

Heavy-quark production in proton-nucleus collisions at the CERN LHC

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A sizable rate of events, with several pairs of heavy quarks produced contemporarily by multiple parton interactions, may be expected at very high energies as a consequence of the large parton luminosities. The production rates are enhanced in hadron-nucleus reactions, which may represent a convenient tool to study the phenomenon. We compare the different contributions to $b\bar{b}b\bar{b}$, $c\bar{c}c\bar{c}$, and $b\bar{b}c\bar{c}$ production due to single- and double-parton scatterings, in collisions of protons with nuclei at the CERN LHC.

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I. INTRODUCTION

The large rates of production of heavy quarks, expected at high energies, may lead to a sizable number of events, at the CERN Large Hadron Collider (LHC), containing different pairs of heavy quarks, generated simultaneously by independent partonic collisions. An inclusive cross section of the order of $10 \mu\text{b}$ may in fact be foreseen for a double-parton collision process, with two $b\bar{b}$ pairs produced in pp interactions at 14 TeV [1], while the cross section to produce two $c\bar{c}$ pairs may be of the order of 1 mb, the contribution of single-parton collisions to the processes being one order of magnitude smaller. All production rates are significantly enhanced in proton-nucleus collisions, which may offer considerable advantages for studying multiparton collisions [2]. Given the large rates expected, the production of multiple pairs of heavy quarks should hence represent a convenient process to study multiparton interactions in pA collisions at the LHC. On the other hand, the mechanism of heavy quark production is not yet understood satisfactorily for the simplest case of nucleon-nucleon collisions, the effects of higher-order corrections in α_s still being a matter of debate. A comprehensive description of the much more structured process of heavy-quark production in hadron-nucleus interactions might therefore be approached after gaining a deeper understanding of the short scale parton-level dynamics of the process. On the other hand, a significant feature of higher-order corrections in α_s is that, for a limited set of physical observables, the whole effect of higher orders reduces to an approximate rescaling of a lowest-order calculation of heavy-quark production in perturbative QCD [3]. Some of the features of the process are therefore effectively described by the simplest parton level dynamics at the lowest order in α_s , which lets one speculate that a similar property might hold also for a much more complex process, as multiple production of heavy quarks.

Taking this optimistic point of view, we will attempt, in the present paper, to obtain indications on some properties of

$b\bar{b}b\bar{b}$, $c\bar{c}c\bar{c}$, and $b\bar{b}c\bar{c}$ production in pA interactions at the LHC. We consider the contributions of the two different interaction mechanisms, the connected $2 \rightarrow 4$ and the disconnected $(2 \rightarrow 2)^2$ parton processes, which will be effectively described by the lowest-order diagrams in perturbative QCD. Higher-order corrections are kept in account by a simple overall rescaling. Obviously, following this philosophy, all considerations are necessarily limited to the restricted class of physical observables falling in the category above.

II. PRODUCTION IN PROTON-NUCLEUS COLLISIONS

In general [4,5], with the only assumption of factorization of the hard component of the interaction, the expression of the double-parton scattering cross section to produce two pairs of heavy quarks is given by

$$\sigma_{(p,A)}^D = \frac{m}{2} \sum_{ij} \int \Gamma_p(x_i, x_j; s_{ij}) \hat{\sigma}(x_i, x'_i) \hat{\sigma}(x_j, x'_j) \times \Gamma_{(N,A)}(x'_i, x'_j; s_{ij}) dx_i dx'_i dx_j dx'_j d^2 s_{ij}, \quad (1)$$

where the index N refers to a nucleon and A to a nucleus, while the indices i, j refer to the different kinds of partons that annihilate to produce a given $q\bar{q}$ pair. The factor $m/2$ is a consequence of the symmetry of the expression for exchanging i and j ; specifically $m=1$ for indistinguishable parton processes and $m=2$ for distinguishable parton processes. The interaction region of a hard process is very small as compared to the hadron scale; hence in the case of a double-parton collision the two elementary interactions are well localized in transverse space, within the two overlapping hadrons. The rate of events where two hard collisions take place simultaneously, in a given inelastic hadron-hadron event, depends therefore on the typical transverse distance between the partons of the pairs undergoing the multiple processes. The main interest is hence that double-parton scatterings may provide information on the typical transverse separation between pairs of partons in the hadron structure. Indeed the nonperturbative input of a double-parton collision is the two-body parton distribution function $\Gamma(x_1, x_2, s_{1,2})$, which depends not only on the fractional momenta $x_{1,2}$, but also on the relative distance in transverse space $s_{1,2}$, besides (al-

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TABLE I. K factor values for different choices of the mass of the heavy quarks. In the k_t factorization approach the cross section has been evaluated by using \hat{s} as factorization scale and the gluon virtuality as renormalization scale; when evaluating the cross section at the lowest order in pQCD the scales have been set equal to the transverse mass of the heavy quarks [8].

