Tetracritical behavior in strongly interacting theories

Francesco Sannino*

NORDITA & The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Kimmo Tuominen†

Department of Physics, P.O. Box 35, FIN-40014 University of Jyväskylä, Finland
and Helsinki Institute of Physics, P.O. Box 64, FIN-00014 University of Helsinki, Finland

We suggest a tetracritical fixed point to naturally occur in strongly interacting theories. As a fundamental example we analyze the temperature-quark chemical potential phase diagram of QCD with fermions in the adjoint representation of the gauge group (i.e., adjoint QCD). Here we show that such a nontrivial multicritical point exists and is due to the interplay between the spontaneous breaking of a global $U(1)$ symmetry and the center group symmetry associated with confinement. Our results demonstrate that taking confinement into account is essential for understanding the critical behavior as well as the full structure of the phase diagram of adjoint QCD. This is in contrast to ordinary QCD where the center group symmetry associated with confinement is explicitly broken when the quarks are part of the theory.

I. INTRODUCTION

Phase diagrams for strongly interacting theories are a topic of past and current interest [1], and the relation between deconfinement and chiral symmetry restoration poses a continuous challenge. In ordinary QCD these problems have been intensively addressed via computer simulations [2]. By investigating such a relation in different strongly interacting theories, one gains insight on the ordinary QCD dynamics as well. We recall that the order parameter for deconfinement is the Polyakov loop [3], while that for chiral symmetry restoration is the quark condensate. The representation of matter with respect to the gauge group is known to play a relevant role in the deconfining dynamics. Much attention in the literature has been given to ordinary QCD with two or three flavors. The presence of quarks in the fundamental representation breaks the center group symmetry explicitly, and for massless quarks only the chiral phase transition remains well defined. The latter is then expected to drive the critical behavior [4]. At nonzero and large quark masses the issue of which transition, i.e., deconfining or chiral symmetry restoring, dominates is a problem that only lattice computations can currently solve.

The situation becomes clearer, at least in principle, for fermions in the adjoint representation of the gauge group. Here one has two well defined and independent order parameters, since the center group symmetry remains intact in the presence of the fermions. Lattice data seem to confirm the independence of the forces driving independently the chiral and deconfining phase transition both for two and three colors [5,6].

However, when two or more orders compete the resulting phase diagram is expected to have a very interesting and rich structure due to the possibility of multicritical behavior. This arises at the intersection of critical lines characterized by different order parameters. Our interest is in the case of two order parameters. If the transition at the multicritical point is continuous, then either bicritical or tetracritical behavior can occur. Bicritical behavior occurs if a first-order line originating from the multicritical point separates two different ordered phases, each separated from the disordered phase by a line of continuous transitions beginning from the multicritical point. Tetracritical behavior, on the other hand, occurs if there exists a mixed phase in which both types of ordering coexist, and which is bounded by two critical lines meeting at the multicritical point. It is also possible that the phase transition at the multicritical point is of first order. This case is similar to the bicritical one, with the distinction that the two lines separating the disordered phase from the ordered ones start from the multicritical point as first-order lines and then turn to second-order lines at tricritical points. A typical condensed matter example of multicritical behavior is the phase diagram of anisotropic antiferromagnets in a uniform magnetic field parallel to the anisotropy axis [7]. Further examples include ^{4}He [8] and high-T_c superconductors [9]. Also, it has been suggested that multicritical behavior might emerge in the phase diagram of hadronic matter at finite baryon chemical potential [10]. For two colors a tetracritical behavior induced by a possible competition between a diquark and a quark-antiquark phase has been investigated in Ref. [11].

In this paper we show that strongly interacting gauge theories with fermions in the adjoint representation may very naturally display a tetracritical behavior. Interestingly, the two competing orders we will consider are confinement and chiral symmetry. The critical behavior arising from two competing orders has a long history. Investigations in anisotropic magnetic systems were carried out at the mean field level in Ref. [8] and subsequently in Ref. [7] to first order in $\epsilon = 4 - D$, where D is the dimension of spacetime. More recently the analysis has been carried up to order $O(\epsilon^5)$ in the ϵ expansion [12].

*Electronic address: francesco.sannino@nbi.dk

†Electronic address: kimmo.tuominen@phys.jyu.fi

In this work we propose that a nontrivial multicritical point exists in the temperature-quark chemical potential phase diagram of QCD with fermions in the adjoint representation of the gauge group (i.e., adjoint QCD). The two competing orders are chiral symmetry and confinement. Our results suggest that taking confinement into account is essential for understanding the critical behavior as well as the full structure of the phase diagram of adjoint QCD. This is in contrast to ordinary QCD where the center group symmetry associated with confinement is explicitly broken when the quarks are part of the theory.

In Sec. II we briefly review the basic classification of the multicritical points [7,8,12] relevant for our discussion. In Sec. III we study Yang-Mills theories with fermions in the adjoint representation of the group at finite temperature. Here we discuss the critical behavior in the hypothetical case in which chiral symmetry and confinement compete for order. The early lattice work [6] seems to exclude the presence of multicritical points. Nevertheless, it is instructive to discuss this regime.

We then introduce, in Sec. IV, a nonzero quark chemical potential for one Dirac flavor in the adjoint representation of two colors. Then we proceed to show that a multicritical point is quite likely to occur in the temperature-chemical-potential phase diagram. The two orders correspond to the Z_2 symmetry [i.e., $O(1)$] and the $U(1) \sim O(2)$, respectively. Z_2 is the center group symmetry associated with confinement, while $O(2)$ is the baryon number that spontaneously breaks due to the formation of diquark condensates. Some analogous theories have been investigated directly via lattice simulations [13] and within the chiral perturbation theory approach [14,15]. We show that the interplay between the two order parameters substantially affects the phase diagram.

The multicritical point is predicted to be in the $O(3)$ Heisenberg universality class, according to the classification in Ref. [7], if the fixed point analysis is performed at one loop in the $\epsilon = 4 - D$ expansion [7]. If higher orders are considered the fixed point is predicted to be a biconical tetracritical point [12]. We finally suggest possible applications of our results to QCD with fermions in the fundamental representation of the gauge group.

