Leptogenesis in the left-right supersymmetric model

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We analyze the effects of the current neutrino data on thermal leptogenesis and $0\nu\beta\beta$ decay in a fully left-right extension of the minimal supersymmetric model. The model has several additional phases compared to the minimal supersymmetric model. These phases appear from *both* the heavy and light neutrino sectors: two CKM-type phases and four Majorana phases which give new contributions to *CP*-violating parameters and leptogenesis. We study observable effects of these phases on leptogenesis in most general neutrino mixing scenarios, with either hierarchical, inverse hierarchical, or quasidegenerate light and heavy neutrinos. We comment on the effects of these scenarios on the $0\nu\beta\beta$ decay. The *CP*-violating phases in both the heavy and light neutrino sectors of the left-right supersymmetric model have unique features, resulting in bounds on heavy neutrino masses different from the minimal scenario in leptogenesis, and which may distinguish the model from other supersymmetric scenarios.

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I. INTRODUCTION

Baryogenesis through leptogenesis is one of the most appealing mechanisms to generate the observed baryon asymmetry in the Universe. Electroweak baryogenesis attempts to generate the observed baryon to entropy ratio through *B*-violating tunneling processes (sphalerons) at the time of the electroweak phase transition. The *CP* violation is due to the asymmetric reflection of the chiral fermions from the bubble wall into the unbroken phase region, where the sphaleron processes convert the chirality into a baryon and lepton asymmetry, with quantum number $B-L$ conserved. This must be a strong first order transition in order to rapidly turn off the sphalerons. The standard model (SM) does not generate enough *CP* violation through the Cabibbo-Kobayashi-Maskawa (CKM) matrix, but various supersymmetric scenarios, such as the minimal supersymmetric standard model (MSSM) and the next-to-MSSM (NMSSM), do.

The interest in leptogenesis has been refueled by the observations of neutrino oscillations. Experimental data in both solar [1] and atmospheric [2] neutrino measurements has provided the first experimental confirmation of physics beyond the standard model (SM). The most commonly accepted explanation for small neutrino masses is provided by the seesaw mechanism [3], in which large Majorana masses for the right-handed neutrinos induce small masses for the light neutrinos. If the neutrinos are massive and mixed, lepton Yukawa interactions are no longer flavor diagonal, and there exists a source of leptonic flavor and *CP* violation, analogous to the CKM mechanism in the quark sector. In the simplest extension of the MSSM, three heavy singlet neutrinos, right-handed neutrinos (RN) are needed [MSSM(RN)] [4].

However MSSM(RN) introduces right-handed singlet neutrinos in a rather *ad hoc* manner, through terms in the Lagrangian, not dictated by any symmetries in the model. Some supersymmetric grand unified theories (SUSY GUT's) alleviate such problems, although again, in SU(5), right-handed neutrinos must be added to the existing theory. The requirement of a viable leptogenesis is a test that any GUT scenario theory must be put through, and such analyses exist for grand unified theories in which the right-handed neutrinos appear naturally, such as SO(10) [5].

In this paper we examine the leptogenesis in the simplest supersymmetric model which accomodates naturally the right-handed neutrinos, the left-right supersymmetric model (LRSUSY) [6–9]. Left-right supersymmetry extends the MSSM gauge group to $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$, which would then break spontaneously to the group $SU(2)_L \times U(1)_Y$ [6]. Originally seen as a natural way to suppress rapid proton decay and as a mechanism for providing small neutrino masses [8], the LRSUSY model can be embedded in a supersymmetric grand unified theory such as SO(10) [10]. Additional support for left-right theories is provided by building realistic brane worlds from type I strings [11].

We have shown previously that LRSUSY provides new sources of leptonic *CP* violation through the flavor structure of the right-handed doublets [12]. Because the model is left-right symmetric, there exists a CKM matrix in the right-handed lepton sector which is in principle different from the corresponding left-handed sector matrix. In addition to a CKM-type *CP*-violating phase, the right-handed neutrino mixing contains Majorana phases.

In this paper we investigate leptogenesis and its constraints on the mass structure and *CP* phases in the light and heavy neutrino sectors. We discuss the parametrization of the leptonic Yukawa couplings, including the effects of *CP*-violating phases and of the renormalization group equations running of soft-breaking terms. We show the results of the analysis in several scenarios, assuming the possibility that either the light or the heavy neutrinos may be hierarchical or (quasi)degenerate, and restrict the phases and the heavy neutrino mass from the requirement of viable leptogenesis in each case.

The paper is organized as follows: We review the LRSUSY model and its sources of leptonic flavor and *CP* violation in Sec. II. In Sec. III we outline each light and heavy neutrino mixings considered and discuss leptogenesis.

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Complementary constraints on the light neutrino sector come from the neutrinoless double beta decay, presented in Sec. IV. In Sec. V we include a numerical analysis and provide some general restrictions on the mass and phase parameters. We conclude in Sec. VI.

II. LEPTONIC PHASES IN THE LEFT-RIGHT SUPERSYMMETRIC MODEL

The left-right supersymmetric model adds supersymmetry to the left-right gauge symmetry group, $SU(2)_L\times SU(2)_R \times U(1)_{B-L}$. It contains three generations of quark and lepton chiral superfields and left and right gauge bosons [8]. The Higgs sector consists of two Higgs bidoublets, $\Phi_1(\frac{1}{2},\frac{1}{2},0)$ and $\Phi_2(\frac{1}{2},\frac{1}{2},0)$, required to give nonvanishing Cabibbo-Kobayashi-Maskawa quark mixing, and of Higgs triplet fields $\Delta_L(1,0,2)$ and $\Delta_R(0,1,2)$, introduced to provide spontaneous symmetry breaking of the group $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$. Triplets rather than doublets are preferred because they provide an implementation of the seesaw mechanism [7]. In supersymmetry, the number of triplets must be doubled. Thus new triplets $\delta_L(1,0,-2)$ and $\delta_R(0,1,-2)$, with quantum number $B-L=-2$, are introduced to ensure anomaly cancellation in the fermionic sector. The superpotential of the LRSUSY model is

$$W=\mathbf{Y}_Q^{(i)}Q^T\tau_2\Phi_i\tau_2Q^c+\mathbf{Y}_L^{(i)}L^T\tau_2\Phi_i\tau_2L^c+i\mathbf{Y}_{LR}(L^T\tau_2\Delta_L L + L^{cT}\tau_2\Delta_R L^c)+M_{LR}[Tr(\Delta_L\delta_L+\Delta_R\delta_R)] + \mu_{ij}Tr(\tau_2\Phi_i^T\tau_2\Phi_j)+W_{NR} \tag{1}$$