$m_b(\text{GeV})$	$\mu_F^2 = \hat{s}$ $K_b = \sigma(b\bar{b})/\sigma_{LO}(b\bar{b})$	$\mu_R^2 = q^2$ (gluon virtuality) $m_c(\text{GeV})$	$K_c = \sigma(c\bar{c})/\sigma_{LO}(c\bar{c})$
4.25	5.9	1.2	7.0
4.5	5.7	1.4	6.6
4.75	5.5	1.6	6.1

though not written explicitly to simplify the notation) on the scales of the two interactions and on the different kinds of partons involved. As a consequence the double-parton scattering cross section is characterized by a linear dependence on dimensional scale factors, which are related directly to the typical transverse distances between the various pairs of partons, contributing to the double scattering process under consideration.

The cross section is simplest when the target is a nucleon and partons are not correlated in fractional momenta, which may be a sensible approximation in the limit of small x . In such a case the two-body parton distribution may be factorized as $\Gamma_p(x_i, x_j; s_{ij}) = G(x_i)G(x_j)F(s_{ij})$, where $G(x)$ is the usual one-body parton distribution and $F(s)$ a function normalized to 1 and representing the parton pair density in transverse space. With this simplifying assumption the inclusive cross section to produce two pairs of heavy quarks is written as [6]

$$\sigma_N^D = \frac{m}{2} \sum_{ij} \Theta^{ij} \sigma_i \sigma_j, \quad (2)$$

where $\sigma_i \sigma_j$ represent the inclusive cross sections to produce a $q\bar{q}$ pair in a hadronic collision, with the indices i, j labeling a definite parton process. The factors Θ^{ij} have the dimension of an inverse cross section and result from integrating the products of the two-body parton distributions in transverse space. In this simplified case the factors Θ^{ij} provide a direct

TABLE II. K factor values for different choices of the masses of the heavy quarks. In the calculation of the cross sections, factorization and renormalization scales have been set equal to the average of the squared transverse masses of the heavy quarks [8].

$m_b(\text{GeV})$	$\mu_F^2 = \mu_R^2 = \mu_0^2 = (m_{t,Q}^2 + m_{b,Q}^2)/2$ $K_b = \sigma(b\bar{b})/\sigma_{LO}(b\bar{b})$	$m_c(\text{GeV})$	$K_c = \sigma(c\bar{c})/\sigma_{LO}(c\bar{c})$
4.25	6.1	1.2	6.2
4.5	5.9	1.4	5.9
4.75	5.8	1.6	5.6

TABLE III. K factor values for different choices of the masses of the heavy quarks. In the calculation of the cross sections, factorization and renormalization scales have been set equal to the mass of the heavy quark [8].

$m_b(\text{GeV})$	$\mu_F^2 = \mu_R^2 = m_Q^2$ $K_b = \sigma(b\bar{b})/\sigma_{LO}(b\bar{b})$	$m_c(\text{GeV})$	$K_c = \sigma(c\bar{c})/\sigma_{LO}(c\bar{c})$
4.25	5.7	1.2	8.1
4.5	5.6	1.4	7.9
4.75	5.4	1.6	7.3

measure of the different average transverse distances between different pairs of partons in the hadron structure [6,7].

The cross section has a more elaborate structure in the case of a nuclear target. The most suitable conditions for studying the phenomenon are those where the nuclear distributions are additive in the nucleon parton distributions. In such a case one may express the nuclear parton pair density, $\Gamma_A(x'_i, x'_j; s_{ij})$, as the sum of two well-defined contributions, where the two partons are originated by either one or by two different parent nucleons:

$$\Gamma_A(x'_i, x'_j; s_{ij}) = \Gamma_A(x'_i, x'_j; s_{ij})|_1 + \Gamma_A(x'_i, x'_j; s_{ij})|_2 \quad (3)$$

and correspondingly $\sigma_D^A = \sigma_D^A|_1 + \sigma_D^A|_2$. The two terms $\Gamma_A|_{1,2}$ are related to the nuclear nucleon's density. Introducing the transverse parton coordinates $B \pm s_{ij}/2$, where B is the impact parameter of the hadron-nucleus collision, one may write

$$\Gamma_A(x'_i, x'_j; s_{ij})|_{1,2} = \int d^2B \gamma_A \left(x'_i, x'_j; B + \frac{s_{ij}}{2}, B - \frac{s_{ij}}{2} \right) \Big|_{1,2}, \quad (4)$$

where $\gamma_A|_{1,2}$ are given by

$$\begin{aligned} \gamma_A \left(x'_i, x'_j; B + \frac{s_{ij}}{2}, B - \frac{s_{ij}}{2} \right) \Big|_1 &= \Gamma_N(x'_i, x'_j; s_{ij}) T(B), \\ \gamma_A \left(x'_i, x'_j; B + \frac{s_{ij}}{2}, B - \frac{s_{ij}}{2} \right) \Big|_2 &= G_N(x'_i) G_N(x'_j) T \left(B + \frac{s_{ij}}{2} \right) T \left(B - \frac{s_{ij}}{2} \right), \end{aligned} \quad (5)$$

with $T(B)$ is the nuclear thickness function, normalized to the atomic mass number A , and G_N is the nuclear parton distributions divided by the atomic mass number.

