From QFT to a disoriented chiral condensate

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A quantum field theoretical model for the dynamics of the disoriented chiral condensate is presented. A unified approach to relate the quantum field theory directly to the formation, decay, and signals of the DCC and its evolution is taken. We use a background field analysis of the $O(4)$ sigma model, keeping one-loop quantum corrections (quadratic order in the fluctuations). An evolution of the quantum fluctuations in an external, expanding metric which simulates the expansion of the plasma is carried out. We examine, in detail, the amplification of the low-momentum pion modes with two competing effects: the expansion rate of the plasma and the transition rate of the vacuum configuration from a metastable state into a stable state. We show the effect of DCC formation on the multiplicity distributions and the Bose-Einstein correlations.

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I. INTRODUCTION

The mechanism of spontaneous symmetry breaking [1] occupies an important role in particle physics, cosmology [2], and condensed matter physics [3]. The understanding of this phenomenon has lead to cross fertilization between these fields and many new ideas have emerged [4]. When a symmetry is spontaneously broken by a vacuum state, it is well known that thermal effects at equilibrium can restore the broken symmetry [5]. Typically, there exists a critical temperature T_c at which the effective potential as a function of some order parameter ϕ develops an absolute minimum at $\phi=0$ and the system undergoes a phase transition and relaxes to this minimum at higher temperature. A manifestation of this, in a second-order phase transition is that at $T=T_c$ the order parameter fluctuates at all scales and long-wavelength oscillations occur [6,7]. For nonequilibrium situations, the dynamics of the transition of a system from a symmetric to a broken symmetry state leads to the formation of new structures and the generation of entropy in the form of particle production [3,8,9]. In high-energy physics, there are two well-known examples of symmetry breaking: electroweak symmetry breaking and chiral symmetry breaking in strong interactions. One of the main features of QCD, the underlying theory of strong interactions, is the spontaneous breaking of its approximate $SU(2)_L \times SU(2)_R$ chiral symmetry. Spontaneous breaking of this approximate symmetry explains the very small pion masses. If the symmetry breaking were exact, pions would be massless Nambu-Goldstone bosons. Another consequence of this symmetry breaking is the presence of a nonvanishing quark condensate in the vacuum. It is believed that at very high temperature a quark-gluon plasma is formed in which chiral symmetry is restored, and much effort is being made to explore such a phase transition by means of high-energy hadron or heavy-ion collisions [10]. The question is, how do we test the spontaneous symmetry breaking mechanism in a nonequilibrium process directly? Can we create a suitable condition so that the vacuum state is disturbed for a small region of space-time and observe different excitations and domain structures in the vacuum? Recently it has been conjectured that this may be possible for the chiral symmetry breaking in strong interactions given the present high-energy physics facilities [11–17,52–54,57].

To describe aspects of QCD related to this symmetry, it is convenient to introduce an effective theory, to use a low-energy effective sigma model as a model for the QCD phase transition because it respects the $SU(2)\times SU(2)$ chiral symmetry of QCD with two light flavors of quarks and it contains a scalar field (Σ) that has the same chiral properties as the quark condensate. The Σ field can thus be used to represent the order parameter of the chiral phase transition. The chiral four vector of fields is $(\Sigma, \boldsymbol{\pi})$, where Σ represents the quark condensate and $\boldsymbol{\pi}$ the three vector of the pion field. In the physical vacuum, $(\Sigma, \boldsymbol{\pi})$ points in the Σ direction. If the chiral symmetry were exact, then there would be a "chiral circle" of states degenerate with this vacuum state. In practice, the symmetry is explicitly broken by the current quark masses and so there is a unique vacuum.

Because of this circle of nearly degenerate field configurations, as the chirally restored plasma cools and returns to the normal phase, the system could form regions in which the chiral fields are misaligned—that is, chirally rotated from their usual orientation along the Σ direction. There has been much recent interest in this phenomenon, which is known as a disoriented chiral condensate (DCC). If such a state were formed, it would lead to anomalously large event-by-event fluctuations in the ratio of charged to neutral pions.

A region of DCC can be thought of as *a cluster of pions of near-identical momentum around zero (coherently produced) with an anomalously large amount of fluctuation of the neutral fraction*. In order to produce such a state in a quark gluon plasma, the hot plasma must evolve far from equilibrium and in particular it must reach an unstable configuration such that the long-wavelength pion modes are amplified ex-

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ponentially when the system relaxes to the stable vacuum state. Thus questions of whether a DCC forms and how it evolves cannot be addressed in the framework of equilibrium thermodynamics. Techniques for applying QCD directly to such situations do not exist at present. To explain these nonequilibrium phenomena, we need to restructure the theory of phase transitions to incorporate the microstructures (or states) instead of macrostructures. For this we need to set up a theory at the quantum level.

In his talk at Trento [18], Bjorken pointed out some of the unresolved issues concerning the DCC called the *DCC trouble list*. They were the following:

(a) Are coherent states the right quantum DCC description, or should one go beyond to squeezed states, etc.

(b) How does one link DCC thinking to quantum effects in the data, especially the Bose-Einstein correlations. Is DCC just another way of talking about the same thing?

(c) How does one enforce quantum number conservation, especially charge [19]?

(d) DCC production may imply anomalous bremsstrahlung, due to the large quantum fluctuations in charge. Can this be calculated from first principles?

(e) Can one really set up the problem at the quantum field theory level?

Some attempts have been made to answer these questions, especially question (e) [17,20–23], but to our knowledge these are incomplete as they do not incorporate the disorientation of the condensate and the evolution of the plasma in one unified picture. In particular, Ref. [17] has developed a quantum field theoretic treatment of the DCC in nonuniform environments by using a space- and time-dependent effective mass function to illustrate the importance of including quantum effects in the dynamical treatment. Although the new aspect of inhomogeneity of the plasma is dealt with (which we will not consider in the present paper, but will deal with in a later communication), an arbitrary disorientation of the condensate in isospin space and treatment of charged and neutral pion distributions and correlations are not included in this quantum treatment.

In this paper we give a unified quantum field theoretic picture of the DCC with and without orientation and give our answers to the questions posed by Bjorken. We also provide additional support to the claim made in Ref. [17], that incorporation of quantum effects has very strong quantitative and qualitative effects on the signals of the DCC.

II. MODEL

In most treatments of DCC formation, the classic Gell-Mann–Levy Lagrangian [24] is used. Our starting point is the $O(4)$ linear sigma model with symmetry breaking, which has become the standard model used for the study of pionic physics, especially in the context of DCC formation [17,25–27]. We study it in the expanding Friedmann-Robertson-Walker metric

$$ds^2=dt^2-a(t)^2d\vec{x}^2, \tag{1}$$

where $a(t)$ is the expansion parameter. The $O(4)$-symmetric sigma model with Lagrangian density

$$L=\left(\frac{1}{2}\dot{\Phi}_i^2-\frac{1}{2a^2}(\nabla\Phi_i)^2-\frac{1}{2}m^2\Phi_i^2-\frac{\lambda}{4}(\Phi_i^2)^2\right), \tag{2}$$

$$\Phi_i=\begin{pmatrix}\Phi_1\\ \Phi_2\\ \Phi_3\\ \Phi_4\end{pmatrix}. \tag{3}$$

Depending upon the sign of m^2 the Lagrangian admits a restored symmetry state $\langle\Phi_i\rangle=0$ and broken symmetries $\langle\Phi_i\rangle=\delta V/\delta\Phi=\pm v$ are two degenerate vacuua.

We note here that the potential we use is not the one traditionally used by the early practitioners of DCC analyses. Thus, for later comparison with the dynamical equations for the pions used as starting points for analyzing the DCC, we will later relate the potential that we have chosen to the traditional potential in the Gell-Mann–Levy model [28]. The Lagrangian density in the Gell-Mann–Levy model is given by

$$L=\frac{1}{2}(\partial_\mu\vec{\pi}\partial^\mu\vec{\pi}+\partial_\mu\Sigma\partial^\mu\Sigma)-\frac{\lambda}{4}(\vec{\pi}^2+\Sigma^2-v^2)^2+H\Sigma. \tag{4}$$

The action is

$$S=\int d^4x\left[\frac{1}{2}\left(\partial_\mu\phi^a\partial_\mu\phi_a-\frac{\lambda}{4}(\phi^a\phi_a-v^2)^2+H\Sigma\right]. \tag{5}$$

The basic object of study is then the chiral field

$$\vec{\phi}(r,t)=\Sigma(r,t)+i\boldsymbol{\tau}\cdot\vec{\pi}(r,t), \tag{6}$$

where $\Sigma(r,t)$ and $\vec{\pi}(r,t)$ are the scalar and vector fields, respectively, of the $O(4)$ vector $\vec{\phi}=(\Sigma,\vec{\pi})$ and τ are the Pauli matrices.

