Probing gluon helicity distribution and quark transversity through hyperon polarization in singly polarized *pp* collisions

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We study the polarization of hyperon in different processes in singly polarized *pp* collisions, in particular its relation to the polarized parton distributions. We show that by measuring hyperon polarization in particularly chosen processes, one can extract useful information on these parton distributions. We show in particular that, by measuring the Σ^+ polarization in high p_T direct photon production process, one can extract information on the gluon helicity distribution; and by measuring the transverse polarization of hyperons with high p_T in singly polarized reactions, one can obtain useful information on the transversity distribution. We present the numerical results obtained for those hyperon polarizations using different models for parton distribution function and those for the spin transfer in fragmentation processes.

I. INTRODUCTION

The spin structure of nucleon is one of the important issues in high energy spin physics. Since the "spin crisis" induced by the EMC data in 1988 [1], the study of it has been an active area of both experimental and theoretical research. Presently, the helicity distributions of quarks, in particular the valence quarks, have been determined with relatively high precision experimentally through inclusive deeply inelastic lepton-nucleon scattering (DIS) processes. However, little is known from experiment about the gluon helicity distribution $\Delta g(x, Q^2)$, except some indirect estimates from the Q^2 evolution of the quark distributions [2]. For the transversely polarized case, i.e., the transversity distribution of quarks in the nucleon, $\delta q(x, Q^2)$, the situation is even worse. Due to its chiral-odd property, it cannot be measured in inclusive DIS process and is up to now unmeasured [3]. It has to be accompanied by another chiral-odd quantity. Excitingly, many interesting ideas to measure $\Delta g(x, Q^2)$ and $\delta q(x, Q^2)$ in polarized *ep* scatterings and *pp* collisions have been proposed [2–4] and the related experiments are being undertaken or are underway.

Hyperon polarization, in particular for Λ, has been widely used to study various aspects of spin effects in different reactions in particular the spin-dependent fragmentation functions for their self-analyzing decay. The spin-dependent fragmentation function is interesting in itself and also in serving as filters for exotic parton distribution functions, such as polarized antiquark distributions and the above-mentioned gluon helicity distribution and transversity of nucleon [5].

In a recent publication [6], we made a detailed study of the longitudinal polarization of hyperons with high transverse momenta p_T in inclusive process in *pp* collisions with one beam longitudinally polarized. We found out in particular that the origin of Λ is usually very complicated and the contribution of decay of other hyperons is very large. On the contrary, the origin of Σ^+ is much cleaner. The contribution from decay of other hyperons is very small. In addition, there is a characteristic feature for Σ^+ production with high p_T in *pp* collisions, i.e., the contributions from those which are directly produced and contain a fragmenting *u* quark from

the hard subprocess play the dominant role. In particular for the region of $\eta > 1.5$ where η is the pseudorapidity of produced hyperons, such contributions are larger than 93% at $p_T > 13$ GeV and $\sqrt{s} = 500$ GeV (cf. Fig. 10 of [6]).

This characteristics of Σ^+ production is remarkable, since in the limiting case that only this kind of contribution is considered, the Σ^+ polarization is $P_L^{\Sigma^+(lim)} = t_{\Sigma^+,u}^F P_L^u$ (where P_L^u is the longitudinal polarization of the *u* quark after the hard scattering and $t_{\Sigma^+,u}^F$ is the spin transfer factor in the fragmentation process from the *u* quark to Σ^+ for the case that Σ^+ contains the *u* quark). There are two different models for the spin transfer in the fragmentation processes, one is based on the SU(6) wave function of the hadrons, the other is based on the DIS data and those of hyperon decay. The spin transfer factor is different in the two pictures but is a constant in both cases. Hence, $P_L^{\Sigma^+}$ is directly proportional to P_L^u. The fragmentation effects come in only through the proportional constant $t_{\Sigma^+,u}^F$. In this case, measuring Σ^+ polarization provides a nice tool to study the polarization of the quarks before fragmentation. Since such quark polarization is determined by the polarized quark distribution functions in the nucleon and the calculable spin transfer factor in the hard scattering, we expect that we can use it to study the exotic distributions, i.e., the gluon helicity distribution and the quark transversity distribution in suitable processes.

In this paper, after a brief summary of the calculation method and main conclusion for hyperon polarization in inclusive production processes in *pp* collisions, we present two examples in this direction, i.e., hyperon polarization in direct photon production process in longitudinally polarized *pp* collisions, and the transverse polarization of hyperons with high p_T in transversely polarized *pp* collisions. We show that they can be used to study the gluon helicity distribution $\Delta g(x, Q^2)$ and the quark transversity distribution $\delta q(x, Q^2)$ respectively. We present the numerical results for the hyperon polarization in these two processes obtained by using different models for $\Delta g(x, Q^2)$ and $\delta q(x, Q^2)$. We show that both of them can be measured at, e.g., BNL RHIC and such measurements should provide important information for $\Delta g(x, Q^2)$ and $\delta q(x, Q^2)$.

II. CALCULATION METHOD OF POLARIZATION OF HYPERONS WITH HIGH p_T IN POLARIZED pp COLLISIONS

The calculation method of the polarization of hyperons with high p_T in longitudinally polarized pp collisions has been given in [6]. The method can be easily extended to other cases. In this section, we summarize the main points of the method.

A. General formulas

We consider inclusive hyperon production with high p_T in pp collisions with one beam longitudinally polarized. The produced hyperons with high p_T are mainly from the fragmentation of a scattered high p_T parton in the hard subprocess, in which one of the initial partons is (longitudinally) polarized. Since the polarization can be transferred to the outgoing parton in the hard scattering, the scattered parton can also be polarized and its polarization can further be transferred to the produced hyperons. Their polarizations can be obtained as follows (the subscripts "+" and "−" below denote helicities):

$$P_L^H = \frac{d\sigma^{(p+p \to H_+X)} - d\sigma^{(p+p \to H_-X)}}{d\sigma^{(p+p \to H_+X)} + d\sigma^{(p+p \to H_-X)}} \equiv \frac{d\Delta\sigma^{(\vec{p}p \to \vec{H}X)}/d\eta}{d\sigma^{(pp \to HX)}/d\eta},$$
(1)