II. CLASSIFICATION OF MULTICRITICAL POINTS

In this paper we will argue that certain strongly interacting theories naturally lead to phase transitions, in the temperature-quark-chemical-potential plane, possessing multicritical points. The novelty is in the fact that this multicritical behavior is a result of the interplay of deconfinement and global symmetry breaking.

One of the most remarkable features of continuous phase transitions is their universal character. There is, indeed, a rich variety of systems that exhibit the same identical critical behavior. When possible, it is convenient to introduce order parameters to describe the phase transition. In our case we will have two order parameters: one associated to deconfinement and the other to a global symmetry. Note, that even though we start from a fermionic theory, near the critical point of interest the relevant effective degrees of freedom are

bosonic, and are naturally identified with the physical fluctuations of the order parameters. This is the standard approach related to the study of phase transitions.

Before moving to the theories of interest to us, we introduce in this section the relevant definitions and the classification of the multicritical behaviors emerging when two order parameters compete for order. We will keep the discussion general.

Following Refs. [7] and [12] when we have two order parameters, ℓ and σ, which compete with symmetries $O(N_1)$ and $O(N_2)$, respectively, one can write the effective theory symmetric under $O(N_1) \oplus O(N_2)$. Up to quartic terms the effective theory containing both order parameters in D Euclidean dimensions is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \ell)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}m_\ell^2 \ell^2 + \frac{1}{2}m_\sigma^2 \sigma^2 + \frac{\lambda}{4!}(\ell^2)^2$$

$$+ \frac{g_4}{4!}(\sigma^2)^2 + \frac{g_2}{4}\ell^2 \sigma^2. \quad (1)$$

Here $\ell^2 = \Sigma_{n=1}^{N_1} \ell_n^2$ and $\sigma^2 = \Sigma_{m=1}^{N_2} \sigma_m^2$. It is possible that for a certain value of the physical parameters, and for $D = 3$, the correlation lengths of the two order parameters diverge simultaneously yielding a multicritical point. At such a point the critical behavior can be determined by tuning the parameters m_ℓ^2 and m_σ^2 to their critical values and studying the stable fixed points of the renormalization group flow.

A. Fixed points and critical behavior at one loop

A first-order analysis in the ϵ expansion [7] for the theory (1) at multicritical point shows that six distinct fixed points exist. Four of them have $g_2 = 0$, and three of these, namely the Gaussian, $O(N_1)$ and $O(N_2)$ symmetric ones, are always unstable against the perturbations away from the $g_2 = 0$ plane, while the fourth one is stable for sufficiently large values of N_1 and N_2. Since $g_2 = 0$, the two fields behave independently and this stable fixed point is termed *decoupled* fixed point. The other two stable fixed points lie at nonzero g_2. The first of them is called the Heisenberg $O(N_1 + N_2)$ fixed point, due to enhanced symmetry, and the second one is called the *biconical* fixed point. The fixed points can be determined by computing the zeros of the beta functions of the theory, which at one loop are

$$\beta(\lambda) = \frac{\lambda}{6} \frac{(N_1 + 8)}{8\pi^2} + 3\frac{g_2^2}{2} \frac{N_2}{8\pi^2} - \lambda \epsilon,$$

$$\beta(g_4) = \frac{g_4}{6} \frac{(N_2 + 8)}{8\pi^2} + 3\frac{g_2^2}{2} \frac{N_1}{8\pi^2} - \lambda \epsilon,$$

$$\beta(g_2) = \frac{\lambda g_2}{6} \frac{(N_1 + 2)}{8\pi^2} + \frac{g_4 g_2}{6} \frac{(N_2 + 2)}{8\pi^2} + 2\frac{g_2^2}{8\pi^2} - g_2 \epsilon.$$

The stability of a generic fixed point is ensured if the matrix

TABLE I. The critical exponents of the correlation length at the various multicritical points of $O(N_1=1)\oplus O(N_2)$ theory.

FP	$n=N_1+N_2$	$\nu_1,\ \nu_2$
Decoupled	$n\geqslant 10$ $(N_1=1)$	$\dfrac{1}{2}+\dfrac{N_2+2}{N_2+8}\dfrac{\epsilon}{4},\ \dfrac{1}{2}+\dfrac{\epsilon}{12}$
Biconical	$4\leqslant n<10$ $(N_1=1)$	
	$N_2=3$	$0.5+0.1250\epsilon,\ 0.5+0.0417\epsilon$
	$N_2=4$	$0.5+0.1336\epsilon,\ 0.5+0.0560\epsilon$
	$N_2=5$	$0.5+0.1403\epsilon,\ 0.5+0.0667\epsilon$
	$N_2=6$	$0.5+0.1460\epsilon,\ 0.5+0.0741\epsilon$
	$N_2=7$	$0.5+0.1515\epsilon,\ 0.5+0.0789\epsilon$
	$N_2=8$	$0.5+0.1568\epsilon,\ 0.5+0.0816\epsilon$
Heisenberg	$n<4$	$\dfrac{1}{2}+\dfrac{n+2}{n+8}\dfrac{\epsilon}{4},\ \dfrac{1}{2}+\dfrac{1}{n+8}\dfrac{\epsilon}{2}$

$$\omega_{ij}=\frac{\partial\beta_i}{\partial g_i}\bigg|_{g*}, \tag{2}$$

evaluated at the fixed point $g*$ has real and positive eigenvalues. Our results agree with those in Ref. [7].

The nature of the multicritical point is determined by the sign of the quantity $\lambda g_4-g_2^2/9$ [8]. This constraint simply tells us, at the level of the effective Lagrangian, if the phase displaying two orders (i.e., nonvanishing condensates for both order parameters) has a higher or lower free energy with respect to the phases in which one of the condensates vanishes [8]. If the sign is positive we expect a tetracritical behavior. For the negative sign the phase with two orders has higher free energy than the phases with only partial order. In this case a simultaneous existence of two orders is unstable and a jump between the phases with partial orders occurs. In the latter case we expect a bicritical behavior.

