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Scalar mesons in lattice QCD

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We explore whether "f$_0$(600) or σ" exists as a pole in QCD, by a full lattice QCD simulation on the $8^3\times 16$ lattice using the plaquette action and Wilson fermions. It is shown that there exists a σ pole, whose mass is as low as that of the ρ, owing to the disconnected diagram of the meson propagator. A discussion is given on the physical content of the σ.

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Recently there has been renewed and growing interest in scalar mesons [1,2]. One of the most noteworthy developments in this field is the reidentification of the $\sigma(600)$ after some 20 years. The results of reanalyses of the π-π scattering phase shift of the scattering (S) matrix strongly suggest the existence of the pole of the σ meson with $I=0$ and $J^{PC}=0^{++}$ in the s channel as well as the ρ pole in the t channel. In confirming the σ pole, it was essential to respect chiral symmetry, analyticity, unitarity, and crossing symmetry in constructing the S matrix [3–5]. The significant contributions of the σ pole were also identified in the decay processes of heavy particles, such as $D\to\pi\pi\pi$ and $\Upsilon(3S)\to\Upsilon\pi\pi$ [1,6–8]. The meson called "$f_0(600)$ or σ" appeared in the 2002 Review of Particle Physics [9]. It should be noted that a recent analysis [10] clearly shows phase motion consistent with resonance behavior.

As stressed above, the significance of the σ pole is intimately related to chiral symmetry in QCD; the σ may be identified as the chiral partner of the π in the linear representation of chiral $\mathrm{SU}(2)\otimes\mathrm{SU}(2)$ symmetry [11]. Here, it is noteworthy that the κ meson with $I=1/2$ is reported to exist with a mass m_κ of about 800 MeV [8,12,13]; the κ is supposed to constitute the nonet scalar states of chiral $\mathrm{SU}(3)\otimes\mathrm{SU}(3)$ symmetry together with the σ.[1]

Even with the confirmed existence of the low-lying scalar mesons, there are long standing controversies on their nature [2]: Are they usual $q\bar{q}$ mesons, four quark states like $qq\bar{q}\bar{q}$ [14], or $\pi\pi$ ($K\bar{K}$) molecules, rather than preexisting resonances? How large is the mixing among them or with glueballs [15]? Although not mentioned in [2], the σ may be a *collective* $q\bar{q}$ state described as a superposition of many *atomic* $q\bar{q}$ states [16,17]. In fact, the dynamical models of chiral symmetry breaking, such as the Nambu–Jona-Lasinio model [17] and the Schwinger-Dyson approaches [18], describe the σ as well as the π as collective states.

Confronting these problems with the σ, it would be intriguing to explore whether QCD accommodates the σ and other possible low-lying scalar mesons as well as to examine their nature by a first-principles calculation of QCD. In fact, studies of scalar hadrons in lattice QCD were important mainly for exploring properties of glueballs until recently [19]. We notice, however, that a different choice of the interpolation field may give a different overlap with the physical states, and thereby reveal a novel state, such as the possible low-mass σ.

DeTar and Kogut [20] were the first to measure the screening mass of the σ meson in lattice QCD, although in the quenched approximation. In the same approximation, Alford and Jaffe explored possible light scalar mesons described as $q^2\bar{q}^2$ states [21], while the masses of the $q\bar{q}$ states and their mixing with glueball states in the scalar channel were investigated by Lee and Weingarten [22]. It was, however, pointed out [23] that the so-called disconnected diagram, which is not properly treated in the quenched approximation, gives almost the same amount of contribution to the propagator as the connected diagram does; this indicates that using the dynamical quarks is essential to study the σ on a lattice.

A simulation with dynamical quarks was carried out by McNeile and Michael [24], who computed the masses of the iso-singlet scalar states given by a superposition of $q\bar{q}$ and glueball states. In their simulation, the σ meson was found to be lighter than the π; hence the relevance of the simulation in reality is obscure.