where η is the pseudorapidity of produced hyperon, $d\Delta\sigma$ and $d\sigma$ are polarized and unpolarized cross sections for inclusive hyperon production with high p_T. For sufficiently high p_T, the factorization theorem can be applied and the (polarized) cross section can be expressed as a convolution of perturbatively calculable partonic cross sections with certain sets of (polarized) parton distribution and fragmentation functions at a proper scale. In this case the relevant polarized cross section can be expressed as [7,18]

$$\frac{d\Delta\sigma^{(\vec{p}p \to \vec{H}X)}}{d\eta} = \int_{p_T^{min}} dp_T \sum_{abcd} \int dx_a dx_b dz \Delta f_a(x_a, \mu^2)$$
$$\times f_b(x_b, \mu^2) \Delta D_c^H(z, \mu^2) \frac{d\Delta\hat{\sigma}^{(\vec{a}b \to \vec{c}d)}}{d\eta},$$
(2)

where the sum runs over all possible subprocesses, and the transverse momenta p_T of hyperons are integrated above a lower limit p_T^{min}; $\Delta f_a(x_a, \mu^2)$ and $f_b(x_b, \mu^2)$ are the longitudinally polarized and unpolarized distribution functions of partons in proton at the scale μ; x_a and x_b are the corresponding momentum fractions carried by parton a and b. $\Delta D_c^H(z, \mu^2)$ is the longitudinally polarized fragmentation function,

$$\Delta D_c^H(z, \mu^2) \equiv D_{c(+)}^{H(+)}(z, \mu^2) - D_{c(+)}^{H(-)}(z, \mu^2),$$
(3)

where $D_{c(+)}^{H(+)}(z, \mu^2)$ and $D_{c(+)}^{H(-)}(z, \mu^2)$ are the probabilities to produce a H with positive and negative helicity in the fragmentation of a parton c with positive helicity, carrying off a

fraction z of parent parton's momentum. $d\Delta\hat{\sigma}$ is defined for the hard scattering subprocess similarly as $d\Delta\sigma$ in Eq. (1), which can be calculated by PQCD.

The unpolarized cross section $d\sigma^{(pp \to HX)}/d\eta$ is given by an expression similar to that in Eq. (2), with all Δ's removed. It can be calculated with considerable precision using the available parametrizations of the unpolarized parton distribution and fragmentation functions.

For the polarized case, we see that from Eq. (2), the unknown parts in the formulas are the polarized fragmentation function $\Delta D_c^H(z, \mu^2)$ and the polarized parton distribution $\Delta f_a(x_a, \mu^2)$. If we know $\Delta f_a(x_a, \mu^2)$, we can study $\Delta D_c^H(z, \mu^2)$ through hyperon polarization. Similarly, if the influence from $\Delta D_c^H(z, \mu^2)$ can be determined, we can use it to study the polarized parton distribution functions.

B. The polarized fragmentation function $\Delta D_c^H(z, \mu^2)$

We consider a general fragmentation process $q_f \to H_i + X$ (subscripts "f" and "i" denote the quark flavor and the type of hyperons) and calculate $\Delta D_{q_f}^{H_i}(z, \mu^2)$ in the following way. We divide the produced H_i's into four groups and consider them separately: (A) directly produced and contain the q_f's; (B) decay products of heavier hyperons which were polarized before their decays; (C) directly produced but do not contain the q_f's; (D) decay products of heavier hyperons which were unpolarized before their decays. In this way, we have

$$D_{q_f}^{H_i}(z, \mu^2) = \sum_\alpha D_{q_f}^{H_i(\alpha)}(z, \mu^2),$$
(4)

$$\Delta D_{q_f}^{H_i}(z, \mu^2) = \sum_\alpha \Delta D_{q_f}^{H_i(\alpha)}(z, \mu^2),$$
(5)

where $D_{q_f}^{H_i(\alpha)}$ and $\Delta D_{q_f}^{H_i(\alpha)}$ are the unpolarized and polarized fragmentation functions for hyperons of group (α), and $\alpha = A, B, C$ or D.

It is clear that hyperons from (A) and (B) can be polarized while those from (C) and (D) are not [8–11]. Hence, if we denote $t_{H_i, f}^{F(\alpha)}$ as the spin transfer factor for group (α) in the fragmentation process, i.e.,

$$t_{H_i, f}^{F(\alpha)}(z, \mu^2) = \frac{\Delta D_{q_f}^{H_i(\alpha)}(z, \mu^2)}{D_{q_f}^{H_i(\alpha)}(z, \mu^2)},$$
(6)

we have

$$t_{H_i, f}^{F(C)}(z, \mu^2) = t_{H_i, f}^{F(D)}(z, \mu^2) = 0.$$
(7)

For those hyperons from group (A), the spin transfer factor $t_{H_i, f}^{F(A)}(z, \mu^2)$ is taken as the fraction of spin carried by the f-flavor quark divided by the number of valence quark of flavor f in H_i. In different pictures, such as the SU(6) picture and DIS picture used in e^+e^- and ep reactions [8–11], the contributions to the hyperon spin from different flavors are

different. In the SU(6) picture, these contributions can be obtained from the SU(6) wave functions of the hyperons. In the DIS picture, for the $J^P = (1/2)^+$ octet hyperons, they are obtained from the DIS data on the spin dependent structure functions and those on hyperon decay. In both pictures, $t_{H_i,f}^{F(A)}$ is a constant independent of z. Clearly, $t_{H_i,f}^{F(A)}$ is the probability for the polarization of q_f to be transferred to the produced hyperon H_i in the case that H_i contains q_f. It is called "fragmentation spin transfer factor" and is usually denoted as $t_{H_i,f}^F$, i.e., $t_{H_i,f}^{F(A)} \equiv t_{H_i,f}^F$. A list of $t_{H_i,f}^F$ for different hyperons can be found in Table I of Ref. [10].

For those from group (B), hyperon H_i's are from the decay of the heavier hyperon H_j's which are from group (A) and polarized before their decays. Here, an additional decay polarization transfer factor t_{H_i,H_j}^D is needed to obtain $t_{H_i,f}^{F(B)}(z,\mu^2)$ and we have

$$t_{H_i,f}^{F(B)} = t_{H_i,H_j}^D t_{H_j,f}^F, \tag{8}$$

where t_{H_i,H_j}^D is the probability for the polarization of H_j to be transferred to H_i in the decay process $H_j \to H_i + X$ and the superscript "D" stands for decay. It is determined by the decay process and is independent of the process in which H_j is produced. For the octet hyperon decays, they are extracted from the materials in Review of Particle Properties [12]. For the decuplet hyperons, we have to use an estimation based on the SU(6) quark model [8]. Thus, as we pointed out in [6], to reduce the uncertainty of calculations, it is important to consider the hyperons to which decay contributions are small.

