

CP violation at a linear collider with transverse polarizationB. Ananthanarayan^{1,2} and Saurabh D. Rindani³¹*Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA*²*Centre for High Energy Physics, Indian Institute of Science, Bangalore 560 012, India**³*Theory Group, Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India*

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We show how transverse beam polarization at e^+e^- colliders can provide a novel means to search for CP violation by observing the distribution of a single final-state particle without measuring its spin. We suggest an azimuthal asymmetry which singles out interference terms between standard model contribution and new-physics scalar or tensor effective interactions in the limit in which the electron mass is neglected. Such terms are inaccessible with unpolarized or longitudinally polarized beams. The asymmetry is sensitive to CP violation when the transverse polarizations of the electron and positron are in opposite senses. The sensitivity of planned future linear colliders to new-physics CP violation in $e^+e^- \rightarrow t\bar{t}$ is estimated in a model-independent parametrization. It would be possible to put a bound of ~ 7 TeV on the new-physics scale Λ at the 90% C.L. for $\sqrt{s}=500$ GeV and $\int dt\mathcal{L}=500$ fb⁻¹, with transverse polarizations of 80% and 60% for the electron and positron beams, respectively.

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I. INTRODUCTION

An e^+e^- linear collider operating at a center-of-mass (c.m.) energy of a few hundred GeV and with an integrated luminosity of several hundred inverse femtobarns is now a distinct possibility. It is likely that the beams can be longitudinally polarized, and there is also the possibility that spin rotators can be used to produce transversely polarized beams. Proposals include the GLC (Global Linear Collider) in Japan [1], the NLC (Next Linear Collider) in the USA [2], and TESLA (TeV-Energy Superconducting Linear Accelerator) in Germany [3]. The physics objectives of these facilities include the precision study of standard model (SM) particles, Higgs discovery and study, and the discovery of physics beyond the standard model.

One important manifestation of new physics would be the observation of CP violation outside the traditional setting of meson systems, since CP violation due to SM interactions is predicted to be unobservably small elsewhere. For instance, one may consider the presence of model independent “weak” and “electric” dipole form factors for heavy particles such as the τ lepton and the top quark. In case of the τ lepton, LEP experiments have constrained their magnitudes from certain CP -violating correlations proposed in [4]. Furthermore, it was pointed out that longitudinal polarization of the electron and/or positron beams dramatically improves the resolving power of other CP -violating correlations in τ -lepton [5] and top-quark pair production [6], and of decay-lepton asymmetries in top-quark pair production [7].

Here we consider exploring new physics via the observation of CP violation in top-quark pair production, by exploiting the transverse polarization (TP) of the beams at these facilities. We rely on completely general and model-independent parametrization of beyond the standard model interactions [8–10] in terms of contact interactions, and on

very general results on the role of TP effects due to Dass and Ross [11]. We demonstrate through explicit computations that only those interactions that transform as tensor or (pseudo)scalar interactions under Lorentz transformations can contribute to CP -violating terms in the differential cross section at leading order when the beams have only TP. By considering realistic energies and integrated luminosities, and some angular-integrated asymmetries in $e^+e^- \rightarrow t\bar{t}$, we find that the scale Λ at which new physics sets in can be probed at the 90% confidence level is $O(10)$ TeV. This effective scale can reach or go beyond what one might expect in popular extensions of the SM such as the minimal supersymmetric model, or extra-dimensional theories. Note that the tensor and (pseudo-)scalar interactions are accessible only at a higher order of perturbation theory without TP, even if longitudinal polarization is available. Also, in the foregoing, effects due to m_e are neglected everywhere.

It may be mentioned that TP in the search of new physics has received sparse attention (for the limited old and recent references with or without CP violation, see [12]). In the CP violation context, the only work of relevance, to our knowledge, is that of Burgess and Robinson [13], who considered pair production of leptons and light quarks in the context of LEP and SLC. Our discussion of top pair production, which is in the context of much higher energies, does have some features in common with the work of Ref. [13], though the numerical analysis is necessarily different. Furthermore, we have included a discussion of CP violation for a general inclusive process.

