16\pi^2} & 2\frac{\alpha_s}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{16\pi^2} & 0 & -\frac{\alpha_s}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{16\pi^2} & 0 & 0 & -4\frac{\alpha_s}{\pi} & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\frac{3}{32\pi^2} & 0 & 0 & 0 & 2\frac{\alpha_s}{\pi} & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{32\pi^2} & 0 & 0 & 0 & 0 & -\frac{\alpha_s}{\pi} & 0 & 0 & 0 & 0 \\
\frac{1}{32\pi^2} & 0 & 0 & 0 & 0 & 0 & -4\frac{\alpha_s}{\pi} & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\frac{\alpha_s}{\pi} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\alpha_s}{\pi} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4\frac{\alpha_s}{\pi}
\end{array}\right). \tag{25}$$

In terms of the "diagonalized" coefficients

$$C^{++}=C_{22}+C_{11}+C_{12}+C_{21}$$

$$C^{--}=C_{22}+C_{11}-C_{12}-C_{21}$$

$$C^{+-}=C_{22}-C_{11}+C_{12}-C_{21}$$

$$C^{-+}=C_{22}-C_{11}-C_{12}+C_{21} \tag{26}$$

the matching conditions (i.e. the starting points for the renormalization-group evolution) are

$$C_{cc}^{++}(M_W^2)=\lambda_c^2,\quad C_{cc}^{+-}(M_W^2)=2\lambda_c^2,\quad C_{cc}^{--}(M_W^2)=\lambda_c^2,$$
$$C_{cu}^{++}(M_W^2)=2\lambda_c\lambda_u,\quad C_{cu}^{+-}(M_W^2)=4\lambda_c\lambda_u,$$
$$C_{cu}^{--}(M_W^2)=2\lambda_c\lambda_u,$$
$$C_{uu}^{++}(M_W^2)=\lambda_u^2,\quad C_{uu}^{+-}(M_W^2)=2\lambda_u^2,\quad C_{uu}^{--}(M_W^2)=\lambda_u^2, \tag{27}$$

where we combined $+-$ and $-+$.

The solution of the renormalization-group equation (at one loop) for the Wilson coefficients of the $\Delta B=1$ operators is well known $[\gamma^{\sigma\sigma'}=\tilde{\gamma}^{\sigma\sigma'}\alpha_s(\mu)/\pi,\tilde{\gamma}^{++,+-,--}=2,-1,-4]$ and yields

$$C_{qq'}^{\sigma\sigma'}(\mu)=C_{qq'}^{\sigma\sigma'}(M_W)\cdot\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-2\tilde{\gamma}^{\sigma\sigma'}/\beta_0}, \tag{28}$$

where $\beta_0=11-2N_f/3$ is as usual the coefficient of the one-loop QCD beta function. Due to the special structure of the anomalous dimension matrix, we obtain the renormalization-group equation for the coefficient C_3 of the operator Q_3

$$\mu\frac{d}{d\mu}C_3(\mu)=-\frac{\alpha_s(\mu)}{\pi}C_3(\mu)-\frac{\lambda_c\lambda_t}{16\pi^2}\sum_{\xi=++,+-,--}\Gamma^{\xi}\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-2\tilde{\gamma}^{\xi}/\beta_0}, \tag{29}$$

where $\Gamma^{++,+-,--}=3,-2,1$ originates from the first column of Eq. (25). Note that there is a factor two for $+-$ due to the equal contribution from $+-$ and $-+$. Furthermore, the flavor dependence of the time-ordered products' mixing into the quasi-local operators appeared within the structure $\lambda_c^2+\frac{1}{2}\cdot 2\lambda_u\lambda_c=-\lambda_c\lambda_t$. The factor of two here stems from the fact that the *uc* and *cu* flavor combinations give the same contribution.

The equation for C_3 can be solved by standard methods; the solution of the homogeneous differential equation is

$$C_3^{(0)}(\mu)=c\cdot\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-2\tilde{\gamma}_3/\beta_0}. \tag{30}$$

By setting $C_3(\mu)=c(\mu)C_3^{(0)}(\mu)$ we get the solution

$$C_3(\mu)=\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-2\tilde{\gamma}_3/\beta_0}\cdot\left\{\hat{C}_3(M_W,x_t)+\frac{\lambda_c\lambda_t}{8\pi\beta_0}\sum_{\xi}\Gamma^{\xi}\left(\frac{2(\tilde{\gamma}_3-\tilde{\gamma}^{\xi})}{\beta_0}-1\right)^{-1}\cdot\left[\frac{1}{\alpha_s(\mu)}\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{2[(\tilde{\gamma}_3-\tilde{\gamma}^{\xi})/\beta_0]}-\frac{1}{\alpha_s(M_W)}\right]\right\}. \tag{31}$$

We shall consider only the evolution from M_W down to m_b, since at m_b we would need to consider again a different set of operators, including static quarks for the b and later also for the c and their renormalization [19]. While this can be done in principle, the corresponding logarithms $\alpha_s\ln(m_b/m_c)$ and $\alpha_s\ln(m_c/\mu)$ are smaller than the ones from the running from M_W to m_b and we shall include here only the leading term.