Obtaining the spin transfer factors for different groups of hyperons, we can get $\Delta D_{q_f}^{H_i}(z,\mu^2)$ if we know the corresponding unpolarized fragmentation functions $D_{q_f}^{H_i(\alpha)}(z,\mu^2)$. The result is given by

$$\Delta D_{q_f}^{H_i}(z,\mu^2) = t_{H_i,f}^F D_{q_f}^{H_i(A)}(z,\mu^2)$$
$$+ \sum_{H_j} t_{H_i,H_j}^D t_{H_j,f}^F D_{q_f}^{H_i(B,H_j)}(z,\mu^2), \tag{9}$$

where $D_{q_f}^{H_i(B,H_j)}(z,\mu^2)$ is the contribution to H_i production from group (B) through $q_f \to H_j + X$ and $H_j \to H_i + X'$.

Using Eqs. (4) and (9), we can obtain the H_i polarization in the general fragmentation process $q_f \to H_i + X$. We note that, one of the characteristics of the model is that, the z-dependence of P^{H_i} comes from the interplay of the different contributions $D_{q_f}^{H_i(\alpha)}(z,\mu^2)$, which are determined by the hadronization mechanism. These different contributions can be calculated using a hadronization model that are well tested in unpolarized reactions. This means that the shape of the z-dependence of the hyperon polarization from the fragmentation process $q_f \to H_i + X$ in this model can be fixed to a very large extent by the results of an unpolarized hadronization model without any new free parameter. It is therefore very crucial to test the model by looking at the shape of the z-dependence of hyperon polarization. In this sense, the best

places to test this model are, e.g., e^+e^- annihilation or deeply inelastic ep scattering, where fragmentation of a polarized quark with a given energy can be studied. The model has been applied to these processes [9–11]. Presently, data on Λ polarization in e^+e^- annihilation are available. It is encouraging to see that the obtained results under both SU(6) and DIS pictures give a good description of the z-dependence of the data. Further tests can be made by future experiments in other reactions such as lepton nucleon deep-inelastic scattering and pp collisions.

C. Different contributions to hyperon production in $pp \to H_i X$

As mentioned earlier, the different contributions $D_{q_f}^{H_i(\alpha)}(z,\mu^2)$'s are determined by the hadronization mechanism, and are independent of the polarization properties and can be obtained from a hadronization model. Presently, the most convenient way to do this is to employ a Monte Carlo event generator. In practice, we calculate hyperon polarization by rewriting $P_L^{H_i}$ in Eq. (1) as

$$P_L^{H_i} = \frac{\sum_f t_{H_i,f}^F P_L^{q_f} \langle n_{H_i,f}^A \rangle + \sum_{j,f} t_{H_i,f}^{F(B)} P_L^{q_f} \langle n_{H_i,H_j,f}^B \rangle}{\langle n_{H_i}^A \rangle + \langle n_{H_i}^B \rangle + \langle n_{H_i}^C \rangle + \langle n_{H_i}^D \rangle}. \tag{10}$$

The quantities $P_L^{q_f}$ and the different $\langle n_{H_i}^\alpha \rangle$'s are related to the parton distribution and the fragmentation functions in the following way. $P_L^{q_f}$ is the polarization of the fragmenting quark q_f, i.e., the polarization of the outgoing quark in the hard scattering, which can be calculated by PQCD and suitable parton distributions. $\langle n_{H_i,f}^A \rangle$ is the average number of H_i's which are directly produced and contain q_f, and $\langle n_{H_i,H_j,f}^B \rangle$ is the average number of H_i coming from the decay of H_j which contains a polarized q_f. $\langle n_{H_i}^A \rangle$ $(\equiv \Sigma_f \langle n_{H_i,f}^A \rangle)$, $\langle n_{H_i}^B \rangle (\equiv \Sigma_{j,f} \langle n_{H_i,H_j,f}^B \rangle)$, $\langle n_{H_i}^C \rangle$, and $\langle n_{H_i}^D \rangle$ are the average numbers of hyperons of group (A), (B), (C), and (D) in $pp \to H_i X$, respectively. They are related to $D_{q_f}^{H_i(\alpha)}$ in $pp \to H_i X$ by

$$\langle n_{H_i}^\alpha \rangle \propto \sum_{abcd} \int dx_a dx_b dz f_a(x_a,\mu^2) f_b(x_b,\mu^2)$$
$$\times D_c^{H_i(\alpha)}(z,\mu^2) d\hat\sigma^{(ab \to cd)}. \tag{11}$$

We use Lund model [13] implemented by the event generator PYTHIA [14] to obtain them in our calculations for pp collisions.

In Ref. [6], with the aid of generator PYTHIA, we calculated the different contributions to the inclusive high p_T hyperon production in pp collisions. We found out that, for Λ production (cf. Fig. 7 of Ref. [6]), the contribution of group (A) is relatively small. The contribution of decay of other hyperons is considerably large. Hence, the uncertainties in the calculations of Λ polarization are relatively large. In contrast, we found out also that, for Σ^+ production (cf. Fig. 10
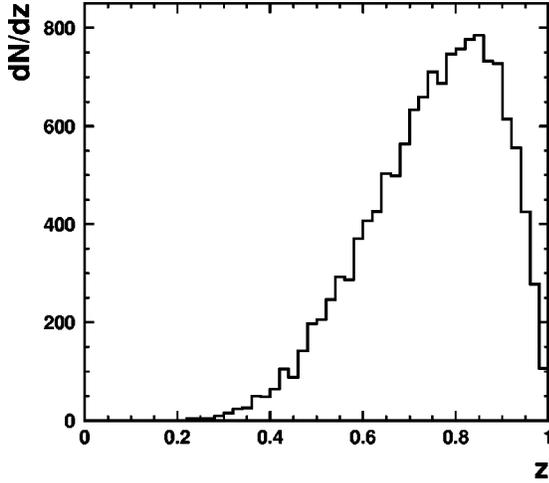
FIG. 1. The distribution of z fraction of Σ^+'s that are directly produced and contain a fragmenting u quark in pp collisions at $\sqrt{s} = 500$ GeV and $p_T^{min} = 13$ GeV.