Decoupled and biconical stable fixed points mentioned above satisfy the criterion of tetracriticality, $\lambda g_4>g_2^2/9$, at the critical point, while for the fixed point corresponding to the isotropic (N_1+N_2)-vector model can, interestingly, be either bicritical or tetracritical [12]. This possibility arises due to the presence of a dangerous irrelevant variable [16].

Defining $n=N_1+N_2$, the low-order ϵ-expansion calculation shows that for $n<4$ the critical behavior is due to the stable fixed point corresponding to an isotropic $O(n)$ Heisenberg model. As n increases, the biconical fixed point (FP) becomes stable and yields a new tetracritical behavior. Finally, for large n, namely for $N_1N_2+2N_1+2N_2\geqslant 32$, the stable fixed point is the decoupled one, which leads to the tetracritical behavior in which the two fields do not affect each other.

We first summarize in Table I the generic results that were obtained in the $\mathcal{O}(\epsilon)$ calculation, and then we discuss them in the context of strong interactions. Each of these fixed points have interesting specific properties: For the Heisen-

berg fixed point the symmetry is enhanced from $O(N_1)\oplus O(N_2)$ to $O(N_1+N_2)$, and the theory (1) becomes that of the isotropic Heisenberg (N_1+N_2)-component model, as can be seen by inspecting the Lagrangian (1) at the fixed point given by $\lambda=g_4=3g_2$. The critical exponents ν_1 and ν_2 quoted in Table I are defined in terms of the eigenvalues λ_ℓ and λ_σ of the corresponding relevant variables m_ℓ^2 and m_σ^2 in the linearized renormalization group recursion relations as

$$\nu_1=\frac{1}{\lambda_\ell}, \quad \nu_2=\frac{1}{\lambda_\sigma}. \tag{3}$$

These describe the divergence of the correlation lengths as a function of suitable scaling fields, for example, the reduced temperature $t=T/T_c-1$ and, say, a new scaling field g. The latter can be a magnetic field, a quark mass parameter, etc.

The effect of the perturbation controlled by g is generally captured by the crossover exponent ϕ defined through the usual scaling formula for, e.g., the correlation length

$$\xi(T,g)\sim t^{-\nu}F(g/t^\phi), \tag{4}$$

and similarly for other thermodynamical quantities. Here $\nu=\nu_1$ corresponds to $g=0$ case and $\phi=\nu/\nu_2$. The crossover scaling function $F(z)$ is finite at $z=0$, but has divergences at specific points and these divergences then modify the $g=0$ behavior $\sim t^{-\nu}$.

Considering magnetic systems as an example, for $n<4$ at $\mathcal{O}(\epsilon)$, the crossover corresponds to the weakly anisotropic n-vector model, where the anisotropy is given by the term $\sim g\ell^2\sigma^2$ in the isotropic Hamiltonian. In other words, ν_1 describes the divergence of the correlation length as $\xi\sim|t|^{-\nu_1}$, where t is the reduced temperature, while the exponent ν_2 describes the divergence of the correlation length in the anisotropy g as $\xi\sim g^{-\nu_1}$ when $g\to 0$. The crossover exponent in this case is given by $\phi=1+n\epsilon/2(n+8)$.

The decoupled fixed point describes a system consisting of effectively independent N_1- and N_2-component Heisenberg subsystems, and therefore the critical indices are the ones of the two independent Heisenberg subsystems and are in that respect trivial. Interestingly, though, the total scaling will break, since a single scaling function cannot properly describe the asymptotic free energy when $N_1\neq N_2$. Finally, the biconical fixed point features completely new critical exponents. However, since they are numerically very close to the corresponding Heinseberg ones, they may be hard to distinguish experimentally.

B. Results from higher-order computations

It is important to note that the numbers quoted in Table I are a result of a first-order calculation in ϵ. Furthermore, these results must be extrapolated to $\epsilon=1$ to be applicable. However, past experience has shown that even in this limit, the ϵ expansion describes the fixed point physics surprisingly well. Already the $\mathcal{O}(\epsilon)$ results show that as a function of n the critical behavior in the case of two competing orders leads to a rich spectrum of possibilities. However, in the present case higher-order contributions are relevant. For the

case of $O(N_1) \oplus O(N_2)$ theory, a remarkable $\mathcal{O}(\epsilon^5)$ calculation exists [12], and we will briefly discuss the improvements for the critical exponents in what follows. First, however, let us note that the higher orders also lead to important changes in the domains of stability of the fixed points in the (N_1, N_2) plane. The $\mathcal{O}(\epsilon^3)$ results for the Heisenberg fixed point [17] lead to the stability for $N_1 + N_2 < 4 - 2\epsilon + \frac{5}{12}[6\zeta(3)-1] + \mathcal{O}(\epsilon^3)$.

A calculation to $\mathcal{O}(\epsilon^5)$ further narrows the domains of stability for the fixed points: The $O(n)$ Heisenberg fixed point is stable only for $n=2$, i.e., only in the case of two intersecting Ising lines. Then, for $n=3$ the stable fixed point is the biconical one, and the decoupled fixed point is stable for all $n \geq 4$ with any values of N_1 and N_2. For further details we refer to the existing literature [7,12].

Since the domain of stability of the Heisenberg fixed point shrinks down to $n=2$ it will not play a role in our strong interaction examples. The biconical fixed point is stable for $n=3$. We will see that this fixed point will be relevant for our investigations. For all of the other combinations of N_1 and N_2 such that $N_1 + N_2 \geq 4$, the stable fixed point is the decoupled one with well known independent $O(N_1)$ and $O(N_2)$ exponents. Therefore, to conclude this section, let us state the high-order values for the critical exponents relative to the biconical fixed point at $\epsilon=1$. Using the general definitions $\nu = \nu_1$ and $\phi = \nu/\nu_2$, the numerical Padé-Borel resummed $\mathcal{O}(\epsilon^5)$ values for the biconical exponents at $N_1 = 1$ and $N_2 = 2$ are $\nu_B = 0.70(3)$, $\phi_B = 1.25(1)$. As already mentioned, these are very close to the corresponding Heisenberg $O(3)$ exponents: $\nu_H = 0.7045(55)$, $\phi_H = 1.260(11)$ at $\mathcal{O}(\epsilon^5)$ [18].