with W_{NR} possible nonrenormalizable terms arising from higher scale physics or Planck scale effects [13]. In Eq. (1), $\mathbf{Y}_Q$ and $\mathbf{Y}_L$ are the Yukawa couplings for the quarks and leptons with bidoublet Higgs bosons, respectively, and $\mathbf{Y}_{LR}$ is the coupling for the leptons and triplet Higgs bosons. Left-right symmetry forces all $\mathbf{Y}$ matrices to be Hermitean in the generation space and $\mathbf{Y}_{LR}$ matrix to be symmetric: $\mathbf{Y}_{Q,L}^i = \mathbf{Y}_{Q,L}^{i\dagger}$, $\mathbf{Y}_{LR}=\mathbf{Y}_{LR}^t$.

Neutral Higgs fields acquire nonzero vacuum expectation values (VEV's) through spontaneous symmetry breaking of both parity and $SU(2)_R$:

$$\langle\Delta\rangle_L=0,\quad \langle\Delta\rangle_R=\begin{pmatrix}0 & 0\\ v_R & 0\end{pmatrix},$$

and

$$\langle\Phi\rangle_{1,2}=\begin{pmatrix}\kappa_{1,2} & 0\\ 0 & \kappa'_{1,2}e^{i\omega}\end{pmatrix}.$$

We describe briefly leptonic mixing in LRSUSY. After symmetry breaking, the neutrino mass Lagrangian becomes

$$-2\mathcal{L}_{\rm mass}=\overline{\nu_L^c}M_\nu\nu_R+\overline{\nu_R^c}M_\nu^*\nu_L^c. \tag{2}$$

There are 6 weak neutrino eigenstates which form

$$N_R=\begin{pmatrix}\nu_R^c\\ \nu_R\end{pmatrix}\qquad \nu_R^c=C\overline{\nu_L}^T. \tag{3}$$

To find the neutrino mass eigenstates, we perform a unitary transformation: $\nu_R=U\hat{\nu}_R$, such that $U^TM_\nu U=\hat{M}_\nu$ is a diagonal, positive mass matrix, and

$$\mathcal{L}_{\rm mass}=-\frac{1}{2}\overline{\hat{\nu}_L^c}\hat{M}_\nu\hat{\nu}_R+h.c\equiv-\frac{1}{2}\overline{N}\hat{M}_\nu N, \tag{4}$$

with $N=\hat{\nu}_R+\hat{\nu}_L^c\equiv\hat{\nu}_R+C\overline{\hat{\nu}_R}^T$, and

$$U=\begin{pmatrix}U_L^*\\ U_R\end{pmatrix}, \tag{5}$$

where $\nu_L=U_L\hat{\nu}_L^c\equiv U_L(P_LN)$ and $\nu_R=U_R\hat{\nu}_R\equiv U_R(P_RN)$. Similarly, the charged lepton mass matrix is formally diagonalized by the unitary transformation: $U_L^{l\dagger}M_lU_R^l=\hat{M}_l$, with $\hat{M}_l$ a diagonal,positive 3×3 mass matrix. The physical lepton fields are $l_{L,R}=U_{L,R}^l\hat{l}_{L,R}$ and we define the leptonic Cabibbo-Kobayashi-Maskawa matrix as

$$K_L^{CKM\ \dagger}=U_L^\dagger U_L^l, \tag{6}$$

$$K_R^{CKM\ \dagger}=U_R^\dagger U_R^l. \tag{7}$$

We work in the basis in which lepton mass matrices are diagonal to a high level of precision; thus U_L^l and U_R^l are proportional to the unit matrix. The mass matrix for the light neutrino states M_ν can then be diagonalized by the unitary matrix U_L:

$$U_L^TM_\nu U_L=M_\nu^D, \tag{8}$$

where $M_\nu^D={\rm diag}(m_{\nu_1},m_{\nu_2},m_{\nu_3})$. Given the fact that neutrinos are Majorana particles, the U_L matrix can be expressed as

$$U_L=K_L^{CKM}(\theta,\delta)P(\phi), \tag{9}$$

where $P(\phi)={\rm diag}(e^{-i\phi_1},e^{-i\phi_2},1)$ and

$$K_L^{CKM}(\theta,\delta)=\begin{pmatrix}c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta}\\ -c_{23}s_{12}-s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12}-s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13}\\ s_{23}s_{12}-c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12}-c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13}\end{pmatrix}, \tag{10}$$

with $c(s)_{ij}=\cos(\sin)\theta_{ij}$. The mixing angles θ_{ij} and the *CP*-violating phase δ can be measured in neutrino oscillation experiments.

A similar analysis for the heavy neutrino sector yields the matrix U_R which can be expressed as

$$U_R=K_R^{CKM}(\beta,\sigma)P(\psi), \tag{11}$$

where $P(\psi)=\mathrm{diag}(1,e^{-i\psi_2},e^{-i\psi_3})$. $K_R^{CKM}(\beta,\sigma)$ has the same form as the CKM matrix in the left-hand sector, but is a function of the independent (unknown) angles β_{ij} and σ. Thus the neutrino sector of the LRSUSY contains four Majorana and two Dirac phases (three in the light and three in the heavy neutrino sector), which is realization of the conventional seesaw model [14]. Intergenerational and left-right slepton mixing are responsible for the off-diagonal nature of the matrices, for flavor and *CP* violation and thus for leptogenesis. Using the Casas and Ibarra parametrization for Yukawa couplings we write [4]

$$\begin{aligned}(Y_L)_{ki}&=\frac{1}{v\sin\beta}\mathrm{diag}(\sqrt{M_{N_1}},\sqrt{M_{N_2}},\sqrt{M_{N_3}})\\&\quad\times R_{kl}\mathrm{diag}(\sqrt{m_1},\sqrt{m_2},\sqrt{m_3})(U_L^\dagger)_{li},\\(Y_{LR})_{ki}&=\frac{1}{2v_R}(U_R)^*_{kl}\mathrm{diag}(M_{N_1},M_{N_2},M_{N_3})(U_R^\dagger)_{li},\end{aligned} \tag{12}$$

with R_{kl} an auxiliary complex orthogonal matrix. Explicit expressions for the Yukawa couplings as functions of neutrino masses and mixings can then be obtained for various assumptions about light and heavy neutrino mixings and hierarchy. We use relationships already established between the mixing in the heavy and light neutrino sectors in models with the seesaw mechanism.