In the simplest additive case, the first term in Eq. (3) obviously gives only a rescaling of the double-parton distribution of an isolated nucleon,

$$\Gamma_A(x'_i, x'_j; s_{ij})|_1 = \Gamma_N(x'_i, x'_j; s_{ij}) \int d^2B T(B), \quad (6)$$

and the resulting contribution to the double-parton scattering cross section is the same as that in a nucleon-nucleon inter-

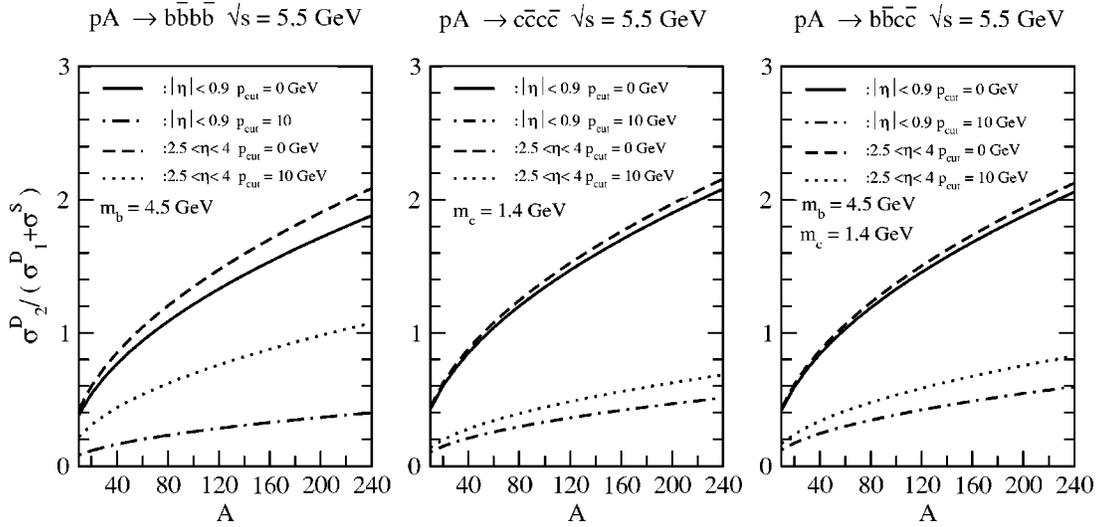


FIG. 1. Relative weights of the terms with “anomalous” and “usual” A dependence in the double scattering cross section for $b\bar{b}b\bar{b}$, $c\bar{c}c\bar{c}$, $b\bar{b}c\bar{c}$ production.

action, apart from an enhancement factor due to the nuclear flux, which is given by the value of the atomic mass number A :

$$\sigma_A^D|_1 = A \sigma_N^D. \quad (7)$$

The $\sigma_A^D|_2$ term has more structure. In this case the integration on the relative transverse distance between the partons of the interacting pairs, s_{ij} , involves both the projectile and two different target nucleons:

$$\int ds_{ij} \Gamma_p(x_i, x_j; s_{ij}) T\left(B + \frac{s_{ij}}{2}\right) T\left(B - \frac{s_{ij}}{2}\right). \quad (8)$$

As one may notice the expression (8) depends on two very different scales: the hadron radius r_p and the nuclear radius R_A . The usual approximation in pA interactions is to consider the limit $r_p \ll R_A$, where one may use the approximation

$$T\left(B \pm \frac{s_{ij}}{2}\right) \approx T(B), \quad (9)$$

which allows one to decouple the integrations on s_{ij} and on B . One hence obtains

$$\sigma_A^D|_2 = \frac{1}{2} \sum_{ij} \int G_p(x_i, x_j) \hat{\sigma}(x_i, x'_i) \hat{\sigma}(x_j, x'_j) G_N(x'_i) \times G_N(x'_j) dx_i dx'_i dx_j dx'_j \int d^2B T^2(B), \quad (10)$$

where

$$G_p(x_i, x_j) = \int d^2s_{ij} \Gamma_p(x_i, x_j; s_{ij}). \quad (11)$$

Remarkably the two terms $\sigma_A^D|_1$ and $\sigma_A^D|_2$ have very different properties. In fact, while the correct dimensionality of

