Nucleon masses and magnetic moments in a finite volume

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We compute finite-size corrections to nucleon masses and magnetic moments in a periodic, spatial box of size L, both in QCD and in partially quenched QCD.

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I. INTRODUCTION

Impressive progress is being achieved in deriving the properties and interactions of hadrons using lattice QCD. In several instances, lattice methods are making predictions of hadronic quantities at the several percent level [1]. Despite remarkable technical advances, current computational limitations continue to necessitate the use of quark masses, m_q, that are significantly larger than the physical values, lattice spacings, a, that are not significantly smaller than the physical scales of interest, and lattice sizes, L, that are not significantly larger than the pion Compton wavelength [2]. Therefore, lattice QCD simulations of hadronic physics require extrapolations in the quark masses, lattice spacing and lattice size, and ultimately it is confidence in these extrapolations that will allow a confrontation between lattice QCD and experiment. Fortunately, in many cases, the dependence of hadronic physics on these parameters can be calculated analytically in the low-energy effective field theory (EFT). Calculability requires maintaining the hierarchy of mass scales,

$$|\vec{p}|, m_\pi \ll \Lambda_\chi \ll a^{-1}, \tag{1}$$

where $|\vec{p}|$ is a typical momentum in the system of interest, m_π is the pion mass and $\Lambda_\chi \sim 2\sqrt{2}\pi f$ is the scale of chiral symmetry breaking ($f=132$ MeV is the pion decay constant). In a spatial box of size L, momenta are quantized such that $\vec{p}=2\pi\vec{n}/L$ with $\vec{n}\in \mathbf{Z}$. The hierarchy of Eq. (1) then requires maintenance of the additional inequality $fL \gg 1$. This bound ensures that (non-pionic) hadronic physics lives inside the box. In addition, the bound $(m_\pi L)^2(fL)^2 \gg 1$ ensures that the box size has no effect on spontaneous chiral symmetry breaking [3,4]. These two bounds, taken together, then imply that we must have $m_\pi L \gtrsim 1$. When $(m_\pi L)^2(fL)^2 \lesssim 1$, and therefore $m_\pi L \ll 1$, momentum zero modes must be treated nonperturbatively [3,4] and one is in the so-called ϵ regime.

Here we will consider the range of pion masses, $139\ \text{MeV} < m_\pi < 300\ \text{MeV}$, and therefore we will take $L \gtrsim 2$ fm, keeping in mind that the EFT may be reaching the limits of its validity when this bound on L is saturated, particularly when the pions are light. For the observables considered here, finite-volume effects tend to be small for $L > 4$ fm. It is therefore of interest to have control over the finite-size dependence of hadronic observables in the range $2\ \text{fm} < L \leq 4$ fm. Chiral perturbation theory (χPT), which provides a systematic description of low-energy QCD near the chiral limit, is the appropriate EFT to exploit the hierarchy of Eq. (1) and to describe the dependence of hadronic observables on L [3,5–7]. Recent work has investigated the finite-volume dependence in the meson [8–16] sector and in the baryon [17–21] sector. In this paper we compute the leading finite-volume dependence of the nucleon masses and magnetic moments in baryon χPT, including the Δ as an explicit degree of freedom.[1] The finite-size dependence of the nucleon mass was first studied in Ref. [7], and has recently been computed to $O(m_\pi^4)$ in baryon χPT (without including the Δ as an explicit degree of freedom) in Ref. [20]. (Some discussion of the effects of the Δ on the finite-size dependence of the nucleon mass appears in Ref. [17].)

We also give expressions for the finite-size dependence of the nucleon masses and magnetic moments in partially quenched QCD (PQQCD), including strong isospin breaking. The cost of simulating dynamical quarks with light masses suggests varying the sea and valence quark masses separately in the lattice QCD partition function, a procedure known as partial quenching. This procedure has important advantages beyond issues of cost; by increasing the dimensionality of the parameter space that is explored, lattice QCD simulations can provide additional "data," which can significantly improve the quality of extrapolations. χPT has been extended to describe both quenched QCD [10,24–27] with quenched chiral perturbation theory (QχPT) and PQQCD [12,28–31] with partially quenched chiral perturbation theory (PQχPT). Recently, meson and baryon properties have been studied extensively in both QχPT [26,27] and PQχPT [32–36]. The effective field theory describing the low-energy dynamics of two-nucleon systems and nucleon-hyperon systems in PQQCD has also been explored [37–39].

This paper is organized as follows: In Sec. II, the leading finite-size corrections to the nucleon masses are computed. The same is done for the nucleon magnetic moments in Sec. III. We conclude in Sec. IV. Mathematical details and the partially quenched extensions of the QCD results (including strong isospin breaking) are left to the Appendixes.

[1]Recent work [22,23] has suggested that for certain observables, a rearrangement of the chiral expansion may improve convergence. We do not utilize these modified chiral expansions in this paper.

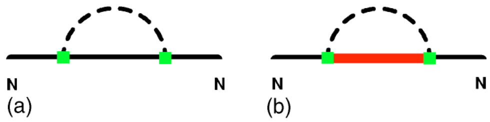

FIG. 1. One-loop graphs that give contributions of the form $\sim m_q^{3/2}$ to the masses of the proton and neutron. A solid, thick solid, or dashed line denotes a nucleon, Δ resonance, or meson, respectively. The solid squares denote an axial coupling.