of Ref. [6]), the decay contribution from heavier hyperons is very small. It takes only a few percents for $p_T > 13$ GeV at $\sqrt{s} = 500$ GeV. The contributions from group (A), i.e., which are directly produced and contain a fragmenting u quark play the dominant role. In fact, they give more than 75% of the whole produced Σ^+'s at $p_T > 13$ GeV. In particular for the region of $\eta > 1.5$, they take more than 93%. The reason for this result is simple: The Σ^+'s of group (A) are usually the leading particles of quark fragmentation. They take the largest fractions of the momenta of the fragmenting quark. To produce a nonleading Σ^+ with the same p_T, one needs a quark of much higher p_T, whose production is much suppressed. Since Σ^+ has two u valence quarks, and u-quark contributes much to the high p_T quark jet, the contribution to Σ^+ from u dominates at large p_T.

If we neglect all other contributions, i.e., consider only the contribution of type (A), we obtain the $P_L^{\Sigma^+}$ in this limiting case, i.e. the $P_L^{\Sigma^+}$ for those only from the origin (A) as

$$P_L^{\Sigma^+(lim)} = P_L^{\Sigma^+(A)} = t_{\Sigma^+,u}^F P_L^u. \tag{12}$$

It is directly proportional to the polarization of the u-quark, P_L^u, after the hard scattering and the proportional constant is just the fragmentation spin transfer factor $t_{\Sigma^+,u}^F$.

We note that, as can be seen from Eq. (6), the factor $t_{\Sigma^+,u}^F$ can in general be z-dependent. But, since the Σ^+'s from group (A) are usually the leading particles in the jet, its z-distribution is expected to be very narrow. To get a feeling of it, we calculate the distribution of z for such Σ^+'s using the MC event generator PYTHIA and the results are shown in Fig. 1. We see that, the distribution is indeed very narrow with a peak at $z \simeq 0.8$. In this region, the fragmentation function do not change very fast with z. Hence, we expect that the z-dependence of $t_{\Sigma^+,u}^F$, if any, should not have much influence on the Σ^+ polarization. In this paper, we only discuss the case that such a z-dependence is neglected. In this case, the factor $t_{\Sigma^+,u}^F$ can be different for different pictures

but it is a constant and can in principle be determined in corresponding experiments by one data point. Hence, Σ^+ polarization provides a nice tool to study the polarization of the quarks. In this way, we can study the polarized parton distributions, such as the gluon helicity distribution and the quark transversity distribution, by measuring Σ^+ polarization in suitable process of pp collisions. In next two sections, we give two of such examples.

III. POLARIZATION OF Σ^+ IN DIRECT PHOTON PRODUCTION PROCESS AND GLUON HELICITY DISTRIBUTION

Among different methods to probe gluon helicity distribution in nucleon, high p_T direct photon production in pp collisions is one of the most direct process since the cross section is directly related to the gluon distribution. The idea to measure gluon helicity distribution through the double spin asymmetry A_{LL} in direct photon production process is one of the most promising methods in this connection [4]. Here, both beams need to be polarized. In this section, we show that useful information can also be extracted on the gluon helicity distribution by measuring polarization of Σ^+ with high p_T in $\vec{p}p \to \gamma \vec{\Sigma}^+ X$ with one beam polarized.

A. Calculation formulas for hyperon polarization in $\vec{p}p \to \gamma\vec{H}X$

We consider the polarization of hyperons with high p_T associated with a single large p_T direct photon in pp collisions with one proton longitudinally polarized. We recall that three kinds of hard scattering, i.e., $qg \to \gamma q$, $\bar{q}g \to \gamma\bar{q}$, and $q\bar{q} \to \gamma g$, contribute to $pp \to \gamma HX$ at high p_T. But, for $pp \to \gamma\Sigma^+X$, the latter two, i.e., $\bar{q}g \to \gamma\bar{q}$, and $q\bar{q} \to \gamma g$, are significantly suppressed. Using the event generator PYTHIA, we can study this explicitly. The results show that their contributions are less than 4% totally at $\sqrt{s} = 200$ GeV and $p_T^{min} = 4$ GeV. This implies that, the hard process $qg \to \gamma q$ play the dominant role in $pp \to \gamma\Sigma^+X$ at high p_T and the contributions of other two subprocesses can be neglected. In this case, the polarized cross section for $pp \to \gamma\Sigma^+X$ is given by

$$\frac{d\Delta\sigma^{(\vec{p}p \to \gamma\vec{\Sigma}^+X)}}{d\eta} = \int_{p_T^{min}} dp_T \sum_f \int dx_a dx_b dz \Delta g(x_a,\mu^2)$$

$$\times q_f(x_b,\mu^2)\Delta D_f^H(z,\mu^2)\frac{d\Delta\hat{\sigma}^{(\vec{g}q_f \to \gamma\vec{q}_f)}}{d\eta}$$

$$+ (g \leftrightarrow q_f), \tag{13}$$

where the sum runs over the different quark flavors. The first term corresponds to the contribution from subprocess $\vec{g}q_f \to \gamma\vec{q}_f$ while the second term $(g \leftrightarrow q_f)$ corresponds to the contribution from $\vec{q}_f g \to \gamma\vec{q}_f$, which has a similar expression as the first term with an exchange of g and q_f in the parton distributions and the hard scattering cross section. We see that the first term is related to $\Delta g(x)$, while the second one

is related to $\Delta q_f(x)$. In our following calculations, we choose the positive axis of pseudorapidity η as the moving direction of the polarized proton. We expect that the first term dominates at the region of $\eta < 0$, while the second one contributes mainly at the region of $\eta > 0$, since the probability for a forward scattering is usually much larger than that for a backward scattering. This will be further illustrated in next subsection using the results from Monte Carlo calculations.