Since the existence and nature of the σ may be related to chiral symmetry and its dynamical breaking, it is desirable to examine the behavior of the chiral limit of the calculated quantities for a proper description of the σ meson. Of course, it is preferable to adopt lattice fermions, which have better chiral properties. Using the domain wall fermions, the RBC (Riken-Brookhaven-Columbia) Collaboration examined the σ propagator on the lattice, unfortunately, however, in the quenched approximation and with an approximate estimate of the disconnected diagram based on the chiral perturbation theory [25].

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[1] See "Meson Particle Listings $f_0(600)$" in Ref. [9], and references therein.

In this paper, we present a lattice calculation of the scalar particles using full QCD with the inclusion of the disconnected diagrams. We employ the most standard lattice QCD numerical techniques and show that there exists a σ pole, whose mass is similar to that of the ρ meson for the quark masses used for the simulation, but tends to become as low as $2m_\pi$ in the chiral limit. We shall show that the incorporation of the disconnected diagram is responsible for realizing such a low mass of the σ meson. We shall briefly discuss the physical content of the σ meson. We shall also mention that our simulation suggests the existence of the κ meson, another member of the nonet of the scalar mesons.

We adopt the following interpolation operator for creating the σ meson with $I=0$ and $J^{PC}=0^{++}$:

$$\hat{\sigma}(x) \equiv \sum_{c=1}^{3} \sum_{\alpha=1}^{4} \frac{\bar{u}_\alpha^c(x)u_\alpha^c(x)+\bar{d}_\alpha^c(x)d_\alpha^c(x)}{\sqrt{2}}, \qquad (1)$$

where u (d) denotes the up-quark (down-quark) operator with c and α being the color and Dirac-spinor indices, respectively. Then the σ meson propagator reads

$$\begin{aligned} G(y,x) &= \langle \mathrm{T}\hat{\sigma}(y)\hat{\sigma}(x)^\dagger \rangle \\ &= \frac{1}{Z}\int DUD\bar{u}DuD\bar{d}Dd \\ &\quad \times \hat{\sigma}(y)\hat{\sigma}(x)^\dagger e^{-S_G-\bar{u}Wu-\bar{d}Wd}. \end{aligned} \qquad (2)$$

Here W^{-1} is the u- and d-quark propagators, U is the link variables of the gluon, and S_G the gauge action. By integration over the quark fields, the σ meson propagator is reduced to

$$\begin{aligned} G(y,x) = &-\langle \mathrm{Tr}\, W^{-1}(x,y)W^{-1}(y,x)\rangle \\ &+ 2\langle(\sigma(y)-\langle\sigma(y)\rangle)(\sigma(x)-\langle\sigma(x)\rangle)\rangle, \end{aligned} \qquad (3)$$

where $\sigma(x) \equiv \mathrm{Tr}\, W^{-1}(x,x)$, with Tr being the trace operation over the color and Dirac indices. Here we set the u- and d-quark propagators to be the same. Equation (3) shows that the σ propagator consists of two terms: one corresponds to a connected diagram and the other to a disconnected diagram. Since the vacuum expectation value $\langle\sigma(x)\rangle$ does not vanish, it should be subtracted from the σ operator. We employ Wilson fermions and the plaquette gauge action. We perform a full QCD simulation in which the disconnected diagrams are included.

As for the simulation parameters, we first note that the CP-PACS Collaboration performed a full-QCD calculation of the light meson spectroscopy with great success [26]. Therefore, we use the same values of the simulation parameters as those used by CP-PACS, i.e., $\beta=4.8$ and the hopping parameter, $\kappa=0.1846$, 0.1874 and 0.1891, except for the lattice size; our lattice size is $8^3\times 16$. We shall use the point source and sink, which together with the smaller lattice size, leads to larger masses due to a mixture of higher mass states. In other words, the masses to be obtained in our simulation should be considered as upper limits. We show in Table I the value of m_π/m_ρ together with that of CP-PACS. They are consistent within the error bars. We generate the gauge configurations in full QCD using the hybrid Monte Carlo (HMC) algorithm. The first 1500 trajectories are updated in quenched QCD, then we switch to a simulation with the dynamical fermion. The next 2000 trajectories of HMC are discarded for thermalization and the σ, π, and ρ propagators are calculated every ten trajectories. The numbers of configurations used for measuring hadron masses are given in Table I ("statistics").