We obtain for the QCD corrections at $\mu\sim m_b$ the explicit formula

$$\Delta\,\mathrm{Im}\left[\frac{M_{12}}{|M_{12}|}\right]=\frac{1}{C_0(x_t)}\frac{m_c^2}{m_t^2}\frac{|V_{cd}||V_{cb}|}{|V_{td}||V_{tb}|}\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-2\tilde{\gamma}_3/\beta_0}\left\{\frac{|V_{cd}||V_{cb}|}{|V_{td}||V_{tb}|}\sin(2\beta)\cos(2\beta)+2\left(C_3(x_t)+\frac{2\pi}{\beta_0}\sum_{\xi}\Gamma^{\xi}\left(\frac{2(\tilde{\gamma}_3-\tilde{\gamma}^{\xi})}{\beta_0}-1\right)^{-1}\cdot\left[\frac{1}{\alpha_s(\mu)}\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{2[(\tilde{\gamma}_3-\tilde{\gamma}^{\xi})/\beta_0]}-\frac{1}{\alpha_s(M_W)}\right]\right)\cdot(\sin\beta-\sin(2\beta)\cos\beta)\right\}, \tag{32}$$

which turns into the simple case [cf. Eq. (20)] as $\alpha_s(M_W) \to 0$. Numerically, one obtains at $\mu = m_b$

$$\Delta\, \mathrm{Im}\left[\frac{M_{12}}{|M_{12}|}\right]_{\mathrm{QCD}} = -2.08\times 10^{-4}, \tag{33}$$

which has to be compared to

$$\Delta\, \mathrm{Im}\left[\frac{M_{12}}{|M_{12}|}\right] = -3.00\times 10^{-4} \tag{34}$$

without QCD corrections. This indicates that QCD can reduce the absolute value of the correction to sin(2β) coming from mixing by about 30%. Although we have not included the mixing among the dim-8 operators, we still take this as the size of the QCD corrections to be expected. We conclude that these contributions are safely below the percent level.

IV. CORRECTIONS TO THE DECAY

The operators contributing to the decay B^0 to $J/\Psi K_S$ are all of the flavor structure $(\bar{b}q)(\bar{q}s)$ where at leading order we have from the current-current operators $q = c, u$. From these two operators $q = c$ is Cabibbo-favored over $q = u$. Furthermore, also the penguin contributions are dominated by the charm quark, such that all contributions to the decay carry the same weak phase and hence one expects a very small direct CP asymmetry [4]. Note that the top quark contribution has been integrated out already at the weak scale for both the QCD and the electroweak penguin contributions.

However, looking for small deviations we have to study the small terms carrying different weak phases which is the $(\bar{b}u)(\bar{u}s)$ contribution. Aside from the QCD penguin contributions also electroweak penguin contributions become important once small corrections are studied [20]. We identify the corresponding matrix elements of the two different contributions—tree and penguin—to the effective Hamiltonian in the numerator and denominator of the ratio of amplitudes

$$\frac{A(\overline{B^0} \to J/\Psi K_S)}{A(B^0 \to J/\Psi K_S)} = \frac{\langle J/\Psi K_S|\mathcal{T}_{\mathrm{eff}}(b\to c\bar{c}s)|\overline{B^0}\rangle + \langle J/\Psi K_S|\mathcal{T}_{\mathrm{eff}}(b\to u\bar{u}s)|\overline{B^0}\rangle |\xi_u/\xi_c| e^{-\mathrm{i}\gamma}}{\langle J/\Psi K_S|\mathcal{T}_{\mathrm{eff}}(b\to c\bar{c}s)|B^0\rangle + \langle J/\Psi K_S|\mathcal{T}_{\mathrm{eff}}(b\to u\bar{u}s)|B^0\rangle |\xi_u/\xi_c| e^{+\mathrm{i}\gamma}}, \tag{35}$$

where $\mathcal{T}_{\mathrm{eff}}(b\to q\bar{q}s)$ is the sum over the operators (multiplied by their Wilson coefficients) with the quark content $(b \to q\bar{q}s)$. It is convenient to define the ratio

$$r := \frac{\langle J/\Psi K_S|\mathcal{T}_{\mathrm{eff}}(b\to u\bar{u}s)|\overline{B^0}\rangle}{\langle J/\Psi K_S|\mathcal{T}_{\mathrm{eff}}(b\to c\bar{c}s)|\overline{B^0}\rangle}\left|\frac{\xi_u}{\xi_c}\right|,$$