For the first term of Eq. (13), the hard scattering is $\vec{g}q_f \rightarrow \gamma \vec{q}_f$. The integrand can be rewritten as

$$\Delta g(x_a,\mu^2)q_f(x_b,\mu^2)\Delta D_f^H(z,\mu^2)d\Delta\hat{\sigma}$$

$$= \sum_\alpha P_L^{q_f} t_{H,f}^{F(\alpha)} g(x_a,\mu^2)q_f(x_b,\mu^2)D_f^{H(\alpha)}(z,\mu^2)d\hat{\sigma}, \tag{14}$$

where $P_L^{q_f}$ is the polarization of outgoing quark in the hard subprocess $\vec{g}q_f \rightarrow \gamma \vec{q}_f$ and is directly related to $\Delta g(x_a,\mu^2)$ by

$$P_L^{q_f} = \frac{d\Delta\hat{\sigma}}{d\hat{\sigma}}\frac{\Delta g(x_a,\mu^2)}{g(x_a,\mu^2)}. \tag{15}$$

$d\Delta\hat{\sigma}/d\hat{\sigma}$ is the spin transfer from g to q_f in the hard scattering $\vec{g}q_f \rightarrow \gamma \vec{q}_f$. To the leading order, it is given by

$$D_L(y) \equiv \frac{d\Delta\hat{\sigma}}{d\hat{\sigma}} = \frac{1-(1-y)^2}{1+(1-y)^2}, \tag{16}$$

where $y \equiv k_q \cdot (k_g - k_\gamma)/k_g \cdot k_q$ and k_g, k_q and k_γ are the four-momenta of incoming gluon g, quark q_f and the produced γ respectively.

For the second term of Eq. (13), the integrand is

$$\Delta q_f(x_a,\mu^2)g(x_b,\mu^2)\Delta D_f^H(z,\mu^2)d\Delta\hat{\sigma}$$

$$= \sum_\alpha P_L^{q_f} t_{H,f}^{F(\alpha)} q_f(x_a,\mu^2)g(x_b,\mu^2)D_f^{H(\alpha)}(z,\mu^2)d\hat{\sigma}, \tag{17}$$

where

$$P_L^{q_f} = \frac{\Delta q_f(x_a,\mu^2)}{q_f(x_a,\mu^2)}, \tag{18}$$

since the spin transfer for $\vec{q}_f g \rightarrow \gamma \vec{q}_f$ is equal to 1.

We further note that the product on the right-hand side (RHS) of Eq. (14) or Eq. (17), $g(x_a,\mu^2)q_f(x_b,\mu^2)D_f^{H(\alpha)}(z,\mu^2)d\hat{\sigma}$, corresponds to the unpolarized hyperon production from group (α) in $pp \rightarrow \gamma HX$. They are proportional to the average numbers $\langle n_H^\alpha \rangle$ of hyperon production in $pp \rightarrow \gamma HX$, which can also be obtained by the generator PYTHIA. Hence, if we can determine the constant $t_{H,f}^{F(\alpha)}$ using a few data points, we will be able to
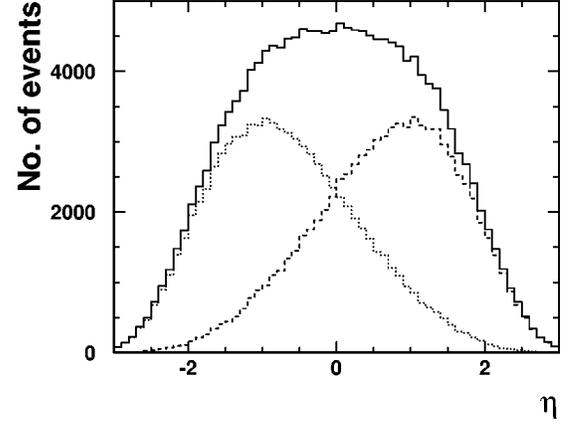
FIG. 2. Different contributions to Σ^+ production at $\sqrt{s} = 200$ GeV and $p_T^{min} = 4$ GeV in the direct photon production in singly polarized pp collisions. Dotted line denotes the contribution from the hard process $\vec{g}q \rightarrow \gamma \vec{q}$, the dashed line denotes that from $\vec{q}g \rightarrow \gamma \vec{q}$, and the solid one is the total of them.

extract information from hyperon polarization on the gluon helicity distribution in direct photon production process in pp collisions.

B. Numerical results for Σ^+ polarization in $\vec{p}p \rightarrow \gamma \vec{\Sigma}^+ X$

As can be seen from the above mentioned discussions, to get information on $\Delta g(x,\mu^2)$, hyperon production from the subprocess $\vec{g}q \rightarrow \gamma \vec{q}$ is useful while that from $\vec{q}g \rightarrow \gamma \vec{q}$ is a background. Fortunately, the former dominates at the region of $\eta < 0$ while the latter dominates at the region of $\eta > 0$. To see this explicitly, with the aid of generator PYTHIA, we calculate these two contributions separately for $pp \rightarrow \gamma\Sigma^+X$ and the results for $\sqrt{s} = 200$ GeV and $p_T^{min} = 4$ GeV are shown in Fig. 2. Here, the cutoff of p_T means that the transverse momenta of the produced photon and hyperon are both larger than 4 GeV. We see that although there are some influences from $\vec{q}g \rightarrow \gamma \vec{q}$ in the region of $\eta < 0$, they are very small and less than 40%. It is possible to use hyperon polarizations in this region to study the gluon helicity distribution.