TABLE I. Summary of the results.

κ	0.1846	0.1874	0.1891
statistics [a]	1110	860	730
m_π/m_ρ [b]	0.8291(12)	0.7715(17)	0.7026(32)
a (fm) [d]	0.289	0.263	0.245
m_q (MeV) [d]	91.6	69.6	52.6
m_π/m_ρ [c]	0.825(2)	0.757(2)	0.693(3)
m_σ/m_ρ [c]	1.6(1)	1.34(8)	1.11(6)
$m_{\mathrm{connect}}/m_\rho$ [c]	2.40(2)	2.44(3)	2.48(4)

[a] Number of configurations separated by 10 trajectories to each other.

[b] CP-PACS.

[c] Our result. m_{connect} is the σ mass estimated only from the connected part.

[d] Estimated from CP-PACS results and $m_V=m_\rho=771(\mathrm{MeV})$.

It is not an easy task to evaluate the disconnected part of the propagator, since one must calculate $\mathrm{Tr}\, W^{-1}(x,x)$ for all lattice sites x. We used the Z_2 noise method to calculate the disconnected diagrams $\sigma(x)$, and the subtraction terms of the vacuum $\langle\sigma\rangle$. Each of these terms is the order of ten, and $\langle(\sigma-\langle\sigma\rangle)(\sigma-\langle\sigma\rangle)\rangle$ becomes less than 10^{-4}, as shown in Fig. 1. Therefore, in order to obtain the signals correctly as the difference between these terms, numerical simulations with a high precision and careful analyses are necessary. One thousand random Z_2 numbers are generated for each configuration: We refer to our previous report [23] for the relationship between the amount of Z_2 noise and the achieved

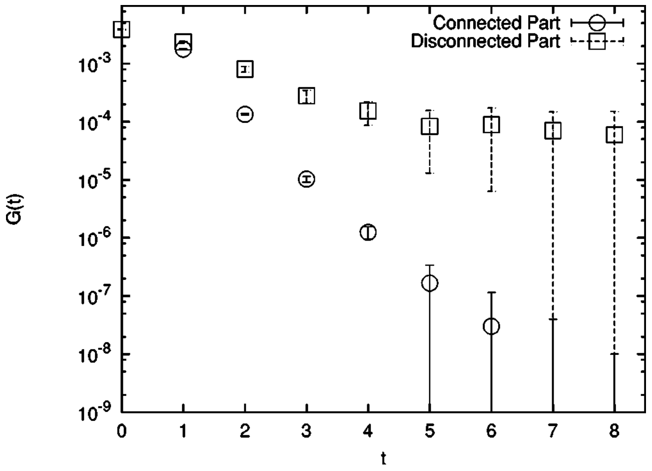

FIG. 1. Propagators of the connected and disconnected diagrams of the σ for $\kappa=0.1874$.

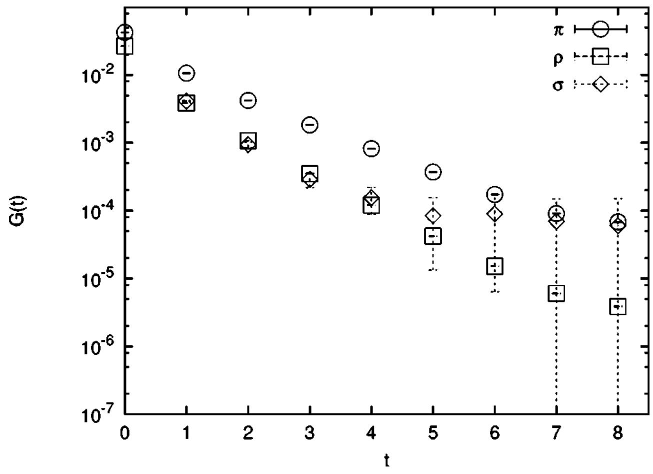

FIG. 2. Propagators of the π, ρ, and σ for $\kappa = 0.1874$.

accuracy. Gauge configurations are created by HMC in a vector supercomputer, while most of the disconnected propagator is calculated in a parallel machine.