II. THE NUCLEON MASSES

A. The infinite-volume limit

For purposes of setting notation, we will begin by reviewing the derivation of leading terms in the chiral expansion of the nucleon mass. The relevant leading baryon mass operators in two-flavor χPT are

$$\mathcal{L} = i\bar{N} v\cdot \mathcal{D} N + 2\alpha_M \bar{N}\mathcal{M}_+ N + 2\sigma_M \bar{N}N\,\mathrm{tr}[\mathcal{M}_+] - i\bar{T}^\mu v\cdot\mathcal{D} T_\mu + \Delta \bar{T}^\mu T_\mu, \tag{2}$$

where the chirally invariant mass operator is $\mathcal{M}_+ = \frac{1}{2}(\xi^\dagger m_q \xi^\dagger + \xi m_q \xi)$, with $m_q = \mathrm{diag}(m_u, m_d)$, and $\xi = \exp(i\pi_a\tau_a/f)$ is the usual two-flavor Goldstone matrix. The relevant leading axial operators are

$$\mathcal{L} = 2g_A \bar{N} S^\mu A_\mu N + g_{\Delta N}[\bar{T}^{abc,\nu} A^d_{a,\nu} N_b \epsilon_{cd} + \mathrm{H.c.}]. \tag{3}$$

The mass of the ith nucleon has a chiral expansion of the form

$$M_i = M_0(\mu) - M_i^{(1)}(\mu) - M_i^{(3/2)}(\mu) + \dots, \tag{4}$$

where a term $M_i^{(\alpha)}$ denotes a contribution of order m_q^α, and $i=p,n$. The nucleon mass is dominated by a term in the χPT Lagrangian, M_0, that is independent of m_q. Here Δ, the Δ-nucleon mass splitting, is assumed to be of the same chiral order as the pion mass [40,41]. Each of the contributions in Eq. (4) depends upon the scale chosen to renormalize the theory. While to $O(m_q)$, the objects M_0 and $M_i^{(1)}$ are scale independent, at one-loop level, $O(m_q^{3/2})$, they are scale dependent. The leading dependence upon m_q, occurring at $O(m_q)$, is due to the operators in Eq. (2) with coefficients α_M and σ_M. The leading non-analytic dependence upon m_q arises from the one-loop diagrams shown in Fig. 1.

In isospin-symmetric QCD,[2] with $m_u, m_d \to \bar{m}$, one finds the nucleon mass at one-loop order in the chiral expansion [42],

$$M_N = M_0(\mu) - 2\bar{m}(\alpha_M + 2\sigma_M)(\mu) - \frac{1}{8\pi f^2}\left[\frac{3}{2}g_A^2 m_\pi^3 + \frac{4g_{\Delta N}^2}{3\pi}F(m_\pi,\Delta,\mu)\right], \tag{5}$$

[2]The nucleon masses, including strong isospin breaking, may be obtained by taking the QCD limit of the partially quenched expressions given in Ref. [33].

where

$$F(m,\Delta,\mu) = (m^2-\Delta^2)\left[\sqrt{\Delta^2-m^2}\,\log\left(\frac{\Delta-\sqrt{\Delta^2-m^2+i\epsilon}}{\Delta+\sqrt{\Delta^2-m^2+i\epsilon}}\right) - \Delta\log\left(\frac{m^2}{\mu^2}\right)\right] - \frac{1}{2}\Delta m^2 \log\left(\frac{m^2}{\mu^2}\right), \tag{6}$$

and Δ is the Δ-nucleon mass splitting. Here we have used dimensional regularization (dim reg) with the modified minimal subtraction $\overline{\mathrm{MS}}$ scheme to define the divergent loop integrals and $M_0(\mu)$ and $M^{(1)}(\mu)$.

B. Finite-size corrections

In the infinite-volume limit, the nucleon mass may be written as

$$M_N = M_0(\mathcal{R}) - 2\bar{m}(\alpha_M + 2\sigma_M)(\mathcal{R}) - i\frac{9g_A^2}{2f^2}\mathcal{I}_\mathcal{R}(\infty,0) - i\frac{4g_{\Delta N}^2}{f^2}\mathcal{I}_\mathcal{R}(\infty,\Delta), \tag{7}$$

where

$$\mathcal{I}_\mathcal{R}(\infty,\Delta) = -\frac{1}{3}\int_\mathcal{R}\frac{d^4k}{(2\pi)^4}\frac{\vec{k}^2}{(k_0-\Delta-i\epsilon)(k_0^2-\vec{k}^2-m_\pi^2+i\epsilon)}. \tag{8}$$

Here $\mathcal{R}$ denotes the choice of ultraviolet regulator and a renormalization scheme.[3]

In a spatial box of size L, $\mathcal{I}_\mathcal{R}$ generalizes to

$$\mathcal{I}_\mathcal{R}(L,\Delta) = i\frac{1}{3}\left(\frac{1}{L^3}\sum_{\vec{k}}^{\mathcal{R}}\int\frac{dk_4}{(2\pi)}\frac{\vec{k}^2}{(ik_4-\Delta)(k_4^2+\vec{k}^2+m_\pi^2)}\right), \tag{9}$$

where we have rotated the integral to Euclidean space and accounted for the quantization of the momentum levels due to the periodic boundary conditions. Feynman parameterizing and explicitly evaluating the k_4 integration lead to

$$\mathcal{I}_\mathcal{R}(L,\Delta) = -i\frac{1}{6}\int_0^\infty d\lambda\left(\frac{1}{L^3}\sum_{\vec{k}}^{\mathcal{R}}\frac{\vec{k}^2}{[\vec{k}^2+\beta_\Delta^2]^{3/2}}\right), \tag{10}$$

where $\beta_\Delta^2 \equiv \lambda^2 + 2\lambda\Delta + m_\pi^2$. We can now write the finite-size corrections to $\mathcal{I}_\mathcal{R}$ as

[3]In dim reg with $\overline{\mathrm{MS}}$,

$$\mathcal{I}_{\overline{\mathrm{MS}}}(\infty,\Delta) = -i\frac{1}{24\pi^2}F(m_\pi,\Delta,\mu),$$

where $F(m_\pi,0,\mu) = \pi m_\pi^3$, which recovers the results of Eq. (5).