The physical constants λ, v, and H are related to the physical quantities f_π (pion form factor) and m_π by the relations

$$m_\pi^2=\frac{H}{f_\pi}=\lambda(f_\pi^2-v^2), \tag{7}$$

$$m_\Sigma^2=3\lambda f_\pi^2-\lambda v^2\approx 2\lambda f_\pi^2=600\ \text{MeV}/c^2,$$

$$\lambda=\frac{m_\Sigma^2c^2-m_\pi^2c^2}{2f_\pi^2}=20.14,$$

$$\langle v\rangle=\left[\frac{m_\Sigma^2-3m_\pi^2}{m_\Sigma^2-m_\pi^2}f_\pi^2\right]^{1/2}=86.71\ \text{MeV},$$

$$H=m_\pi^2c^2f_\pi=(120.55\ \text{MeV})^3. \tag{8}$$

Note that the fermionic part is neglected here, since the focus is on the condensate formed in the symmetry-broken phase where quark degrees of freedom are already confined. With the transformation $\Sigma\rightarrow\Sigma-v$, the minimum of potential is at $\langle \phi^2 \rangle = \langle \Sigma^2 + \pi^2 \rangle = \langle v \rangle^2 = f_\pi^2$ and the usual equilibrium vacuum state is an ordered state $\langle \Sigma \rangle = \langle v \rangle = f_\pi$ and $\langle \pi \rangle = 0$.

Now consider the dynamic evolution of the system in a hot quark gluon plasma. When the temperature $T \geqslant T_c$ the system reaches a state of restored symmetry $\langle \phi^2 \rangle = 0$. If the subsequent expansion of the plasma is adiabatic, the $\langle \phi \rangle$ field gradually relaxes to the equilibrium state as the system cools to below T_c. This is called the "annealing" or adiabatic scenario. It was first pointed out in Ref. [13] that if the cooling process is very rapid and the system is out of equilibrium—i.e., in the event of a sudden quench from a state of restored symmetry to a state of broken symmetry such as that occurring in a rapidly cooling expanding plasma—the configuration of the ϕ field will lag behind the expansion of the plasma and there is a mismatch of the configurations and their evolution. After a quench, the high-temperature configuration does not have time to react to the sudden change of the environment; thus, the vacuum expectation value $\langle \phi \rangle$ would stay what it was at high temperature for a while and then relax to its equilibrium value. This results in the formation of DCC domains and is known as the *baked Alaska* [29] scenario.

Another nonequilibrium situation that can arise is one where the system can go through a metastable-disordered vacuum $\langle \Sigma \rangle = f_\pi \cos(\theta)\, \langle \pi \rangle = f_\pi \overline{n} \sin(\theta)$ and then relax by quantum fluctuations to an equilibrium configuration. Here θ measures the degree of disorientation of the condensate in isospin space.

In both these nonequilibrium situations, the canonical approach to the full quantum evolution of the fields is extremely difficult to carry out explicitly. However, we may study the quantum evolution of the mean field and include fluctuations. In calculating the effective Hamiltonian we use the $O(4)$-symmetric linear sigma model, whose Lagrangian density is given by Eq. (2). It is easy to see that with the identification $\Phi^2 = (\Sigma^2 + \pi^2 - f_\pi^2)$ and $\Sigma \rightarrow \Sigma - v$, the effective potential in the $O(4)$ linear sigma model is the same as the Gell-Mann–Levy model. We will use this identification when considering the evolution of the pion field.

The choice of metric is dictated by the simple spherical geometry of the problem. Assuming a homogeneous expansion of the plasma, a Robertson-Walker type of metric is chosen with an expansion rate $a(t)$. This will allow us to examine the competing rates leading to "rolling down rate" of the vacuum and the expansion rate $a(t)$. The line element describing the expansion of the plasma bubble is chosen to be

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2, \tag{9}$$

where $a(t)$ is the expansion parameter.

The action of the $O(4)$ sigma model in this metric becomes

$$S = \int d^3x dt a(t)^3 \left(\frac{1}{2}\dot{\Phi}_i^2 - \frac{1}{2a^2}(\nabla \Phi_i)^2 - \frac{1}{2}m^2\Phi_i^2 - \frac{\lambda}{4}(\Phi_i^2)^2 \right), \tag{10}$$

with

$$\Phi_i = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \end{pmatrix}, \tag{11}$$

where Φ_i, $i = 1, \ldots, 4$ are real scalar fields.

We now use a background field analysis to study the quantum effects. Assume Φ_i has a background classical component ϕ_i which satisfies the classical equations of motion

$$\left.\frac{\delta S}{\delta \Phi_i}\right|_{\Phi_i = \phi_i} = 0. \tag{12}$$

Treat the quantum field $\widehat{\phi_i}$ as a fluctuation around a classical solution,

$$\Phi_i \rightarrow \phi_i + \widehat{\phi_i}, \tag{13}$$

since ϕ_i satisfies the classical equations of motion:

$$S = S[\phi_i] + \frac{1}{2}\widehat{\phi_i} \left.\frac{\delta^2 S}{\delta \Phi_i \delta \Phi_j}\right|_{\Phi = \phi} \widehat{\phi_j} + \cdots. \tag{14}$$

We shall restrict our analysis to quadratic fluctuations only. In addition, we shall also drop the term $S[\phi_i]$ as it is just a constant additive term to the quadratic action. Therefore we shall deal with a quadratic fluctuation action given simply by

$$S_2 = \frac{1}{2}\widehat{\phi_i} \left.\frac{\delta^2 S}{\delta \Phi_i \delta \Phi_j}\right|_{\Phi = \phi} \widehat{\phi_j}. \tag{15}$$

For the particular scalar field action given above, assuming all fields vanish at infinity,

$$\frac{\delta S}{\delta \Phi_j} = -\partial_\mu (a^3 g^{\mu\nu} \partial_\nu \Phi_j) - a^3 m^2 \Phi_j - a^3 \frac{\delta V}{\delta \Phi_j}. \tag{16}$$

Imposing the classical equations of motion, we find that

$$\partial_\mu (a^3 g^{\mu\nu} \partial_\nu \phi_i) + a^3 m^2 \phi_i + a^3 \frac{\partial V}{\partial \phi_i} = 0, \tag{17}$$

where $\partial V / \partial \phi_i \equiv \partial V / \partial \Phi_i |_{\phi_i}$.

The equations of motion in this metric are

$$3\frac{\dot{a}}{a}\dot{\phi}_i + \dot{\phi}_i{}^2 - \frac{1}{a^2}\nabla^2 \phi_i + m^2 \phi_i + \frac{\partial V}{\partial \phi_i} = 0. \tag{18}$$

Since we are interested in the dynamics of the fluctuation field, we shall treat the fluctuation field in S_2 as a classical field and S_2 itself as the classical action for its dynamics. The quadratic part in the fluctuations is reduced to

$$S_2=\int d^3\underline{x}dt\frac{a^3}{2}\left(\dot{\hat{\phi}}_i^2-\frac{1}{a^2}(\nabla\underline{\hat{\phi}}_i)^2-m^2\hat{\phi}_i^2-\hat{\phi}_i\frac{\delta^2 V}{\delta\hat{\Phi}_i\delta\hat{\Phi}_j}\bigg|_\phi\hat{\phi}_j\right). \tag{19}$$

We can define a Lagrangian density for studying the dynamics of the fluctuations, $\mathcal{L}$, as follows:

$$\mathcal{L}=\frac{a^3}{2}\left(\dot{\hat{\phi}}_i^2-\frac{1}{a^2}(\nabla\underline{\hat{\phi}}_i)^2-m^2\hat{\phi}_i^2-\hat{\phi}_i\frac{\partial^2 V}{\partial\Phi_i\partial\Phi_j}\bigg|_\phi\hat{\phi}_j\right). \tag{20}$$

Carrying out a Legendre transformation, it is easy to write down the Hamiltonian density

$$\mathcal{H}=\frac{1}{2a^3}\hat{p}_i^2+\frac{a}{2}(\nabla\underline{\hat{\phi}}_i)^2+\frac{a^3m^2}{2}\hat{\phi}_i^2+\frac{a^3}{2}\left(\hat{\phi}_i\frac{\partial^2 V}{\partial\Phi_i\partial\Phi_j}\bigg|_\phi\hat{\phi}_j\right), \tag{21}$$

where

$$\hat{p}_i=\frac{\delta L}{\delta\dot{\hat{\phi}}_i}=a^3\dot{\hat{\phi}}_i. \tag{22}$$

We also have

$$\frac{\partial^2 V}{\partial\Phi_i\partial\Phi_j}\bigg|_\phi=2\lambda\phi_i\phi_j+\lambda\phi_k^2\delta_{ij}. \tag{23}$$

Assume that the fluctuation field Φ_i decomposes into its constituents as

$$\hat{\phi}=\langle\Phi\rangle-\phi=\begin{pmatrix}\pi_1\\ \pi_2\\ \pi_0\\ \Sigma\end{pmatrix}. \tag{24}$$

We have started with an $O(4)$ quartet of scalar fields in order that we can construct the dynamics of quenched pions in the formation of a disoriented chiral condensate. The physical fields are defined so that

$$\pi_+=\frac{1}{\sqrt{2}}(\pi_1+i\pi_2),\quad \pi_-=\frac{1}{\sqrt{2}}(\pi_1-i\pi_2). \tag{25}$$

Analogously, we define the classical background fields as

$$v_+=\frac{1}{\sqrt{2}}(v_1+iv_2),\quad v_-=\frac{1}{\sqrt{2}}(v_1-iv_2), \tag{26}$$

following the identification

$$\phi=\begin{pmatrix}v_1\\ v_2\\ v_3=v\\ \sigma\end{pmatrix}\equiv\langle\Phi\rangle. \tag{27}$$

