PHYSICAL REVIEW D **70**, 034017 (2004)

Parton model in Lorentz invariant noncommutative space

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(Received 19 April 2004; published 19 August 2004)

We consider the Lorentz invariant noncommutative QED and complete the Feynman rules for the theory up to the order θ^2. In the Lorentz invariant version of the noncommutative QED the particles with fractional charges can be also considered. We show that in the parton model, even at the lowest order, the Bjorken scaling violates as $\sim\theta^2 Q^4$.

DOI: 10.1103/PhysRevD.70.034017 PACS number(s): 13.60.Hb

I. INTRODUCTION

Noncommutative field theories and its phenomenological aspects has been, recently, considered by many authors [1–8]. Such theories are mostly characterized on a noncommutative space-time with the noncommutativity parameter $\theta_{\mu\nu}$. In the canonical version of the noncommutative space-time one has

$$\theta^{\mu\nu} = -i[\hat{x}^{\mu},\hat{x}^{\nu}], \tag{1}$$

where a hat indicates a noncommutative coordinate and $\theta_{\mu\nu}$ is a real, constant antisymmetric matrix. Obviously, the constant vectors θ_{0i} and θ_{ij} imply preferred directions in a given Lorentz frame which leads to violation of the Lorentz symmetry. Since the Lorentz symmetry is an almost exact symmetry of nature, it is natural to explore the noncommutative (NC) field theories that are Lorentz invariant from the beginning. In this new class of NC theories the parameter of noncommutativity is not a constant but is an operator which transforms as a Lorentz tensor [9,10]. Of course in this way one needs to generalize the star product and operator trace for functions of both x^{μ} and $\theta_{\mu\nu}$, appropriately. However, in both cases experiment should confirm the theories. The obtained upper bounds for the violating Lorentz noncommutative field theory are two folds: the first one comes from bound states such as the hydrogen atom or the positronium [6,7] and the second one is obtained by scattering processes for example the electron-electron and the electron-photon scattering and so on [5,8]; see Table I. In the case of Lorentz conserving noncommutative (LCNC) field theory scattering process is only investigated. The dimensional quantity θ in the noncommutative space imply new aspects in the parton model as well. In this paper we explore parton model in the lowest order in which a virtual photon interacts with partons inside a nucleon in a LCNC space. For this purpose one should consider LCNCQED to find the effect of noncommutativity on the form factors.

In Sec. II we introduce Feynman rules for LCNCQED. In Sec. III we study the parton model in the noncommutative space in the lowest order and show how the form factors in the electron-nucleon scattering depend on Q^2 and the Bjorken scaling is violated. Finally, we compare our results with the experimental data and give an upper bound on the parameter of noncommutativity.

II. LORENTZ CONSERVING NCQED

To construct the noncommutative field theories that are Lorentz invariant one needs to generalize the parameter of noncommutativity. Now we review the formalism of the Lorentz conserving NCQED introduced by Carlson, Carone and Zobin (CCZ) [9]. In the CCZ approach of NCQED $\hat{\theta}^{\mu\nu}$ is an operator and satisfies the following algebra:

$$\begin{aligned}
&[\hat{x}^{\mu},\hat{x}^{\nu}] = i\hat{\theta}^{\mu\nu},\\
&[\hat{\theta}^{\mu\nu},\hat{x}^{\lambda}] = 0,\\
&[\hat{\theta}^{\mu\nu},\hat{\theta}^{\alpha\beta}] = 0,
\end{aligned} \tag{2}$$

where $\hat{\theta}^{\mu\nu}$ is antisymmetric tensor that is not constant but transforms as a Lorentz tensor. The action for field theories on noncommutative spaces is then obtained using the Weyl-Moyal correspondence, according to that, in order to find the noncommutative action, the usual product of fields should be replaced by the star product:

$$f*g(x,\theta) = f(x,\theta)\exp\left(\frac{1}{2}\overleftarrow{\partial}_{\mu}\theta^{\mu\nu}\overrightarrow{\partial}_{\nu}\right)g(x,\theta). \tag{3}$$