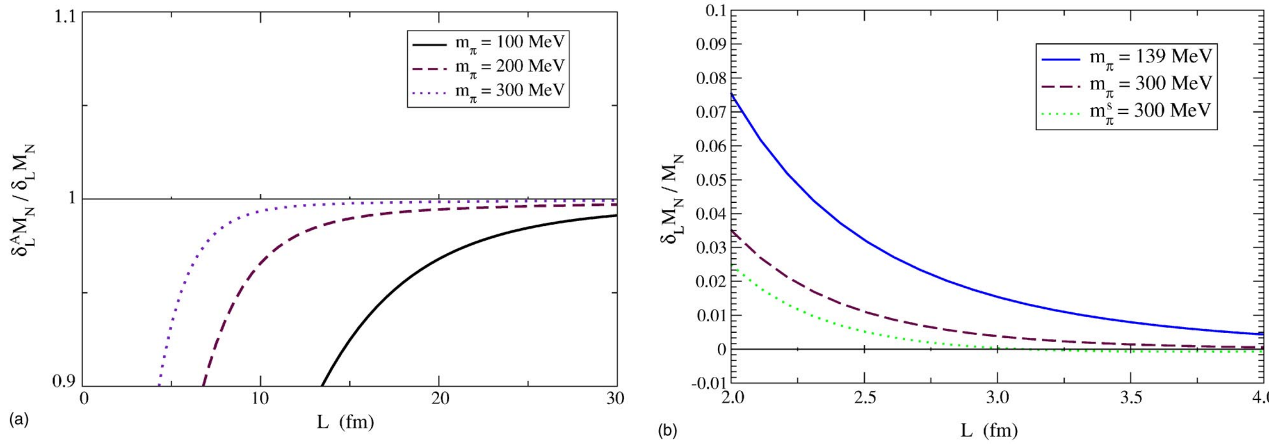

FIG. 2. Left panel: The ratio of the asymptotic formula, Eq. (17), to the exact formula, Eq. (16), as a function of L for various values of m_π. The solid, dashed and dotted lines correspond to $m_\pi = 100$, 200, and 300 MeV, respectively. Right panel: The ratio of the nucleon mass size dependence to the (infinite volume) nucleon mass vs L. The solid and dashed lines correspond to QCD with $m_\pi = 139$ and 300 MeV, respectively. The dotted line corresponds to PQQCD with $m_\pi = 139$ MeV and $m_\pi^s = 300$ MeV.

$$\delta_L \mathcal{I}(\Delta) \equiv \mathcal{I}_{\mathcal{R}}(L,\Delta) - \mathcal{I}_{\mathcal{R}}(\infty,\Delta)$$

$$= -\frac{i}{6}\int_0^\infty d\lambda \left[\delta_L\left(\frac{1}{[\vec{k}^2+\beta_\Delta^2]^{1/2}}\right) - \beta_\Delta^2 \delta_L\left(\frac{1}{[\vec{k}^2+\beta_\Delta^2]^{3/2}}\right)\right], \qquad (11)$$

where $\delta_L(f(|\vec{k}|))$ is defined in Eq. (A1) of Appendix A. Notice that $\delta_L \mathcal{I}(\Delta)$ is a purely infrared quantity and, as such, is independent of $\mathcal{R}$.[4] Using Eqs. (7) and (11), the finite-size corrections to the nucleon mass may then be expressed as

$$\delta_L M_N \equiv M_N(L) - M_N(\infty)$$

$$= -i\frac{9g_A^2}{2f^2}\delta_L \mathcal{I}(0) - i\frac{4g_{\Delta N}^2}{f^2}\delta_L \mathcal{I}(\Delta). \qquad (12)$$

Using the "master" formula, Eq. (A6), derived in Appendix A, we find

$$\delta_L \mathcal{I}(\Delta) = -\frac{i}{12\pi^2}\mathcal{K}(\Delta), \qquad (13)$$

where

$$\mathcal{K}(\Delta) \equiv \int_0^\infty d\lambda\, \beta_\Delta \sum_{\vec{n}\neq 0} [(L|\vec{n}|)^{-1}K_1(L\beta_\Delta|\vec{n}|) - \beta_\Delta K_0(L\beta_\Delta|\vec{n}|)]. \qquad (14)$$

[4]This implies that finite-volume effects should be independent of the lattice spacing a, which appears implicitly as the ultraviolet cutoff π/a in all sums and integrals.

Here $K_n(z)$ is a modified Bessel function of the second kind. With $\Delta = 0$ the integral over λ can be carried out explicitly (see Appendix A) and one has

$$\mathcal{K}(0) = -\frac{\pi}{2}m_\pi^2 \sum_{\vec{n}\neq 0}(L|\vec{n}|)^{-1}\exp(-L|\vec{n}|m_\pi). \qquad (15)$$

Notice that this function contains no power-law corrections. Finally, we have

$$\delta_L M_N = -\frac{3g_A^2}{8\pi^2 f^2}\mathcal{K}(0) - \frac{g_{\Delta N}^2}{3\pi^2 f^2}\mathcal{K}(\Delta). \qquad (16)$$

This is the exact formula for the finite-size corrections to the nucleon mass at leading order in baryon χPT. In Fig. 2 (right panel), the ratio of the nucleon mass size dependence to the (infinite volume) nucleon mass has been plotted against the box size L for various pion masses. The solid and dashed lines correspond to the QCD formula of Eq. (16) with $m_\pi = 139$ MeV and 300 MeV, respectively. The dotted line corresponds to PQQCD in the isospin limit taken from Eq. (B7) with $m_\pi = 139$ MeV and $m_\pi^s = 300$ MeV. We use the parameter set $f = 132$ MeV, $g_A = 1.26$ and $g_{\Delta N} = 1.4$.

C. The asymptotic limit

Using Eqs. (15) and (A11), in the large-L expansion one has

$$\delta_L^A M_N = \left(\frac{9g_A^2 m_\pi^2}{8\pi f^2} + \frac{4g_{\Delta N}^2 m_\pi^{5/2}}{(2\pi)^{3/2} f^2}\frac{1}{\Delta L^{1/2}}\right)\frac{1}{L}\exp(-m_\pi L), \qquad (17)$$

where $\delta_L M_N - \delta_L^A M_N = O[\exp(-m_\pi L)/L^{5/2}]$. In the $M_N \to \infty$ limit, the leading term in the large-L expansion is in agree-

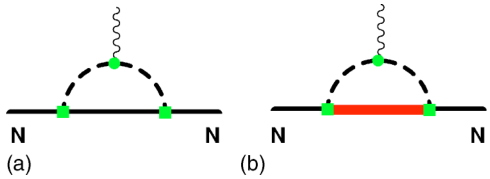

FIG. 3. One-loop graphs that contribute to the proton and neutron magnetic moments. A solid, thick solid, or dashed line denotes a nucleon, Δ resonance, or a meson, respectively. The solid squares denote axial coupling and the solid circles denote a leading-order electromagnetic interaction.

ment with Ref. [20] and in disagreement with Ref. [7].[5] Figure 2 (left panel) plots the ratio $\delta_L^A M_N/\delta_L M_N$ as a function of L for various pion masses. Clearly the utility of Eq. (17) is purely aesthetic; even with heavy pions, the asymptotic formula Eq. (17) is not accurate for $L<10$ fm. This points to the importance of exponential corrections; for $m_\pi L \gtrsim 1$ convergence of the momentum sums requires keeping terms with $|\vec{n}|>1$, i.e. one must include corrections of $O[\exp(-|\vec{n}|m_\pi L)]$.