It is easy to see that the action takes the form

$$S=\int d^3\vec{x}dta^3\Big\{\dot{\pi}_+\dot{\pi}_--\frac{1}{a^2}(\nabla\pi_+)(\nabla\pi_-)-[m^2+(4\lambda)v_+v_-+\lambda v_3^2+\lambda\sigma^2]\pi_+\pi_-+\frac{1}{2}\dot{\pi}_0^2-\frac{1}{2a^2}(\nabla\pi_0)^2$$
$$-\frac{1}{2}[m^2+(2\lambda)v_+v_-+3\lambda v_3^2+\lambda\sigma^2]\pi_0^2+\frac{1}{2}\dot{\Sigma}^2-\frac{1}{2a^2}(\nabla\Sigma)^2-\frac{1}{2}[m^2+(2\lambda)v_+v_-+3\lambda\sigma^2+\lambda v_3^2]\Sigma^2$$
$$-\lambda(v_-^2\pi_+^2+v_+^2\pi_-^2+2\sigma v_3\pi_0\Sigma+2v_-v_3\pi_0\pi_++2v_+v_3\pi_0\pi_-+2v_-\sigma\pi_+\Sigma+2v_+\sigma\pi_-\Sigma)\Big\}. \tag{28}$$

The Hamiltonian for this action can be written as

$$H=H_{neutral}+H_{charged}+H_{mixed}, \tag{29}$$

where

$$H_{neutral}=\int d^3xdta^3\Big\{\frac{(p_{0\pi_0}^2)}{2a^6}+\frac{1}{2a^2}(\nabla\pi_0^2)+\frac{1}{2}(m_\pi^2)(\pi_0)^2+\frac{(\Omega_\pi^2-\omega_\pi^2)}{2a^6}\pi_0^2+\frac{(p_{0\Sigma}^2)}{2a^6}+\frac{1}{2a^2}(\nabla\Sigma^2)$$
$$+\frac{1}{2}(m_\Sigma^2)(\Sigma)^2+\frac{1}{2a^6}(\Omega_\Sigma^2-\omega_\Sigma^2)(\Sigma)^2\Big\}, \tag{30}$$

$$H_{charged}=\int d^3x dt a^3\left\{\frac{p_{0+}p_{0-}}{a^6}+\frac{1}{a^2}(\nabla\pi_-)(\nabla\pi_+)+(m_\pi^2)(\pi_+\pi_-)+\frac{1}{a^6}(\Omega_{\pi_\pm}^2-\omega_{\pi_\pm}^2)(\pi_+\pi_-)\right\}, \tag{31}$$

$$H_{mixed}=\int d^3x a^3 dt\{\lambda(v_+^2\pi_-^2+v_-^2\pi_+^2)+(2\lambda)(v_+v_3\pi_-\pi_0+v_-v_3\pi_+\pi_0+\sigma v_3\Sigma\pi_0+v_+\sigma\pi_-\Sigma+\sigma v_-\Sigma\pi_+)\}, \tag{32}$$

where we have also put

$$\frac{\Omega_{\pi_0}^2-\omega_{\pi_0}^2}{a^6}=\lambda[v^2+2v_3^2],\quad \frac{\Omega_{\pi\pm}^2-\omega_{\pi_\pm}^2}{a^6}=\lambda[v^2+2v_+v_-],\quad \frac{\Omega_\Sigma^2-\omega_\Sigma^2}{a^6}=\lambda[v^2+2\sigma^2], \tag{33}$$

and

$$2v_+v_-+v_3^2+\sigma^2=v^2. \tag{34}$$

This is the most general Hamiltonian for the pion sigma system in the background field formalism. The background field can now be parametrized through three angles:

$$\phi_i=\begin{pmatrix} v\cos(\rho)\sin(\theta)\sin(\alpha)\\ v\cos(\rho)\sin(\theta)\cos(\alpha)\\ v\sin(\rho)\sin(\theta)\\ v\cos(\theta)\end{pmatrix}. \tag{35}$$

In order to consider all the special cases that are possible in a transparent way, we shall simplify the parametrization of the possible form for the background field to two angles θ and ρ by letting $\alpha=\pi/4$: then, $v\pm=(v/\sqrt{2})\cos(\rho)\sin(\theta)$, $v_3=v\sin(\rho)\sin(\theta)$, and $\sigma=v\cos(\theta)$.

III. QUANTIZATION

Using standard canonical quantization techniques the mode decomposition of the Hamiltonian is

$$H_{neutral}=\int\frac{d^3k}{(2\pi)^3}\frac{1}{2}\left\{\frac{\omega_\pi}{a^3}(a_k^\dagger a_k+a_k a_k^\dagger)+\frac{\omega_\pi}{2a^3}\left(\frac{\Omega_\pi^2}{\omega_\pi^2}-1\right)(a_k^\dagger a_k+a_k a_k^\dagger+a_{-k}a_k+a_{-k}^\dagger a_k^\dagger)+\frac{\omega_\Sigma}{a^3}(d_k^\dagger d_k+d_k d_k^\dagger)\right.$$
$$\left.+\frac{\omega_\Sigma}{2a^3}\left(\frac{\Omega_\Sigma^2}{\omega_\Sigma^2}-1\right)(d_k^\dagger d_k+d_k d_k^\dagger+d_{-k}d_k+d_{-k}^\dagger d_k^\dagger)\right\}, \tag{36}$$

$$H_{charged}=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{\omega_\pi}{a^3}(b_k^\dagger b_k+c_k c_k^\dagger)+\frac{\omega_\pi}{2a^3}\left(\frac{\Omega_{\pi_\pm}^2}{\omega_\pi^2}-1\right)(b_k^\dagger b_k+c_k c_k^\dagger+b_{-k}c_k+c_{-k}^\dagger b_k^\dagger)\right\}, \tag{37}$$

$$H_{mixed}=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{\lambda a^3 v^2\cos^2(\rho)\sin^2(\theta)}{4\omega_\pi}(b_k b_{-k}+b_k c_k^\dagger+c_k^\dagger b_k+c_k c_{-k}+c_k^\dagger c_{-k}^\dagger+c_k b_k^\dagger+b_k^\dagger c_k+b_k^\dagger b_{-k}^\dagger)\right.$$
$$+\frac{\lambda a^3 v^2\cos(\rho)\sin(\rho)\sin^2(\theta)}{\sqrt{\omega_\Sigma\omega_\pi}}(b_k a_{-k}+b_k a_k^\dagger+c_k^\dagger a_k+c_k a_{-k}+c_k^\dagger a_{-k}^\dagger+c_k a_k^\dagger+b_k^\dagger a_k+b_k^\dagger a_{-k}^\dagger)$$
$$+\frac{\lambda a^3 v^2\sin(\rho)\sin(\theta)\cos(\theta)}{\sqrt{\omega_\pi\omega_\Sigma}}(d_k a_{-k}+d_k a_k^\dagger+d_k^\dagger a_k+d_k^\dagger a_{-k}^\dagger)+\frac{\lambda a^3 v^2\cos(\rho)\sin(\theta)\cos(\theta)}{\sqrt{\omega_\pi\omega_\Sigma}}(b_k d_{-k}+b_k d_k^\dagger$$
$$\left.+c_k^\dagger d_k+c_k d_{-k}+c_k^\dagger d_{-k}^\dagger+c_k d_k^\dagger+b_k^\dagger d_k+b_k^\dagger d_{-k}^\dagger)\right\}, \tag{38}$$

where

$$\frac{\omega_\pi^2(k)}{a^6}\equiv\frac{\omega_{\pi_0}^2(k)}{a^6}=\frac{\omega_{\pi_\pm}^2(k)}{a^6}=\left(m_\pi^2+\frac{k^2}{a^2}\right),$$

$$\frac{\omega_\Sigma^2(k)}{a^6}=\left(m_\Sigma^2+\frac{k^2}{a^2}\right) \tag{39}$$

and

$$\frac{\Omega_\pi^2(k)}{a^6}=\frac{k^2}{a^2}+m_\pi^2+\lambda(v^2+2v_3^2), \tag{40}$$

$$\frac{\Omega_{\pi_\pm}^2(k)}{a^6}=\frac{k^2}{a^2}+m_{\pi_\pm}^2+\lambda(v^2+2v_+v_-). \tag{41}$$

An important point to note here is that although $\Omega_\pi(k)$ and $\omega_\pi(k)$ are momentum-dependent quantities, for ease of notation we will drop the k dependence for further calculations in this section and revive it when necessary for the description of the physical processes.

It is very interesting to note that if either of $v_\pm$ or v_3 is zero, then we obtain the usual expected dynamics for the pions with back-to-back correlations. But if we allow for either $v_\pm$ or v_3 to be nonzero as may be envisaged in the highly nonequilibrium dynamics involving the formation of a metastable DCC, then we have terms which involve mixed interactions of the pions and sigma.

Clearly, there are two interesting cases to be considered here: $v_\pm=0$ and $v_3=0$ (equivalently, $\theta=0$) and the second case occurs with the formation of a metastable, misaligned vacuum when $\rho=\pi/2$.