The propagators of the π, ρ, and σ for $\kappa = 0.1874$ are shown in Fig. 2. From our results, we estimate the critical value of the hopping parameter and the lattice space as $\kappa_c = 0.195(3)$ and $a = 0.207(9)$ fm, respectively, which are to be compared with the CP-PACS values, $\kappa_c = 0.19286(14)$ and $a = 0.197(2)$ fm. Thus our lattice size is $(1.6\ \text{fm})^3$. The mass ratios of m_π/m_ρ and m_σ/m_ρ together with $m_{\text{connect}}/m_\rho$ are summarized in Table I, where m_{connect} is the mass evaluated only from the connected diagram. We have extracted the π and ρ masses using two-pole formula from their propagators. The small errors of the π and ρ propagators indicate the high precision of our simulation. Our result for the mass ratios m_π/m_ρ in Table I is in good agreement with that of CP-PACS, with unavoidable small differences owing to the different lattice sizes. To extract the σ meson mass, we have used the one-pole ansatz because its propagator has large error bars at large $t \sim 6$ in spite of our high statistics, which implies that the obtained value gives an upper limit of the scalar meson mass. Figure 2 and Table I show that the mass of the σ is of the same order as the mass of the ρ; we remark that the 2π threshold is higher than the σ in our simulation. We have checked that the effective mass method gives essentially the same result as that given above.

Figure 1 shows the individual contributions of the connected and disconnected parts of the σ propagator, which reaches a plateau at $t \geqslant 5$, as the precision of our calculation is limited to $O[G(t)] \sim 10^{-4}$. The connected part shows clear signals of a rapid damping with small error bars. We note that the connected part of the σ propagator can be regarded as the one of the a_0 meson, provided that the difference between the u-quark and the d-quark is neglected. Therefore, the rapid damping of the connected part of the σ propagator is in accordance with the fact that the a_0 is heavier than the ρ. Figure 1 shows that the disconnected part dominates the σ propagator. By comparing Fig. 2 with Fig. 1, we see that the σ as a light meson results from the disconnected part of the σ propagator with the background vacuum condensate subtracted, which are odd with the constituent quark model. In the naive sense, the connected quark diagram corresponds to the constituent quarks in the SU(3) nonrelativistic quark model where the OZI rule is satisfied. This may give a clue to clarify the physical content of the σ meson, as will be discussed later. The mass of the connected σ (the a_0) shown in Table I is almost 2.5 times of the ρ mass and exhibits only a weak dependence on the hopping parameter, suggesting an irrelevance of chiral symmetry to the a_0 meson.

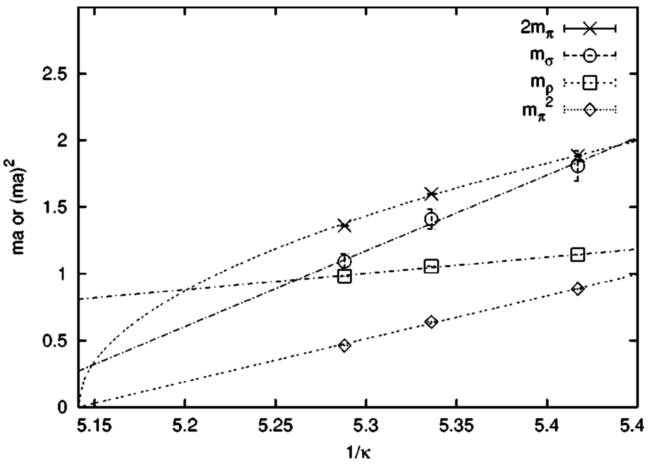

FIG. 3. m_π^2, m_ρ, m_σ, and $2m_\pi$ in the lattice unit as a function of the inverse hopping parameter. The chiral limit is given by $\kappa_c = 0.195(3)$.