III. LEPTOGENESIS IN LRSUSY

Leptogenesis is generated by the *CP* asymmetry which appears in the interference between tree-level and one-loop heavy Majorana neutrino decays. The *CP* asymmetry produced by the out-of-equilibrium decays of each of the heavy neutrinos N_i in LRSUSY is given by [15]

$$\begin{aligned}\epsilon_i&=\sum_k\frac{\Gamma(N_i\to l_k\Phi^\star)-\Gamma(N_i\to\overline{l}_k\Phi)}{\Gamma(N_i\to l_k\Phi^\star)+\Gamma(N_i\to\overline{l}_k\Phi)}\\&=-\frac{1}{8\pi}\sum_k\frac{\mathrm{Im}(Y_NY_N^\dagger)^2_{ik}}{|Y_N^{ik}|^2}\sqrt{x_k}\left[\ln\left(1+\frac{1}{x_k}\right)+\frac{2}{x_k-1}\right]\\&\quad-\frac{1}{2\pi}\sum_{k,l}\frac{\mathrm{Im}[(Y_N)^{ik}(Y_N)^{il}(Y^*_{LR})^{kl}\mu]}{|(Y_N)^{ik}|^2M_{N_i}}\\&\quad\times\left[1-\frac{1}{x_{k\Delta}}\ln(1+x_{k\Delta})\right],\end{aligned} \tag{13}$$

where $x_k=(M_{N_k}/M_{N_i})^2$, $x_{k\Delta}=(M_{N_k}/M_\Delta)^2$, and $Y_N=(\kappa_1\mathbf{Y}_L^1+\kappa_2'\mathbf{Y}_L^2)/(\kappa_1^2+\kappa_2'^2)$ is the Dirac neutrino Yukawa coupling. The first term of the asymmetry is given by the interference of tree and one loop level of the ordinary vertex and self-energy diagrams involving another (virtual) right-handed neutrino. The Yukawa coupling Y_N appearing in the formula is the one responsible for the neutrino Dirac mass. The second term in the contribution is a LRSUSY specific term and comes from a virtual scalar triplet Higgs field in the loop [16] coupling to the leptons and is a new contribution to lepton asymmetry in LRSUSY. Additionally in LRSUSY, one can obtain a lepton *CP* asymmetry from the decay of each of the triplet Higgs Δ_L bosons [15]:

$$\begin{aligned}\epsilon_\Delta&=2\sum_k\frac{\Gamma(\Delta_L^*\to l_k+l_k)-\Gamma(\Delta_L\to\overline{l}_k+l_k)}{(\Delta_L^*\to l_k+l_k)+\Gamma(\Delta_L\to\overline{l}_k+l_k)}\\&=\frac{1}{8\pi}\sum_k M_{N_k}\frac{\sum_{il}\mathrm{Im}[(Y_N^*)_{ik}(Y_N^*)_{lk}(Y_{LR})_{li}\mu^*]}{\sum_{ij}|(Y_{LR})_{ij}|^2M_\Delta^2+|\mu|^2}\\&\quad\times\ln\left(1+\frac{1}{x_{k\Delta}}\right).\end{aligned} \tag{14}$$

Note that in LRSUSY the leptogenesis depends on a different set of Yukawa couplings from the lepton flavor violating decays, unlike MSSM [17].

The above formula is valid for the case in which the heavy neutrinos have all different masses. If the right-handed neutrinos are completely degenerate, the generated lepton asymmetry is zero [18]. But if only two of the heavy neutrinos are approximately degenerate in mass $M_{N_i}\simeq M_{N_j}$, the *CP* asymmetry is enhanced due to self-energy contributions and the above formula must be modified to include these contributions [19,20]:

$$\begin{aligned}\epsilon_i&\simeq\left\{-\frac{1}{8\pi}\sum_{k\neq i}\frac{\mathrm{Im}(Y_NY_N^\dagger)^2_{ik}}{|Y_N^{ik}|^2}\right.\\&\quad\left.-\frac{1}{2\pi}\sum_{k,l\neq i}\frac{\mathrm{Im}[(Y_N)^{ik}(Y_N)^{il}(Y^*_{LR})^{kl}\mu]}{|(Y_N)^{ik}|^2M_{N_i}}\right\}\\&\quad\times\frac{\Delta M_N^2M_{N_i}\Gamma_{N_k}}{(\Delta M_N^2)^2+M_{N_i}^2\Gamma_{N_k}^2},\end{aligned} \tag{15}$$

where $\Delta M_N^2=M_{N_i}^2-M_{N_k}^2$ and $\Gamma_{N_i}=[(\Sigma_{k\neq i}|Y_N^{ik}|^2)/8\pi]M_{N_i}$ is the decay width of the ith heavy neutrino. The lepton asymmetry y_L is related to the *CP* asymmetry through the relation

$$y_L = \frac{n_L - n_{\bar{L}}}{s} = d\frac{\epsilon_1}{g_\star} \tag{16}$$

with $g_\star$ the effective number of degrees of freedom ($g_\star \simeq 220$ in LRSUSY) and d the dilution factor which accounts for the washout processes. The produced lepton asymmetry y_L is converted into a net baryon asymmetry y_B through $(B+L)$-violating sphaleron processes [14]:

$$y_B = \xi y_{B-L} = \frac{\xi}{\xi - 1} y_L, \quad \xi = \frac{8N_f + 4N_\Phi}{22N_f + 13N_\Phi} \tag{17}$$

with N_f and N_Φ the number of fermion families and Higgs doublets, respectively, which gives

$$y_B \simeq -\frac{1}{2} y_L \tag{18}$$

The determination of the dilution factor involves the integration of the full set of Boltzmann equations [21].