Away from the tetracritical points the second-order lines have independent critical behaviors and the two order parameters do not compete. We have now the basic terminology and tools to analyze and make predictions for strongly interacting theories exhibiting multicritical behavior.

III. FINITE TEMPERATURE ADJOINT QCD

Let us now turn to the possibility of tetracritical behavior in the theories of strong interactions. To be specific, consider two-color QCD with $N_f \leq 2$ massless Dirac flavors in the adjoint representation of the gauge group. One of the main motivations for studying the phase diagram of gauge theories with fermions in the adjoint representation (adjoint QCD) is that, contrary to ordinary QCD, in adjoint QCD there is a well defined symmetry associated to confinement. The symmetry is identified with the center of the gauge group which for a generic $SU(N)$ gauge theory is Z_N. Here we consider explicitly the case $N=2$. The breaking of this symmetry is monitored by the expectation value of the Polyakov loop [3], which is the order parameter of the theory.

Besides the center group symmetry, and in absence of quark masses, adjoint QCD possesses a global quantum symmetry which for N_f Dirac fermions is $SU(2N_f)$ [19]. The fact that the symmetry group here is $SU(2N_f)$ rather than $SU(N_f) \times SU(N_f) \times U(1)$ is due to the fact that the fermions belong to a real representation of the gauge group. We note that the ordinary baryon number is one of the diagonal generators of $SU(2N_f)$. If a democratic Dirac mass term is added into the theory, $SU(2N_f)$ breaks explicitly to $SU(N_f) \times U(1)$, with $U(1)$ the baryon number of the theory. In this section we consider the massless limit, but note that the introduction of a small mass term for the fermions in the theory can be introduced and studied in a straightforward way. At low temperatures the global symmetry is expected to break to the maximum diagonal subgroup $O(2N_f)$ leaving behind a number of Goldstone bosons, some of them charged under the ordinary baryon number. We will collectively refer to the Goldstone bosons as pions and will also use, at times, chiral symmetry to indicate the global symmetry of the theory. In the next section, and for the specific case of two colors and one Dirac flavor, we will work out in detail the global symmetry properties for massless and massive fermions. We will also discuss the breaking patterns of the global symmetry, and consider old and new arguments supporting these patterns. At high temperatures it is natural to expect a global symmetry restoration. Such a global symmetry restoration is also termed, at times, chiral symmetry restoration.

We now naturally have two well defined order parameters: The Polyakov loop and the fermion condensate. It is interesting to consider the possibility that they may compete for order when considering a temperature driven phase transition. The hope being, as already mentioned in the Introduction, that by studying strongly interacting theories such as adjoint QCD, one might shed light on ordinary QCD.

Having outlined the general behavior, symmetries and defined the order parameters it is now natural to use the results and methodology presented in the preceding section to make predictions for the critical exponents related to the phase transitions of adjoint QCD.

For two colors the center group is Z_2, which is equivalent to a $O(1)$ symmetry and the associated order parameter is denoted by ℓ. The flavor groups $SU(4)$ for $N_f = 2$ and $SU(2)$ for $N_f = 1$ are locally isomorphic, respectively, to $O(6)$ and $O(3)$, and the order parameter with such symmetry is denoted by σ.

Here the results of Ref. [12], denoted by $O(1) \oplus O(6)$ and $O(1) \oplus O(3)$, are directly applicable. The first phase diagram we draw is the one in which the temperature drives the phase transition at zero quark chemical potential. We know [3] that at high temperatures we have center group order and at low temperatures chiral order. This sorts for us the orientation for a possible phase structure with respect to the condensed matter ones [12].

Besides the temperature, which can be tuned, we also have two independent and dynamically generated scales in the problem: the deconfining scale Λ_d and the chiral symmetry restoration scale Λ_c. These two scales are intimately related to the number of colors and flavors of the theory.

However, it is the relative magnitude of these scales that is of importance for the phase diagram. One might argue that in strong interactions only one scale is dynamically generated. On the other hand, it is quite reasonable to imagine the dynamics driving chiral symmetry breaking to be different than the one for center group breaking.

There are also theoretical arguments [20] suggesting that $\Lambda_d \lesssim \Lambda_c$ (see the next section for a more detailed discussion).
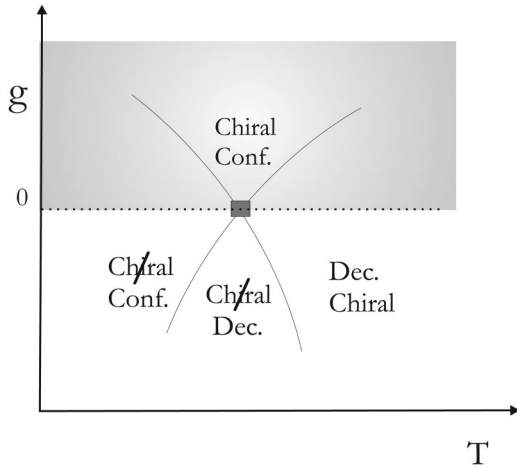
FIG. 1. Phase diagram displaying a tetracritical point. The physically allowed part of the phase diagram lies beneath the $g = 0$ line.

It is then natural to define a new parameter:

$$g = \frac{\Lambda_d - \Lambda_c}{\Lambda_d} \lesssim 0. \qquad (5)$$

Differently from the condensed matter cases, here g cannot be tuned but rather defines the theory. A possible phase diagram in the (g, T) plane is the one shown in Fig. 1. We stress that the expectation $\Lambda_d \lesssim \Lambda_c$ forces the physically allowed part of the phase diagram to lie below the $g = 0$ line.