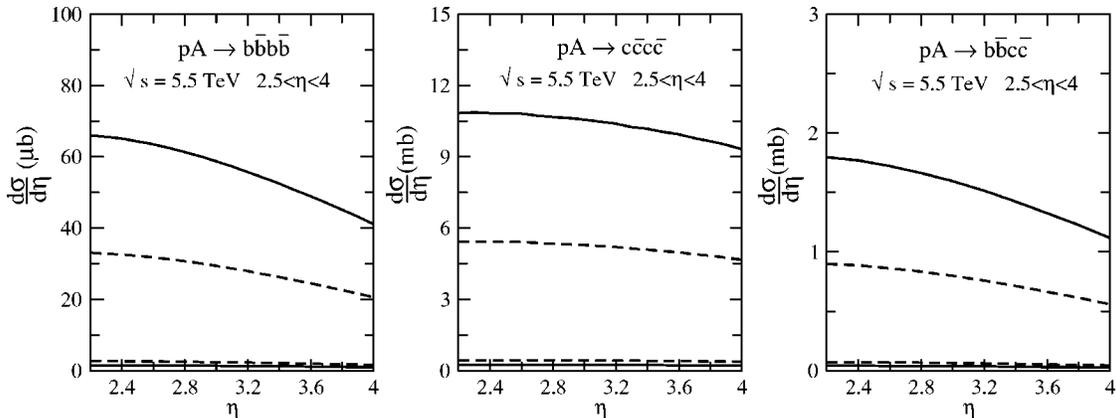


FIG. 2. Pseudorapidity distribution in a forward calorimeter of a heavy quark produced in events with $b\bar{b}b\bar{b}$, $c\bar{c}c\bar{c}$, and $b\bar{b}c\bar{c}$: one-nucleon (dashed lines) and two-nucleon contributions (continuous lines) in the case of a heavy (higher curves) and of a light nucleus (lower curves).

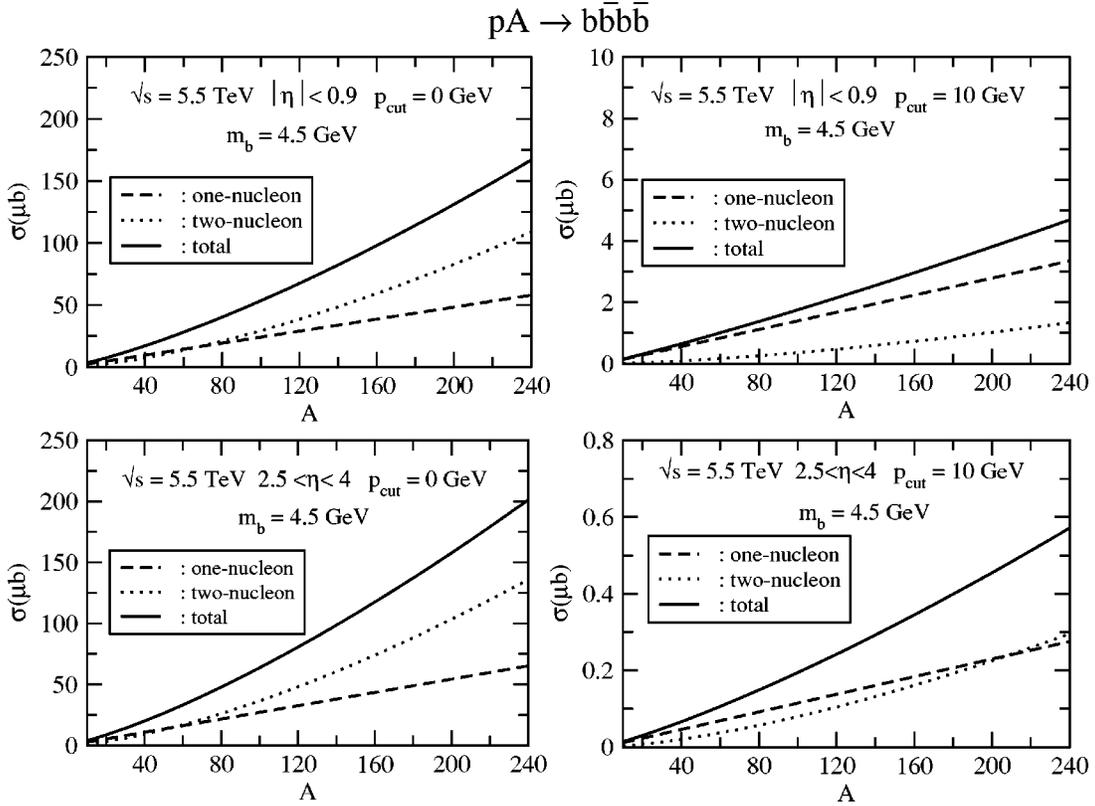


FIG. 3. Different contributions to the cross section of $b\bar{b}b\bar{b}$ production in a central and in a forward calorimeter as a function of A . Cross sections without any cut in p_t (left figures) and after applying a cut of 10 GeV in the transverse momenta of each produced heavy quark (right figure): one-nucleon (dashed lines) and two-nucleon contributions (dotted lines) and total cross section (continuous lines).

$\sigma_A^D|_1$ is provided by transverse scale factors related to the *nucleon* scale, Eqs. (2), (7), the analogous dimensional factor is provided in $\sigma_A^D|_2$ by the *nuclear* thickness function, which is at the second power in Eq. (10), two representing the target nucleons involved in the interaction.

As pointed out in Ref. [2], on general grounds $\sigma_{(p,A)}^D$ depends both on the longitudinal and transverse parton correlations. However, the $\sigma_A^D|_2$ term depends solely on the longitudinal momentum fractions x_i, x_j so that, when the $\sigma_A^D|_2$ term is isolated, one has the capability of measuring the longitudinal and, *a fortiori*, also the transverse parton correlations of the hadron structure in a model-independent way.