In the process $e^+e^- \rightarrow f\bar{f}$, where \bar{f} is different from f , testing CP violation needs more than just the momenta of the particles to be measured. In the c.m. frame, there are only two vectors, $\vec{p}_{e^-} - \vec{p}_{e^+}$ and $\vec{p}_f - \vec{p}_{\bar{f}}$. The only scalar observable one can construct out of these is $(\vec{p}_{e^-} - \vec{p}_{e^+}) \cdot (\vec{p}_f - \vec{p}_{\bar{f}})$. This is even under CP . Hence one needs either initial spin or final spin to be observed. Observing the final spin in the case of the top quark is feasible because of the

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fact that the top quark decays before it hadronizes. Several studies have been undertaken to make predictions for the polarization, and for the distributions of the decay distributions in the presence of CP violation in top production and decay.

On the other hand, the presence of TP of the beams would provide one more vector, making it possible to observe CP violating asymmetries without the need to observe final-state polarization. This would mean gain in statistics. Thus, possible CP -odd scalars which can be constructed out of the available momenta and TP are $(\vec{p}_{e^-} - \vec{p}_{e^+}) \times (\vec{s}_{e^-} - \vec{s}_{e^+}) \cdot (\vec{p}_f - \vec{p}_{\bar{f}})$ and $(\vec{s}_{e^-} - \vec{s}_{e^+}) \cdot (\vec{p}_f - \vec{p}_{\bar{f}})$, together with combinations of the above with CP -even scalar products of vectors.

It may be noted that if $\bar{f}=f$, that is, if f is a self-conjugate boson, or a Majorana fermion, possible CP -odd scalars in a process $e^+e^- \rightarrow f+X$ are $(\vec{p}_{e^-} - \vec{p}_{e^+}) \cdot \vec{p}_f$, $(\vec{s}_{e^-} + \vec{s}_{e^+}) \cdot \vec{p}_f$ and $(\vec{p}_{e^-} - \vec{p}_{e^+}) \times (\vec{s}_{e^-} + \vec{s}_{e^+}) \cdot \vec{p}_f$. Of these, observation of the first does not need initial-state polarization, observation of the second is possible with either longitudinally or transversely polarized beams, and the third requires e^+ and e^- transverse polarizations.

We investigate below how new physics could give rise to such CP -odd observables in the presence of TP of the beams, and how the sensitivity of such measurements would compare with the sensitivity to other observables involving TP, or final-state polarization. While our general considerations are valid for any one-particle inclusive final state A in $e^+e^- \rightarrow A+X$, for a concrete illustration we consider the specific process $e^+e^- \rightarrow t\bar{t}$.

One may gain an insight from the elegant and general results of Dass and Ross [11], who listed all possible single-particle distributions from the interference of the electromagnetic contribution with S (scalar), P (pseudoscalar), T (tensor), V (vector) and A (axial-vector) type of neutral current interactions in the presence of arbitrary beam polarization. It may be concluded from the tables in [11] that with only TP, V and A coupling at the e^+e^- vertex cannot give rise to CP -violating asymmetries. Even on generalization to include interference of the Z contribution, we have checked that the same negative result holds. This is true so long as the e^+e^- couple to a vector or axial vector current, even though the coupling of the final state is more general, as for example, of the dipole type. However, S, P and T can give CP -odd contributions like the ones mentioned earlier. These results may also be deduced from some general results for azimuthal distributions given by Hikasa [14].

For vanishing electron mass, S, P, and T couplings at the e^+e^- vertex are helicity violating, whereas V and A couplings are helicity conserving. So with arbitrary longitudinal polarizations, they do not give any interference. Hence new physics appears only in terms quadratic in the new coupling. However, with TP, these interference terms are non-vanishing, and can be studied. Thus, TP has the distinct advantage that it would be able to probe first-order contributions to new physics appearing as S, P and T couplings, in contrast with the case of no polarization, or longitudinal polarization, which can probe only second order contribution from new physics.