It should be noted that here the mapping to c-number coordinates involve $\theta^{\mu\nu}$ as a c-number due to the presence of the operator $\hat{\theta}^{\mu\nu}$ in the Lorentz-conserving case. In this formulation, the operator trace that is a map from operator space to numbers, is defined as

$$\mathrm{Tr}\,\hat{f} = \int d^4x\,d^6\theta\,W(\theta)f(x,\theta), \tag{4}$$

where $W(\theta)$ is a Lorentz invariant weight function and is assumed to be positive and even function of θ therefore one has

$$\int d^6\theta\,W(\theta)\,\theta^{\mu\nu} = 0. \tag{5}$$

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TABLE I. Upper bounds on the noncommutativity parameter. $\Lambda_{NC}=1/\sqrt{\Theta}$ describes the parameter of noncommutativity in the standard NC space while Λ_{LCNC} shows NC parameter for LCNC space.

NC parameter	Bound state	Scattering
Λ_{NC}	$\sim$100 GeV	0.1–2.5 TeV
Λ_{LCNC}	?	0.1–1 TeV

Furthermore, the weight function is assumed to fall sufficiently fast so that all integrals are well defined. Now the properties of $W(\theta)$ and the definition of the operator trace Eq. (4) allows one to extract the interactions in the Lorentz conserving noncommutative field theory. To this end the action can be written as follows

$$s=\int d^4x d^6\theta W(\theta)\mathcal{L}(\phi,\partial\phi)_*, \tag{6}$$

where $\mathcal{L}(\phi,\partial\phi)_*$ depends on both x and θ and its subscript indicates the $*$-product which is defined in Eq. (3). For a $U(1)$ gauge theory the gauge invariant Lagrangian is

$$\mathcal{L}=\int d^6\theta W(\theta)\left[-\frac{1}{4}F_{\mu\nu}*F^{\mu\nu}+\bar{\psi}*(i\not{D}-m)*\psi\right], \tag{7}$$

where ψ is a matter field with charge q and for a gauge field A

$$D_\mu=\partial_\mu+iqA_\mu, \tag{8}$$

and

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+iq[A_\mu,A_\nu]_*. \tag{9}$$

The matter field and the gauge field as well as the parameter of the gauge transformation $\Lambda(x,\theta)$ are functions of both x and θ. Therefore, using the same method as applied in the construction of $SU(N)$ noncommutative gauge theories [11], one should expand the fields as a power series in the variable θ as follows

$$\begin{aligned}\Lambda_\alpha(x,\theta)&=\alpha(x)+\theta^{\mu\nu}\Lambda^{(1)}_{\mu\nu}(x,\alpha)+\theta^{\mu\nu}\theta^{\eta\delta}\Lambda^{(2)}_{\mu\nu\eta\delta}(x,\alpha)\\&\quad+\cdots,\\ A_\rho(x,\theta)&=A_\rho(x)+\theta^{\mu\nu}A^{(1)}_{\mu\nu\rho}(x)+\theta^{\mu\nu}\theta^{\eta\delta}A^{(2)}_{\mu\nu\eta\delta\rho}(x)\\&\quad+\cdots,\\ \psi(x,\theta)&=\psi(x)+\theta^{\mu\nu}\psi^{(1)}_{\mu\nu}(x,\alpha)+\theta^{\mu\nu}\theta^{\eta\delta}\psi^{(2)}_{\mu\nu\eta\delta}(x)\\&\quad+\cdots,\end{aligned} \tag{10}$$

where $\alpha(x)$, $A(x)$ and $\psi(x)$ are gauge parameter, gauge field and matter field in the commutative space, respectively, and the coefficients of $\theta_{\mu\nu}$ can be obtained as