For the purpose of investigating leptogenesis, we consider the most general cases of hierarchical, inversely hierarchical and degenerate neutrinos, and we distinguish the following cases:

(1) Nondegenerate ν_R and hierarchical ν_L:

$$m_1 \approx 0, \quad m_2 \approx \sqrt{\Delta m^2_{sol}}, \quad m_3 \approx \sqrt{\Delta m^2_{atm}},$$

$$M_{N_1} : M_{N_2} : M_{N_3} = \epsilon_N^3 : \epsilon_N^2 : 1. \tag{19}$$

(2) Nondegenerate ν_R and inversely hierarchical ν_L:

$$m_3 \approx 0, \quad m_1 \simeq m_2 \simeq \sqrt{\Delta m^2_{atm}},$$

$$M_{N_1} : M_{N_2} : M_{N_3} = \epsilon_N^3 : \epsilon_N^2 : 1. \tag{20}$$

(3) Nondegenerate ν_R and (quasi)degenerate ν_L:

$$m_1, \quad m_2 \approx m_1 + \frac{1}{2m_1}\Delta m^2_{sol}, \quad m_3 \approx m_1 + \frac{1}{2m_1}\Delta m^2_{atm},$$

$$M_{N_1} : M_{N_2} : M_{N_3} = \epsilon_N^3 : \epsilon_N^2 : 1. \tag{21}$$

(4) Quasidegenerate ν_R and hierarchical ν_L:

$$m_1 \approx 0, \quad m_2 \approx \sqrt{\Delta m^2_{sol}}, \quad m_3 \approx \sqrt{\Delta m^2_{atm}},$$

$$M_{N_1} \approx M_{N_2} < M_{N_3}. \tag{22}$$

(5) Quasidegenerate ν_R and inversely hierarchical ν_L:

$$m_3 \approx 0, \quad m_1 \simeq m_2 \simeq \sqrt{\Delta m^2_{atm}},$$

$$M_{N_1} \approx M_{N_2} < M_{N_3}. \tag{23}$$

(6) Quasidegenerate ν_R and (quasi)degenerate ν_L:

$$m_1, \quad m_2 \approx m_1 + \frac{1}{2m_1}\Delta m^2_{sol}, \quad m_3 \approx m_1 + \frac{1}{2m_1}\Delta m^2_{atm},$$

$$M_{N_1} \approx M_{N_2} < M_{N_3}. \tag{24}$$

with ϵ_N the hierarchy parameter in the heavy neutrino sector. By considering $\epsilon_N < 1 (>1)$ one could analyze the case for hierarchical (and inverse hierarchical) ordering of heavy neutrinos.

Relationships between light and heavy neutrino masses and mixings have been obtained by inverting the relationships for the seesaw mechanism, and their effects were analyzed previously in lepton flavor conserving and violating decays [12]. We use these relationships to express matrix elements of $(m^{D\dagger} m^D)$ in terms of the light and heavy neutrino masses and mixings. These cases correspond to the following expressions for the Yukawa couplings which appear in the asymmetry:

$$(1)(Y_N Y_N^\dagger)_{ij} \approx \frac{M_{N_3}}{v^2 \sin^2\beta}\sqrt{\Delta m^2_{atm}}\left(\sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}(U_L)_{i2}(U_L)^*_{j2}\frac{M_{N_2}}{M_{N_3}} + (U_L)_{i3}(U_L)^*_{j3}\right),$$

$$[(Y_N)_{ik}(Y_N)_{il}(Y^*_{LR})_{kl}] \approx \frac{M^2_{N_3}}{2v_R v^2 \sin^2\beta}\sqrt{\Delta m^2_{atm}}\left(\sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}(U_L)_{2k}(U_L)_{2l}(U_R)^*_{kl}\frac{M^2_{N_2}}{M^2_{N_3}} + (U_L)_{3k}(U_L)_{3l}(U_R)^*_{kl}\right),$$

$$(2)(Y_N Y_N^\dagger)_{ij} \approx \frac{M_{N_3}}{v^2 \sin^2\beta}\Delta m^2_{atm}\left((U_L)_{i2}(U_L)^*_{j2}\frac{M_{N_2}}{M_{N_3}} + (U_L)_{i3}(U_L)^*_{j3}\right),$$

$$[(Y_N)_{ik}(Y_N)_{il}(Y^*_{LR})_{kl}] \approx \frac{M^2_{N_3}}{2v_R v^2 \sin^2\beta}\Delta m^2_{atm}\left((U_L)_{2k}(U_L)_{2l}(U_R)^*_{kl}\frac{M^2_{N_2}}{M^2_{N_3}} + (U_L)_{3k}(U_L)_{3l}(U_R)^*_{kl}\right),$$

$$(3)(Y_N Y_N^\dagger)_{ij} \approx \frac{M_{N_3}}{v^2 \sin^2\beta}\left[m_1\delta_{ij} + \frac{\Delta m^2_{atm}}{2m_1}\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}(U_L)_{i2}(U_L)^*_{j2}\frac{M_{N_1}}{M_{N_3}} + (U_L)_{i3}(U_L)^*_{j3}\right)\right],$$

$$[(Y_N)_{ik}(Y_N)_{il}(Y^*_{LR})_{kl}] \approx \frac{M^2_{N_3}}{v_R v^2 \sin^2\beta}\left[m_1 \delta_{kl}(U_R)^*_{kl}\frac{M_{N_1}}{M_{N_3}} + \frac{\Delta m^2_{atm}}{2m_1}\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}(U_L)_{2k}(U_L)_{2l}(U_R)^*_{kl}\frac{M^2_{N_2}}{M^2_{N_3}} + (U_L)_{3k}(U_L)_{3l}(U_R)^*_{kl}\right)\right],$$

$$(4)(Y_N Y^\dagger_N)_{ij} \approx \frac{M_{N_3}}{v^2\sin^2\beta}\sqrt{\Delta m^2_{atm}}\left(\sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}(U_L)_{i2}(U_L)^*_{j2}\frac{M_{N_2}}{M_{N_3}} + (U_L)_{i3}(U_L)^*_{j3}\right), \tag{25}$$

$$[(Y_N)_{ik}(Y_N)_{il}(Y^*_{LR})_{kl}] \approx \frac{M_{N_3}M_{N_1}}{2v_R v^2\sin^2\beta}\sqrt{\Delta m^2_{atm}}\left(\sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}(U_L)_{2k}(U_L)_{2l}(U_R)^*_{kl} + (U_L)_{3k}(U_L)_{3l}(U_R)^*_{kl}\frac{M_{N_3}}{M_{N_1}}\right),$$