II. THE PROCESS $e^+e^- \rightarrow t\bar{t}$

We now consider the specific process $e^+e^- \rightarrow t\bar{t}$. For our purposes, we have found it economical to employ the discussion of Ref. [10], based on the notation and formalism of [9]. The following operators contribute to this process [10]:

$$\begin{aligned} \mathcal{O}_{\ell q}^{(1)} &= \frac{1}{2} (\bar{\ell} \gamma_\mu \ell) (\bar{q} \gamma^\mu q), \\ \mathcal{O}_{\ell q}^{(3)} &= \frac{1}{2} (\bar{\ell} \gamma_\mu \tau^I \ell) (\bar{q} \gamma^\mu \tau^I q), \\ \mathcal{O}_{eu} &= \frac{1}{2} (\bar{e} \gamma_\mu e) (\bar{u} \gamma_\mu u), \\ \mathcal{O}_{\ell u} &= (\bar{\ell} u) (\bar{u} \ell), \\ \mathcal{O}_{qe} &= (\bar{q} e) (\bar{e} q), \\ \mathcal{O}_{\ell q} &= (\bar{\ell} e) \epsilon (\bar{q} u), \\ \mathcal{O}_{\ell q'} &= (\bar{\ell} u) \epsilon (\bar{q} e), \end{aligned} \quad (1)$$

where l, q denote respectively the left-handed electroweak $SU(2)$ lepton and quark doublets, and e and u denote $SU(2)$ singlet charged-lepton and up-quark right-handed fields. τ^I ($I=1,2,3$) are the usual Pauli matrices, and ϵ is the 2×2 anti-symmetric matrix, $\epsilon_{12} = -\epsilon_{21} = 1$. Generation indices are suppressed. The Lagrangian which we use in our following calculations is written in terms of the above operators as [10]:

$$\mathcal{L} = \mathcal{L}^{SM} + \frac{1}{\Lambda^2} \sum_i (\alpha_i \mathcal{O}_i + \text{H.c.}), \quad (2)$$

where α 's are the coefficients which parameterize nonstandard interactions. Such an effective interaction could arise in extensions of SM like multi-Higgs doublet models, supersymmetric standard model through loops involving heavy particles or theories with large extra dimensions.

After Fierz transformation the Lagrangian containing the new-physics four-Fermi operators takes the form

$$\begin{aligned} \mathcal{L}^{4F} &= \sum_{i,j=L,R} \left[S_{ij} (\bar{e} P_i e) (\bar{t} P_j t) + V_{ij} (\bar{e} \gamma_\mu P_i e) (\bar{t} \gamma^\mu P_j t) \right. \\ &\quad \left. + T_{ij} \left(\bar{e} \frac{\sigma^{\mu\nu}}{\sqrt{2}} P_i e \right) \left(\bar{t} \frac{\sigma^{\mu\nu}}{\sqrt{2}} P_j t \right) \right], \end{aligned} \quad (3)$$

with the coefficients satisfying

$$\begin{aligned} S_{RR} &= S_{LL}^*, & S_{LR} &= S_{RL} = 0, \\ V_{ij} &= V_{ij}^*, \\ T_{RR} &= T_{LL}^*, & T_{LR} &= T_{RL} = 0. \end{aligned}$$

In Eq. (3), $P_{L,R}$ are respectively the left- and right-chirality projection matrices. The relation between the coefficients in Eq. (3) and the coefficients α_i of Eq. (2) may be found in [10]. In the above scalar as well as pseudo-scalar interactions are included in a definite combination. Henceforth, we will simply use the term scalar to refer to this combination of scalar and pseudoscalar couplings.

As mentioned earlier, interference between scalar-tensor and SM interactions can only arise in the presence of TP. SM amplitude can, of course, interfere with contributions from vector four-Fermi operators, leading to terms of order $\alpha_i s/\Lambda^2$. However, as far as CP violation is concerned, this interference between SM amplitude and vector amplitude from new physics does not give CP -odd terms in the distribution, even when TP is present. On the other hand, in the presence of TP, the interference between SM amplitude and scalar or tensor contribution does produce CP -odd variables.