Although the two contributions may be defined in a more general case, the separation of the cross section in the two terms $\sigma_A^D|_1$ and $\sigma_A^D|_2$ is most useful in the regime of additivity of the nuclear structure functions, which may not be a bad approximation for a sizable part of the kinematical regime of heavy-quark production at the LHC. In the case of a central calorimeter with the acceptance of the ALICE detector ($|\eta| < 0.9$), the average value of the momentum fraction of the initial state partons, in a $pp \rightarrow b\bar{b}b\bar{b}$ process, is $\langle x \rangle \approx 6 \times 10^{-3}$ while for $pp \rightarrow c\bar{c}c\bar{c}$ one finds $\langle x \rangle \approx 2 \times 10^{-3}$. With a cut of 5 GeV in the transverse momenta of the b, c quarks one obtains $\langle x \rangle \approx 10^{-2}$ and $\langle x \rangle \approx 8 \times 10^{-3}$, respectively, while a cut of 20 GeV gives $\langle x \rangle \approx 3 \times 10^{-2}$. In the case of

$2.5 < \eta < 4$ and without any cut in p_t , the average values of momentum fraction are $\langle x \rangle \approx 3 \times 10^{-2}$ and $\langle x \rangle \approx 9 \times 10^{-3}$ [8]. Deviations from additivity at low x are less than 10% for $x \geq 2 \times 10^{-2}$ [10] and, although increasing with the atomic mass number, nonadditive corrections should hence be at most a 20% effect, in above the kinematical regimes.

As mentioned in the Introduction, heavy-quark production is characterized by nontrivial dynamics, in such a way that also next-to-leading corrections to the lowest-order term in perturbative QCD are not sufficient for an exhaustive description of the inclusive spectra, which most likely need an infinite resummation to be evaluated. After comparing the results of different approaches to heavy-quark production (as next to leading order QCD and k_t factorization), one nevertheless finds that, in a few cases, the whole effect of higher orders is to a large extent just an approximate rescaling of the results obtained by a lowest-order evaluation in perturbative QCD. When limiting the discussion to an accordingly restricted set of physical observables, the whole effect of higher-order corrections to heavy quark production is hence summarized by a single number, the value of the so-called K factor defined as

$$K = \frac{\sigma(q\bar{q})}{\sigma_{LO}(q\bar{q})}, \quad (12)$$

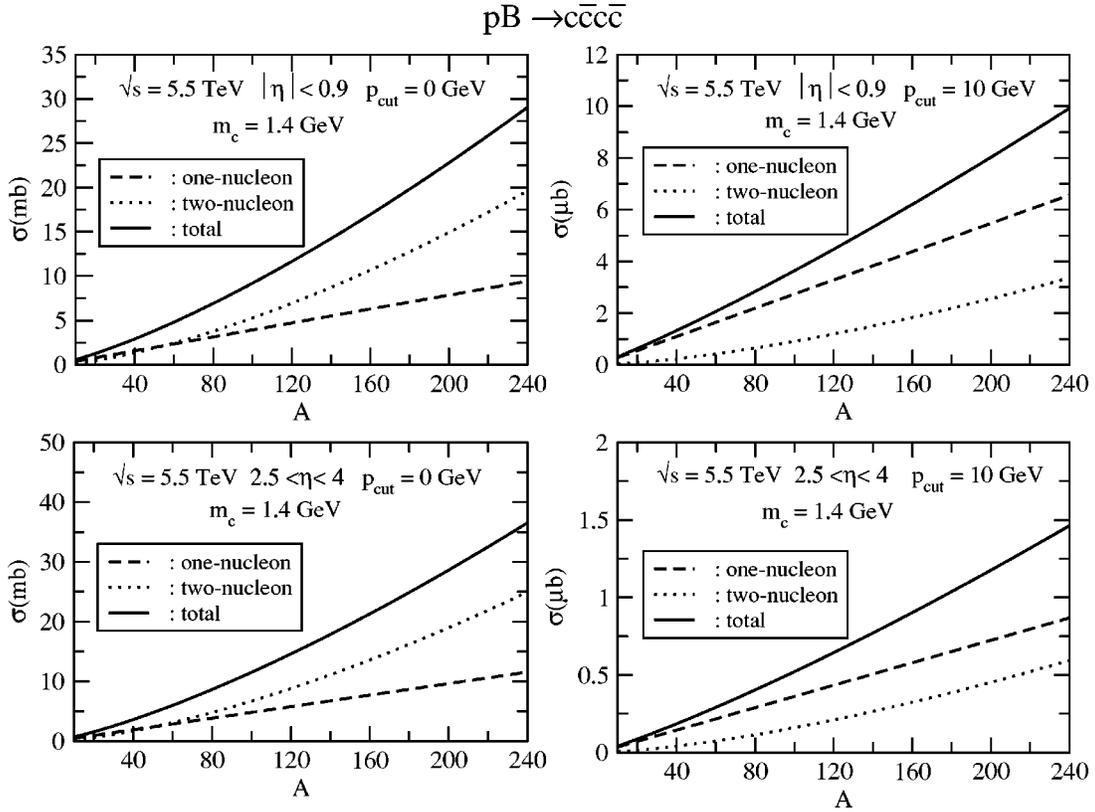


FIG. 4. Different contributions to the cross section of $c\bar{c}c\bar{c}$ production in a central and in a forward calorimeter as a function of A . Cross sections without any cut in p_t (left figures) and after applying a cut of 10 GeV in the transverse momenta of each produced heavy quark (right figure): one-nucleon (dashed lines) and two-nucleon contributions (dotted lines) and total cross section (continuous lines).