$$(5)(Y_N Y^\dagger_N)_{ij} \approx \frac{M_{N_3}}{v^2\sin^2\beta}\Delta m^2_{atm}\left((U_L)_{i2}(U_L)^*_{j2}\frac{M_{N_2}}{M_{N_3}} + (U_L)_{i3}(U_L)^*_{j3}\right),$$

$$[(Y_N)_{ik}(Y_N)_{il}(Y^*_{LR})_{kl}] \approx \frac{M_{N_3}M_{N_1}}{2v_R v^2\sin^2\beta}\Delta m^2_{atm}\left((U_L)_{2k}(U_L)_{2l}(U_R)^*_{kl} + (U_L)_{3k}(U_L)_{3l}(U_R)^*_{kl}\frac{M_{N_3}}{M_{N_1}}\right),$$

$$(6)(Y_N Y^\dagger_N)_{ij} \approx \frac{M_{N_3}}{v^2\sin^2\beta}\left[m_1\delta_{ij} + \frac{\Delta m^2_{atm}}{2m_1}\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}(U_L)_{i2}(U_L)^*_{j2}\frac{M_{N_1}}{M_{N_3}}(U_L)_{i3}(U_L)^*_{j3}\right)\right],$$

$$[(Y_N)_{ik}(Y_N)_{il}(Y^*_{LR})_{kl}] \approx \frac{M_{N_3}M_{N_1}}{v_R v^2\sin^2\beta}\left[m_1\delta_{kl}(U_R)^*_{kl} + \frac{\Delta m^2_{atm}}{2m_1}\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}(U_L)_{2k}(U_L)_{2l}(U_R)^*_{kl} + (U_L)_{3k}(U_L)_{3l}(U_R)^*_{kl}\frac{M_{N_3}}{M_{N_1}}\right)\right],$$

with $v^2 = \kappa_1^2 + \kappa_2^2$, $\tan\beta = \kappa_1/\kappa_2$, and where the ratios of the right-handed neutrino masses and mixing elements can be approximated as in Ref. [12].

Examining the combinations of $(Y_N Y^\dagger_N)_{ij}$ $[(Y_N)_{ik}(Y_N)_{il}(Y^*_{LR})_{kl}]$, we notice that several complex phases from both the heavy neutrino and light lepton sectors are responsible for leptogenesis. In general, in the cases where the triplet Higgs boson and the lightest heavy neutrino have the same mass and same order of magnitude couplings, all three types of asymmetries are important. In what follows we investigate the consequences of leptogenesis in each of the proposed schemes, assuming one process dominates over the others. We are looking in particular for restrictions on the *CP* phases in the model and the mass of the lightest heavy neutrino M_{N_1} [22]. It is known that the *CP* asymmetry ϵ_1 depends on the mass of the lightest heavy neutrino. This is true for this model, though we expect the dependence to be different in LRSUSY and in each of the scenarios studied, as we will show in the analysis.

IV. NEUTRINOLESS DOUBLE BETA DECAY IN LRSUSY

While neutrino oscillations measure the differences in mass squares of neutrinos, and their mixing angles, the neutrinoless double beta ($\beta\beta_{0\nu}$) decay measures a particular combination of neutrino masses and mixing elements, the so-called effective Majorana mass parameter:

$$|m_{ee}| \equiv |U^\star_{Lei} m_{\nu_i} U^\dagger_{Lie}|. \tag{26}$$

The neutrinoless double beta decay is an important process since it would provide direct evidence for total lepton number violation. A direct observation of such a process would imply that neutrinos are Majorana particles. The size of the double beta decay depends on the presence of Majorana *CP*-violating phases in the light sector only; thus it could set independent constraints on the size these phases. Currently, the most stringent constraint on $|m_{ee}|$ comes from the ^{76}Ge Heidelberg-Moscow experiment [23]:

$$|m_{ee}| < 0.35 \text{ eV}. \tag{27}$$

In the following section, we investigate the constraints coming from the neutrinoless double beta decay in addition to the leptogenesis.

V. NUMERICAL ANALYSIS

Due to the lepton doublet structure in LRSUSY several mixing angles are introduced: three light neutrino mixing angles θ_{ij}, three *CP*-violating light neutrino mixing angles δ and $\phi_{1,2}$, and similarly, for the heavy neutrino sector, the mixing angles β_{ij} and the *CP*-violating angles σ and $\psi_{2,3}$. In addition to these, there are three charged lepton, three Higgs triplet bosons, three light neutrino, and three heavy neutrino mass parameters. We study the effects of the masses and mixings in the neutrino sector on leptogenesis within the ordering scenarios outlined in the previous section.

In numerical evaluations, we fix the gauge and Yukawa couplings at M_Z, then use the renormalization group equations (RGE's) up to the scale M_{W_R}, where we introduce the heavy neutrinos, fix their masses, and the light neutrino masses and mixing. We assume universal soft-symmetry breaking at the GUT scale $M_{GUT}\sim 2\times 10^{16}$ GeV. We then run all Yukawa coupling matrices from M_{W_R} to M_{GUT} using the renormalization group equations for LRSUSY [24]. We assume universality of the soft-supersymmetry breaking terms, then run all parameters back to M_{W_R}, where the heavy neutrinos and sneutrinos decouple and LRSUSY breaks to MSSM. We do not consider any intermediate scales between M_{W_R} and M_Z. We then obtain all parameters by running the RGE's to M_Z.

For the light neutrinos masses and mixings, we fix the parameters using the large-mixing angle (LMA-MSW) solution as [25]

$$\tan^2\theta_{12}=0.36, \quad \tan^2\theta_{23}=1.4, \quad \text{and} \quad \tan^2\theta_{13}=0.005, \tag{28}$$

and, correspondingly, the median values for Δm^2_{atm} and Δm^2_{sol}:

$$\Delta m^2_{atm}\approx\Delta m^2_{23}=3\times 10^{-3}\ \text{eV}^2,$$
$$\Delta m^2_{sol}\approx\Delta m^2_{12}=3\times 10^{-5}\ \text{eV}^2. \tag{29}$$

In what follows we assume $M_\Delta > M_{N_1}$, a scenario favored by the seesaw mechanism. In the case in which $M_{N_1}\ll M_\Delta$ and the triplet couplings to the leptons are negligible with respect to the leading right-handed neutrino contributions, the triplet has a negligible effect for both neutrino masses and leptogenesis. The situation is similar to the MSSM with right-handed neutrinos and has been studied extensively in the literature [17].