Here we concentrate on the process $e^+e^- \rightarrow t\bar{t}$ and examine the CP -violating contribution in the interference of the

SM amplitude with the scalar and tensor four-Fermi amplitudes. We will take the electron TP to be 100% and along the positive or negative x axis, and the positron polarization to be 100%, parallel or antiparallel to the electron polarization. The z axis is chosen along the direction of the e^- . The differential cross sections for $e^+e^- \rightarrow t\bar{t}$, with the superscripts denoting the respective signs of the e^- and e^+ TP, are

$$\frac{d\sigma^{\pm\pm}}{d\Omega} = \frac{d\sigma_{SM}^{\pm\pm}}{d\Omega} \pm \frac{3\alpha\beta^2}{4\pi} \frac{m_t\sqrt{s}}{s-m_Z^2} (c_V^t c_A^e \text{Re } S) \sin\theta \cos\phi, \quad (4)$$

$$\frac{d\sigma^{\pm\mp}}{d\Omega} = \frac{d\sigma_{SM}^{\pm\mp}}{d\Omega} \pm \frac{3\alpha\beta^2}{4\pi} \frac{m_t\sqrt{s}}{s-m_Z^2} (c_V^t c_A^e \text{Im } S) \sin\theta \sin\phi, \quad (5)$$

where

$$\begin{aligned} \frac{d\sigma_{SM}^{+\pm}}{d\Omega} &= \frac{d\sigma_{SM}^{-\mp}}{d\Omega} \\ &= \frac{3\alpha^2\beta}{4s} \left[\frac{4}{9} \left\{ 1 + \cos^2\theta + \frac{4m_t^2}{s} \sin^2\theta \pm \beta^2 \sin^2\theta \cos 2\phi \right\} - \frac{s}{s-m_Z^2} \frac{4}{3} \left\{ c_V^e c_V^t \left(1 + \cos^2\theta + \frac{4m_t^2}{s} \sin^2\theta \pm \beta^2 \sin^2\theta \cos 2\phi \right) \right. \right. \\ &\quad \left. \left. + 2c_A^e c_A^t \beta \cos\theta \right\} + \frac{s^2}{(s-m_Z^2)^2} \left\{ (c_V^e{}^2 + c_A^e{}^2) \left[(c_V^t{}^2 + c_A^t{}^2) \beta^2 (1 + \cos^2\theta) + c_V^t{}^2 \frac{8m_t^2}{s} \right] + 8c_V^e c_A^e c_V^t c_A^t \beta \cos\theta \right. \right. \\ &\quad \left. \left. \pm (c_V^e{}^2 - c_A^e{}^2) (c_V^t{}^2 + c_A^t{}^2) \beta^2 \sin^2\theta \cos 2\phi \right\} \right]. \end{aligned} \quad (6)$$

Here $\beta = \sqrt{1 - 4m_t^2/s}$, and we have defined

$$S \equiv S_{RR} + \frac{2c_A^t c_V^e}{c_V^t c_A^e} T_{RR}, \quad (7)$$

where c_V^i, c_A^i are the couplings of Z to e^-e^+ and $t\bar{t}$, and where we have retained the new couplings to linear order only. In Eq. (7) the contribution of the tensor term relative to the scalar term is suppressed by a factor $2c_A^t c_V^e/c_V^t c_A^e \approx 0.36$. In what follows, we will consider only the combination S , and not S_{RR} and T_{RR} separately.

The differential cross section corresponding to antiparallel e^- and e^+ polarizations, Eq. (5), has the CP -odd quantity

$$\sin\theta \sin\phi \equiv \frac{(\vec{p}_{e^-} - \vec{p}_{e^+}) \times (\vec{s}_{e^-} - \vec{s}_{e^+}) \cdot (\vec{p}_t - \vec{p}_{\bar{t}})}{|\vec{p}_{e^-} - \vec{p}_{e^+}| |\vec{s}_{e^-} - \vec{s}_{e^+}| |\vec{p}_t - \vec{p}_{\bar{t}}|},$$

while the interference term in the case with parallel e^- and e^+ polarizations, Eq. (4), has the CP -even quantity

$$\sin\theta \cos\phi \equiv \frac{(\vec{p}_t - \vec{p}_{\bar{t}}) \cdot (\vec{s}_{e^-} + \vec{s}_{e^+})}{2|\vec{p}_t - \vec{p}_{\bar{t}}|}.$$