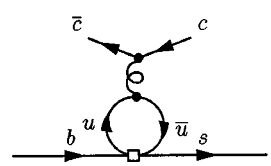
$$\left|\frac{\xi_u}{\xi_c}\right| = \frac{|V_{ub}||V_{us}|}{|V_{cb}||V_{cs}|} = 0.0203 \pm 0.0066 \tag{36}$$

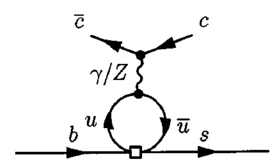
as an expansion parameter. In this way we get

$$\eta_{J/\Psi K_S}\cdot \frac{A(\overline{B^0} \to J/\Psi K_S)}{A(B^0 \to J/\Psi K_S)} = \frac{1 + r e^{-\mathrm{i}\gamma}}{1 + r e^{+\mathrm{i}\gamma}} \approx 1 - 2\,\mathrm{i} r \sin\gamma, \tag{37}$$

where $\eta_{J/\Psi K_S} = -1$ is the CP eigenvalue.

The main obstacle to obtain a reliable quantitative estimate is the evaluation of these hadronic matrix elements. Some time ago the so-called BSS mechanism [21] has been suggested, where the $u\bar{u}$-loops are evaluated perturbatively, assuming a sufficiently large momentum transfer through this loop. In fact, this approach has been supported recently by QCD factorization [22], indicating that the loop is indeed perturbative.





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The evaluation of the loop requires to insert a typical momentum transfer k^2 passing through the up-quark loop [21]; furthermore, the loop also depends on the typical scale μ of the problem, which will be the mass of the b quark. Since the loop involves only scales well above the hadronic scale, one obtains again local operators and thus one may use as an estimate

$$\mathcal{H}_{\mathrm{eff}}^{\mathrm{Peng.}}(b\to c\bar{c}s) = -\frac{G_F}{\sqrt{2}}\left\{\frac{\alpha}{3\pi}(\bar{s}b)_{V-A}(\bar{c}c)_V\cdot\left[1 + \mathcal{O}\left(\frac{M_\Psi^2}{M_Z^2}\right)\right] + \frac{\alpha_s}{3\pi}(\bar{s}T^a b)_{V-A}(\bar{c}T^a c)_V\right\}\cdot\left[\frac{5}{3} - \ln\left(\frac{k^2}{\mu^2}\right) + \mathrm{i}\pi\right], \tag{38}$$

where the first term originates from the electroweak penguin contribution and the second one from the QCD penguin contribution.

The remaining problem is to estimate the color-singlet and the color-octet matrix element. We shall take a simple-minded approach and estimate the sizes of these matrix elements from the total rate of the decay $B^0 \to J/\Psi K_S$. From the measured B^0 lifetime, $\tau = (1.537 \pm 0.015)$ ps [23], and the branching ratio $\mathrm{BR}(B^0 \to J/\Psi K_S) = (4.25 \pm 0.25) \times 10^{-4}$ [23] one can calculate the matrix element for the $\bar{c}c$-contribution by taking the square root

$$|\langle J/\Psi K_S|(\bar{s}b)(\bar{c}c)|\bar{B}^0\rangle|_{\mathrm{exp.}} = (8.36 \pm 0.66) \times 10^8 \ \ \mathrm{MeV}^3. \tag{39}$$

The matrix element with the charm pair being in the octet is estimated by splitting the effective Hamiltonian in singlet and octet contributions. In naive factorization only the singlet piece survives which, however, yields a rate roughly a factor three to four too low compared to experiment. The difference comes from nonfactorizable contributions which we shall ascribe completely to the octet term. To obtain an estimate of this matrix element from the measured total rate we also need the relative phase of the two contributions, which we take from QCD light-cone sum rule estimates in [24] (see also [25]). The relative phase turns out to be small and so we may simply add the two parts.