III. THE NUCLEON MAGNETIC MOMENTS

A. The infinite-volume limit

With the finite-size corrections for the masses, it is straightforward to get the magnetic moments. The leading operators contributing to the nucleon magnetic moments are

$$\mathcal{L}=\frac{e}{4M_N}F_{\mu\nu}(\mu_0\bar{N}\sigma^{\mu\nu}N+\mu_1\bar{N}\sigma^{\mu\nu}\tau^3_{\xi+}N), \qquad (18)$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor, M_N is the physical value of the nucleon mass, μ_0 is the isoscalar nucleon magnetic moment, μ_1 is the isovector nucleon magnetic moment and $\tau^a_{\xi+}=\frac{1}{2}(\xi^\dagger\tau^a\xi+\xi\tau^a\xi^\dagger)$.

In isospin-symmetric QCD one finds the nucleon magnetic moment matrix at one-loop order in the chiral expansion [43–45] as

$$\hat{\mu}=\mu_0+\mu_1\hat{\tau}_3-\frac{M_N}{4\pi f^2}\left[g_A^2 m_{\pi^+}+\frac{2}{9}g_{\Delta N}^2\mathcal{F}_{\pi^+}\right]\hat{\tau}_3. \qquad (19)$$

The scale dependence is left implicit. The proton and neutron magnetic moments are the diagonal elements of $\hat{\mu}$. The first term within the brackets is from Fig. 3(a) while the second term is from Fig. 3(b). The function $\mathcal{F}_i=\mathcal{F}(m_i,\Delta,\mu)$ is

$$\pi\mathcal{F}(m,\Delta,\mu)=\sqrt{\Delta^2-m^2}\log\left(\frac{\Delta-\sqrt{\Delta^2-m^2+i\epsilon}}{\Delta+\sqrt{\Delta^2-m^2+i\epsilon}}\right)-\Delta\log\left(\frac{m^2}{\mu^2}\right). \qquad (20)$$

[5]For a detailed discussion of this disagreement, see Ref. [20].

Here again we have used dim reg with $\overline{\mathrm{MS}}$. In the limit $\Delta\to 0$, $\mathcal{F}(m,0,\mu)=m$.

B. Finite-size corrections

In the infinite-volume limit, the nucleon magnetic moments may be written as

$$\hat{\mu}=\mu_0+\mu_1\hat{\tau}_3-\frac{4iM_N}{f^2}\left[g_A^2\mathcal{J}_{\mathcal{R}}(\infty,0)+\frac{2}{9}g_{\Delta N}^2\mathcal{J}_{\mathcal{R}}(\infty,\Delta)\right]\hat{\tau}_3, \qquad (21)$$

where

$$\mathcal{J}_{\mathcal{R}}(\infty,\Delta)=\frac{\partial}{\partial m_\pi^2}\mathcal{I}_{\mathcal{R}}(\infty,\Delta). \qquad (22)$$

Therefore, the finite-size corrections to $\hat{\mu}$ are

$$\delta_L\hat{\mu}\equiv\hat{\mu}(L)-\hat{\mu}(\infty)=-\frac{4iM_N}{f^2}\left[g_A^2\delta_L\mathcal{J}(0)+\frac{2}{9}g_{\Delta N}^2\delta_L\mathcal{J}(\Delta)\right]\hat{\tau}_3. \qquad (23)$$

Using Eqs. (13), (14) and (22), and the properties of modified Bessel functions, it is straightforward to find

$$\delta_L\mathcal{J}(\Delta)=\frac{i}{24\pi^2}\mathcal{Y}(\Delta), \qquad (24)$$

where

$$\mathcal{Y}(\Delta)\equiv\int_0^\infty d\lambda\sum_{\vec{n}\neq 0}[3K_0(L\beta_\Delta|\vec{n}|)-(L\beta_\Delta|\vec{n}|)K_1(L\beta_\Delta|\vec{n}|)]. \qquad (25)$$

With $\Delta=0$ one has

$$\mathcal{Y}(0)=-\frac{\pi}{2}m_\pi\sum_{\vec{n}\neq 0}(1-2(L|\vec{n}|m_\pi)^{-1})\exp(-L|\vec{n}|m_\pi). \qquad (26)$$

Finally one arrives at

$$\delta_L\hat{\mu}=\frac{M_N}{6\pi^2 f^2}\left[g_A^2\mathcal{Y}(0)+\frac{2}{9}g_{\Delta N}^2\mathcal{Y}(\Delta)\right]\hat{\tau}_3. \qquad (27)$$

This is the exact formula for the finite-size corrections to the nucleon magnetic moments at leading order in baryon χPT. In Fig. 4 (right panel), the ratio of the proton magnetic moment size dependence to the (infinite volume) magnetic moment has been plotted against the box size L. The solid and dashed lines correspond to the QCD formula of Eq. (27) with $m_\pi=139$ MeV and 300 MeV, respectively. The dotted line corresponds to PQQCD in the isospin limit taken from Eq. (B11) with $m_\pi=139$ MeV and $m_\pi^s=300$ MeV.