We construct the CP -odd asymmetry, which we call the up-down asymmetry as

$$A(\theta) = \frac{\int_0^\pi \frac{d\sigma^{+-}}{d\Omega} d\phi - \int_\pi^{2\pi} \frac{d\sigma^{+-}}{d\Omega} d\phi}{\int_0^\pi \frac{d\sigma^{+-}}{d\Omega} d\phi + \int_\pi^{2\pi} \frac{d\sigma^{+-}}{d\Omega} d\phi} \quad (8)$$

and also the θ -integrated version,

$$A(\theta_0) = \frac{\int_{-\cos \theta_0}^{\cos \theta_0} \int_0^\pi \frac{d\sigma^{+-}}{d\Omega} d \cos \theta d\phi - \int_{-\cos \theta_0}^{\cos \theta_0} \int_\pi^{2\pi} \frac{d\sigma^{+-}}{d\Omega} d \cos \theta d\phi}{\int_{-\cos \theta_0}^{\cos \theta_0} \int_0^\pi \frac{d\sigma^{+-}}{d\Omega} d \cos \theta d\phi + \int_{-\cos \theta_0}^{\cos \theta_0} \int_\pi^{2\pi} \frac{d\sigma^{+-}}{d\Omega} d \cos \theta d\phi}. \tag{9}$$

In the latter, a cutoff on θ has been introduced, so that the limits of integration for θ are $\theta_0 < \theta < \pi - \theta_0$. Using our expressions for the differential cross sections, it is easy to obtain expressions for these asymmetries, and we do not present them here. Such a cutoff in the forward and backward directions is indeed needed for practical reasons to be away from the beam pipe. We can further choose the cutoff to optimize the sensitivity of the measurement.

III. NUMERICAL RESULTS

We now proceed with a numerical study of these asymmetries and the limits that can be put on the parameters using the integrated asymmetry $A(\theta_0)$. We assume that a linear collider operating at $\sqrt{s} = 500$ GeV and the ideal condition of 100% beam polarizations for e^- as well as e^+ . We will comment later on about the result for more realistic polarizations.

In Fig. 1, we plot the SM differential cross section integrated over ϕ , as well as the numerator of $A(\theta)$ of Eq. (8), which is two times the contribution of the interference term (the CP -violating contribution coming from $\text{Im} S$) integrated over ϕ from 0 to π for a value of $\text{Im} S = 1 \text{ TeV}^{-2}$. In Fig. 2 we show the asymmetry $A(\theta)$ for $\text{Im} S = 1 \text{ TeV}^{-2}$ as a function of θ . The asymmetry peaks at about $\theta = 120^\circ$, and takes values as high as 30–40%.

In Fig. 3 we plot, as functions of the cutoff angle θ_0 , the θ -integrated versions of the quantities that are plotted in Fig. 1. The limits of integration are θ_0 and $\pi - \theta_0$. Figure 4 shows the integrated up-down asymmetry $A(\theta_0)$ as a function of θ_0 . The value of $A(\theta_0)$ increases with the cutoff,

because the SM cross section in the denominator of Eq. (9) decreases with cutoff faster than the numerator.

Figure 5 shows the 90% confidence level (C.L.) limits that could be placed on $\text{Im} S$ for an integrated luminosity of $L = 500 \text{ fb}^{-1}$. The limit is the value of $\text{Im} S$ which would give rise to an asymmetry $A_{\text{lim}} = 1.64/\sqrt{L\Delta\sigma}$, where $\Delta\sigma$ is the SM cross section. The limit is relatively insensitive to the cutoff θ_0 until about $\theta_0 = 60^\circ$, after which it increases. A cutoff could be chosen anywhere upto this value. The corresponding limit is about $1.6 \times 10^{-8} \text{ GeV}^{-2}$, after which it gets worse. This limit translates to a value of Λ of the order of 8 TeV, assuming that the coefficients α_i in Eq. (2) are of order 1. The corresponding limit for \sqrt{s} of 800 GeV with the same integrated luminosity is ~ 9.5 TeV.