where $\sigma(q\bar{q})$ is the inclusive cross section for $q\bar{q}$ production and $\sigma_{LO}(q\bar{q})$ the result of the lowest-order calculation in perturbative QCD (pQCD). By evaluating, with the k_t -factorization approach, rapidity and pseudorapidity distributions of $b\bar{b}$ and of $c\bar{c}$ production, in pp collisions at the center-of-mass energy of $\sqrt{s}=5.5$ TeV, within $|\eta|<0.9$ and $2.5<\eta<4$, with different choices of heavy-quark masses and of the factorization and renormalization scales, one finds a result not incompatible with a lowest-order calculation in perturbative QCD rescaled by the factors K shown in Tables I–III.

To obtain the values of the K factor in Table I, in the evaluation of the cross section with the k_t -factorization approach, we have set the factorization scale equal to the invariant mass of the parton process \hat{s} and the renormalization scale equal to the virtuality of the initial state gluons, while, for the cross section at the lowest order in pQCD, we used as a scale factor the transverse mass of the produced quarks. In Tables II and III we used instead in both cross sections the average of the squared transverse masses of the produced quarks and the heavy-quark mass as scale factors [8].

With a fixed choice of factorization and renormalization scales, one obtains a substantial increase of the value of the K factor when decreasing the value of the heavy-quark mass, the increase being larger for $c\bar{c}$ than for $b\bar{b}$ production. With our different choices for the scale factors we obtain varia-

tions of the K factor within 4% for $b\bar{b}$ and 15% for $c\bar{c}$ production.

To discuss $q\bar{q}q\bar{q}$ production in pA collisions, while remaining in a kinematical regime where non additive corrections to the nuclear structure functions are not a major effect, we have limited all considerations to physical observables, where higher orders may be taken into account by a simple rescaling of the lowest-order calculation. For $q\bar{q}q\bar{q}$ production, where only results of three-level calculations are up to now available, we have further assumed that the value of the K factor in the $2\rightarrow 4$ parton process is the same as in the $2\rightarrow 2$ process [1,9]. We have hence evaluated the various contributions to the cross section in the case of a central, $|\eta|<0.9$, and of a forward calorimeter, $2.5<\eta<4$, where the approximate expression of the cross section in Eq. (2) may not be an unreasonable approximation. As the process is dominated by gluon fusion, the expression of the cross section in Eq. (2) may be limited to a single term only. For the corresponding dimensional scale factor we used the value reported by the CDF measurement of double-parton collisions [11,12] and to evaluate the cross section at the lowest order in pQCD we used the MRS99 parton distributions [13], with factorization and renormalization scale equal to the transverse mass of the produced quarks. The cross sections of the $2\rightarrow 4$ processes have been evaluated generating the matrix elements of the partonic amplitudes with MADGRAPH