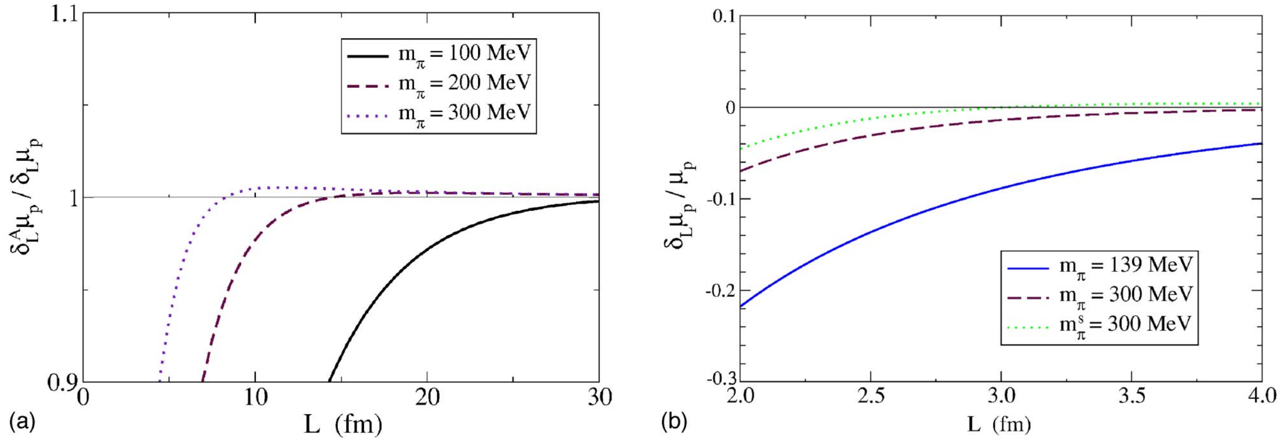

FIG. 4. Left panel: The ratio of the asymptotic formula, Eq. (28), to the exact formula, Eq. (27), as a function of L for various values of m_π. The solid, dashed and dotted lines correspond to $m_\pi = 100$, 200, and 300 MeV, respectively. Right panel: The ratio of the nucleon magnetic moment size dependence to the (infinite volume) nucleon magnetic moment vs L. The solid and dashed lines correspond to QCD with $m_\pi = 139$ and 300 MeV, respectively. The dotted line corresponds to PQQCD with $m_\pi = 139$ MeV and $m_\pi^s = 300$ MeV.

C. The asymptotic limit

Again using Eqs. (13), (14) and (22), together with Eqs. (26) and (A11), in the large-L expansion one has

$$\delta_L \hat{\mu} = -\left[\frac{M_N g_A^2 m_\pi}{2\pi f^2}\left(1 - \frac{2}{m_\pi L}\right) + \frac{4 M_N g_{\Delta N}^2 m_\pi^{3/2}}{9(2\pi)^{3/2} f^2} \frac{1}{\Delta L^{1/2}} \right] \times \exp(-m_\pi L)\hat{\tau}_3 + \dots \tag{28}$$

where the ellipses denotes contributions of $O[\exp(-m_\pi L)/L^{3/2}]$. Figure 4 (left panel) plots the ratio $\delta_L^A \mu / \delta_L \mu$ as a function of L for various pion masses. The curves are similar to those of the nucleon mass in Fig. 2, as is the conclusion about the practical utility of Eq. (28).

IV. DISCUSSION AND CONCLUSION

It is hoped that in the near future lattice (PQ)QCD simulations of baryon properties will encounter the chiral regime, where the quark masses are sufficiently small to allow a meaningful chiral expansion in quark masses, box size and lattice spacing. It is likely that this regime has been encountered in recent work on heavy-meson systems [1].

The results of this paper, together with the results of Refs. [33] and [46], give the dependence of the nucleon masses and magnetic moments on the sea and valence quark masses *and* on the lattice spacing, a, and size, L, to leading order in the chiral expansion. We eagerly await lattice (PQ)QCD simulations within the chiral regime where this parameter space may be fruitfully explored.

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APPENDIX A: FINITE SIZE CORRECTIONS

1. The master formula

We wish to evaluate

$$\delta_L\left(\frac{1}{[\vec{l}^2 + \mathcal{M}^2]^m}\right) \equiv \frac{1}{L^3}\sum_{\vec{l}} \frac{1}{(\vec{l}^2 + \mathcal{M}^2)^m} - \int \frac{d^3 l}{(2\pi)^3} \frac{1}{(\vec{l}^2 + \mathcal{M}^2)^m}. \tag{A1}$$

As this difference is ultraviolet finite, we omit the label denoting the scheme dependence of the individual sum and integral. This expression has been evaluated in many places.[6] Using the identity

$$D^{-m} = \frac{1}{\Gamma(m)}\int_0^\infty d\eta\, \eta^{m-1} e^{-\eta D} \tag{A2}$$

one finds

$$\delta_L\left(\frac{1}{[\vec{l}^2 + \mathcal{M}^2]^m}\right) = \frac{1}{(4\pi)^{3/2}\Gamma(m)}\int_0^\infty d\eta\, \eta^{m-5/2} e^{-\eta \mathcal{M}^2} \times \left[\frac{(4\pi\eta)^{3/2}}{L^3}\sum_{\vec{l}} e^{-\eta \vec{l}^2} - 1\right]. \tag{A3}$$

Expressing the momentum as $\vec{l} = 2\pi \vec{n}/L$ and using the Jacobi identity [47]

$$\mathcal{S}(z) \equiv \sum_{n=-\infty}^{\infty} e^{-zn^2}, \quad \mathcal{S}(z) = \sqrt{\frac{\pi}{z}}\,\mathcal{S}\left(\frac{\pi^2}{z}\right) \tag{A4}$$

[6] References that have been of use to the author include Refs. [15,47,48].

leads to

$$\delta_L\left(\frac{1}{[\vec{l}^2+\mathcal{M}^2]^m}\right)=\frac{1}{(4\pi)^{3/2}\Gamma(m)}\times\sum_{\vec{n}\neq 0}\int_0^\infty d\eta\,\eta^{m-5/2}e^{-\eta\mathcal{M}^2}e^{-L^2\vec{n}^2/4\eta}. \tag{A5}$$

Performing the integral over η then gives the "master" formula

$$\delta_L\left(\frac{1}{[\vec{l}^2+\mathcal{M}^2]^m}\right)=\frac{2^{-1/2-m}\mathcal{M}^{3-2m}}{\pi^{3/2}\Gamma(m)}\times\sum_{\vec{n}\neq 0}(L\mathcal{M}|\vec{n}|)^{-3/2+m}K_{3/2-m}(L\mathcal{M}|\vec{n}|), \tag{A6}$$

where $K_n(z)$ is a modified Bessel function of the second kind.