A. Case 1

For this case we use $v_\pm=0$ and $v_3=0$ (or, equivalently, $\theta=0$) showing that H reduces to

$$H=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{\omega_\pi}{2a^3}(a_k^\dagger a_k+a_ka_k^\dagger)+\frac{\omega_\pi}{4a^3}\left(\frac{\Omega_\pi^2}{\omega_\pi^2}-1\right)(a_k^\dagger a_k+a_ka_k^\dagger+a_{-k}a_k+a_{-k}^\dagger a_k^\dagger)+\frac{\omega_\Sigma}{2a^3}(d_k^\dagger d_k+d_kd_k^\dagger)+\frac{\omega_\Sigma}{4a^3}\left(\frac{\Omega_\Sigma^2}{\omega_\Sigma^2}-1\right)\right.$$
$$\left.\times(d_k^\dagger d_k+d_kd_k^\dagger+d_{-k}d_k+d_{-k}^\dagger d_k^\dagger)\right\}+\left\{\frac{\omega_\pi}{a^3}(b_k^\dagger b_k+c_kc_k^\dagger)+\frac{\omega_\pi}{2a^3}\left(\frac{\Omega_\pi^2}{\omega_\pi^2}-1\right)(b_k^\dagger b_k+c_kc_k^\dagger+b_{-k}c_k+c_{-k}^\dagger b_k^\dagger)\right\}. \tag{42}$$

This Hamiltonian has an $su(1,1)$ symmetry and can be diagonalized by a series of Bogolubov (squeezing) transformations simply given as follows: in the neutral sector, writing

$$A_k(t,r)=\mu(r,t)a_k+\nu(r,t)a_{-k}^\dagger=U^{-1}(r,t)a_kU(r,t),$$

$$A_k^\dagger(t,r)=\nu(r,t)a_{-k}+\mu(r,t)a_k^\dagger=U^{-1}(r,t)a_k^\dagger U(r,t). \tag{43}$$

A similar expansion for diagonalization is done for the sigma field, with operators $D_k(t,r')$ and $D_k^\dagger(t,r')$ similar to the definition of $A_k(t,r)$ and $A_k^\dagger(t,r)$ with the d's replacing the a's. For the charged sector,

$$C_k(t,r)=\mu c_k+\nu b_{-k}^\dagger=U^{-1}(r,t)c_kU(r,t),\quad C_k^\dagger(t,r)=\mu c_k^\dagger+\nu b_{-k}=U^{-1}(r,t)c_{-k}^\dagger U(r,t), \tag{44}$$

$$B_k(t,r)=\mu c_{-k}+\nu b_k^\dagger=U^{-1}(r,t)b_kU(r,t),\quad B_k^\dagger(t,r)=\mu c_{-k}^\dagger+\nu b_k=U^{-1}(r,t)b_k^\dagger U(r,t), \tag{45}$$

where

$$\mu=\cosh(r)=\sqrt{\frac{1}{2}\left[\left(\frac{\Omega_\pi}{\omega_\pi}+\frac{\omega_\pi}{\Omega_\pi}\right)+1\right]},\quad \nu=\sinh(r)=\sqrt{\frac{1}{2}\left[\left(\frac{\Omega_\pi}{\omega_\pi}+\frac{\omega_\pi}{\Omega_\pi}\right)-1\right]}, \tag{46}$$

and r is the squeezing parameter, it is easily seen to give the usual result that $\mu^2-\nu^2=1$ for a squeezing transformation. The complete unitary matrix accomplishing the squeezing transformation may be written down as

$$U(r,t)=\exp\left(\int\frac{d^3\underline{k}}{(2\pi)^3}r(k,t)\{(a_k^\dagger a_{-k}^\dagger-a_ka_{-k})+(d_k^\dagger d_{-k}^\dagger-d_kd_{-k})+(c_kb_{-k}+b_kc_{-k})-(c_k^\dagger b_{-k}^\dagger+b_k^\dagger c_{-k}^\dagger)\}\right). \tag{47}$$

It should be noted here that putting together all our results for the neutral and charged sectors, the total diagonalized Hamiltonian is written in terms of various creation and annihilation operators as

$$H=\int\frac{d^3\underline{k}}{(2\pi)^3}\frac{1}{2a^3}\left\{\Omega_\pi\left[\left(A_k^\dagger A_k+\frac{1}{2}\right)+(C_k^\dagger C_k+B_k^\dagger B_k+1)\right]+\Omega_\Sigma\left(D_k^\dagger D_k+\frac{1}{2}\right)\right\}. \tag{48}$$

Since the Σ field decouples, we drop all terms associated with it whenever it is not essential to our arguments, allowing us to write the total dynamical Hamiltonian for the pion fields in terms of the observed pion creation and annihilation operators (a, $a^\dagger$, c, $c^\dagger$, b, and $b^\dagger$):

$$H_0=\int\frac{d^3\underline{k}}{(2\pi)^3}\frac{\Omega_\pi}{a^3}\{2(\mu^2+\nu^2)(c_k^\dagger c_k+b_k^\dagger b_k+1)+\mu\nu[(c_kb_{-k}+b_kc_{-k})+(c_k^\dagger b_{-k}^\dagger+b_k^\dagger c_{-k}^\dagger)]\}+\frac{\Omega_\pi}{a^3}[(\mu^2+\nu^2)(a_k^\dagger a_k+1)$$
$$+\nu\mu(a_{-k}^\dagger a_k^\dagger+a_ka_{-k})]. \tag{49}$$

This completes the analysis for the case when $\theta=0$.

B. Case 2

The second case of interest is when $\rho=\pi/2$. The Hamiltonian for this case reduces to the following:

$$H_{neutral}=\int\frac{d^3\underline{k}}{(2\pi)^3}\frac{1}{2}\left\{\frac{\omega_\pi}{a^3}(a_k^\dagger a_k+a_ka_k^\dagger)+\frac{a^3v^2\lambda}{2\omega_\pi}[1+2\sin^2(\theta)](a_k^\dagger a_k+a_ka_k^\dagger+a_{-k}a_k+a_{-k}^\dagger a_k^\dagger)+\frac{\omega_\Sigma}{a^3}(d_k^\dagger d_k+d_kd_k^\dagger)\right.$$
$$\left.+\frac{a^3v^2\lambda}{2\omega_\Sigma}[1+2\cos^2(\theta)](d_k^\dagger d_k+d_kd_k^\dagger+d_{-k}d_k+d_{-k}^\dagger d_k^\dagger)\right\}, \tag{50}$$

$$H_{charged}=\int\frac{d^3\underline{k}}{(2\pi)^3}\left\{\frac{\omega_\pi}{a^3}(b_k^\dagger b_k+c_kc_k^\dagger)+\frac{\omega_\pi}{2a^3}\left(\frac{\Omega_{\pi_\pm}^2}{\omega_\pi^2}-1\right)(b_k^\dagger b_k+c_kc_k^\dagger+b_{-k}c_k+c_{-k}^\dagger b_k^\dagger)\right\}, \tag{51}$$

$$H_{mixed}=\int\frac{d^3\underline{k}}{(2\pi)^3}\left\{\frac{\lambda a^3v^2}{2}\left(\frac{2\sin(\theta)\cos(\theta)}{\sqrt{\omega_\pi\omega_\Sigma}}\right)(d_ka_{-k}+d_ka_k^\dagger+d_k^\dagger a_k+d_k^\dagger a_{-k}^\dagger)\right\}, \tag{52}$$

where (for $\rho=\pi/2$)

$$\frac{\Omega_\pi^2}{a^6}=\frac{k^2}{a^2}+m_\pi^2+\lambda v^2,$$

$$\frac{\Omega_{\pi_\pm}^2}{a^6}=\frac{k^2}{a^2}+m_{\pi_\pm}^2+\lambda v^2[1+\sin^2(\theta)]. \tag{53}$$

Unlike the case considered above, we now have a nonzero mixing term coming from the π_0-Σ sector. Since both of these are neutral sectors, we can combine the $H_{neutral}$ and H_{mixed} terms to form a single neutral sector Hamiltonian which we will again call $H_{neutral}$ while $H_{charged}$ remains unchanged:

$$H_{neutral}=\int\frac{d^3\underline{k}}{(2\pi)^3}\Bigg\{\begin{pmatrix}\frac{a_k^\dagger}{\sqrt{\omega_\pi}} & \frac{d_k^\dagger}{\sqrt{\omega_\Sigma}}\end{pmatrix}\begin{pmatrix}\frac{a^3}{4}\left(\frac{\Omega_\pi^2+\omega_\pi^2}{a^6}\right) & \frac{\lambda v^2a^3}{4}\sin(2\theta)\\ \frac{\lambda v^2a^3}{4}\sin(2\theta) & \frac{a^3}{4}\left(\frac{\Omega_\Sigma^2+\omega_\Sigma^2}{a^6}\right)\end{pmatrix}\begin{pmatrix}\frac{a_k}{\sqrt{\omega_\pi}}\\ \frac{d_k}{\sqrt{\omega_\Sigma}}\end{pmatrix}$$

$$+\begin{pmatrix}\frac{a_k}{\sqrt{\omega_\pi}} & \frac{d_k}{\sqrt{\omega_\Sigma}}\end{pmatrix}\begin{pmatrix}\frac{a^3}{4}\left(\frac{\Omega_\pi^2+\omega_\pi^2}{a^6}\right) & \frac{\lambda v^2a^3}{4}\sin(2\theta)\\ \frac{\lambda v^2a^3}{4}\sin(2\theta) & \frac{a^3}{4}\left(\frac{\Omega_\Sigma^2+\omega_\Sigma^2}{a^6}\right)\end{pmatrix}\begin{pmatrix}\frac{a_k^\dagger}{\sqrt{\omega_\pi}}\\ \frac{d_k^\dagger}{\sqrt{\omega_\Sigma}}\end{pmatrix}$$

$$+\begin{pmatrix}\frac{a_{-k}}{\sqrt{\omega_\pi}} & \frac{d_{-k}}{\sqrt{\omega_\Sigma}}\end{pmatrix}\begin{pmatrix}\frac{\lambda v^2a^3}{4}[1+2\sin^2(\theta)] & \frac{\lambda v^2a^3}{4}\sin(2\theta)\\ \frac{\lambda v^2a^3}{4}\sin(2\theta) & \frac{\lambda v^2a^3}{4}[1+2\cos^2(\theta)]\end{pmatrix}\begin{pmatrix}\frac{a_k}{\sqrt{\omega_\pi}}\\ \frac{d_k}{\sqrt{\omega_\Sigma}}\end{pmatrix}$$

$$+\begin{pmatrix}\frac{a_{-k}^\dagger}{\sqrt{\omega_\pi}} & \frac{d_{-k}}{\sqrt{\omega_\Sigma}}^\dagger\end{pmatrix}\begin{pmatrix}\frac{\lambda v^2a^3}{4}[1+2\sin^2(\theta)] & \frac{\lambda v^2a^3}{4}\sin(2\theta)\\ \frac{\lambda v^2a^3}{4}\sin(2\theta) & \frac{\lambda v^2a^3}{4}[1+2\cos^2(\theta)]\end{pmatrix}\begin{pmatrix}\frac{a_k^\dagger}{\sqrt{\omega_\pi}}\\ \frac{d_k^\dagger}{\sqrt{\omega_\Sigma}}\end{pmatrix}\Bigg\}. \quad (54)$$