For the singlet matrix element calculated by means of factorization we get

$$|\langle J/\Psi K_S|(\bar{s}b)(\bar{c}c)|\bar{B}^0\rangle|_{\mathrm{fact.}} = (3.96 \pm 0.36) \times 10^9 \ \ \mathrm{MeV}^3. \tag{40}$$

Inserting for the Wilson coefficients the values [31]

$$C^{(1)} = C_1 + \frac{1}{N_c} C_2 = 0.10 \pm 0.03, \quad C^{(8)} = 2C_2 = 2.24 \pm 0.04, \tag{41}$$

we get as an estimate for the octet matrix element

$$|\langle J/\Psi K_S|(\bar{s}T^a b)(\bar{c}T^a c)|\bar{B}^0\rangle| = (1.97 \pm 0.64) \times 10^8 \ \ \mathrm{MeV}^3. \tag{42}$$

Taking the scale to be m_b and momentum transfer as $m_{J/\Psi}$ gives (the two contributions are the electroweak and the QCD penguins, respectively)

$$\begin{aligned} r &= [-(0.16+1.27) \pm 0.66] \times 10^{-4} \cdot \left[\frac{5}{3} - \ln\left(\frac{k^2}{\mu^2}\right) + \ i\pi\right] \\ &\Rightarrow \mathrm{Re}[r] = (-3.62 \pm 1.55) \times 10^{-4}, \\ &\quad\ \mathrm{Im}[r] = (-4.48 \pm 1.92) \times 10^{-4}. \end{aligned} \tag{43}$$

We have to point out that the estimates of these matrix elements are extremely difficult and hence quite uncertain; the electroweak penguin contributions have been estimated in a recent paper with a similar approach [26].

V. EFFECTS ON THE *CP* ASYMMETRY

Now we are ready to collect all elements for the *CP* violation terms within the asymmetry. The quantity $|\lambda|^2$ is close to unity, the deviation being a small quantity of the same order as r. Thus we write $|\lambda|^2 =: 1 + \delta$ and get

$$C_{B \to J/\Psi K_S} \approx -\frac{\delta}{2}. \tag{44}$$

Therein δ is given by

$$\delta = \Delta A + \epsilon = 4\ \mathrm{Im}[r] \sin\gamma - \mathrm{Im}\left[\frac{\Gamma_{12}}{M_{12}}\right], \tag{45}$$

where ΔA is the deviation of the $|\bar{A}/A|^2$ from 1 and ϵ has been defined in Eq. (4).

For the ratio between the off-diagonal width and mass matrix elements one can take e.g. the calculation in [27] (this quantity has also been calculated at NLO [29]) [32]:

$$\begin{aligned} \epsilon &= -\mathrm{Im}\left[\frac{\Gamma_{12}}{M_{12}}\right] = -\frac{4\pi}{C_0(x_t)} \frac{m_c^2}{m_t^2} \mathrm{Im}\left[\frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*}\right] \\ &\approx +(5.18 \pm 2.96) \times 10^{-4}, \end{aligned} \tag{46}$$

from which we obtain a numerical value for δ

$$\delta = -(1.02 \pm 0.75) \times 10^{-3}. \tag{47}$$

Including all small corrections, the time-dependent *CP* asymmetry in $B^0 \to J/\Psi K_S$ has to be fitted to

$$\begin{aligned} a_{\mathrm{CP}}(t) &= -[\sin(2\beta) + \Delta S_{B \to J/\Psi K_S}] \cdot \sin(\Delta m t) - \frac{\delta}{2} \cos(\Delta m t) \\ &\quad + \frac{\epsilon}{2} - \frac{\epsilon}{2} \sin^2(\Delta m t) \sin^2(2\beta) \\ &\quad + \sin(4\beta) \frac{\Delta\Gamma t}{4} \sin(\Delta m t). \end{aligned} \tag{48}$$

Besides the correction to the leading $\sin(2\beta)$ term and the cosine term there is now a small constant contribution to the *CP* asymmetry as well as terms proportional to $\sin^2(\Delta mt)$ and $t\ \sin(\Delta mt)$.