At exactly $g = 0$ tetracritical behavior would be expected, and for this point we can translate the critical behavior discussed in the preceding sections for strong interactions. The deconfinement order parameter symmetry fixes $N_1 = 1$, and we now consider different flavors in turn.

Let us start with quenched super Yang-Mills theory. In this case we have only one Majorana fermion in the adjoint representation of the gauge group. The only global symmetry associated is an axial symmetry that is affected by the Adler-Bell-Jackiw anomaly. However, in the quenched limit such a symmetry is restored. The chiral symmetry is then $U(1)$ (which is also an R symmetry from the supersymmetry transformations point of view), which breaks spontaneously. Here $N_2 = 2$ and if a tetracritical point would exist it would be a biconical one. Away from the quenched limit the $U(1)$-R symmetry is explicitly broken by an anomaly and it might still be interesting to study what happens if one considers this symmetry almost restorable at large T.

In the case of two Majorana fermions in the adjoint (i.e., one Dirac flavor) the chiral symmetry group, after having taken into account anomalies, is $SU(2)$, i.e., $O(3)$ with $N_2 = 3$. The physics of the tetracritical point, according to high-order calculations, is the one for which the critical behaviors of the two order parameters are unaffected by each other, i.e., we have a decoupled fixed point. Finally for two Dirac flavors we have $N_2 = 6$, and again a decoupled fixed point is expected.

Lattice simulations can determine how far we are from the tetracritical point. For two colors with fermions in the adjoint, there are numerical computations [6] that indicate that the chiral and deconfinement phase transitions happen at different temperatures. This corresponds to $g \neq 0$ and the two transitions have the expected independent critical behavior. It would be interesting if more recent simulations might further investigate how competing the two orders actually are.

The main problem for not being able to reach a tetracritical point here is that in order to change g one has to change theory. In condensed matter physics one can usually tune parameters, via other scaling fields than the temperature, e.g., external magnetic fields. The freedom to tune different quantities in the theory allows, on one hand, to test the theory of critical phenomena and to shape our understanding of phase transitions, on the other.

Since the parameter we have defined for the adjoint QCD is dimensionless, one would expect it to be proportional to some combination of number of colors and flavors. Then, in numerical experiments, it might be possible to use, e.g., number of flavors, N_f, as a scaling field. Tuning the value of N_f would affect the relative magnitude of Λ_d and Λ_c and allow, perhaps, the two transitions to close on each other. The existing numerical investigations [6] show the strong dependence on the number of flavors for the chiral phase transition. As already emphasized, it would be interesting to have an up to date study of these matters.

We shall shortly see how we can achieve a multicritical point in strong interactions with diquark condensation and confinement as competing orders by introducing a more practical scaling field into the problem, i.e., the quark mass.

IV. DECONFINEMENT−CHIRAL-SYMMETRY TETRACRITICAL POINT

In this section we investigate in some detail the two color gauge theory with one Dirac fermion in the adjoint representation of the gauge group. This is a theory with a number of fascinating properties. A relevant one, for our purposes, being that when adding a nonzero quark chemical potential, one observes, at sufficiently large baryon chemical potential, a color superfluid transition rather than a color superconductive one [14]. This is so since we have some Goldstone bosons (pions) carrying baryonic charge.

We have divided this section into a number of subsections to help the reader concentrate on one problem at the time, and to build up the relevant knowledge. We will first describe the symmetries of the fermionic action of the underlying theory, and then explore the symmetry breaking pattern first at zero temperature and baryon chemical potential of the theory. We briefly review the temperature (zero-baryon chemical potential) phase transition scenario, which has essentially been studied in the preceding section. Subsequently, we describe the deconfining phase transition at nonzero temperature and baryon chemical potential, while ignoring the possible superfluid phase transition. We then describe the superfluid phase transition at nonzero temperature and quark chemical potential neglecting the deconfining phase transition. We will consider both transitions simultaneously in the next section. It is important to observe that both, the introduction of the chemical potential as well as the presence of a

Dirac mass for the theory break explicitly the underlying global $SU(2)$ symmetry group while preserving the $U(1)$ baryon symmetry of the theory, as we will explicitly see below. At nonzero temperature and nonzero baryon chemical potential we will then consider only the exact symmetries of the problem, i.e., the center group and the $U(1)$ symmetries. It interesting to note that when this theory has been investigated in the literature at finite temperature and chemical potential, so far attention has been paid only to the global symmetry of the theory.

A. Symmetries of the underlying theory

Consider one massless Dirac flavor in the adjoint representation of two colors. The flavor group is $SU(2)$, which spontaneously breaks to $O(2)$. The latter is the conserved quark number. In order to elucidate all of the symmetries of the problem in detail we write the underlying tree level Lagrangian for the fermionic part [21] in presence of the mass term and quark chemical potential:

$$+i\bar{Q}^A\bar{\sigma}^\mu D_\mu^{AB}Q^B - \mu\bar{Q}^A\bar{\sigma}^0BQ^A - \frac{m}{2}[Q^A\tau^1Q^A + \text{H.c.}].$$

Here $D_\mu^{AB}Q^B = \partial_\mu\delta^{AB}Q^B - if^{ABC}G_\mu^BQ^C$ and f^{ABC} are the structure constants of the gauge group. The matrices τ^a are the Pauli matrices with the baryon number $B = \tau^3$ acting in the flavor space, and $A = 1,2,3$ is the gauge index for the fermions in the adjoint representation. The Weyl spinor $Q_{\alpha,f}^A$ with $\alpha = 1,2$ the spin index and $f = 1,2$ the flavor index can be represented as a vector as follows:

$$Q_\alpha^A = \begin{pmatrix} \chi_\alpha^A \\ \psi_\alpha^A \end{pmatrix}, \tag{6}$$

while in the Dirac representation we have

$$\Psi_D^A = \begin{pmatrix} \chi_\alpha^A \\ \bar{\psi}^{\dot{\alpha}A} \end{pmatrix}. \tag{7}$$

At zero quark mass and chemical potential the $SU(2)$ symmetry is evident. The extra classical $U_A(1)$ symmetry is anomalous. The baryon number here is the τ^3 generator of $SU(2)$. At nonzero baryon chemical potential and nonzero Dirac quark mass the baryon symmetry is the only symmetry left unbroken at the fundamental level.