Figure 3 shows the different contributions to Σ^+ production in $\vec{p}p \rightarrow \gamma \vec{\Sigma}^+ X$ for $\sqrt{s} = 200$ GeV and $p_T^{min} = 4$ GeV. We see that, the contribution from group (A), i.e., which are directly produced and contain a fragmenting u quark is larger than 80%, even in the small $|\eta|$ region. This percentage is even larger than that for Σ^+ production at high p_T in $pp \rightarrow \Sigma^+X$. In particular in the large $|\eta|$ region, the u quark fragmentation plays the dominant role. Since u quark carries most of the hyperon Σ^+'s spin, the resulting Σ^+ polarization in $\vec{p}p \rightarrow \gamma \vec{\Sigma}^+ X$ should be much larger and more sensitive to $\Delta g(x)$ than that of Λ. Furthermore, also due to the dominance of the u quark fragmentation, the production rate of Σ^+ should be comparable with that of Λ, which implies that the statistics for studying Σ^+ should be similar to that of Λ. We first calculate $P_L^{\Sigma^+}$ in region of $-1 < \eta < 0$ as a function of p_T at $\sqrt{s} = 200$ GeV under the SU(6) and DIS pictures. We use three different sets of $\Delta g(x)$ parametrization as inputs
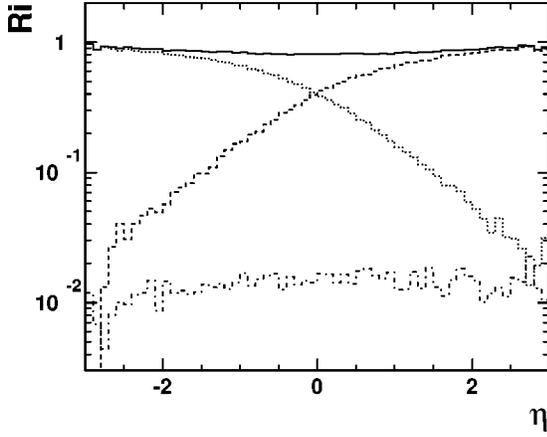
FIG. 3. Different contributions to Σ^+ production in $pp \rightarrow \gamma \Sigma^+ X$ as a function of η at $\sqrt{s} = 200$ GeV and $p_T^{min} = 4$ GeV. The solid line denotes the contribution of group (A), i.e., those which are directly produced and contain a fragmenting quark in the hard scattering; the dash-dotted line corresponds to the contribution of decay of other hyperons; the dotted and the dashed lines denote those which contain a fragmenting quark in the subprocess $\vec{g}q \rightarrow \gamma \vec{q}$ and $\vec{q}g \rightarrow \gamma \vec{q}$ respectively.

and the results are shown in Fig. 4. The obtained results from different Δg parametrizations differ significantly from each other. We also show the results of $P_L^{\Sigma^+}$ in Fig. 5 for $p_T^{min} = 8$ GeV at $\sqrt{s} = 200$ GeV as a function of η. We see that, $P_L^{\Sigma^+}$ increases with increasing η to the order of 0.3 at $\eta = 1$. $P_L^{\Sigma^+}$ can select among different sets of $\Delta g(x, \mu^2)$ in the region of $\eta < 0$. Such differences could be distinguished at RHIC or other future experiments. Therefore, by measuring $P_L^{\Sigma^+}$ in high p_T direct photon production process, one can get useful information [15] to distinguish between different sets of $\Delta g(x, \mu^2)$.
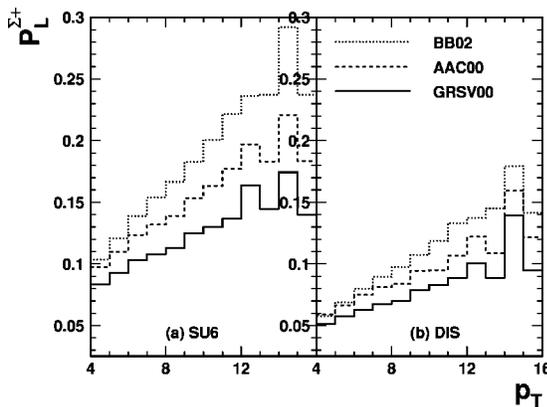
FIG. 4. Polarization of Σ^+ for different sets of $\Delta g(x, \mu^2)$ in $\vec{p}p \rightarrow \gamma \vec{\Sigma}^+ X$ as a function of p_T in the region of $-1 < \eta < 0$ at $\sqrt{s} = 200$ GeV under (a) SU6 picture and (b) DIS picture for the spin transfer factor. "GRSV2000" denotes the standard LO GRSV2000 [23] parametrization of gluon distribution; "BB02" denotes the LO BB02 parametrization [24]; "AAC00" denotes the LO AAC00 parametrization [25]. GRV98 [26] for unpolarized parton distribution is used in all the calculations.
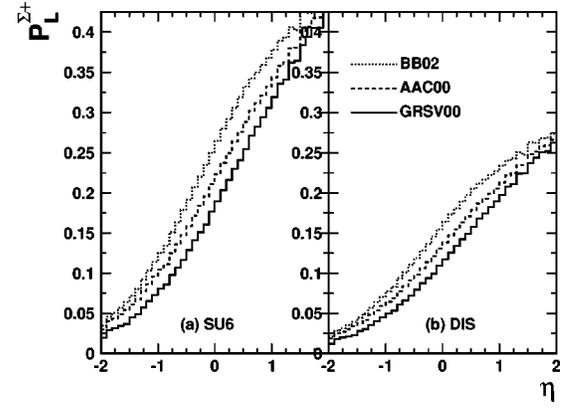
FIG. 5. Polarization of Σ^+ for different sets of $\Delta g(x, \mu^2)$ in $\vec{p}p \rightarrow \gamma \vec{\Sigma}^+ X$ as a function of η at $\sqrt{s} = 200$ GeV and $p_T^{min} = 8$ GeV under (a) SU6 picture and (b) DIS picture for the spin transfer factor.

As we mentioned in Sec. II C, the origin of Λ inclusively produced in pp collisions is very complicated and the decay contribution is very large. Also, because of the dominance of u quark fragmentation, the obtained Λ polarization is expected to be very small and contain a large uncertainty. We do similar calculations and find that the obtained P_L^Λ is smaller than 3% in all cases and thus it cannot be used to study gluon helicity distribution.

IV. TRANSVERSE POLARIZATION OF HYPERONS WITH HIGH p_T

When one of the proton beam is transversely polarized, the hyperons with high p_T can also be transversely polarized. In this section, we show that by measuring the polarization of Σ^+ with high p_T in transversely polarized pp collisions, one can get useful information on the quark transversity distributions in nucleon.