$$\begin{aligned}\Lambda^{(1)}_{\mu\nu}(x,\alpha)&=\frac{-q}{2}\partial_\mu\alpha(x)A_\mu(x),\\ \Lambda^{(2)}_{\mu\nu\eta\delta}(x,\alpha)&=-\frac{q^2}{2}\partial_\mu\alpha(x)A_\eta(x)\partial_\delta A_\nu(x),\\ A^{(1)}_{\mu\nu\rho}(x)&=\frac{q}{2}A_\mu(\partial_\nu A_\rho+\mathcal{F}^0_{\nu\rho}),\\ A^{(2)}_{\mu\nu\eta\delta\rho}(x)&=\frac{q^2}{2}(A_\mu A_\eta\partial_\delta\mathcal{F}^0_{\nu\rho}-\partial_\nu A_\rho\partial_\eta A_\mu A_\delta\\&\quad+A_\mu\mathcal{F}^0_{\nu\eta}\mathcal{F}^0_{\delta\rho}),\\ \psi^{(1)}_{\mu\nu}(x)&=\frac{q}{2}A_\mu\partial_\nu\psi,\\ \psi^{(2)}_{\mu\nu\eta\delta}(x)&=-\frac{q}{8}\bigg(-i\partial_\mu A_\eta\partial_\nu\partial_\delta\psi-qA_\mu A_\eta\partial_\nu\partial_\delta\psi\\&\quad-2qA_\mu\partial_\nu A_\eta\partial_\delta\psi-qA_\mu\mathcal{F}^0_{\nu\eta}\partial_\delta\psi\\&\quad+\frac{q}{2}\partial_\mu A_\eta\partial_\nu A_\delta\psi+iq^2A_\mu A_\delta\partial_\eta A_\nu\psi\bigg),\end{aligned} \tag{11}$$

where $\mathcal{F}^0_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. Now we need $\mathcal{L}(x)$, to extract the interactions, that can be obtained by inserting the above relations into Eq. (7) and performing the integration on θ using the weighted average

$$\int d^6\theta W(\theta)\theta^{\mu\nu}\theta^{\eta\rho}=\frac{\langle\theta^2\rangle}{12}(g^{\mu\eta}g^{\nu\rho}-g^{\mu\rho}g^{\eta\nu}), \tag{12}$$

where

$$\langle\theta^2\rangle=\int d^6\theta W(\theta)\theta^{\mu\nu}\theta_{\mu\nu}. \tag{13}$$

Therefore up to the order θ^2 the second term in the Lagrangian density Eq. (7) can be rewritten as follows

$$\bar{\psi}*(i\not{D}-m)*\psi=\mathcal{L}_0+\mathcal{L}_{qq\gamma}+\mathcal{L}_{qq\gamma\gamma}+\mathcal{L}_{qq\gamma\gamma\gamma}, \tag{14}$$

where

$$\begin{aligned}\mathcal{L}_0&=\bar{\psi}^{(0)}(i\not{\partial}-m)\psi^{(0)},\\ \mathcal{L}_{qq\gamma}&=-q(\bar{\psi}^{(0)}*\not{A}^{(0)})\psi^{(0)}+\bar{\psi}^{(0)}(i\not{\partial}-m)\psi^{(2)}\\&\quad+\bar{\psi}^{(2)}(i\not{\partial}-m)\psi^{(0)},\\ \mathcal{L}_{qq\gamma\gamma}&=-q(\bar{\psi}^{(0)}*\not{A}^{(0)})\psi^{(1)}-q(\bar{\psi}^{(1)}*\not{A}^{(0)})\psi^{(0)}\\&\quad-q\bar{\psi}^{(0)}\not{A}^{(0)}\psi^{(2)}-q\bar{\psi}^{(2)}\not{A}^{(0)}\psi^{(0)}\\&\quad-q(\bar{\psi}^{(0)}*\not{A}^{(1)})\psi^{(0)}+\bar{\psi}^{(1)}(i\not{\partial}-m)\psi^{(1)},\\ \mathcal{L}_{qq\gamma\gamma\gamma}&=-q\bar{\psi}^{(0)}\not{A}^{(1)}\psi^{(1)}-q\bar{\psi}^{(1)}\not{A}^{(1)}\psi^{(0)}\\&\quad-q\bar{\psi}^{(0)}\not{A}^{(2)}\psi^{(0)}-q\bar{\psi}^{(1)}\not{A}^{(0)}\psi^{(1)}.\end{aligned} \tag{15}$$

The Feynman rules can be extracted from Eq. (7), for example $\mathcal{L}_0$ in Eq. (15) leads to the ordinary propagator for a fermion field while the rest terms contain a variety of vertices. Comparing Eqs. (15) and (11) we see that, up to the order θ^2, $\mathcal{L}_{qq\gamma}$ contributes to 2-fermion-1-photon ($qq\gamma$), 2-fermion-2-photon ($qq\gamma\gamma$) and 2-fermion-3-photon ($qq\gamma\gamma\gamma$) vertices while $\mathcal{L}_{qq\gamma\gamma\gamma}$ has just contribution on $qq\gamma\gamma\gamma$-vertex and $\mathcal{L}_{qq\gamma\gamma}$ has terms which contains two fermions and two, three and four photons. In [12] the Feynman rules for $qq\gamma$-vertex in general case and $qq\gamma\gamma$-vertex when all fermions and photons are on the mass shell is obtained. Here we complete the Feynman rules for the interaction between fermions and photons up to the order θ^2 as follows.