B. Chiral symmetry breaking: No anomaly matching but entropy counting

We set, for the moment, the fermion mass term and the baryon chemical potential to zero. Usually one of the powerful methods to discover if, in strongly interacting gauge theories, a global symmetry breaks at low energies, is to require the global anomaly matching conditions [22] among the ultraviolet and the infrared realization of the theory. Unfortunately, for this theory the global anomalies vanishes, since the flavor group is $SU(2)$, and hence we cannot invoke

the 't Hooft anomaly matching conditions [22] to suggest that chiral symmetry must break at low temperatures. Indeed, we can well imagine a low-temperature phase in which chiral symmetry is not broken. Although in principle we do not need massless composite fermions, the simplest fermions we can construct are composite objects of the type $\lambda_{\alpha,F} \sim \bar{Q}_F^{\dot{\alpha}A}\bar{\sigma}_{\alpha\dot{\alpha}}^\mu G_\mu^A$. Due to the Vafa-Witten theorem [23], vector symmetries cannot break spontaneously, which, in turn, means that the fermions do not develop dynamically generated Majorana masses. However, a Dirac mass term is of the form $\lambda_{F=1}^\alpha\lambda_{\alpha F=2}$ and breaks the global $SU(2)$ symmetry to the baryon number $U(1)$.

Therefore, in the absence of 't Hooft anomaly matching conditions two possible scenarios arise: We can either have spontaneous chiral symmetry breaking, with associated two Goldstone bosons, or chiral symmetry intact but a massless composite Dirac fermion. This is very similar to the case of ordinary QCD with two flavors. According to the guide suggested in Ref. [24], the most likely phase in the infrared is the one for which the degrees of freedom counted according to the entropy factor, $f = $ No. of real bosons $+ (7/4)$No. of Weyl fermions, are minimized. Here, the spontaneously broken phase has $f = 2$ and the chiral-symmetry-preserving phase has $f = 7/2$. Chiral symmetry, here the $SU(2)$, is therefore predicted to break at low temperatures. Clearly these results do not depend on the number of colors. In the case of larger number of fermion flavors the 't Hooft anomaly conditions are nontrivial and single out the infrared phase in which chiral symmetry is broken. 't Hooft anomaly conditions have been generalized, first, at nonzero temperature [25] and more recently at nonzero quark chemical potential [26].

A possible scalar condensate must be of the form:

$$\epsilon^{\alpha\beta}\langle Q_{\alpha,f}^AQ_{\beta,f'}^A\rangle \propto E_{ff'}. \tag{8}$$

The subgroup that leaves the condensate invariant is given by the generators of $SU(2)$ satisfying the condition:

$$\tau^aE + E\tau^{aT} = 0. \tag{9}$$

Since the condensate is symmetric in color and antisymmetric in spin, it must be symmetric in flavor (i.e., $E = E^T$). Requiring the $SU(2)$ symmetry to break to its maximal orthogonal subgroup [i.e., $O(2)$] [27], we can have, for example, E proportional to the 2×2 identity matrix or to τ^1. If we choose the identity, then the unbroken generator is τ^2, but if we choose τ^1, then the unbroken generator is τ^3. Since we have identified the $O(2)$ generator corresponding to the baryon number with τ^3, the condensate must be proportional to τ^1, i.e.,

$$\epsilon^{\alpha\beta}\langle Q_{\alpha,f}^AQ_{\beta,f'}^A\rangle \propto \tau_{ff'}^1. \tag{10}$$

Two Goldstone bosons are present and are associated with the generators $X^a = \tau^a/2$ with $a = 1,2$. Note, that since the pions here are associated with the generators that do not commute with the baryon generator τ^3 they are automatically

charged under the baryon number. The low-energy effective theory in absence of quark chemical potential is

$$\mathcal{L}_{eff} = F_\pi^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U] + F_\pi^2 m_\pi^2 \text{Tr}[U + U^\dagger] \quad (11)$$

with

$$U = e^{i\pi^a X^a / F_\pi}, \quad a = 1,2 \quad (12)$$

where we have also introduced a Dirac mass m in the underlying theory. Such a mass appears in the effective Lagrangian as a nonzero mass for the pions, and one expects $m_\pi^2 \propto m$. U transforms as $g\tau^1 U g^T$ for $g \in SU(2)$. The previous effective Lagrangian still preserves the $U(1)$ baryon symmetry.

Besides chiral symmetry we also have deconfinement. Here the order parameter is the Polyakov loop, which is associated with the center group symmetry Z_2 for two colors. Note that the previous analysis is completely independent on the number of colors, which becomes a relevant parameter only when considering the center group symmetry as well.

C. The temperature-driven phase transition

We have discussed the nonzero temperature case in Sec. III. Here we recall the salient information needed when endowing the quarks with a nonzero mass and chemical potential. At zero quark chemical potential, the $SU(2)$ symmetry is restored at a given temperature T_c, while the Z_2 deconfinig phase transition is indicated with T_d. The latter is expected to be somewhat lower than T_c. If the two-phase transitions are independent, no tetracritical point is expected to occur in this case. As soon as we add a quark mass, we expect a crossover behavior for the $SU(2)$ phase transition. This is true also at nonzero chemical potential, since both the mass term and the chemical potential term explicitly break the $SU(2)$ global symmetry. It is also worth emphasizing again that at zero quark chemical potential and quark mass, and due to the absence of the 't Hooft anomaly conditions to satisfy, in principle, a chiral symmetry restoring phase transition before deconfinement might have been possible. However, this is not allowed according to the guide in Ref. [24], which selects the chiral symmetry breaking confined phase as the preferred ground state even in absence of 't Hooft anomaly conditions. Summarizing, the $SU(2)$ symmetry is always broken at nonzero baryon chemical potential and Dirac mass. If a crossover phenomenon exists, it is expected to happen, for fixed chemical potential and quark mass, at a temperature larger or at most equal to the critical temperature for deconfinement. As we increase the chemical potential, the explicit breaking of the $SU(2)$ symmetry becomes severe. We will then neglect the $SU(2)$ symmetry and analyze the fate of the $U(1)$ baryon symmetry, the only global symmetry left unbroken.