These terms clearly show the mixings between the forward and backward pions as well as the mixing between the π_0 and Σ fields. The misalignment of the vacuum through an angle θ induces a mixing of the two fields. The mixed fields are

$$\begin{pmatrix}A_{\theta(k)}\\ D_{\theta(k)}\end{pmatrix}=\begin{pmatrix}\cos(\theta) & \sin(\theta)\\ -\sin(\theta) & \cos(\theta)\end{pmatrix}\begin{pmatrix}a_k\\ d_k\end{pmatrix}. \quad (55)$$

Then $H_{neutral}$ can be expressed as

$$H_{neutral}=(A_{\theta(k)}^\dagger \quad D_{\theta(k)}^\dagger)\begin{pmatrix}m_1 & 0\\ 0 & m_2\end{pmatrix}\begin{pmatrix}A_{\theta(k)}\\ D_{\theta(k)}\end{pmatrix}+(A_{\theta(k)} \quad D_{\theta(k)})\begin{pmatrix}m_1 & 0\\ 0 & m_2\end{pmatrix}\begin{pmatrix}A_{\theta(k)}^\dagger\\ D_{\theta(k)}^\dagger\end{pmatrix}$$
$$+(A_{\theta(-k)}k \quad D_{\theta(-k)}k)\begin{pmatrix}n_1 & 0\\ 0 & n_2\end{pmatrix}\begin{pmatrix}A_{\theta(k)}\\ D_{\theta(k)}\end{pmatrix}+(A_{\theta(-k)}^\dagger \quad D_{\theta(-k)}^\dagger)\begin{pmatrix}n_1 & 0\\ 0 & n_2\end{pmatrix}\begin{pmatrix}A_{\theta(k)}^\dagger\\ D_{\theta(k)}^\dagger\end{pmatrix}. \quad (56)$$

Here

$$2m_2=\frac{\sqrt{\Omega_\pi\Omega_\Sigma}}{4a^3}\left\{\sqrt{\frac{\Omega_\pi}{\Omega_\Sigma}}\left[\frac{\Omega_\pi}{\omega_\pi}\left(1+\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)+\frac{\omega_\pi}{\Omega_\pi}\left(1-\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)\right]\right\}+\frac{\sqrt{\Omega_\pi\Omega_\Sigma}}{4a^3}$$
$$\times\left\{\sqrt{\frac{\Omega_\Sigma}{\Omega_\pi}}\left[\frac{\Omega_\Sigma}{\omega_\Sigma}\left(1+\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)+\frac{\omega_\Sigma}{\Omega_\Sigma}\left(1-\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)\right]\right\}, \quad (57)$$

$$2m_1=\frac{\sqrt{\Omega_\pi\Omega_\Sigma}}{4a^3}\left\{\sqrt{\frac{\Omega_\pi}{\Omega_\Sigma}}\left[\frac{\Omega_\pi}{\omega_\pi}\left(1-\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)+\frac{\omega_\pi}{\Omega_\pi}\left(1+\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)\right]\right\}+\frac{\sqrt{\Omega_\pi\Omega_\Sigma}}{4a^3}$$
$$\times\left\{\sqrt{\frac{\Omega_\Sigma}{\Omega_\pi}}\left[\frac{\Omega_\Sigma}{\omega_\Sigma}\left(1-\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)+\frac{\omega_\Sigma}{\Omega_\Sigma}\left(1+\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)\right]\right\}, \quad (58)$$

while

$$2n_2=\frac{\sqrt{\Omega_\pi\Omega_\Sigma}}{4a^3}\left[\sqrt{\frac{\Omega_\pi}{\Omega_\Sigma}}\left(\frac{\Omega_\pi}{\omega_\pi}-\frac{\omega_\pi}{\Omega_\pi}\right)\left(1+\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)+\sqrt{\frac{\Omega_\Sigma}{\Omega_\pi}}\left(\frac{\Omega_\Sigma}{\omega_\Sigma}-\frac{\omega_\Sigma}{\Omega_\Sigma}\right)\left(1+\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)\right] \quad (59)$$

and

$$2n_1=\frac{\sqrt{\Omega_\pi\Omega_\Sigma}}{4a^3}\left[\sqrt{\frac{\Omega_\pi}{\Omega_\Sigma}}\left(\frac{\Omega_\pi}{\omega_\pi}-\frac{\omega_\pi}{\Omega_\pi}\right)\left(1-\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)+\sqrt{\frac{\Omega_\Sigma}{\Omega_\pi}}\left(\frac{\Omega_\Sigma}{\omega_\Sigma}-\frac{\omega_\Sigma}{\Omega_\Sigma}\right)\left(1-\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)\right]. \tag{60}$$

The diagonalization procedure for this case along with the dynamical consequences of this Hamiltonian will be discussed in [30].

We finally get

$$H_{neutral}=m_1(A^\dagger_{\theta(k)}A_{\theta(k)}+A_{\theta(k)}A^\dagger_{\theta(k)})+n_1(A_{\theta(k)}A_{\theta(-k)}+A^\dagger_{\theta(k)}A^\dagger_{\theta(-k)})+m_2(D^\dagger_{\theta(k)}D_{\theta(k)}+D_{\theta(k)}D^\dagger_{\theta(k)})+n_2(D_{\theta(k)}D_{\theta(-k)}+D^\dagger_{\theta(k)}D^\dagger_{\theta(-k)}). \tag{61}$$

We now apply two squeezing transformations

$$F_k=\mu A_{\theta(k)}+\nu A^\dagger_{\theta(-k)},\quad F^\dagger_k=\mu A^\dagger_{\theta(k)}+\nu A_{\theta(-k)}k \tag{62}$$

and

$$G_k=\rho D_{\theta(k)}+\sigma D^\dagger_{\theta(-k)},\quad G^\dagger_k=\rho D^\dagger_{\theta(k)}+\sigma D_{\theta(-k)}. \tag{63}$$

Then,

$$[F_k,F^\dagger_k]=1 \tag{64}$$

and

$$[G_k,G^\dagger_k]=1 \tag{65}$$

imply that the transformations are indeed Bogolyubov transformations as

$$\mu^2-\nu^2=1 \tag{66}$$

and

$$\rho^2-\sigma^2=1. \tag{67}$$

The squeezing parameters μ,ν,ρ,σ are given by

$$2\mu^2=\sqrt{\frac{\Omega_\pi}{\Omega_\Sigma}}\left[\frac{\Omega_\pi}{\omega_\pi}\left(1-\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)+\frac{\omega_\pi}{\Omega_\pi}\left(1+\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)\right]+\sqrt{\frac{\Omega_\Sigma}{\Omega_\pi}}\left[\frac{\Omega_\Sigma}{\omega_\Sigma}\left(1-\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)+\frac{\omega_\Sigma}{\Omega_\Sigma}\left(1+\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)\right]+1, \tag{68}$$

$$2\nu^2=\sqrt{\frac{\Omega_\pi}{\Omega_\Sigma}}\left[\frac{\Omega_\pi}{\omega_\pi}\left(1-\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)+\frac{\omega_\pi}{\Omega_\pi}\left(1+\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)\right]+\sqrt{\frac{\Omega_\Sigma}{\Omega_\pi}}\left[\frac{\Omega_\Sigma}{\omega_\Sigma}\left(1-\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)+\frac{\omega_\Sigma}{\Omega_\Sigma}\left(1+\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)\right]-1, \tag{69}$$

$$2\rho^2=\sqrt{\frac{\Omega_\pi}{\Omega_\Sigma}}\left[\frac{\Omega_\pi}{\omega_\pi}\left(1+\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)+\frac{\omega_\pi}{\Omega_\pi}\left(1-\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)\right]+\sqrt{\frac{\Omega_\Sigma}{\Omega_\pi}}\left[\frac{\Omega_\Sigma}{\omega_\Sigma}\left(1+\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)+\frac{\omega_\Sigma}{\Omega_\Sigma}\left(1-\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)\right]+1, \tag{70}$$

$$2\sigma^2=\sqrt{\frac{\Omega_\pi}{\Omega_\Sigma}}\left[\frac{\Omega_\pi}{\omega_\pi}\left(1+\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)+\frac{\omega_\pi}{\Omega_\pi}\left(1-\frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}}\right)\right]+\sqrt{\frac{\Omega_\Sigma}{\Omega_\pi}}\left[\frac{\Omega_\Sigma}{\omega_\Sigma}\left(1+\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)+\frac{\omega_\Sigma}{\Omega_\Sigma}\left(1-\frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}}\right)\right]-1. \tag{71}$$