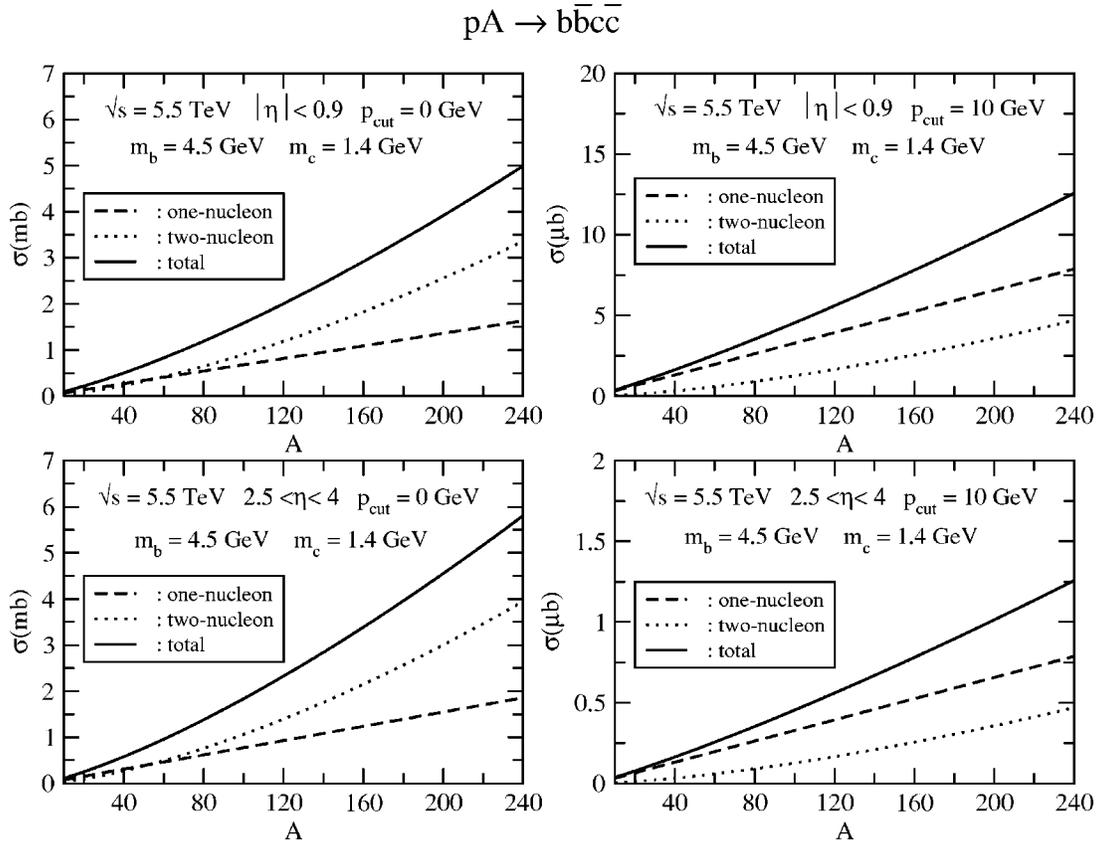


FIG. 5. Different contributions to the cross section of $b\bar{b}c\bar{c}$ production in a central and in a forward calorimeter as a function of A . Cross sections without any cut in p_t (left figures) and after applying a cut of 10 GeV in the transverse momenta of each produced heavy quark (right figure): one-nucleon (dashed lines) and two-nucleon contributions (dotted lines) and total cross section (continuous lines).

[14] and HELAS [15]. For the mass of the bottom quark we used the central values $m_b = 4.6$ and $m_c = 1.4$ GeV. The multidimensional integrations have been performed by VEGAS [16] and the lowest-order pQCD cross sections have been finally multiplied by the K factors of Table I.

III. RESULTS

The major result of the present analysis is that the effects induced by the presence of the nucleonic degrees of freedom, in double-parton scatterings with a nuclear target, cannot be reduced to the simple shadowing corrections of the nuclear parton structure functions, whose effect is to decrease the cross section as a function of A . In the case of double-parton collisions, the main effect of the nuclear structure is represented by the presence of the $\sigma_A^D|_2$ term in the cross section, which scales with a different power of A as compared to the single-scattering contribution, producing an additive correction to the cross section.

To emphasize the resulting “anomalous” dependence of the double-parton scattering cross section as a function of the atomic mass number, we have plotted in Fig. 1 the ratio $\sigma_2^D/(\sigma_1^D + \sigma^S)$ as a function of A . The ratio represents the contribution to the cross section of the processes where two different nuclear target nucleons are involved in the interaction, scaled with the contribution where only a single target

nucleon is involved. The dependence on the atomic mass number of the latter terms of the cross section, namely the single- (σ^S) and the double- (σ_1^D) parton scattering terms against a single nucleon in the nucleus, is the same for all hard processes usually considered, where nuclear effects may be wholly absorbed in the shadowing corrections to the nuclear structure functions. The contribution to the cross section of the σ_2^D term is, on the contrary, “anomalous,” involving two different target nucleons in the interaction process. The ratio above hence represents the relative weights of the anomalous to the usual contributions to the double-parton scattering cross section on a nuclear target.

The plots in Fig. 2 show the rapidity distribution in a forward calorimeter ($2.5 \leq \eta \leq 4$) of a heavy quark produced in an event with $b\bar{b}b\bar{b}$, $c\bar{c}c\bar{c}$, and $b\bar{b}c\bar{c}$, respectively: one-nucleon (dashed lines) and two-nucleon contributions (continuous lines) in the case of a heavy, Au (higher curves), and of a light nucleus, O (lower curves). In each case considered continuous and dashed curves differ essentially only by a rescaling, showing that the effect of the single-parton scattering term (the $2 \rightarrow 4$ parton process) is negligible in this kinematical regime. The different contributions to the cross section for $b\bar{b}b\bar{b}$, $c\bar{c}c\bar{c}$, and $b\bar{b}c\bar{c}$ production, due to interactions with a single (dashed line) or with two different target nucleons (dotted line), are shown in Figs. 3–5 as a