$qq\gamma$ vertex in general case [12]:

$$-iq\bigg[\gamma_\mu+\frac{\langle\theta^2\rangle}{96}\bigg\{(\not{p}_1-m)p_2^2p_3^\mu-(\not{p}_2-m)p_1^2p_3^\mu +(\not{p}_2-m)p_1\cdot p_3p_1^\mu-(\not{p}_1-m)p_2\cdot p_3p_2^\mu +\frac{1}{2}((p_1\cdot p_3)^2+(p_2\cdot p_3)^2)\gamma^\mu\bigg\}\bigg].$$

$qq\gamma\gamma$ vertex with all fermions and photons are on the mass shell [12]:

$$iq^2\frac{\langle\theta^2\rangle}{96}\{p_1\cdot p_3[p_2^\rho\gamma^\eta-p_1^\eta\gamma^\rho]+p_1\cdot p_4[p_2^\eta\gamma^\rho-p_1^\rho\gamma^\eta] +(\not{p}_3-\not{p}_4)[p_1^\rho p_2^\eta-p_1^\eta p_2^\rho\}.$$

$qq\gamma\gamma\gamma$ vertex with all fermions and photons on shell:

$$\frac{q^3\langle\theta^2\rangle}{96}[(k_1+k_2)^\tau(\not{k}_1+\not{k}_2)g^{\sigma\rho}+(k_1+k_3)^\sigma(\not{k}_1+\not{k}_3)g^{\tau\rho} +(k_2+k_3)^\rho(\not{k}_2+\not{k}_3)g^{\tau\sigma}+(k_2+k_1)\cdot k_3\gamma^\tau g^{\sigma\rho} +(k_3+k_1)\cdot k_3\gamma^\sigma g^{\tau\rho}+(k_3+k_2)\cdot k_1\gamma^\rho g^{\tau\sigma}], \tag{16}$$

where $k_i, i=1,2,3,$ are defined in Fig. 1.

$qq\gamma\gamma\gamma\gamma$ vertex in general case is zero because in Eq. (15) terms which contain two fermions and four photons are $-q\overline{\psi}^{(0)}\not{A}^{(0)}\psi^{(2)}$ and $-q\overline{\psi}^{(2)}\not{A}^{(0)}\psi^{(0)}$ or

$$\frac{iq^4}{8}(\overline{\psi}^{(0)}\not{A}A_\mu A_{\nu'}\partial_{\mu'}A_\nu\psi^{(0)}-\overline{\psi}^{(0)}A_\mu A_{\nu'}\partial_{\mu'}A_\nu\not{A}\psi^{(0)}),$$

which is equal to zero. For the pure gauge vertices up to the order θ^2 there is only four photon vertex which has been already obtained in [9]. These rules for vertices is relevant to study the phenomenology of LCNCQED. For instance at the lowest order (i.e., tree level) the correction to $qq\gamma$ vertex can be obtained for on shell fermions as

$$-iq\bigg\{1+\frac{\langle\theta^2\rangle}{384}k^4\bigg\}\gamma^\mu, \tag{17}$$

where k is the photon momentum. For the cross section of the process $e^-e^+\rightarrow\mu^+\mu^-$ such a correction results in