With these definitions the neutral sector Hamiltonian is simply

$$H_{neutral}=\frac{\sqrt{\Omega_\pi\Omega_\Sigma}}{4a^3}\left[\left(F^\dagger_kF_k+\frac{1}{2}\right)+\left(G^\dagger_kG_k+\frac{1}{2}\right)\right]. \tag{72}$$

Combining with the charged sector

$$H_{charged}=\frac{\Omega_{\pi_\pm}}{a^3}\{C^\dagger_kC_k+B^\dagger_kB_k+1\}, \tag{73}$$

the total Hamiltonian for the case $\rho=\pi/2$ is

$$H_{\pi/2}=\int \frac{d^3\underline{k}}{(2\pi)^3}\left\{\frac{\Omega_{\pi_\pm}}{a^3}(C_k^\dagger C_k+B_k^\dagger B_k+1)+\frac{\sqrt{\Omega_\pi\Omega_\Sigma}}{4a^3}\left[\left(F_k^\dagger F_k+\frac{1}{2}\right)+\left(G_k^\dagger G_k+\frac{1}{2}\right)\right]\right\}, \tag{74}$$

with

$$\frac{\Omega_\pi^2}{a^6}=\frac{k^2}{a^2}+m_\pi^2+\lambda v^2, \quad \frac{\Omega_{\pi_\pm}^2}{a^6}=\frac{k^2}{a^2}+m_{\pi_\pm}^2+\lambda v^2[1+\sin^2(\theta)]. \tag{75}$$

This completes our analysis of the quantization of the Hamiltonian for the two cases mentioned above. We have taken an $O(4)$ sigma model and succeeded in quantizing the quadratic fluctuations to arrive at two Hamiltonians which provide all the required ingredients for analyzing the formation, evolution, and eventual decay of the DCC. Indeed, in case 2, the Hamiltonian shows explicit mixing with an angle (θ), providing a mixing parameter and therefore a misalignment parameter in isospin space.

IV. EVOLUTION OF THE FLUCTUATIONS AND PARAMETRIC AMPLIFICATIONS

In the last section we have constructed the quantum Hamiltonians for two cases ($\theta=0$ and $\rho=\pi/2$) of the $O(4)$ sigma model. We have also shown the diagonalization of the Hamiltonians so that they can be written in terms of the appropriate quantum fluctuation fields as purely quadratic Hamiltonians. In this section we shall explicitly consider the case when $\theta=\mathbf{0}$ (the dynamics of the case $\rho=\pi/2$ will be given in a subsequent paper [30]). We notice that H has the form of a decoupled Hamiltonian. This is easy to understand from the $SO(4)$ parent. The $SO(4)$ vector has been decomposed into four fields $\pi_\pm$, π_0, and Σ being, respectively, the charged pions, the neutral pion, and the sigma fields.

It is the sigma field which decouples in this particular Hamiltonian and therefore it can be analyzed independently of the pion fields. As our interest is in the pion fields, we can write the total dynamical Hamiltonian for the pion fields in terms of the observed pion creation and annihilation operators (a, $a^\dagger$, c, $c^\dagger$, b, and $b^\dagger$) in terms of the squeezing parameters as

$$H=\int \frac{d^3\underline{k}}{(2\pi)^3}\frac{\Omega_\pi}{a^3}\Bigg\{2(\mu^2+\nu^2)\{c_k^\dagger c_k+b_k^\dagger b_k+1\} + \mu\nu\{(c_k b_{-k}+b_k c_{-k})+(c_k^\dagger b_{-k}^\dagger+b_k^\dagger c_{-k}^\dagger)\} + \frac{\Omega_\pi}{a^3}\{(\mu^2+\nu^2)\{a_k^\dagger a_k+1\}+(\nu\mu)\{a_{-k}^\dagger a_k^\dagger+a_k a_{-k}\}\}\Bigg\}. \tag{76}$$

We now define the bilinear operators

$$\mathcal{D}=a_k a_{-k}+b_k c_{-k}+c_k b_{-k}=K_1^-+K_2^-+K_3^-,$$

$$\mathcal{D}^\dagger=a_{-k}^\dagger a_k^\dagger+c_{-k}^\dagger b_k^\dagger+b_{-k}^\dagger c_k^\dagger=K_1^++K_2^++K_3^+,$$

$$N=\frac{1}{2}\{a_k^\dagger a_k+a_{-k}^\dagger a_{-k}+b_k^\dagger b_k+b_{-k}^\dagger b_{-k}+c_k^\dagger c_k + c_{-k}^\dagger c_{-k}+3\}=K_1^0+K_2^0+K_3^0, \tag{77}$$

and it is easy to see that they satisfy an $su(1,1)$ algebra

$$[N,\mathcal{D}]=-\mathcal{D}, \quad [N,\mathcal{D}^\dagger]=\mathcal{D}^\dagger, \quad [\mathcal{D}^\dagger,\mathcal{D}]=-2N. \tag{78}$$

The $su(1,1)$-invariant Hamiltonian for the pion fields assumes the form

$$H=\int \frac{d^3\underline{k}}{(2\pi)^3}\frac{1}{a^3}2\Omega_\pi(k,t)(\mu^2+\nu^2)N + 2\Omega_\pi(k,t)\mu\nu(\mathcal{D}+\mathcal{D}^\dagger). \tag{79}$$

The time-dependent evolution equation for the eigenstates of H is given by

$$H(t)|\psi(t)\rangle=i\frac{d}{dt}|\psi(t)\rangle. \tag{80}$$

The particular $su(1,1)$ structure elucidated above provides us the solution

$$|\psi(t)\rangle=\exp\left(\int \frac{d^3\underline{k}}{(2\pi)^3}r_k(\mathcal{D}_k^\dagger-\mathcal{D}_k)\right)|\psi(0\rangle \tag{81}$$

for the evolution of the wave function immediately [31]. Here r_k is the squeezing parameter related to the physical variables $\Omega_\pi(k,t)$ and $\omega_\pi(k)$ through

$$\tanh(2r_k)=\frac{\left(\frac{\Omega_\pi(k,t)}{\omega_\pi}\right)^2-1}{\left(\frac{\Omega_\pi(k,t)}{\omega_\pi}\right)^2+1}, \tag{82}$$

where $\Omega_\pi(k,t\to\infty)=\omega_\pi(k)$. Thus, in the evolution of the condensate, it is the frequency changes which bring about squeezing [32].

The diagonalized Hamiltonian H_0 can be converted into a Hamiltonian in terms of quantum fields corresponding to the operators A,B,C and their adjoints to obtain a purely quadratic Hamiltonian (the starting point of many early works on the subject of the DCC) [33,34].

We can, for example, write

$$\frac{\Omega_\pi}{a^3}\left(A_k^\dagger A_k+\frac{1}{2}\right)=\frac{\Omega_\pi}{a^3}(A_k^\dagger A_k+A_kA_k^\dagger)=\left(\frac{\Omega_\pi}{a^3}\right)^2\Pi_A^2(k,t)+P_{\Pi_A}^2(k,t). \quad (83)$$

Similarly, for B and C, we write

$$\frac{\Omega_\pi}{a^3}\left(B_k^\dagger B_k+\frac{1}{2}\right)=\frac{\Omega_\pi}{a^3}(B_k^\dagger B_k+B_kB_k^\dagger)=\left(\frac{\Omega_\pi}{a^3}\right)^2\Pi_B^2(k,t)+P_{\Pi_B}^2(k,t), \quad (84)$$

$$\frac{\Omega_\pi}{a^3}\left(C_k^\dagger C_k+\frac{1}{2}\right)=\frac{\Omega_\pi}{a^3}(C_k^\dagger C_k+C_kC_k^\dagger)=\left(\frac{\Omega_\pi}{a^3}\right)^2\Pi_C^2(k,t)+P_{\Pi_C}^2(k,t). \quad (85)$$

The Hamiltonian H_0 can then be written as

$$H_0(t)=\int\frac{d^3k}{(2\pi)^3}\sum_{i=A,B,C}\frac{1}{2}\left[\left(\frac{\Omega_\pi}{a^3}\right)^2\Pi_i^2(k,t)+P_{\Pi_i}^2(k,t)\right]. \quad (86)$$

The Schrödinger equation for each momentum mode is simply

$$H_0(k,t)\psi(k,t)=i\frac{d}{dt}\psi(k,t). \quad (87)$$

If we use the Π representation (coordinate space representation) for $\psi(k,t)$, then, the $su(1,1)$ symmetry of the Hamiltonian tells us that the solution for $\psi(k,t)$ is just a Gaussian. For simplicity we work with a Gaussian of the form

$$\langle\pi_0,\pi_+,\pi_-|\psi\rangle(t)=\prod_{k,i}L_i(t)e^{[-W_i(t)\Pi_i^2]}, \quad (88)$$

while the complete wave function is

$$\psi(t)=\int\frac{d^3k}{(2\pi)^3}\psi(k,t). \quad (89)$$

Then for each mode A,B,C,D the wave function $\psi_{k,i}(t)$ evolves as

$$i\frac{\partial\psi_{k,i}(t)}{\partial t}=\left(i\frac{\dot{L}_i}{L_i}-i\Pi_i^2(k)\dot{W}_i(k,t)\right)\psi_{k,i}(t), \quad (90)$$

while

$$H_{k,i}\psi_{k,i}(t)=\frac{1}{2a^3}\left\{\Omega_{\pi_i}^2\Pi_i^2(k,t)-\frac{\partial^2\psi_{k,i}(t)}{\partial\Pi_i^2}\right\}. \quad (91)$$