So far we have assumed 100% TP for both e^+ and e^- beams. We now discuss the effect of realistic TP. Since longitudinal polarizations of 80% and 60% are likely to be feasible respectively for e^- and e^+ beams, we will assume that the same degree of TP will also be possible. We are assuming here that spin rotators that convert longitudinal polarization to TP will not deplete the degree of polarization significantly. Since we use differential cross sections integrated over ϕ at least over the range 0 to π , the polarization dependent terms in the SM contribution, being proportional to $\cos 2\phi$, drop out, as does the second term on the right-hand side of Eq. (4). So far as the up-down asymmetry $A(\theta)$ or $A(\theta_0)$ is concerned, it gets multiplied by a factor $\frac{1}{2}(P_1 - P_2)$ in the presence of degrees of TP P_1 and P_2 for e^- and e^+ beams respectively. For $P_1 = 0.8$ and $P_2 = -0.6$, this means a reduction of the asymmetry by a factor of 0.7. Since the SM

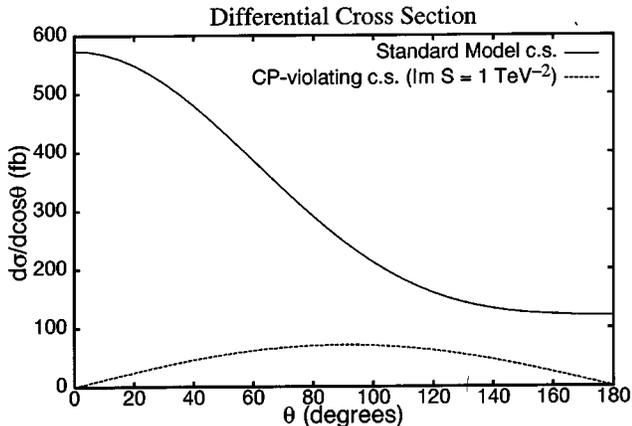


FIG. 1. The SM differential cross section $d\sigma/d \cos \theta$ (fb) (solid line) and the numerator of the asymmetry $A(\theta)$ in Eq. (8) (broken line) as a function of θ . The latter is for $\text{Im} S = 1 \text{ TeV}^{-2}$.

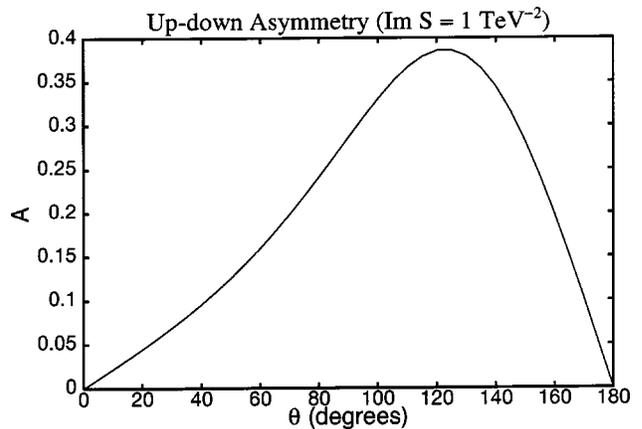


FIG. 2. The asymmetry $A(\theta)$ defined in Eq. (8) as a function of θ for a value of $\text{Im} S = 1 \text{ TeV}^{-2}$.

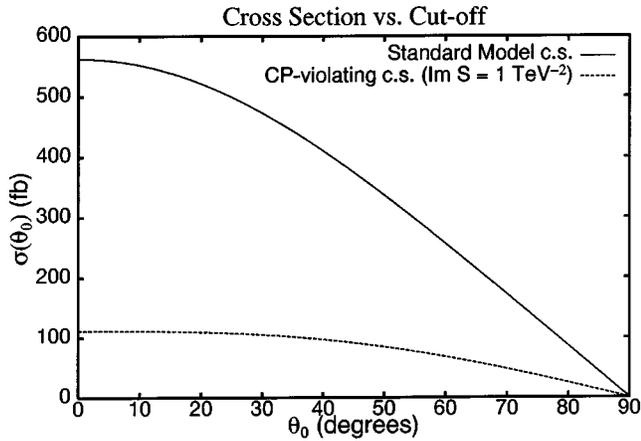


FIG. 3. As in Fig. 1, but now for quantities integrated over θ with a cutoff θ_0 , plotted as a function of θ_0 .

cross section does not change, this also means that the limit on the parameter $\text{Im } S$ goes up by a factor of $1/0.7 \approx 1.4$, and the limit on Λ goes down by a factor of $\sqrt{0.7} \approx 0.84$, to about 6.7 TeV. If the positron beam is unpolarized, however, the sensitivity goes down further.