The case in which $M_{N_1}\ll M_\Delta$, but the triplet couplings to the leptons are significant, the contribution of the triplet is important, and comparable to the right-handed neutrino contribution. We will consider this case below, and look for differences between this case and MSSM with right-handed neutrinos. In order to obtain reasonable values for the triplet Yukawa coupling $\mathbf{Y}_{LR}$, we choose $v_R=3.2\times 10^{14}$ GeV. This ensures that the largest Yukawa coupling of the triplet Δ_R^{--} is within experimental bounds, $(Y_{LR})_{\tau\tau}\leq 0.8$. The drawback of such heavy scales is that the W_R boson is superheavy, $M_{W_R}\approx 1.5\times 10^{11}$ TeV, but this is known to be the scenario favored by the seesaw mechanism. We take $M_\Delta\approx\mathcal{O}(M_{W_R})$.

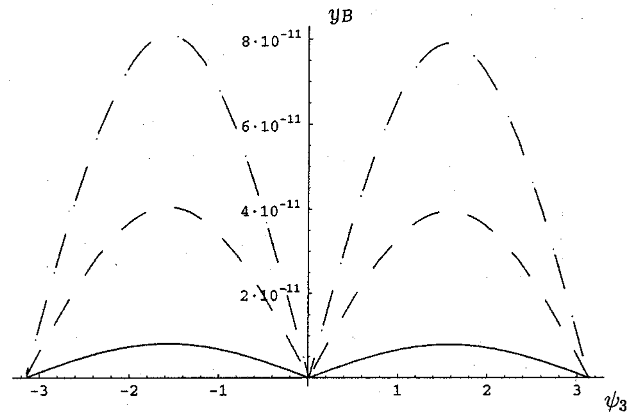

FIG. 1. The dependence of baryon asymmetry y_B on the Majorana phase ψ_3 (from the heavy right-handed neutrino mixing) for $\delta=\pi/4$ (the CKM-like phase from the left-handed light neutrino matrix), for several values of the lightest heavy neutrino mass $M_{N_1}=1\times 10^{12}$ GeV (solid curve), $M_{N_1}=5\times 10^{12}$ GeV (dash curve), and $M_{N_1}=10^{11}$ GeV (dot-dashed curve), for scenario (1).

In all scenarios explored, the angles θ_{12}, θ_{23}, θ_{13} from the light neutrino mixing matrix, and β_{12}, β_{23}, β_{13} from the heavy neutrino mixing matrix are set by the solar and atmospheric neutrino constraints and the inverse seesaw relationships.

We first study the cases for nondegenerate heavy neutrino masses, where we assume $M_{N_1}<M_{N_2}<M_{N_3}$ [scenarios (1)–(3)]. We perform a numerical computation of the baryon asymmetry induced by the lepton asymmetry, including all the *CP*-violating phases in the light and heavy neutrino sectors. The dependence on the certain phases is more pronounced in some scenarios and less in others. For the hierarchical light spectrum case (1), we have $m_1\ll m_2 \simeq\sqrt{\Delta m^2_{sol}}\ll m_3\simeq\sqrt{\Delta m^2_{atm}}$. We show the dependence of the baryon asymmetry y_B on phase ψ_3 (Majorana phase from the heavy right-handed neutrino mixing) for $\delta=\pi/4$ (the CKM-like phase from the left-handed light neutrino matrix) in Fig. 1, for several values of the lightest heavy neutrino mass $M_{N_1}=(1,5,10)\times 10^{12}$ GeV. The asymmetry is practically independent of the other phases. The induced baryon asymmetry is required to be in the range [26]

$$1.7\times 10^{-11}\leq y_B\leq 8.1\times 10^{-11}. \tag{30}$$

Imposing these limits, the mass of the lightest heavy neutrino is constrained to be $M_{N_1}\geq 2.3\times 10^{12}$ GeV, about one order of magnitude higher than the value obtained in MSSM with right-handed neutrinos [27].

For the inverse hierarchical light spectrum, case (2), we assume $m_3\ll m_1\simeq m_2\simeq\sqrt{\Delta m^2_{atm}}$. In this case the *CP* asymmetry depends on the Majorana phase ϕ_2 from the left-handed neutrino sector, in addition to dependence on the

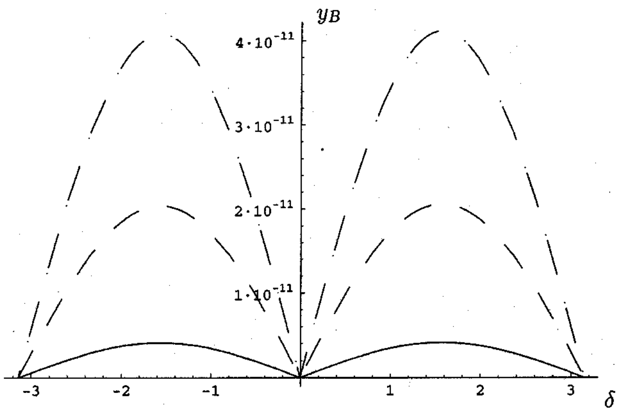

FIG. 2. The dependence of the induced baryon asymmetry y_B on the light CKM phase δ for $\phi_2 = \psi_3 = \pi/4$ for several values of the lightest heavy neutrino mass $M_{N_1} = 10^6$ GeV (solid curve), $M_{N_1} = 5\times10^6$ GeV (dash curve), and $M_{N_1} = 10^7$ GeV (dot-dashed curve), for scenario (3).

phases δ and ψ_3 from the hierarchical case. Imposing the limits for the acceptable range of the baryon asymmetry sets a limit $M_{N_1} \geqslant 7\times10^{12}$ GeV, about a factor of 3 higher than the limit obtained for the normally ordered light neutrino spectrum. The same phenomenon is observed for the minimal seesaw mechanism and in SO(10) GUT scenario [6], though there the factor is closer to 10, because the dependence on phases is less complicated than in LRSUSY. The dependence on the phases is similar to case (1); thus we do not show it in a separate plot.