Combining the above we find that $W_i(t)$ and $\psi_{k,i}(t)$ evolve as

$$W_i(t)=-ia^3/2\frac{\dot{\psi}_{k,i}(t)}{\psi_{k,i}(t)}, \quad (92)$$

$$\ddot{\psi}_{i,k}(t)+3\frac{\dot{a}}{a}\dot{\psi}_{i,k}(t)-\frac{\Omega_{\pi_i}^2}{a^3}\psi_{i,k}(t)=0. \quad (93)$$

The equation satisfied by the wave functions for each mode are then given by

$$\ddot{\psi}_A(k,t)+\frac{3\dot{a}}{a}\dot{\psi}_A+\left(\frac{\Omega_\pi}{a^3}\right)^2(k,t)\psi_A(k,t)=0,$$

$$\ddot{\psi}_B(k,t)+\frac{3\dot{a}}{a}\dot{\psi}_B+\left(\frac{\Omega_\pi}{a^3}\right)^2(k,t)\psi_B(k,t)=0,$$

$$\ddot{\psi}_C(k,t)+\frac{3\dot{a}}{a}\dot{\psi}_C+\left(\frac{\Omega_\pi}{a^3}\right)^2(k,t)\psi_C(k,t)=0, \quad (94)$$

where

$$\left(\frac{\Omega_\pi}{a^3}\right)^2(k,t)=\left(\frac{k^2}{a^2}\right)+m_\pi^2+\lambda v^2, \quad (95)$$

where it may be recalled that $A_k(t)$, $B_k(t)$, and $C_k(t)$ are related to a_k, b_k, and c_k (the physical pion operators) by the squeezing transformation given by

$$A_k(t,r)=U^{-1}(r,t)a_kU(r,t), \quad (96a)$$

$$B_k(t,r)=U^{-1}(r,t)b_kU(r,t), \quad (96b)$$

$$C_k(t,r)=U^{-1}(r,t)c_kU(r,t), \quad (96c)$$

$$\cosh(r)=\sqrt{\frac{1}{2}\left[\left(\frac{\Omega_\pi}{\omega_\pi}+\frac{\omega_\pi}{\Omega_\pi}\right)+1\right]},$$

$$\nu=\sinh(r)=\sqrt{\frac{1}{2}\left[\left(\frac{\Omega_\pi}{\omega_\pi}+\frac{\omega_\pi}{\Omega_\pi}\right)-1\right]} \quad (97)$$

and r is the squeezing parameter:

$$U(r,t)=\exp\left(\int\frac{d^3\underline{k}}{(2\pi)^3}r(k,t)[(a_k^\dagger a_{-k}^\dagger-a_ka_{-k})+(d_k^\dagger d_{-k}^\dagger-d_kd_{-k})+(c_kb_{-k}+b_kc_{-k})-(c_k^\dagger b_{-k}^\dagger+b_k^\dagger c_{-k}^\dagger)]\right). \tag{98}$$

The expectation values of the number operator for the neutral pions for each momentum k are given by

$$\langle\psi_k(t)|a_k^\dagger a_k|\psi_k(t)\rangle=\sinh^2(r)=\langle\psi_k|A_k^\dagger(t)A_k(t)|\psi_k\rangle. \tag{99}$$

A similar expression may be obtained for the charged pions.

Since we are considering the expansion of the plasma in a spherically symmetric manner as emphasized by the Robertson-Walker-type metric $ds^2=dt^2-a(t)^2d\underline{x}^2$, it is possible to scale the time so as to provide a conformally flat metric: we let

$$d\eta=a(t)^{-1}dt, \tag{100}$$

so that

$$ds^2=a(\eta)^2(d\eta^2-d\underline{x}^2). \tag{101}$$

The equations of motion given above can be transformed into ones that resemble a harmonic oscillator with time-dependent frequencies. We shall write only the generic form of the above equations: In terms of the scaled time η, we have

$$\psi''+\frac{2a'}{a}\psi'+\vec{k}^2+[m_\pi^2+\lambda(\langle\Phi^2\rangle-f_\pi^2)]a^2\psi=0, \tag{102}$$

where a prime denotes a differential with respect to η. Here we have a few remarks: First, we note that to make contact with the dynamical equations for the pions used as starting points for analyzing the DCC, we return to the point about the potential that we have chosen versus the traditional potential chosen in such studies. The traditional potential is $(\Phi_i^2-f_\pi^2)^2$ while we have taken a potential $(\Phi_i^2)^2$. Therefore, the relationship between our work and early studies is accomplished through a replacement $v^2\rightarrow[\langle\Phi^2\rangle(\eta)-f_\pi^2]$. This explains the equation written above and the further analysis of this paper. Last, let us scale ψ:

$$\xi=a\psi, \tag{103}$$

so that the equation becomes

$$-\xi''+V(\eta)\xi=(k^2+m_\pi^2)\xi, \tag{104}$$

where

$$V(\eta)=a^{-1}\frac{d^2a}{d\eta^2}+m_\pi^2(1-a^2)-\lambda(\langle\Phi^2\rangle-f_\pi^2). \tag{105}$$

Thus in the symmetry-broken stage $\langle\Phi^2\rangle=f_\pi^2$,

$$V_b(\eta)=a^{-1}\frac{d^2a}{d\eta^2}+m_\pi^2(1-a^2), \tag{106}$$

and in the symmetry-restored stage $\langle\Phi^2\rangle=0$,

$$V_r(\eta)=a^{-1}\frac{d^2a}{d\eta^2}+m_\pi^2(1-a^2)+\lambda a^2f_\pi^2. \tag{107}$$

These equations have a dual nature: on the one hand, they are Schrödinger-like equations with η corresponding to the "spatial"-like variable and $E=\omega_\pi^2$. Therefore, they allow calculation of the reflection and transmission coefficients over the "potential barrier" provided by the $V(\eta)$ term. On the other hand, they can also be looked upon as equations for time-dependent harmonic oscillators with time-dependent frequencies given by Ω_π^2 and ω_π^2. These two pictures enable us to calculate the squeezing-parameter-dependent number operator $N(k)=\sinh^2[r(k)]$ involved in the evolution of the plasma as can be seen in the Appendix.

We also note that the expansion coefficient $a(\eta)$ provides us with a control parameter on the expansion rate of the plasma while the system provides us with another parameter, which we call τ for the "rolling down" of the fields in the potential. According to the pioneering work of papers [29,13], when the expansion rate is greater than the relaxation rate, the DCC forms.

We now consider two cases, the first being a toy model [28] without the effects of expansion and the second being a more realistic "baked Alaska situation."

In the first case, let us suppose that $a(\eta)=1$ so that

$$V(\eta)=-\lambda(\langle\Phi^2\rangle-f_\pi^2). \tag{108}$$

Then the equation for ξ is

$$\xi''+(k^2+m_\pi^2+\lambda(\langle\Phi\rangle^2-f_\pi^2))\xi=0. \tag{109}$$

If τ is the time the state spends in the symmetry-restored phase, a quench can be modeled by assuming that for $-\tau/2\leqslant\eta\leqslant\tau/2$ the vacuum expectation value $\langle\Phi^2\rangle\langle\Phi^2\rangle\langle\Phi^2\rangle=0$ and for $\eta>\tau/2$ the vacuum relaxes to its value $\langle\Phi^2\rangle=\langle v^2\rangle\approx f_\pi^2$ (since the potential is translationally invariant, this could easily be mapped to the interval $\{0,\tau\}$, but our choice makes the potential symmetric). For this sudden quench the problem reduces to that of transmission through a symmetric rectangular potential barrier of height $(\lambda f_\pi^2)=m_\Sigma^2/2$ and width τ.

The transmission coefficient for such a barrier is easily calculable [35,36] and in terms of physically known values is

$$T=\frac{1}{1+\left(\frac{(m_\Sigma^2/2)^2}{(\omega_k^2)(m_\Sigma^2/2-\omega_k^2)}\right)\sinh^2\left[\left(\frac{m_\Sigma^2}{2}-\omega_k^2\right)^{1/2}\tau\right]}. \tag{110}$$

From the Appendix we get

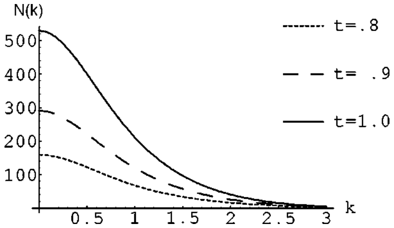

FIG. 1. Variation of $N(k)$ and k (in units of m_π) with $\tau = 0.8, 0.9, 1.0$ in units of m_π^{-1} for the quenched limit.

$$N(k)=\frac{1-T}{T}=\frac{(m_\Sigma^2/2)^2}{\omega_k^2(m_\Sigma^2/2-\omega_k^2)}\sinh^2\left[\left(\frac{m_\Sigma^2}{2}-\omega_k^2\right)^{1/2}\tau\right]. \tag{111}$$

The dependence of $N(k)$ on k for different values of τ is shown in Fig. 1, clearly exhibiting the amplification of the low-momentum modes.