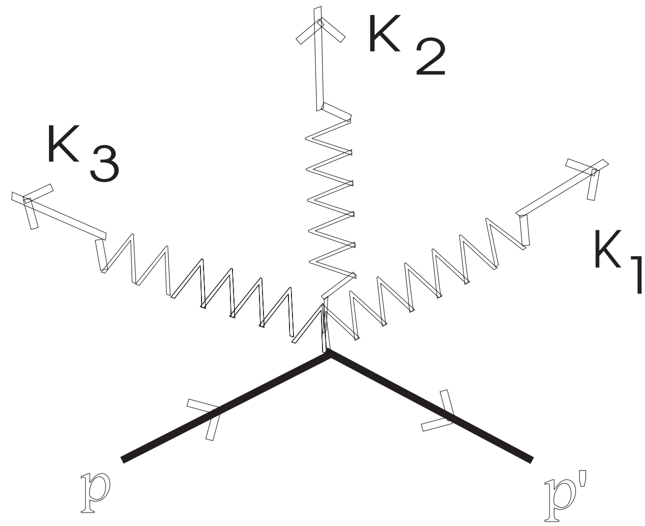

FIG. 1. $qq\gamma\gamma\gamma$ vertex with all fermions and photons on mass shell.

$$\frac{d\sigma}{d\cos\theta}=\left(\frac{d\sigma}{d\cos\theta}\right)_{QED}\left(1+\frac{\langle\theta^2\rangle}{96}s^2\right), \tag{18}$$

and for the cross process $e^-\mu^-\rightarrow e^-\mu^-s$ should be replaced by t in Eq. (18).

III. PARTON MODEL AT THE LOWEST ORDER IN LCNC SPACE

In the canonical version of NCQED the matter fields with charges 0 or ±1 are allowed, i.e., charged leptons and photon. But in the LCNCQED the quarks as well as the leptons and photon, can be accommodated in the theory. Therefore we can examine the NC effects for the processes which contain quarks. For this purpose we consider inclusive inelastic electron-nucleon scattering. In this process the electron, at the lowest order of the parton model, interacts with free charged partons via one photon exchange therefore modification in the obtained results with respect to the usual space can be expected. To this end we explore the differential cross section for the unpolarized e-N scattering as follows

$$\left.\frac{d^2\sigma}{dE'd\Omega}\right|_{eN}=\frac{E'\alpha^2}{EQ^4}L^{\mu\nu}W_{\mu\nu}, \tag{19}$$

where E, E', $\sqrt{-Q^2}$, $L^{\mu\nu}$ and $W_{\mu\nu}$ are initial and final energy of electron, the momentum transfer, the electron and the nucleon scattering tensor, respectively. The $ee\gamma$ vertex in the LCNCQED is given in Eq. (17) therefore $L^{\mu\nu}$ can be easily obtained as

$$L_{\mu\nu}=\frac{1}{2}\left(1+\frac{\langle\theta^2\rangle}{384}Q^4\right)^2\mathrm{tr}((\not{P}+m)\gamma_\mu(\not{P}+m)\gamma_\nu) =2\left(1+\frac{\langle\theta^2\rangle}{384}Q^4\right)^2(p_\mu p'_\nu+p_\nu p'_\mu-g_{\mu\nu}p\cdot p'+g_{\mu\nu}m^2). \tag{20}$$

The inelastic nucleon scattering tensor is proportional to the absolute square of the nucleonic current therefore we need the vertex function (Γ_μ) for the nucleonic current in the NC space. Since it is a Lorentz vector therefore the most general form of Γ_μ can be written as

$$\Gamma_\mu = A\gamma_\mu + BP'_\mu + CP_\mu + iDP'^{\nu}\sigma_{\mu\nu} + iEP'^{\nu}\sigma_{\mu\nu} + FP^\nu\theta_{\mu\nu} + GP'^{\nu}\theta_{\mu\nu}, \tag{21}$$

where A,B, . . . ,G depend on the Lorentz invariant quantity. The gauge invariance and using the Gordon identity leads to

$$\overline{u}(P')\Gamma_\mu(P',P)u(P) = \overline{u}(P')[A\gamma_\mu + iB(P'-P)^\nu\sigma_{\mu\nu} + C(P-P')^\nu\theta_{\mu\nu}]u(p). \tag{22}$$