A. Calculation formulas for hyperon polarization in $\vec{p}_\perp p \rightarrow \vec{H}_\perp X$

The calculation formulas for hyperon polarization in the transversely polarized case are completely similar to those in the longitudinally polarized case. We summarize them in the following. Similar to Eq. (1), the polarization of hyperon in $\vec{p}_\perp p \rightarrow \vec{H}_\perp X$ is given by

$$P_T^H = \frac{d\sigma^{(p_\uparrow p \rightarrow H_\uparrow X)} - d\sigma^{(p_\uparrow p \rightarrow H_\downarrow X)}}{d\sigma^{(p_\uparrow p \rightarrow H_\uparrow X)} + d\sigma^{(p_\uparrow p \rightarrow H_\downarrow X)}} \equiv \frac{d\Delta_T \sigma^{(\vec{p}_\perp p \rightarrow \vec{H}_\perp X)}/d\eta}{d\sigma^{(pp \rightarrow HX)}/d\eta}.$$

(19)

Here, as well as in the following, the subscripts "↑" and "↓" denote the transverse polarization of the particle. The polarized cross section has a similar expression as Eq. (2),

$$\frac{d\Delta_T\sigma^{(\vec{p}_\perp p\to\vec{H}_\perp X)}}{d\eta}=\int_{p_T^{min}}dp_T\sum_{abcd}\int dx_a dx_b dz\,\delta f_a(x_a,\mu^2)$$

$$\times f_b(x_b,\mu^2)\Delta_T D_c^H(z,\mu^2)\frac{d\Delta_T\hat{\sigma}^{(\vec{a}_\perp b\to\vec{c}_\perp d)}}{d\eta},$$

(20)

where $\Delta_T D_c^H(z,\mu^2)\equiv D_{c(\uparrow)}^{H(\uparrow)}(z,\mu^2)-D_{c(\uparrow)}^{H(\downarrow)}(z,\mu^2)$ is the transversely polarized fragmentation function; $d\Delta_T\hat{\sigma}$ is defined for the hard subprocess in case of transverse polarization, which can also be calculated by PQCD. Now, similarly to Eq. (14), the integrand of Eq. (20) can be rewritten as

$$\delta f_a(x_a,\mu^2)f_b(x_b,\mu^2)\Delta_T D_c^H(z,\mu^2)d\Delta_T\hat{\sigma}$$

$$=\sum_\alpha P_T^{q_c}T_{H,c}^{F(\alpha)}f_a(x_a,\mu^2)f_b(x_b,\mu^2)D_c^{H(\alpha)}(z,\mu^2)d\hat{\sigma}.$$

(21)

The only differences are that $P_T^{q_c}$ and $T_{H,c}^{F(\alpha)}=\Delta_T D_c^{H(\alpha)}(z,\mu^2)/D_c^{H(\alpha)}(z,\mu^2)$ are the transverse polarization of the scattering quark c and the transverse spin transfer factor, which take the place of $P_L^{q_c}$ and $t_{H,c}^{F(\alpha)}$ in the longitudinally polarized case. $P_T^{q_c}$ is related to the transversity distribution by

$$P_T^{q_c}=D_T\frac{\delta f_a(x_a,\mu^2)}{f_a(x_a,\mu^2)},$$

(22)

where $D_T\equiv d\Delta_T\hat{\sigma}/d\hat{\sigma}$ is the transverse spin transfer in the hard subprocess $\vec{a}_\perp b\to\vec{c}_\perp d$ from the incoming parton a to the outgoing parton c. To the leading order, it is only a function of y defined in last section, and the results for different hard subprocesses can be found in different publications [16,17].

The product $f_a(x_a,\mu^2)f_b(x_b,\mu^2)D_c^{H(\alpha)}(z,\mu^2)d\hat{\sigma}$ in the RHS of Eq. (21), is again proportional to the average numbers of hyperons from different groups (A), (B), (C) and (D) in $pp\to HX$, which can be calculated by using, e.g., PYTHIA [14]. The unknown factors left to obtain P_T^H are the spin transfer factor $T_{H,c}^{F(\alpha)}$ and $\delta f_a(x_a,\mu^2)$. Hence, if we know one of them, one can study the other by measuring P_T^H.

B. Numerical estimation of transverse polarization of hyperons with high p_T

As we emphasized in Sec. II, the relative weights of the different contributions to hyperons in the final states are determined by the hadronization mechanism and are independent of the polarization properties. This means that, if we consider $\vec{p}_\perp p\to\vec{\Sigma}_\perp^+ X$, we have the same conclusion that Σ^+'s of origin (A), i.e., those are directly produced and contain a scattered u quark, play the dominant role. In this case, for the Σ^+ polarization, the fragmentation effects come in mainly through the transverse spin transfer factor $T_{H,c}^{F(\alpha)}$, which is a constant. Similarly, we denote $T_{H_i,f}^{F(\alpha)}$ as the trans-
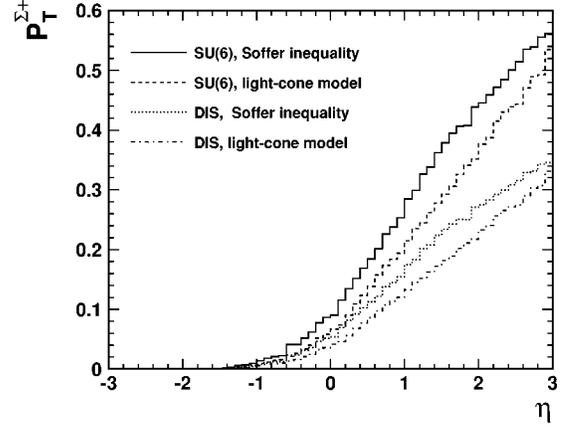
FIG. 6. Σ^+ polarization with $p_T^{min}=13$ GeV in $\vec{p}_\perp p\to\vec{\Sigma}_\perp^+ X$ as a function of η at $\sqrt{s}=500$ GeV.

verse spin transfer factor for the quark q_f to hyperon H_i. So, in experiments, if we can determine $T_{H_i,f}^{F(\alpha)}$ using a few data points, we can extract information on $\delta q(x)$ by measuring $P_T^{\Sigma^+}$ in pp collisions.