In Fig. 3, we display m_π^2, m_ρ, m_σ, and $2m_\pi$ in lattice units as a function of the inverse hopping parameter. As the chiral limit is approached, the σ meson mass obtained from the full σ propagator decreases and eventually becomes smaller than the ρ meson mass in the chiral limit.

We have also calculated the κ meson using the same configurations. We take the same hopping parameter values for u and d quarks and use the following three values for the s-quark hopping parameter, $\kappa_s = 0.1835$, 0.1840, and 0.1845. The ratios m_κ/m_{K*} and m_K/m_{K*} at the chiral limit are listed in Table II. One may notice that all three values of κ_s fairly reproduce the physical mass ratio m_K/m_{K*} in the chiral limit of the u- and d-quarks. A more notable point is that the resultant κ mass hardly changes with the variation of κ_s and is almost twice as heavy as the K^*; i.e. the κ is not degenerated with the σ, although they should belong to the same nonet. The origin to lift the octet-singlet degeneracy may be attributed the following facts; because of the strangeness content, the κ propagator is solely composed of a connected diagram and contains no disconnected part, the latter of which was the origin of the light mass of the σ. In fact, the

TABLE II. The mass ratios m_κ/m_{K*} versus m_K/m_{K*} at $\kappa_c = 0.1945(29)$. The s quark hopping parameters were taken to be 0.1835, 0.1840, and 0.1845.

s-quark hopping parameter	0.1835	0.1840	0.1845
m_K/m_{K*}	0.639(6)	0.631(6)	0.623(6)
m_κ/m_{K*}	2.039(43)	2.037(43)	2.044(44)

κ mass listed in Table II is almost the same as the a_0 mass estimated using the connected part of the σ with the corresponding hopping parameters. Thus we can understand why the σ is light while the κ is heavy, consistently.

The dominance of the disconnected diagram in the σ meson propagator in contrast to the octet scalar mesons suggests possible physical contents of the light σ: The disconnected diagram includes the process $q\bar{q}\leftrightarrow G$ where G denotes a glueball state. This might suggest an importance of the glueball mixing in the light σ if the glueball states were not heavy. Instead, we notice that such a fluctuation process of the $q\bar{q}$ states with the heavy G giving an effective vertex for $q\bar{q}\rightarrow q\bar{q}$ may form a collective state mentioned before. One may also notice that the disconnected diagram contains four quark states composed of diquark and anti-diquark states, i.e., qq-$\overline{qq}$. Clearly more works are needed to elucidate the physical content of the scalar mesons.

We have reported the first study of the scalar mesons based on the full QCD lattice simulation with dynamical fermions, including analysis of the disconnected diagram effects. We have used the most standard lattice QCD techniques, which have worked well for the established mesons and baryons, and clarified the accuracy and statistics required to obtain signals in the scalar channel.

Our results indicate the existence of a light isoscalar $J^{PC}=0^{++}$ scalar meson, i.e., the σ meson with a mass of almost the same order as that of the ρ meson (see Table I). We have also shown that the κ meson belonging to the flavor octet is almost twice as heavy as the K^*.

Of course, the results reported here are in a far-from-final quantitative stage because our quark masses are much heavier than those in the real world, and our scalar mesons cannot decay on our lattice. Although we have got reasonable π and ρ mass values, $1/a=1$ GeV is not high enough to avoid Wilson fermions scaling violation. Calculations at weaker coupling are highly desirable. Nevertheless, it is now clear that the σ meson as well as other scalar mesons can be studied in the lattice QCD. We hope that the present work prompts other lattice studies that will give a deeper and more quantitative understanding of the scalar mesons, especially the σ.

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