We can now construct $W_{\mu\nu}$ as follows:

$$\begin{aligned} W_{\mu\nu} &= \frac{1}{2}\sum_{spin}[\overline{u}(P')\Gamma_\mu u(P)]^*[\overline{u}(P')\Gamma_\nu u(P)] \\ &= \frac{1}{2}\mathrm{tr}\{(A\gamma_\mu - iB(P-P')^\lambda\sigma_{\mu\lambda} + C(P-P')^\lambda \\ &\quad \times \hat{\theta}_{\mu\lambda}(\not{P}'+M))(A\gamma_\nu + iB(P-P')^\rho\sigma_{\nu\rho} \\ &\quad + C(P-P')^\rho\hat{\theta}_{\nu\rho}(\not{P}+M))\}. \end{aligned} \tag{23}$$

Since the weight function is an even function of $\theta^{\mu\nu}$ the odd functions of $\theta^{\mu\nu}$ have not any contribution on the cross section thus Eq. (23) can be cast into

$$W_{\mu\nu} = W^0_{\mu\nu} + 4P\cdot P'\frac{\langle\theta^2\rangle}{12}C(q^2 g_{\mu\nu} - q_\mu q_\nu), \tag{24}$$

where $q_\mu = (P-P')_\mu$, $q^2 = -Q^2$ and $W^0_{\mu\nu}$ is the commutative counterpart of the scattering tensor and is given as

$$W^0_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)W_1 + \left(P_\mu - q_\mu\frac{P\cdot q}{q^2}\right) \times \left(P_\nu - q_\nu\frac{P\cdot q}{q^2}\right)\frac{W_2}{M_N^2}. \tag{25}$$

W_1 and W_2 are the structure functions and depend on the Lorentz invariant quantity such as Q^2, $\nu = P.Q$ and $\langle\theta^2\rangle$. One can see that the second term in Eq. (24) can be absorbed in W_1 and $W_{\mu\nu}$ can be generally written as

$$W^{inel}_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)W_1(Q^2,\nu,\langle\theta^2\rangle) + \left(P_\mu - q_\mu\frac{P\cdot q}{q^2}\right)\left(P_\nu - q_\nu\frac{P\cdot q}{q^2}\right)\frac{W_2(Q^2,\nu,\langle\theta^2\rangle)}{M_N^2}, \tag{26}$$

where

$$\begin{aligned} W_1(Q^2,\nu,\langle\theta^2\rangle) &= W_1 + 4CP\cdot P'\frac{\langle\theta^2\rangle}{12}Q^2, \\ W_2(Q^2,\nu,\langle\theta^2\rangle) &= W_2. \end{aligned} \tag{27}$$

In the high energy limit the mass of the electron can be neglected and the differential cross section for the inclusive e-N scattering, in terms of Bjorken variable $x = Q^2/2\nu$ and the inelasticity parameter $y = (E-E')/E$, can be cast into

$$\left.\frac{d^2\sigma}{dxdy}\right|_{eN} = \frac{4\pi s\alpha^2}{Q^4}\left(1 + \frac{\langle\theta^2\rangle}{384}Q^4\right)^2\left[xy^2F_1^{eN}(Q^2,x,\langle\theta^2\rangle) + \left(1 - y - \frac{xyM_N^2}{s}\right)F_2^{eN}(Q^2,x,\langle\theta^2\rangle)\right], \tag{28}$$

where M_N is the nucleon mass, $F_1^{eN} = M_N W_1^{eN}$, $F_2^{eN} = (\nu/M_N)W_2^{eN}$ and s is the Mandelstam variable. In the parton model at the lowest order, one consider the elastic scattering of the electron off a free point charged parton with mass M_i, momentum P_i and charge $q_i e$. Therefore the cross section for this scattering can be easily constructed from the results for the electron-muon scattering [see Eq. (18)] as

$$\frac{d\sigma_i}{dQ^2} = \left(\frac{d\sigma_i}{dQ^2}\right)_{QED}\left(1 + \frac{\langle\theta^2\rangle t_i^2}{96}\right), \tag{29}$$

where t_i is the Mandelstam variable for the parton i. If we neglect the electron and the partons masses in the Briet frame of reference

$$\begin{aligned} q_\mu &= (0,0,0,\sqrt{-q^2}) = (0,0,0,\sqrt{Q^2}) \\ P_\mu &= (\tilde{P},0,0,-\tilde{P}), \quad \tilde{P} \gg M_N, \\ P^2 &= \tilde{P}^2 - \tilde{P}^2 = 0 \approx M_N^2, \end{aligned} \tag{30}$$