A well-chosen change of integration variable and the properties of modified Bessel functions allow one to write Eq. (14) as

$$\mathcal{K}(\Delta)=\sum_{\vec{n}\neq 0}(L|\vec{n}|)^{-1}\frac{1}{L}\frac{d}{dL}\left(L^2\int_{m_\pi}^\infty d\xi\,\xi^2(\xi^2-m_\pi^2+\Delta^2)^{-1/2}K_1(L\xi|\vec{n}|)\right). \tag{A7}$$

We find no useful simplification of this formula in the general case. With $\Delta=0$ one directly finds

$$\mathcal{K}(0)=-\sqrt{\frac{\pi}{2}}m_\pi^3\sum_{\vec{n}\neq 0}(L|\vec{n}|m_\pi)^{-1/2}K_{1/2}(L|\vec{n}|m_\pi), \tag{A8}$$

which gives Eq. (15).

2. The asymptotic limit

In the large-L limit, using the expansion of the modified Bessel function for large argument, one finds from Eq. (A7)

$$\mathcal{K}(\Delta)=3\sqrt{2\pi}\frac{1}{L^2}\frac{d}{dL}\left(L^{3/2}\int_{m_\pi}^\infty d\xi\,\xi^{3/2}(\xi^2-m_\pi^2+\Delta^2)^{-1/2}\exp(-L\xi)\right)+\dots \tag{A9}$$

where the ellipses denotes contributions of $O[\exp(-m_\pi L)/L^{5/2}]$. Observe that one can expand the integrand in powers of $\alpha^2\equiv\Delta^2-m_\pi^2$,

$$\begin{aligned}\int_{m_\pi}^\infty d\xi\,\xi^{\ell/2}(\xi^2-m_\pi^2+\Delta^2)^{-1/2}e^{-L\xi}&=\sum_{n=0}^\infty\binom{-\frac{1}{2}}{n}\alpha^{2n}\int_{m_\pi}^\infty d\xi\,\xi^{\ell/2-1-2n}e^{-L\xi}\\&=m_\pi^{\ell/2-1}\frac{1}{L}e^{-m_\pi L}\sum_{n=0}^\infty\binom{-\frac{1}{2}}{n}\frac{\alpha^{2n}}{m_\pi^{2n}}+\dots\\&=m_\pi^{\ell/2}\frac{1}{\Delta L}\exp(-Lm_\pi)+\dots\end{aligned} \tag{A10}$$

where the ellipses denote contributions of $O[\exp(-m_\pi L)/L^2]$. (Similar technology has been developed in Ref. [16] in the context of heavy-meson systems.) Plugging this back into Eq. (A9) one finds, in the asymptotic limit,

$$\mathcal{K}(\Delta)=-3\sqrt{2\pi}m_\pi^{5/2}\frac{1}{L^{3/2}\Delta}\exp(-Lm_\pi)+\dots \tag{A11}$$

where the ellipses denotes contributions of $O[\exp(-m_\pi L)/L^{5/2}]$.

APPENDIX B: PARTIALLY QUENCHED QCD

1. Nucleon masses

We work in PQQCD including isospin breaking, but with electromagnetism turned off.[7] The Lagrangian describing the interactions of $\mathcal{B}_{ijk}$ (containing the nucleon) and $\mathcal{T}_{ijk}$ (containing Δ), which transform in the **70** and **44** of $SU(4|2)_V$, respectively, with the pseudo Goldstone bosons at leading order in the chiral expansion is [26]

$$\mathcal{L}=2\alpha(\overline{\mathcal{B}}S^\mu\mathcal{B}A_\mu)+2\beta(\overline{\mathcal{B}}S^\mu A_\mu\mathcal{B})+\sqrt{\frac{3}{2}}\mathcal{C}[(\overline{\mathcal{T}}^\nu A_\nu\mathcal{B})+(\overline{\mathcal{B}}A_\nu\mathcal{T}^\nu)]. \tag{B1}$$

Here the axial-vector field A_μ is a 6×6 matrix. Matching to the QCD effective Lagrangian of Eq. (3) and to the additional operator

$$\mathcal{L}=g_1\overline{N}S^\mu N\,\mathrm{tr}[A_\mu], \tag{B2}$$

one finds that at the tree level,

$$\alpha=\frac{4}{3}g_A+\frac{1}{3}g_1,\quad \beta=\frac{2}{3}g_1-\frac{1}{3}g_A,\quad \mathcal{C}=-g_{\Delta N}. \tag{B3}$$

The finite-size corrections to the proton mass are given by

[7]For details we refer the reader to Ref. [33].

$$\delta_L M_p = -\frac{1}{8\pi f^2}\Bigg(\frac{2g_A^2}{3\pi}[\mathcal{K}(m_{uu},0)+\mathcal{K}(m_{ud},0)+2\mathcal{K}(m_{ju},0)+2\mathcal{K}(m_{lu},0)+3\mathcal{G}_{\eta_u,\eta_u}(0)]+\frac{g_1^2}{6\pi}[\mathcal{K}(m_{uu},0)-5\mathcal{K}(m_{ud},0)$$
$$+3\mathcal{K}(m_{jd},0)+2\mathcal{K}(m_{ju},0)+3\mathcal{K}(m_{ld},0)+2\mathcal{K}(m_{lu},0)+3\mathcal{G}_{\eta_u,\eta_u}(0)+6\mathcal{G}_{\eta_u,\eta_d}(0)+3\mathcal{G}_{\eta_d,\eta_d}(0)]+\frac{2g_A g_1}{3\pi}[\mathcal{K}(m_{ju},0)$$
$$+\mathcal{K}(m_{lu},0)-\mathcal{K}(m_{ud},0)+2\mathcal{K}(m_{uu},0)+3\mathcal{G}_{\eta_u,\eta_d}(0)+3\mathcal{G}_{\eta_u,\eta_u}(0)]+\frac{2g_{\Delta N}^2}{9\pi}[5\mathcal{K}(m_{ud},\Delta)+\mathcal{K}(m_{uu},\Delta)+\mathcal{K}(m_{ju},\Delta)$$
$$+\mathcal{K}(m_{lu},\Delta)+2\mathcal{K}(m_{jd},\Delta)+2\mathcal{K}(m_{ld},\Delta)+2\mathcal{G}_{\eta_d,\eta_d}(\Delta)+2\mathcal{G}_{\eta_u,\eta_u}(\Delta)-4\mathcal{G}_{\eta_u,\eta_d}(\Delta)]\Bigg), \tag{B4}$$