Next we analyze the case in which the heavy neutrinos are nondegenerate, but the light neutrinos are (quasi)degenerate $m_1; m_2 \approx m_1 + (1/2m_1)\Delta m^2_{sol}$; $m_3 \approx m_1 + (1/2m_1)\Delta m^2_{atm}$. In this case, the observed leptonic *CP* asymmetry depends nontrivially on the CKM-like phase δ, the left Majorana phase ϕ_2, and the corresponding phases in the right-handed sector σ and ψ_3. The bound on the lightest heavy neutrino mass is considerably relaxed, $M_{N_1} \geqslant 3.6\times10^6$ GeV, largely due to the smallness of the Yukawa couplings for this case. We show in Fig. 2 the dependence of the induced baryon asymmetry on the light CKM phase δ for $\phi_2 = \psi_3 = \pi/4$ for several values of the lightest heavy neutrino mass $M_{N_1} = (1,5,10)\times10^6$ GeV. In Fig. 3 we plot the baryon asymmetry in terms of both light-sector phases δ and ϕ_2 (the dominant phase parameters) for $M_{N_1} = 10^7$ GeV. Asymmetries of the required magnitude can be easily achieved in this scenario with relatively light heavy neutrinos.

In the context of thermal leptogenesis, the bound on the lightest heavy neutrino mass usually obtained is $M_{N_1} \geqslant 10^8$ GeV. This bound is barely compatible with the reheating temperature bound, $T_R \leqslant 10^8 - 10^9$ GeV, required in several supergravity models in order to avoid gravitino overproduction [28]. This bound is satisfied in the scenario (3), but not in the other two cases in which the heavy neutrinos are nondegenerate. It has been suggested that, to overcome this problem, one should consider the decays of two heavy neutrinos which are quasidegenerate in mass $M_{N_1} \approx M_{N_2}$. In this

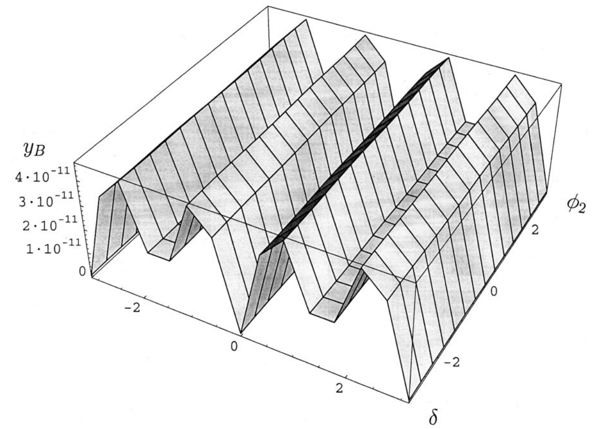

FIG. 3. The dependence of the baryon asymmetry y_B on the light CKM phase δ and the Majorana phase ϕ_2 for $M_{N_1} = 10^7$ GeV, for scenario (3).

case, the required baryon asymmetry can be produced by moderately heavy right-handed neutrino $M_{N_1} \approx M_{N_2} \leqslant 10^8$ GeV [19] in MSSM with right-handed neutrinos. In the next three cases we investigate this scenario in LRSUSY.

In the case of quasidegenerate heavy neutrinos $M_{N_1} \simeq M_{N_2} < M_{N_3}$, the Yukawa coupling which spans the 33 element in the heavy neutrino mass matrix differs from the ones in the 11, 22 elements, giving rise to a different mass for the N_3 neutrino than the N_1, N_2 neutrino masses. In general, the dilution factor entering the Boltzmann equation can differ from the one in the nondegenerate case. We assume, following Ref. [20], that the same formula holds for the washout factor. We analyze first the scenario in which the left-handed neutrinos are hierarchical, case (4). The *CP* asymmetry depends on the CKM-like phases in the light and heavy neutrino sector δ and σ, as well as on the left-handed Majorana phase ϕ_2. We plot the dependence of the baryon asymmetry on the heavy CKM-phase σ for $M_{N_1} \simeq M_{N_2} = (1,5,10)\times10^{11}$ GeV in Fig. 4. The lightest heavy neutrino mass is constrained by the bound on baryon asymmetry to be $M_{N_1} \simeq M_{N_2} \geqslant 2.4\times10^{11}$ GeV. Though this value is an order of magnitude smaller than the one obtained in scenario (1), it

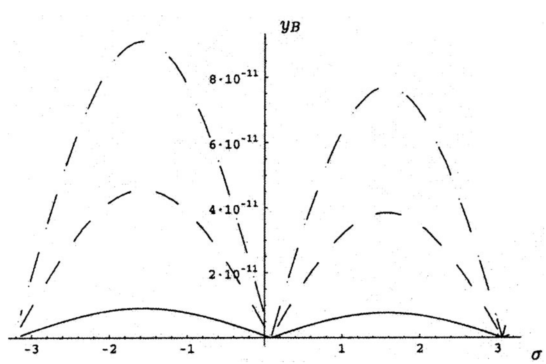

FIG. 4. The dependence of the baryon asymmetry y_B on the heavy CKM phase σ for $M_{N_1} \simeq M_{N_2} = 10^{11}$ GeV (solid curve), $M_{N_1} = 5\times10^{11}$ GeV (dash curve), and $M_{N_1} = 10^{12}$ GeV (dot-dashed curve), for scenario (4).

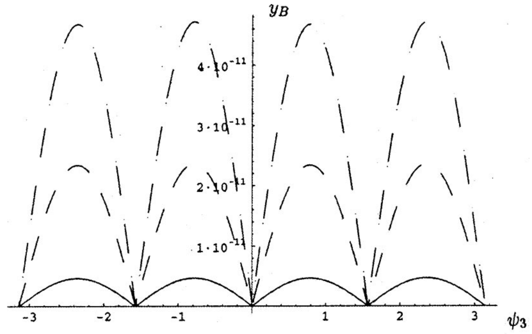

FIG. 5. The dependence of the induced baryon asymmetry y_B on the Majorana phase from the heavy neutrino sector ψ_3 for $\delta=\phi_2=\pi/4$. for several values of the lightest heavy neutrino mass $M_{N_1}\simeq M_{N_2}=10^8$ GeV (solid curve), $M_{N_1}=5\times 10^8$ GeV (dash curve), and $M_{N_1}=10^9$ GeV (dot-dashed curve), for scenario (6).