D. The $U(1)$ baryon superfluid phase transition at nonzero μ and T

As we increase the baryon chemical potential the $U(1)$ baryon symmetry may break spontaneously. In QCD with
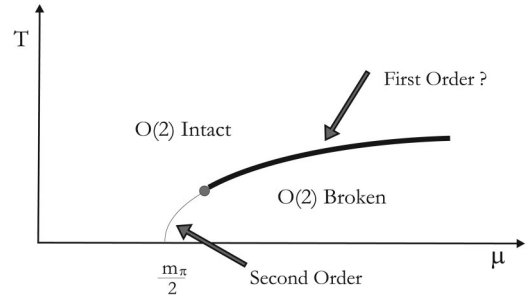

FIG. 2. A schematic (T, μ) phase diagram when only the diquark condensation is considered.

three massless quarks in the fundamental representation the breaking is due to a Cooper pairing phenomenon, i.e., color superconductivity.

For adjoint QCD the situation is different. The spontaneous breaking of the $U(1)$ baryon symmetry is a superfluid phenomenon [14]. This is so since the pions, in this theory, are charged under the baryon number. We have already proven this statement in Sec. IV B. Actually they have baryon number two with respect to the quarks, which we have defined to have unit baryon number. One can easily show that the chemical potential couples directly to the pions via

$$\partial_0 U \to D_0 U = \partial_0 - i\mu[U, B]. \quad (13)$$

After having substituted this covariant derivative in the effective Lagrangian, a negative mass-squared term proportional to μ^2 is induced. For $\mu > m_\pi/2$ the $U(1) \sim O(2)$ breaks spontaneously. On general grounds we expect two regions on the phase diagram, one with intact $O(2)$ and the other where $O(2)$ is spontaneously broken. This is schematically represented in Fig. 2. The second-order line starts at $m_\pi/2$ at zero T. In literature it is argued that, by computing the effective action within the chiral perturbation theory approach [15], such a second-order line ends in a tricritical point, and continues as a first-order line. There is a simple way to understand why the phase transition line must curve to the right in the T-μ plane: By increasing the chemical potential, we effectively increase the negative mass squared of the Goldstone boson. On the other hand, the temperature contribution to the mass of the Goldstone boson is positive and tries to compensate the negative contribution of the chemical potential to the squared-mass term. The larger the chemical potential, the higher the temperature must be to restore the symmetry. This is, in a nutshell, the relativistic Bose-Einstein condensation phenomenon pioneered by Haber and Weldon in Ref. [28].

Both the critical temperature and the critical chemical potential of the tricritical point increase with the pion mass [15]. What is relevant for us is that (i) two well separated regions exist and (ii) we have a second-order phase transition near $\mu = m_\pi/2$.

E. Deconfinement at nonzero μ and T

As already stated, the presence of quarks in the adjoint representation of the gauge group does not break the center
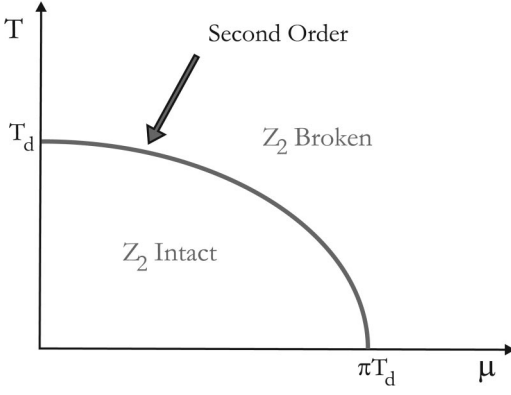
FIG. 3. A schematic (T, μ) phase diagram where only deconfinement is considered.

group symmetry. Note also that up to now the color played little role. In other words, whether the center group is Z_2 or Z_3, one expects the chiral symmetry part of the analysis [here also the $U(1)$ baryon symmetry is termed chiral symmetry] to be to a large extent unaffected. This, however, is not true, as we will demonstrate below. In this section we only consider the pure deconfinement phase transition. Two distinct regions in the phase diagram occur: in one we have center group order (i.e., deconfinement) and in the other we have disorder (i.e., confinement). If the number of colors is larger than two we expect a first-order line, while if the number of colors is two, a second-order line is most likely to occur. Let us consider the two-color case: Then a possible phase diagram (for deconfinement only) is provided in Fig. 3.

We have not considered the possibility of a tricritical point, but here the important point is that there are two well-separated regions. We have simply estimated the critical chemical potential for deconfinement to be of the order of $\sim \pi T_d$, with T_d the deconfinement temperature at zero chemical potential. This value is meant only to guide our intuition, and it has been obtained using the bag model. However, we do expect the correct value to be near the one predicted. More specifically, the contributions to the pressure from free gluons and quarks in the adjoint representation are, respectively,

$$P_g = g_g \frac{\pi^2 T^4}{90}, \tag{14}$$

$$P_q = g_q T^4 \left[\frac{7 \pi^2}{180} + \frac{1}{6} \frac{\mu^2}{T^2} + \frac{1}{12 \pi^2} \frac{\mu^4}{T^4} \right], \tag{15}$$

where generally $g_g = (N_c^2 - 1)$ and $g_q = N_f(N_c^2 - 1)$, and we set $N_c = 2$ and $N_f = 1$. The phase transition line in the (T, μ) plane is determined through

$$P_g + P_q = B, \tag{16}$$

where B is the bag constant. We determine B at zero chemical potential, and using the value so obtained, we find at zero T that $\mu_d = 0.9 \pi T_d$ for the deconfinement transition. Ulti-
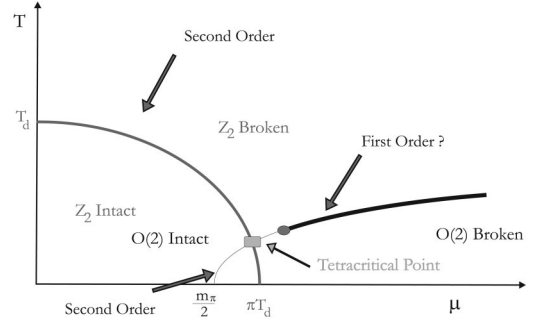
mately this value will have to be determined via lattice simulations. The above computation is meant to be just a rough estimate. If we take the number of colors larger than two, the second-order deconfinement line is replaced by a first-order one.