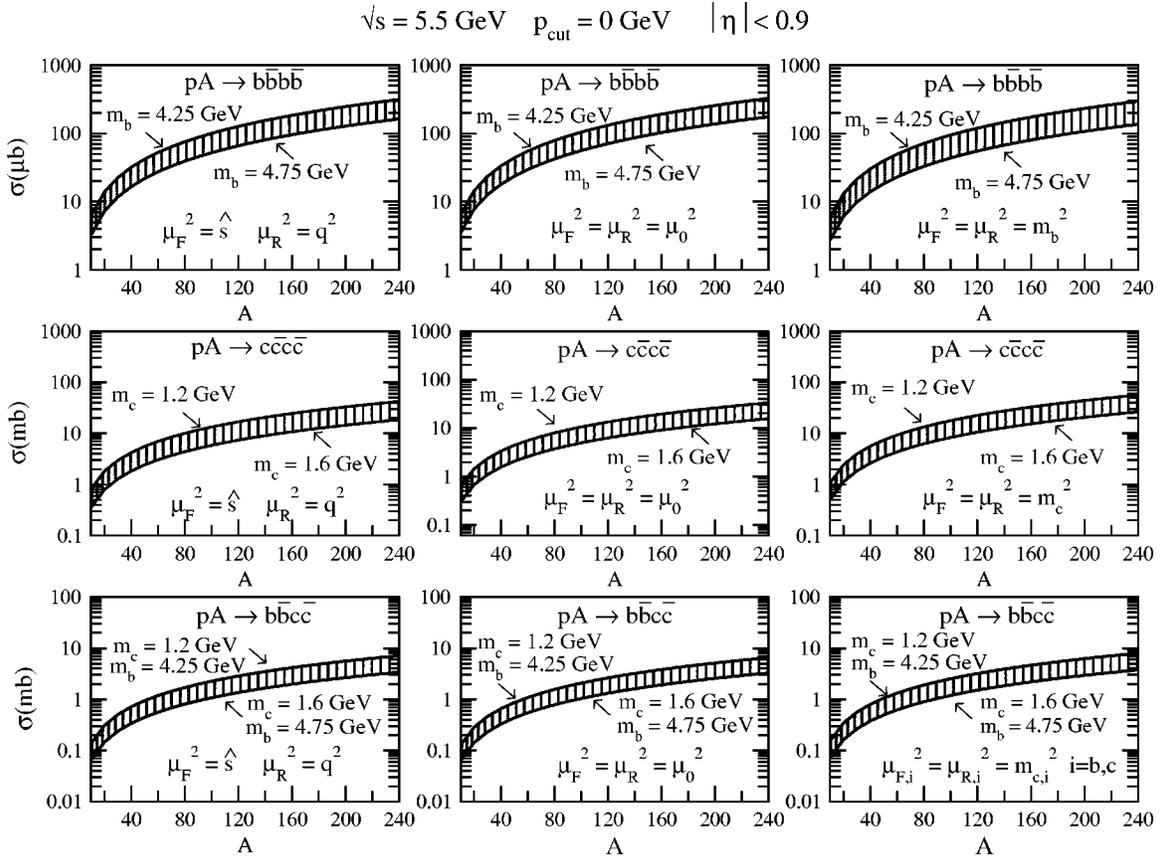


FIG. 6. Production cross section of $b\bar{b}b\bar{b}$, $c\bar{c}c\bar{c}$, and $b\bar{b}c\bar{c}$ in a central calorimeter for different choices of quark masses and factorization scales. Lower and upper curves correspond, respectively, to the lower and upper values of the heavy-quark masses considered in our calculation.

function of the atomic mass number in the case of a central and of a forward calorimeter, with two different choices of cuts ($p_t=0,10 \text{ GeV}$) on the outgoing quark-transverse momenta. The continuous line is the sum of the two contributions. As one may see, by introducing a cutoff in p_t one enhances the single-scattering contribution and the relative role of σ_2^D is decreased.

To have some indication of the overall uncertainty of our estimates we have plotted in Fig. 6 the range of values obtained for the integrated cross sections, for $b\bar{b}b\bar{b}$, $c\bar{c}c\bar{c}$, and $b\bar{b}c\bar{c}$ production, within $|\eta| \leq 0.9$, as a function of the atomic mass number when making the different choices described above. In each panel we report the different choices corresponding to the extreme values of the cross section. The indication obtained in this way is that the overall cross section is roughly determined within a factor of three.

Summarizing the large size of the cross section of heavy-quark production in hadron-nucleus collisions at the LHC (one may expect values of the order of 5–10 mb for charm production) suggests that the production of multiple pairs of heavy quarks is fairly typical at high energies, hence representing a convenient channel to study multiple-parton interactions. A rather direct feature, which is a simplest prediction and then a test of the interaction mechanism described

above, is the “anomalous” dependence on A . The effects induced by the presence of the nucleonic degrees of freedom in the nuclear structure are in fact not limited, in this case, to the shadowing corrections to the nuclear structure functions usually considered, which cause a limited *decrease* (not larger than 20%, in the kinematical regime considered here) of the cross section for a hard interaction in hadron-nucleus collisions. When considering double-parton scatterings, all-nuclear effects can be exhausted in the shadowing corrections to the nuclear structure functions in the $\sigma_A^D|_1$ term only. The dominant effect of the nuclear structure is on the contrary due to the presence of the $\sigma_A^D|_2$ term in the cross section, which scales with a different power of A as compared to the single-scattering term, giving rise to a sizably larger correction with opposite sign as compared to the shadowing correction, namely, to an *increase* of the cross section, which may become larger than 100% for a heavy nucleus.

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