IV. CONCLUSIONS

In summary, TP can be used to study CP -violating asymmetry arising from the interference of new-physics scalar and tensor interactions with the SM interactions. These interference terms cannot be seen with longitudinally polarized or unpolarized beams. Moreover, such an asymmetry would not be sensitive to new vector and axial-vector interactions (as for example, from an extra Z' neutral boson), or even electric or “weak” dipole interactions of heavy particles, since the asymmetry vanishes in such a case in the limit of vanishing electron mass. Since the asymmetry we consider does not involve the polarization of final-state particles, one expects better statistics as compared to the case when measurement of final-state polarization is necessary.

We have studied the CP -violating up-down asymmetry in the case of $e^+e^- \rightarrow t\bar{t}$ in detail using a model-independent

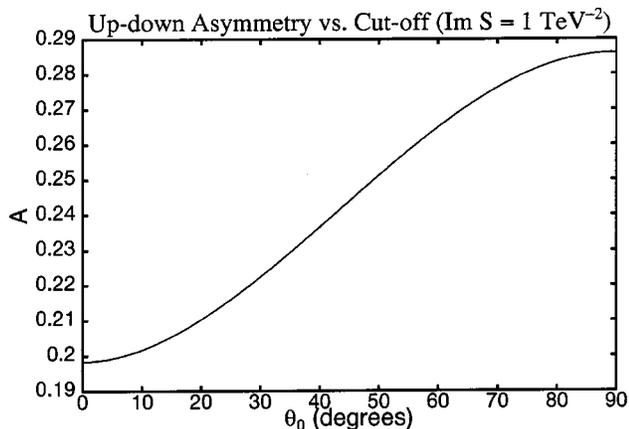


FIG. 4. The asymmetry $A(\theta_0)$ defined in Eq. (9) plotted as a function of θ_0 for $\text{Im } S = 1 \text{ TeV}^{-2}$.

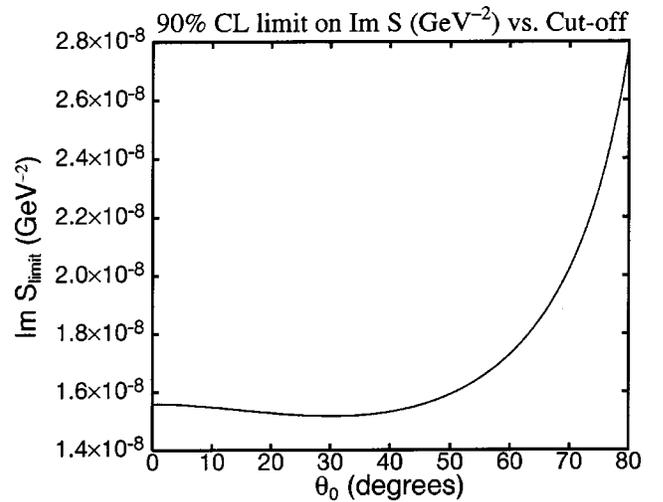


FIG. 5. The 90% C.L. limit that can be obtained on $\text{Im } S$ with an integrated luminosity of 500 fb^{-1} plotted as a function of the cutoff angle θ_0 .

parametrization of new interactions in terms of a four-Fermi effective Lagrangian. We find that a linear collider operating at $\sqrt{s} = 500 \text{ GeV}$ with an integrated luminosity of 500 fb^{-1} would be sensitive to CP -violating new physics scale of about 8 TeV corresponding to a four-Fermi coupling of about $1.6 \times 10^{-8} \text{ GeV}^{-2}$ with fully polarized beams, and somewhat lower scales if the polarization is not 100%.