where $\mathcal{G}_{\eta_a,\eta_b}(\Delta)\equiv\mathcal{H}_{\eta_a\eta_b}(\mathcal{K}(m_{\eta_a},\Delta),\mathcal{K}(m_{\eta_b},\Delta),\mathcal{K}(m_X,\Delta))$, $\mathcal{H}_{\eta_a\eta_b}$ is given by

$$\mathcal{H}_{\eta_a\eta_b}(A,B,C) = -\frac{1}{2}\left[\frac{(m_{jj}^2-m_{\eta_a}^2)(m_{ll}^2-m_{\eta_a}^2)}{(m_{\eta_a}^2-m_{\eta_b}^2)(m_{\eta_a}^2-m_X^2)}A-\frac{(m_{jj}^2-m_{\eta_b}^2)(m_{ll}^2-m_{\eta_b}^2)}{(m_{\eta_a}^2-m_{\eta_b}^2)(m_{\eta_b}^2-m_X^2)}B+\frac{(m_X^2-m_{jj}^2)(m_X^2-m_{ll}^2)}{(m_X^2-m_{\eta_a}^2)(m_X^2-m_{\eta_b}^2)}C\right], \tag{B5}$$

the mass, m_X, is given by $m_X^2=\frac{1}{2}(m_{jj}^2+m_{ll}^2)$, and $\mathcal{K}(m_\pi,\Delta)$ is defined in Eq. (14) (where now the m_π dependence is made explicit). Note that m_{ab} refers to the Goldstone-boson mass with quark content a and b (hence $m_{\pi^\pm}=m_{ud}$, *etc.*); j and l label the sea quark masses.

The finite-size corrections to the neutron mass are given by

$$\delta_L M_n = -\frac{1}{8\pi f^2}\Bigg(\frac{2g_A^2}{3\pi}[\mathcal{K}(m_{dd},0)+\mathcal{K}(m_{ud},0)+2\mathcal{K}(m_{jd},0)+2\mathcal{K}(m_{ld},0)+3\mathcal{G}_{\eta_d,\eta_d}(0)]+\frac{g_1^2}{6\pi}[\mathcal{K}(m_{dd},0)-5\mathcal{K}(m_{ud},0)$$
$$+2\mathcal{K}(m_{jd},0)+3\mathcal{K}(m_{ju},0)+2\mathcal{K}(m_{ld},0)+3\mathcal{K}(m_{lu},0)+3\mathcal{G}_{\eta_u,\eta_u}(0)+6\mathcal{G}_{\eta_u,\eta_d}(0)+3\mathcal{G}_{\eta_d,\eta_d}(0)]+\frac{2g_A g_1}{3\pi}[2\mathcal{K}(m_{dd},0)$$
$$+\mathcal{K}(m_{jd},0)+\mathcal{K}(m_{ld},0)-\mathcal{K}(m_{ud},0)+3\mathcal{G}_{\eta_d,\eta_d}(0)+3\mathcal{G}_{\eta_u,\eta_d}(0)]+\frac{2g_{\Delta N}^2}{9\pi}[5\mathcal{K}(m_{ud},\Delta)+\mathcal{K}(m_{dd},\Delta)+\mathcal{K}(m_{jd},\Delta)$$
$$+\mathcal{K}(m_{ld},\Delta)+2\mathcal{K}(m_{ju},\Delta)+2\mathcal{K}(m_{lu},\Delta)+2\mathcal{G}_{\eta_d,\eta_d}(\Delta)+2\mathcal{G}_{\eta_u,\eta_u}(\Delta)-4\mathcal{G}_{\eta_u,\eta_d}(\Delta)]\Bigg). \tag{B6}$$

In the isospin limit, one has

$$\delta_L M_N = -\frac{g_A^2}{24\pi^2 f^2}[\mathcal{K}(m_\pi,0)+8\mathcal{K}(m_\pi^s,0)]-\frac{g_{\Delta N}^2}{6\pi^2 f^2}[\mathcal{K}(m_\pi,\Delta)+\mathcal{K}(m_\pi^s,\Delta)]+\frac{g_1}{24\pi^2 f^2}(5g_1+4g_A)[\mathcal{K}(m_\pi,0)-\mathcal{K}(m_\pi^s,0)], \tag{B7}$$

where we have used the fact that $\mathcal{G}_{\eta_d,\eta_d}(\Delta)\to-\frac{1}{2}\mathcal{K}(m_\pi,\Delta)$ in the isospin limit. Here m_π^s denotes the mass of a pion made of one valence quark and one sea quark. These expressions further collapse down to isospin-symmetric QCD in the limit $m_\pi^s \to m_\pi$.

2. Nucleon magnetic moments

The most general charge matrix whose matrix elements reduce to those of QCD (keeping the valence-quark charges fixed) is [33]

$$\mathcal{Q}^{(PQ)} = \mathrm{diag}\left(+\frac{2}{3},\ -\frac{1}{3},\ q_j,\ q_l,\ q_j,\ q_l\right). \tag{B8}$$