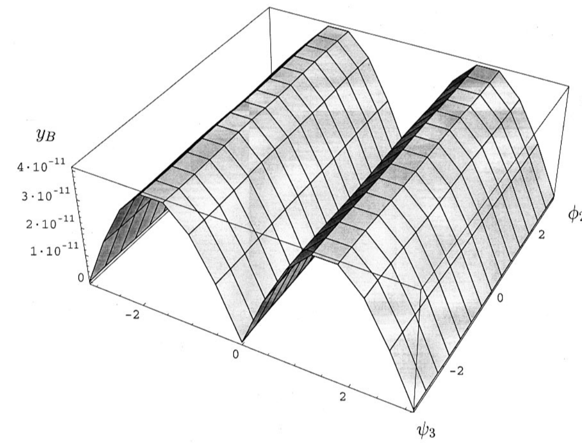

FIG. 6. The dependence of the baryon asymmetry y_B on the Majorana phases from the light and heavy neutrino sector ϕ_2 and ψ_3 for $M_{N_1}\simeq M_{N_2}=10^9$ GeV, for scenario (6).

does not sufficiently alleviate the high neutrino mass problem.

The situation is practically the same in scenario (5), in which we consider the case of quasidegenerate heavy neutrinos and inversely hierarchical light neutrinos. Though the leptonic *CP* asymmetry depends significantly on a slightly different set of phases (δ, ϕ_2, and ψ_3), the lightest heavy neutrino mass is bound by $M_{N_1}\simeq M_{N_2}\geqslant 2.2\times 10^{11}$ GeV, practically the same as for the hierarchical light neutrinos case. The dependence on the phases is similar to case (4); thus we do not show it in a separate plot.

The case in which the heavy neutrinos are inversely hierarchical can be explored by setting the ordering parameter in the heavy neutrino sector $\epsilon_N>1$. This analysis produces similar results to the cases studied, thus not requiring separate investigation.

In the case in which both the light and heavy neutrino spectra are quasidegenerate, (6), the observed leptonic *CP* asymmetry depends nontrivially on the CKM-like phase δ, the left Majorana phase ϕ_2, and the corresponding phases in the right-handed sector, σ and ψ_3. In this case, the lightest heavy neutrino mass is bound by $M_{N_1}\simeq M_{N_2}\geqslant 3.5\times 10^8$ GeV, alleviating somewhat the compatibility problem with the reheating temperature. We show in Fig. 5 the dependence of the induced baryon asymmetry on the heavy sector Majorana phase ψ_3 for $\delta=\phi_2=\pi/4$ for several values of the lightest heavy neutrino mass $M_{N_1}=(1,5,10)\times 10^8$ GeV. We plot the dependence of the baryon asymmetry on the Majorana phases ϕ_2 and ψ_3 for $M_{N_1}=M_{N_2}=10^9$ GeV in Fig. 6.

Finally, we analyze the constraints arising from the neutrinoless double beta decays. These test only the light neutrino mixing matrix elements and masses, and thus cannot distinguish between cases with quasidegenerate or hierarchical heavy neutrinos. For the case of hierarchical left-handed neutrinos, scenarios (1) and (4), $|m_{ee}|$ depends predominantly on the CKM angle δ, but for every value of the angle δ, $|m_{ee}|\leqslant 10^{-3}$ eV, due to the LMA-MSW constraint coming from the solar neutrino mixing. For inversely hierarchical light neutrinos, scenarios (2) and (5), the effective Majorana mass parameter depends only on the angle ϕ_1, and $2.9\times 10^{-2}\leqslant|m_{ee}|\leqslant 5.3\times 10^{-2}$, a much larger value, since it is dominated by atmospheric neutrino constraints. Finally, for degenerate light neutrinos, scenarios (3) and (6), $|m_{ee}|$ depends on the angle ϕ_1, but the here the most stringent dependence is on the values of the common neutrino mass m_1, so that to a good extent, $|m_{ee}|\approx m_1$. As long as $m_1\leqslant 0.1$ eV (a fairly conservative estimate), we obtain $|m_{ee}|<0.1$ eV, which is the sensitivity expected to be achieved by the new generation of neutrinoless double beta decay experiments.

VI. CONCLUSION

We have presented an analysis of the effects of Majorana and CKM-like phases in the heavy and light neutrino sectors, as well as of heavy neutrino masses, on leptogenesis in a fully left-right supersymmetric model. In this model, leptonic *CP* violation is introduced by two CKM-type phases and four Majorana phases. We express the heavy neutrino mixings as functions of the light neutrino parameters using present constraints from atmospheric and solar neutrinos within the LMA of the MSW. In LRSUSY, leptogenesis is further enhanced by the contribution of the Yukawa coupling of the Higgs triplet bosons, in addition to the usual leptonic Yukawa couplings.

We use these parameters to study leptogenesis and comment on neutrinoless double beta decay in LRSUSY. We look at general scenarios with either nondegenerate, or quasidegenerate heavy neutrinos, distinguishing in each between cases with normally ordered, inversely ordered and quasidegenerate light neutrinos. In general, sufficient lepton *CP* asymmetry necessary for explaining the baryon asymmetry of the Universe (BAU) through leptogenesis can be obtained in LRSUSY, but for rather large values for the heavy neutrino masses, usually an order of magnitude larger than in MSSM(RN). This is the distinguishing feature of all scenarios, though they each differ in the detailed dependence on the CKM- and Majorana-like phases in the heavy and light neutrino sector. Imposing the large mixing-angle scenario of the MSW, we obtain the smallest value of the mass of the lightest right-handed neutrino for the case of nondegenerate heavy neutrino and quasidegenerate light neutrino.

If one analyzes all six scenarios presented here, it appears that in LRSUSY quasidegeneracy in the light neutrino sector plays a more important role than quasidegeneracy in the heavy neutrino sector, the latter of which fails to provide the enhancement needed for compatibility between the heavy neutrino mass and the reheating temperature. For the quasidegenerate light neutrino spectrum, values of M_{N_1} of $\mathcal{O}(10^7-10^9)$ GeV are possible, while in all other cases we expect that $M_{N_1} \geqslant 10^{11}$ GeV.

For the neutrinoless double beta decay, we find that, as long as $m_1 \leqslant 0.1$ eV (a fairly conservative estimate), we obtain $|m_{ee}| < 0.1$ eV, which is the sensitivity expected to be achieved by the new generation of experiments.

In conclusion, the dependence on the light and heavy neutrino masses, mixings, and *CP*-violating phases in the LRSUSY model exhibit different features from the MSSM with right-handed singlet neutrinos. It can be used to test the heavy neutrino sector masses and mixings within the context of leptogenesis, but also to serve as a distinguishing signal of supersymmetric left-right models.

ACKNOWLEDGMENT

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