The correction to the mixing-induced *CP* violation is (note that the term containing the imaginary part of r cancels in the expression for the correction)

$$\begin{aligned} \Delta S_{B \to J/\Psi K_S} &= 2\ \mathrm{Im}[r] \sin\gamma \sin(2\beta) - \Delta\ \mathrm{Im}\left[\frac{M_{12}}{|M_{12}|}\right] \\ &\quad + 2 \sin\gamma\, \mathrm{Re}\left[r \frac{M_{12}^*}{|M_{12}|}\right] \\ &= 2 \sin\gamma\, \mathrm{Re}[r] \cos(2\beta) - \Delta\ \mathrm{Im}\left[\frac{M_{12}}{|M_{12}|}\right]. \end{aligned} \tag{49}$$

With the most recent value of $\sin(2\beta)=0.736\pm0.049$ [1], the corrections in $\Delta S_{B\rightarrow J/\Psi K_S}$ have the size of

$$\Delta\,\mathrm{Im}\left[\frac{M_{12}}{|M_{12}|}\right]=-(2.08\pm1.23)\times10^{-4} \tag{50a}$$

$$2\sin\gamma\,\mathrm{Re}[r]\cos(2\beta)=-(4.24\pm1.94)\times10^{-4} \tag{50b}$$

and sum up to

$$\Delta S_{B\rightarrow J/\Psi K_S}=-(2.16\pm2.23)\times10^{-4}, \tag{51}$$

which is a correction of roughly a third of a per mil with respect to the measured value for sin(2β), but due to the large uncertainty could also be much smaller.

From the mass difference as measured quantity and the approximate calculation for

$$\Delta\Gamma\approx-\frac{3\pi}{2C_0(x_t)}\frac{m_b^2}{m_t^2}\left[1-\frac{8}{3}\frac{m_c^2}{m_b^2}\frac{|V_{cb}||V_{cd}|}{|V_{tb}||V_{td}|}\cos\beta\right]\Delta M_B, \tag{52}$$

one can determine the term leading to a linear dependence on t in Eq. (48)

$$\Delta\Gamma\approx-(1.773\pm0.249)\times10^{-12}\ \mathrm{MeV}$$
$$\approx-(2.694\pm0.378)\times10^{-3}\ \mathrm{ps}^{-1}. \tag{53}$$

Hence, this term has a typical size of the order of

$$\sin(4\beta)\frac{\Delta\Gamma\,\tau_{B^0}}{4}\approx-(1.03\pm0.15)\times10^{-3}, \tag{54}$$

again a very small contribution, which, in addition, will not show up at small times since its time dependence is $(t/\tau_{B^0})\sin(\Delta mt)$.

VI. DISCUSSION

We have reinvestigated the well known fact that the mixing-induced *CP* asymmetry of $B^0\rightarrow J/\Psi K_S$ provides us with a very clean measurement of sin(2β). Already in the original paper [4] it was argued that in the standard model the contamination from "wrong" weak phases is tiny in this decay, but in the meantime the measurements became so precise that we considered it worthwhile to attempt again to quantitatively analyze these small standard-model contributions. In particular, we have used an effective field theory ansatz for the analysis of $\Delta B=2$, which has not been employed before to this process.

Our motivation was twofold. First of all, it has been argued that the *CP* asymmetry could give a hint to new physics, which has been analyzed generically in [5]. It turns out that the general picture conjectured in [5] is supported by the present analysis. Secondly, the *B* factories are doing very well and produce a large amount of data. Already now the measurement of the *CP* asymmetry in the gold-plated mode is a precision measurement with uncertainties at the level of less than ten percent. In the near future this measurement will improve further.

We have shown that the corrections to be expected in the standard model can partially be calculated systematically, namely the part originating from corrections to the mixing. Unfortunately, the second contribution, which is the one from the decay matrix element, is much harder to access, so we still cannot obtain a reliable estimate for the corrections to the *CP* asymmetry of the gold-plated mode. This situation could be improved once data on the decay $B_s\rightarrow J/\Psi K_S$ becomes available, since some of the uncertainties could be eliminated using these data [28]. However, currently one has to use the methods that have been proposed by different authors and one can infer that the corrections will be very small, in the range of a few per mil. At least we can conclude from our analysis that there is still room for new physics in this observable, since a deviation from the standard-model prediction at the level of percents (which is not yet the experimental accuracy) would indicate the presence of new physics.

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[30] If the complete matching at $\mu = M_W$ would be performed, many more dim-8 operators would be induced; however, their coefficients would not be enhanced by a large logarithm; large logarithms at order $(\alpha_s)^0$ are only induced by the mixing with the non-local terms, which induces only Q_1 and Q_2.
[31] We take the Wilson coefficients at Leading-Log order at the scale m_b; unfortunately, the scale dependence is still large at leading order, but we are aiming only at an estimate.
[32] Here we are assuming the solution for β in accord with the unitarity triangle.