The finite-size corrections to the proton magnetic moment in PQQCD are

$$\delta_L\mu_p = -\frac{M_N}{6\pi^2 f^2}\Bigg\{\frac{g_A^2}{9}[4\mathcal{Y}(m_{uu},0)-5\mathcal{Y}(m_{ud},0)-4\mathcal{Y}(m_{ju},0)-4\mathcal{Y}(m_{lu},0)]+\frac{2g_1g_A}{9}[\mathcal{Y}(m_{ud},0)+\mathcal{Y}(m_{uu},0)-\mathcal{Y}(m_{ju},0)$$
$$-\mathcal{Y}(m_{lu},0)]+\frac{g_1^2}{36}[\mathcal{Y}(m_{ud},0)+4\mathcal{Y}(m_{uu},0)-3\mathcal{Y}(m_{dd},0)+3\mathcal{Y}(m_{jd},0)-4\mathcal{Y}(m_{ju},0)+3\mathcal{Y}(m_{ld},0)-4\mathcal{Y}(m_{lu},0)]$$
$$+q_j\Bigg[\frac{2g_A^2}{3}[\mathcal{Y}(m_{ju},0)-\mathcal{Y}(m_{uu},0)]+\frac{g_1g_A}{3}[\mathcal{Y}(m_{ju},0)-\mathcal{Y}(m_{uu},0)]+\frac{g_1^2}{6}\Bigg(\mathcal{Y}(m_{ju},0)-\mathcal{Y}(m_{uu},0)+\frac{3}{2}\mathcal{Y}(m_{jd},0)$$
$$-\frac{3}{2}\mathcal{Y}(m_{ud},0)\Bigg)\Bigg]+q_l\Bigg[\frac{2g_A^2}{3}[\mathcal{Y}(m_{lu},0)-\mathcal{Y}(m_{ud},0)]+\frac{g_1g_A}{3}[\mathcal{Y}(m_{lu},0)-\mathcal{Y}(m_{ud},0)]+\frac{g_1^2}{6}\Bigg(\mathcal{Y}(m_{lu},0)-\mathcal{Y}(m_{ud},0)$$
$$+\frac{3}{2}\mathcal{Y}(m_{ld},0)-\frac{3}{2}\mathcal{Y}(m_{dd},0)\Bigg)\Bigg]+\frac{g_{\Delta N}^2}{27}\Bigg(\mathcal{Y}(m_{dd},\Delta)-\mathcal{Y}(m_{uu},\Delta)-6\mathcal{Y}(m_{ud},\Delta)-\mathcal{Y}(m_{jd},\Delta)+\mathcal{Y}(m_{ju},\Delta)-\mathcal{Y}(m_{ld},\Delta)$$
$$+\mathcal{Y}(m_{lu},\Delta)+\frac{3}{2}q_j[\mathcal{Y}(m_{uu},\Delta)+2\mathcal{Y}(m_{ud},\Delta)-\mathcal{Y}(m_{ju},\Delta)-2\mathcal{Y}(m_{jd},\Delta)]+\frac{3}{2}q_l[\mathcal{Y}(m_{ud},\Delta)+2\mathcal{Y}(m_{dd},\Delta)$$
$$-\mathcal{Y}(m_{lu},\Delta)-2\mathcal{Y}(m_{ld},\Delta)]\Bigg)\Bigg\}, \tag{B9}$$

where $\mathcal{Y}(m_\pi,\Delta)$ is defined in Eq. (25) (where now the m_π dependence is made explicit).

The finite-size corrections to the neutron magnetic moment are

$$\delta_L\mu_n = -\frac{M_N}{6\pi^2 f^2}\Bigg\{\frac{g_A^2}{9}[7\mathcal{Y}(m_{ud},0)+2\mathcal{Y}(m_{ld},0)+2\mathcal{Y}(m_{jd},0)-2\mathcal{Y}(m_{dd},0)]+\frac{g_1g_A}{9}[\mathcal{Y}(m_{jd},0)+\mathcal{Y}(m_{ld},0)-\mathcal{Y}(m_{ud},0)$$
$$-\mathcal{Y}(m_{dd},0)]+\frac{g_1^2}{18}[3\mathcal{Y}(m_{uu},0)+2\mathcal{Y}(m_{ud},0)-\mathcal{Y}(m_{dd},0)+\mathcal{Y}(m_{jd},0)-3\mathcal{Y}(m_{ju},0)+\mathcal{Y}(m_{ld},0)-3\mathcal{Y}(m_{lu},0)]$$
$$+q_j\Bigg[\frac{2g_A^2}{3}[\mathcal{Y}(m_{jd},0)-\mathcal{Y}(m_{ud},0)]+\frac{g_1g_A}{3}[\mathcal{Y}(m_{jd},0)-\mathcal{Y}(m_{ud},0)]+\frac{g_1^2}{6}\Bigg(\mathcal{Y}(m_{jd},0)-\mathcal{Y}(m_{ud},0)+\frac{3}{2}\mathcal{Y}(m_{ju},0)$$
$$-\frac{3}{2}\mathcal{Y}(m_{uu},0)\Bigg)\Bigg]+q_l\Bigg[\frac{2g_A^2}{3}[\mathcal{Y}(m_{ld},0)-\mathcal{Y}(m_{dd},0)]+\frac{g_1g_A}{3}[\mathcal{Y}(m_{ld},0)-\mathcal{Y}(m_{dd},0)]+\frac{g_1^2}{6}\Bigg(\mathcal{Y}(m_{ld},0)-\mathcal{Y}(m_{dd},0)$$
$$+\frac{3}{2}\mathcal{Y}(m_{lu},0)-\frac{3}{2}\mathcal{Y}(m_{ud},0)\Bigg)\Bigg]+\frac{g_{\Delta N}^2}{54}\{\mathcal{Y}(m_{dd},\Delta)-4\mathcal{Y}(m_{uu},\Delta)+9\mathcal{Y}(m_{ud},\Delta)-\mathcal{Y}(m_{jd},\Delta)+4\mathcal{Y}(m_{ju},\Delta)-\mathcal{Y}(m_{ld},\Delta)$$
$$+4\mathcal{Y}(m_{lu},\Delta)+3q_j[\mathcal{Y}(m_{ud},\Delta)+2\mathcal{Y}(m_{uu},\Delta)-\mathcal{Y}(m_{jd},\Delta)-2\mathcal{Y}(m_{ju},\Delta)]+3q_l[\mathcal{Y}(m_{dd},\Delta)+2\mathcal{Y}(m_{ud},\Delta)-\mathcal{Y}(m_{ld},\Delta)$$
$$-2\mathcal{Y}(m_{lu},\Delta)]\}\Bigg\}. \tag{B10}$$

In the isospin limit (with $q_j = q_l = 0$), one has

$$\delta_L\hat{\mu} = \frac{M_N}{6\pi^2 f^2}\left(\frac{g_A^2}{9}[\mathcal{Y}(m_\pi,0)+8\mathcal{Y}(m_\pi^s,0)]+\frac{2g_{\Delta N}^2}{9}\mathcal{Y}(m_\pi,\Delta)-\frac{g_1}{18}(g_1+8g_A)[\mathcal{Y}(m_\pi,0)-\mathcal{Y}(m_\pi^s,0)]\right)\hat{\tau}_3. \tag{B11}$$

These expressions further collapse down to isospin-symmetric QCD in the limit $m_\pi^s \to m_\pi$.

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