To get a feeling of how large $P_T^{\Sigma^+}$ can be and how strongly it depends on the different choices of $\delta q(x,Q^2)$, we make some numerical estimations by using different inputs for $T_{H_i,f}^{F(\alpha)}$ and $\delta q(x,Q^2)$ in the following. In general, the transverse spin transfer factor $T_{H_i,f}^{F(\alpha)}$ can be different from $t_{H_i,f}^{F(\alpha)}$, the spin transfer factor in the longitudinally polarized case. This is similar to the difference between the helicity and the transversity distributions of the quarks in nucleon [3]. Because of the relativistic effects, the magnitudes and/or shapes of them are in general different from each other. On the other hand, it seems that the qualitative features, in particular the signs of them, are the same, especially in the large momentum fraction region [3]. Similarly, we may expect that $T_{H_i,f}^{F(\alpha)}$ has the same qualitative behavior as $t_{H_i,f}^{F(\alpha)}$. Hence, we use the same results as those for $t_{H_i,f}^{F(\alpha)}$ obtained in the SU(6) and DIS pictures in the following estimations. For the transversity distribution $\delta q(x,Q^2)$, we use the simple form obtained in the light-cone SU(6) quark-spectator model [20,21], and for comparison, the upper limit of $\delta q(x,Q^2)$ in Soffer's inequality [22],

$$|\delta q(x,Q^2)|\leq\frac{1}{2}[\Delta q(x,Q^2)+q(x,Q^2)].$$

(23)

We first calculate the transverse polarization of Σ^+ as a function of η at $\sqrt{s}=500$ GeV and $p_T^{min}=13$ GeV in pp collisions with one beam transversely polarized. Here, the p_T^{min} we have chosen not only guarantees the applicability of PQCD [18], but also ensures that the quark involved partonic subprocesses dominate others [6,19] in $pp\to\Sigma^+ X$. The results are shown in Fig. 6. We see that, the magnitude of $P_T^{\Sigma^+}$ can be quite large. It increases to about 0.5 with increasing η. Though not very large, the difference between the results of Soffer inequality and the light-cone model is obvious. In
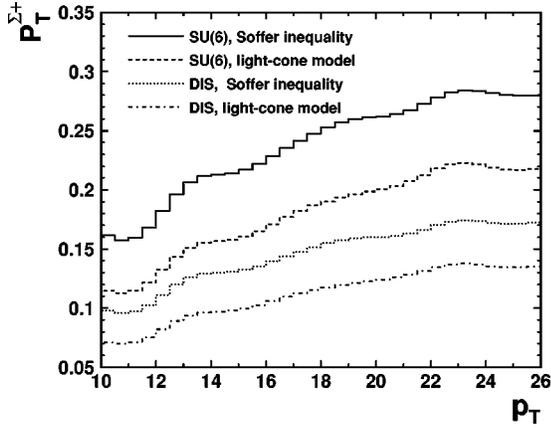
FIG. 7. Σ^+ polarization at $0 < \eta < 1.5$ as a function of p_T in $\vec{p}_\perp p \to \vec{\Sigma}_\perp^+ X$ at $\sqrt{s} = 500$ GeV.

FIG. 8. Polarizations for Λ, Σ^-, Ξ^0 and Ξ^- with $p_T^{min} = 13$ GeV in $\vec{p}_\perp p \to \vec{H}_\perp X$ as a function of η at $\sqrt{s} = 500$ GeV.

addition, for the convenience of comparison with future experimental data, we also calculate $P_T^{\Sigma^+}$ for the region of $0 < \eta < 1.5$ as a function of p_T at $\sqrt{s} = 500$ GeV. The results are shown in Fig. 7. We see that, $P_T^{\Sigma^+}$'s in different cases increase with p_T and the differences among different models are clear. $P_T^{\Sigma^+}$ could be measured at RHIC or other future experiments and can be used as a complementary method to obtain $\delta u(x, Q^2)$.

We should note that, the transverse polarization of the produced hyperons discussed above is defined with respect to the direction of motion of the quark before fragmentation, or the jet axis in the final state. It refers to the transverse polarization direction of the quark after the hard subprocess. This polarization direction is determined by the polarization direction of the incoming quark and the scattering process which is calculable by using PQCD. The PQCD calculations show that the direction of transverse polarization of the incoming and that of the outgoing quark are related to each other by a rotation around the normal of the scattering plane, which changes the moving direction of the quark from the incoming to the outgoing direction (cf. Fig. 2 of Ref. [16]). In practise, since the hyperons that we consider are mainly the leading particles in the jet, the jet axis can approximately be replaced by the direction of motion of the hyperons.

As a comparison, we also calculate the transverse polarizations for Λ, Σ^-, Ξ^0 and Ξ^- with $p_T > 13$ GeV in polarized $pp \to HX$ as a function of η at $\sqrt{s} = 500$ GeV. The results are shown in Fig. 8. We can see that, they are also transversely polarized with different signs. P_T^Λ is very small either for light-cone model or Soffer inequality. This is because, the spin transfer from the u and d quark to the produced Λ is zero in SU(6) picture and very small in DIS
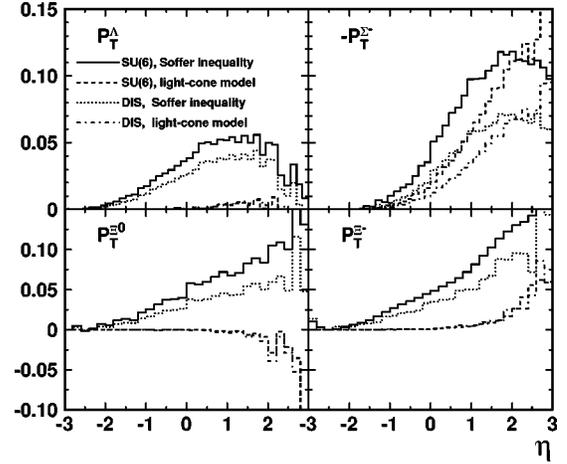
picture and it is right u and d quark fragmentation that dominate the high p_T hyperon production in pp collisions.

V. SUMMARY

In this paper, we study the polarization of hyperon in different processes in polarized pp collisions and its relation to the polarized parton distributions, in particular the gluon helicity distribution and the quark transversity distribution. We show that by measuring the Σ^+ polarization in high p_T direct photon production in singly polarized pp collisions at high energy, one can extract the gluon helicity distribution; and present the numerical results for Σ^+ polarization in $\vec{p}p \to \gamma\vec{\Sigma}^+ X$ at RHIC energy obtained using different inputs of $\Delta g(x, Q^2)$. The results show that Σ^+ polarization is in general large and sensitive to $\Delta g(x, Q^2)$ in the region of $-2 < \eta < 0$. We also find out that the measurement of transverse polarization of Σ^+ with high p_T in $\vec{p}_\perp p \to \vec{\Sigma}_\perp^+ X$ can provide useful information for the transversity distribution of nucleon, and present the numerical results obtained using different inputs for the transversity distributions and the spin transfer factor in fragmentation of transversely polarized quark. Such measurements can be carried out, e.g., at RHIC and should provide useful information on the gluon helicity distribution and quark transversity distribution in nucleon.

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