Present experimental limits on the scale of CP -conserving new physics interactions are of the same order or better than those obtainable for CP -violating interactions as described above. Limits of order 10–20 TeV have been obtained for production of light quarks [15]. The limits are somewhat lower for rare flavor-violating processes [16]. Recently Rizzo [17] has discussed the dependence on linear collider energies and luminosities and on positron polarization of the reach of future experiments on contact interaction searches. Our discussion, while not as exhaustive, extends this in another direction, namely, that of CP violation in the presence of TP.

While it is clear that scalar and tensor effective four-fermion interactions can arise in many extensions of the standard model, definite predictions of their magnitudes are, to our knowledge, not available. However, it is likely that CP -violating box diagrams, which seem to contribute significantly in supersymmetric theory (see for example [18]), may lead to such effective interactions in many extensions of SM. One obvious case where a tensor contribution occurs is when one includes a CP -violating dipole coupling of the electron to γ and Z , and one does expect azimuthal asymmetries in the presence of transverse polarization [19]. However, in view of the strong limits on the electric dipole moment of the electron, the effect will be tiny, and we have not considered it here.

One may ask if there are any naturalness constraints on the parameters of the Lagrangian \mathcal{L} . It is possible to conclude that for the effective theory a constraint may arise from requiring that the one-loop contribution δm_e to the electron mass due to scalar interactions is small compared to m_e . The

electron mass shift would be proportional to the square of the cutoff:

$$\delta m_e \sim \frac{m_t}{8\pi^2} \text{Re } S_{RR} \Lambda^2. \quad (10)$$

It should be noted that such a contribution, if the underlying theory is renormalizable, would be renormalized to zero, and so would not arise in the underlying theory. Secondly, this contribution is independent of $\text{Im } S_{RR}$, which is sought to be constrained from CP -odd asymmetry. However, if from the point of view of naturalness one requires $\delta m_e < m_e$, then the conclusion would be that the $\text{Re } S_{RR}$ must be suppressed by an additional factor $8\pi^2 m_e / m_t \sim 10^{-4}$, independent of the new physics scale Λ .

Constraints could arise on the magnitude of the tensor coupling due to possible contributions to the electron electric and magnetic dipole moments. For example, the real part of the tensor coupling would contribute an amount

$$\frac{m_e m_t}{16\pi^2} \text{Re } T_{RR} \log(\Lambda^2/m_t^2) \quad (11)$$

to the $g-2$ of the electron, which would give a constraint

$$\text{Re } T_{ij} \lesssim 10^{-3} \text{ TeV}^{-2}$$

when we impose the requirement that the additional contribution is less than the experimental uncertainty of about 8×10^{-12} . The imaginary part of the tensor coupling would contribute an amount

$$\frac{m_t}{16\pi^2} \text{Im } T_{RR} \log(\Lambda^2/m_t^2) \quad (12)$$

to the electric dipole moment d_e of the electron, giving a constraint

$$\text{Im } T_{ij} \lesssim 10^{-8} \text{ TeV}^{-2}$$

when we impose the experimental constraint of $d_e \lesssim 10^{-27} e \text{ cm}$. However, there is always the possibility of cancellations between contributions from different four-Fermion couplings, only one of which (viz., the one corresponding to the $t\bar{t}$ final state) contributes to $e^+e^- \rightarrow t\bar{t}$. While such a cancellation may seem ‘‘unnatural,’’ we take the point of view that constraints on individual couplings can only be obtained from direct experimental study of processes like $e^+e^- \rightarrow t\bar{t}$, including the proposal discussed in this work. Note, however, that from all the above, $\text{Im } S_{RR}$ remains completely unconstrained.

We have restricted ourselves mainly to the \sqrt{s} value of 500 GeV. A linear collider operating at other energies would give similar results. In terms of the new physics scale Λ , it is expected that colliders at higher energies would be able to put a better limit on Λ , since the new interactions would be enhanced relative to SM for larger \sqrt{s} , indeed as we have illustrated for the case of $\sqrt{s} = 800$ GeV. In this work, we have combined many simple principles which in our opinion make the results of this investigation particularly compelling.

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