and after a little algebra the cross section in terms of x and y variables becomes

$$\frac{d^2\sigma_i}{dxdy} = \frac{4\pi\alpha^2 q_i^2 x}{Q^2}\left(\frac{s^2+u^2}{2s^2}\right)\delta(\xi_i - x)\left(1 + \frac{\langle\theta^2\rangle t^2}{96}\right), \tag{31}$$

where ξ_i is a fraction of the nucleon's total momentum carried by the ith parton (i.e., $P_i^\mu = \xi_i P^\mu$) and the Mandelstam variables for ith parton in terms of the Mandelstam variables for the whole nucleon are

$$s_i=(p+P_i)^2=\xi_i s,$$
$$t_i=(P_i-P_i')^2=t=-Q^2, \tag{32}$$
$$u_i=(p'-P_i)^2=\xi_i u.$$

Various types of partons carry a different fraction of the parent nucleon's momentum therefore for the parton momentum distribution function $f_i(\xi_i)$ with the appropriate normalization:

$$\int_0^1 d\xi_i f_i(\xi_i)=1, \tag{33}$$

one has

$$\frac{d^2\sigma}{dxdy}=\frac{2\pi\alpha^2 s}{Q^4}[(y-1)^2+1]\sum_i f_i(x)q_i^2 x\left(1+\frac{\langle\theta^2\rangle t^2}{96}\right). \tag{34}$$

Now comparing Eq. (34) with Eq. (28), where $M_N=0$, reads

$$2xy^2F_1^{eN}(Q^2,x,\langle\theta^2\rangle)+2(1-y)F_2^{eN}(Q^2,x,\langle\theta^2\rangle)$$
$$=(y^2+2(1-y))\sum_i f_i(x)q_i^2 x\left(1+\frac{\langle\theta^2\rangle Q^4}{192}\right). \tag{35}$$

Equation (35) shows that

$$F_2^{eN}(Q^2,x,\langle\theta^2\rangle)=\sum_i f_i(x)q_i^2 x\left(1+\frac{\langle\theta^2\rangle Q^4}{192}\right). \tag{36}$$

In other words, the parton model at the lowest order in LCNCQED violates the Bjorken scaling but Callan-Gross relation still holds:

$$F_2^{eN}(Q^2,x,\langle\theta^2\rangle)=2xF_1^{eN}(Q^2,x,\langle\theta^2\rangle). \tag{37}$$

One can use the data on the deep inelastic e-N scattering to obtain the upper bound on the value of the parameter of noncommutativity in the LCNCQED, Λ_{LCNC}. Equation (36) shows that

$$\frac{F_2^{NC}-F_2^0}{F_2^0}=\frac{\langle\theta^2\rangle Q^4}{192}=\frac{Q^4}{16\Lambda_{LCNC}^4}, \tag{38}$$

where $\Lambda_{LCNC}=(12/\langle\theta^2\rangle)^{1/4}$. The measurements of F_2 structure function in deep inelastic scattering can provide a test on the noncommutativity of space. For example F_2 in positron-proton neutral current scattering has been measured with statistical and systematic uncertainties below 2% [13]. Therefore, as an estimation, one percent error in the experimental value of the structure function for $\sqrt{Q^2}=200$ GeV results in $\Lambda_{LCNC}\sim 300$ GeV.

IV. SUMMARY

We completed the Feynman rules for the Lorentz conserving noncommutative QED up to the order $\langle\theta^2\rangle$. Besides two fermions and one and two photons vertices which has already introduced in [12] there is a two fermions and three photon vertex given in Eq. (16). The parameter of noncommutativity is a dimensional quantity therefore the dimensionless form factors in lepton-nucleon scattering should depend on $\langle\theta^2\rangle Q^4$ and violate the Bjorken scaling. We explicitly obtained this violation in Eqs. (36) and (38) while the Callan-Gross relation still holds as is shown in Eq. (37). The obtained results provide an experimental tool to find if the nature can be described by LCNC-space or not, for $Q\sim 1.1\Lambda_{LCNC}-2\Lambda_{LCNC}$ there is 10–100 % correction which can be easily verified in comparison with the experimental data.

ACKNOWLEDGMENT

The financial support of IUT research council and IPM are acknowledged.

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