V. EMERGENCE OF A TETRACRITICAL POINT

The previous analysis neglects the fact that the two order parameters (i.e., the Polyakov loop and the diquark condensate) can and will compete. To argue that a tetracritical point is a natural outcome, take the pion mass to be lighter than twice the critical chemical potential (near zero temperature) for deconfinement, $m_\pi \lesssim 2 \pi T_d$. Now the two curves, i.e., the one for deconfinement and the one for the $U(1)$ baryon (or chiral) symmetry breaking, meet at a tetracritical point as qualitatively illustrated in the Fig. 4.

We have chosen, in plotting the curves, the pion mass to be such that the tetracritical point occurs when the two second-order lines meet. A tetracritical point is a very intriguing possibility and the two order parameters here will influence each other. So, the naive expectation that in the adjoint representation chiral symmetry and deconfinement do not communicate is misleading.

By tuning the value of m_π one can tune the position of the diquark condensation line with respect to the deconfinement one. Here the pion mass plays the role of the anisotropy parameter.

Near the tetracritical point one can apply the results of a standard ϵ expansion analysis as discussed earlier. The tetracritical fixed point in adjoint QCD with single Dirac flavor, when the two second-order lines meet, is in the universality class of the $O(1) \oplus O(2)$ theory. The effective potential contains the Polyakov loop ℓ and the matrix U, which corresponds in practice to a complex scalar field or two-component real field. Due to such a group structure, using the results of Refs. [7,12], we predict the tetracritical point to be a nontrivial (i.e., nondecoupled) biconical one. The critical exponents are provided in Sec. II.

Other interesting phase diagrams can be considered: For example, by tuning the quark mass the first-order chiral line can meet the second-order deconfinement transition. As another alternative, while we have assumed here the deconfinement transition to be second order over the whole T-μ plane, we cannot generally exclude the possibility that the decon-

FIG. 4. A possible (T, μ) phase diagram when both possible phase transitions, chiral and deconfinement, are taken into account.
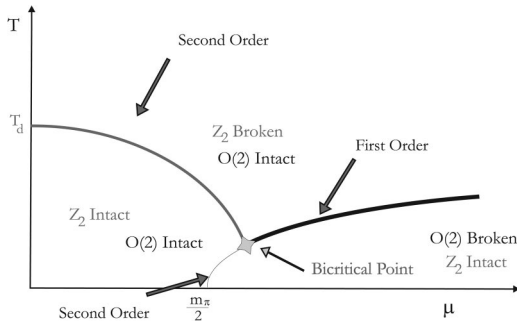
FIG. 5. A possible (T,μ) phase diagram when both possible phase transitions, chiral and confinement, are taken into account and meet at a bicritical point.

finement line develops a tricritical point before meeting the chiral line. Also, when the number of colors is larger than 2, the deconfinement line is always first order. We do not exclude the possibility that for similar theories one could observe the appearance of a bicritical point. In this case a typical phase diagram is depicted in Fig. 5. If the pion mass is sufficiently large, deconfinement is expected to occur before spontaneous breaking of the baryon number. In this regime the two order parameters do not compete anymore. Clearly all of these possibilities are intriguing and deserve to be investigated.

VI. CONCLUSIONS AND SUGGESTIONS

We have shown that when the fermions are in the adjoint representation of the gauge group, a tetracritical fixed point naturally emerges. This is possible since the Z_N symmetry associated with deconfinement is well defined in this theory. The tetracritical point lies in the T-μ plane and for two colors may be biconical with a suitable choice of the quark mass. What is interesting is that in this way we can quantitatively test the effects of confinement, or center group symmetry, on a chiral symmetry type phase transition and vice versa.

For quarks in the fundamental representation of the gauge group the possibility of a tetracritical point is not expected, since the center group symmetry is explicitly broken. Besides, the breaking of the Z_N symmetry was used to explain in Refs. [4,29], via a simple effective Lagrangian, how deconfinement and chiral symmetry are intertwined not only at the level of susceptibilities but also at the level of condensates. The results in our earlier investigations were able to provide a general qualitative understanding of the lattice data. It is, however, still possible, although unlikely (see the discussion in Ref. [30]), that the breaking of the center group symmetry (due to the quarks in the fundamental representation of the center group symmetry) is dynamically suppressed. Such a breaking is much attenuated, for example, when considering a small ratio of the number of flavors over the number of colors. If such a dynamical suppression of the center symmetry breaking occurs in the chiral limit, a (quasi)tetracritical point may be observed in lattice simulations. Unfortunately, it is very hard to disentangle such a behavior if the phase transitions are of first order, and hence this behavior might be better tested in two color QCD with two Dirac flavors in the fundamental representation. The tetracritical point on the temperature axis would be characterized by a $O(1)\oplus O(6)$ symmetry. A decoupled tetracritical point would emerge with independent Ising and Heisenberg behaviors. Considering this scenario at any nonzero quark masses, the $O(1)$ symmetry would be (quasi)exact, and the chiral transition would be then induced [4]. The critical exponents are well known here. Departures from these limiting behaviors is a measure of the amount of center symmetry breaking induced by the presence of the quarks in the fundamental representation of the gauge group.

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