Figure 1 also shows us that the longer the system stays in the state of broken symmetry, the larger the DCC domains. The dependence of $N(k)$ on τ for different values of k is shown in Fig. 2. Recall that, by definition, the amplification of the zero modes constitutes DCC formation. Since we have a ‘‘Schrödinger’’ wave equation that is exactly solvable, we can also calculate the size of the DCC domains. We shall leave that for a subsequent paper [30].

This case is similar to that considered by Ref. [28].

For a more realistic scenario, the expansion must be included to show that the enhancement of the low-energy modes and the squeezing parameter are dependent on the rate of the expansion mechanism by which symmetry is restored. To produce substantial squeezing, we require a quenched scenario. To show this we have to compare the situation of a sudden quench with a slow adiabatic relaxation of the system from the symmetry-restored stage to the symmetry-broken stage. The transition between quenching and adiabaticity can be modeled in two ways in view of the dual nature of Eq. (105).

We consider the expansion coefficient $a(\eta)$ to be of the form

$$a^2(\eta)=\Theta(-\eta)\tanh\left[-\frac{b}{2}\left(\eta+\frac{\tau}{2}\right)\right] + \Theta(\eta)\tanh\left[\frac{b}{2}\left(\eta-\frac{\tau}{2}\right)\right]. \tag{112}$$

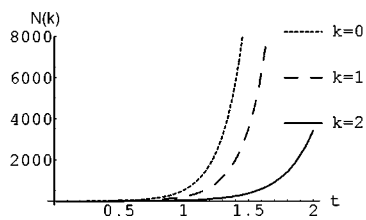

FIG. 2. Variation of $N(k)$ with τ (units of $m_\pi^- 1$) for $k=0$, 1, and 2 (units of m_π) for the quenched limit.

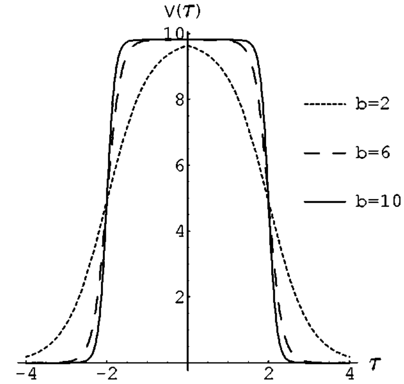

FIG. 3. Variation of the Woods-Saxon potential barrier of width $\tau=4$ with b. Large $b\tau$ gives the rectangular potential barrier (quenched limit).

Here $1/b$ measures the rate of the expansion and we choose b to be in units of m_π. When $b\tau \gg 1$ we have the quenched limit (fast expansion) and enhancement of low-momentum modes should occur (DCC formation). When $b\tau \ll 1$ we have the adiabatic limit and no enhancement of low-momentum modes should occur (no DCC formation).

When viewed as a time-dependent oscillator equation in an expanding metric we may write the equation for ξ in the symmetry-restored phase as an oscillator equation

$$\xi''+\omega_\pi^2\xi-V_r(\eta)\xi=0 \tag{113}$$

and in the symmetry-broken stage as

$$\xi''+\omega_\pi^2\xi-V_b(\eta)\xi=0, \tag{114}$$

where $V_b(\eta)$and $V_r(\eta)$ are given by Eqs. (107) and (106) with the particular choice of the expansion parameter $a(\eta)$ given by Eq. (112). Thus the change in frequency from the restored to the broken stage is given by

$$V_r(\eta)-V_b(\eta)=+\frac{m_\Sigma^2}{2}a(\eta)^2. \tag{115}$$

Given that we have to calculate the transmission coefficients we need to look at the wave functions ξ in the limit $\eta\to\pm\infty$. This is possible by solving the oscillator equation with a variable frequency:

$$\xi''+\omega_\pi^2+\frac{m_\Sigma^2}{2}[1-a^2(\eta)]\xi=0. \tag{116}$$

We see that Eq. (116) satisfies the required limits $\Omega(\eta)^2 = \omega_\pi^2$ as $\eta\to\pm\infty$ and between these limits it gradually goes to a maximum at $\Omega(\eta)=\Omega_\pi^2$. Equation (116) can be converted into a Schrödinger wave equation for a Woods-Saxon potential barrier given by

$$V(\eta)=V_0\left[\Theta(-\eta)\left(\frac{1}{1+e^{-b(\eta+\tau/2)}}\right)+\Theta(\eta)\left(\frac{1}{1+e^{b(\eta-\tau/2)}}\right)\right], \tag{117}$$

where $V_0=m_\Sigma^2/2$. The comparison of the rectangular potential barrier with this barrier is shown in Fig. 3, which reveals it to be a good approximation in the adiabatic limit.

We take the values of the variables $E=(k^2+\omega_\pi^2)^{1/2}$ and $k'^2=E-V_0$. The transmission coefficient for this barrier is given by

$$T_{ws}=\frac{\sinh^2(\pi 2k/b)\sinh^2(\pi 2k'/b)}{\sinh^4[\pi(k-k')/b]\{4|C|\sin^2(k'b)+(|C|^2-1)\}},\quad E\geqslant V_0, \tag{118}$$

and the same with $k'\to ik'$ for $E<V_0$ where

$$C=\frac{\sinh^2[\pi 2(k+k')/b]}{\sinh^2[\pi 2(k-k')/b]}.$$

Note that for b large this reduces exactly to the rectangular potential barrier (quenched limit) transmission coefficient and for b very small this goes over to the Poschl-Teller (Eckart) barrier transmission coefficient.

The number of particles of mode k equals

$$N(k)=\frac{1-(\sinh^4[\pi(k-k')/b]\{4|C|\sin^2(k'b)+(|C|^2-1)\})}{\sinh^2(\pi 2k/b)\sinh^2(\pi 2k'/b)}. \tag{119}$$

Here b measures the duration of the quench. $N(k)$ vs k is plotted in Fig. 4. We see that in the adiabatic limit of small $b\tau$, $N(k)$ is exponentially suppressed so that there is no enhancement of low-momentum modes.

Figure 5 shows the variation of $N(k)$ with k for large values of b, showing the enhancement of low-momentum modes in the quenched limit.

From the above we conclude that if the expansion time $(1/b)$ is faster than the rolling time (τ), we get the quenched limit, while if the expansion time is slower than the rolling time, we get the adiabatic limit. This provides an analytic supplement of earlier numerical simulations of quenching versus annealing with regards to DCC formation [37–41]. We see that since in the squeezed-state description $N(k)=\sinh^2 r_k$, where $r_k=r(k)$ is the squeezing parameter, in the quenched limit the squeezing parameter is much greater than in the adiabatic limit. This demonstrates clearly the connection between the rate of expansion and squeezing and the formation of the DCC, as characterized by the enhancement of low-momentum pions. If squeezing is large, we have formed a DCC; if not, then there is no DCC. This will now enable us to give signatures of DCC formation which are related to the formation process of the DCC.

V. PION RADIATION FROM DCC'S

Having shown that the dynamics of the evolution of the DCC suggests a squeezed-state treatment of emerging pion waves and that this effect is more pronounced in a quenched scenario than an adiabatic one, we now proceed to show the possible experimental signals that would result. A complete treatment, with the incorporation of isospin and disorientation in isospin space for arbirary momenta, is given in [42] and [30]. For the present, however, since we see an amplification of the low-momentum modes, we consider only the case when $k=0$. In such case for pions near zero momentum, $k\to 0$, the state (81) factors into a squeezed state for the neutral pions and a Caves-Schumaker state for the charged pions. In this limit the bilinear operators of Eq. (77) are

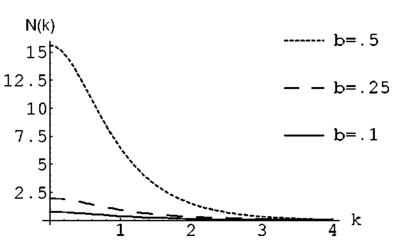

FIG. 4. Variation $N(k)$ with k for values of b in the *adiabatic limit* for the Woods-Saxon barrier.

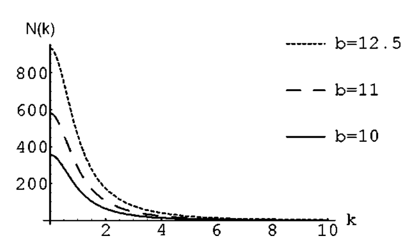

FIG. 5. Variation $N(k)$ with k for values of b in the *quenched limit* for the Woods-Saxon barrier.

$$\mathcal{D}=a_0^2+b_0c_0+c_0b_0,$$
$$\mathcal{D}^\dagger=a_0^{\dagger 2}+c_0^\dagger b_0^\dagger+b_0^\dagger c_0^\dagger,$$
$$N=a_0^\dagger a_0++b_0^\dagger b_0+c_0^\dagger c_0+3/2. \tag{120}$$

Thus the $k=0$ wave function is

$$|\psi\rangle=e^{r_0(\mathcal{D}^\dagger-\mathcal{D})}|\psi_0(0)\rangle$$
$$=e^{r_0(a_0^{\dagger 2}+c_0^\dagger b_0^\dagger+b_0^\dagger c_0^\dagger-a_0^2+b_0c_0+c_0b_0)}|\psi_0(0)\rangle, \tag{121}$$

where r_0 is the squeezing parameter at zero momentum—i.e.,

$$\tanh(2r_0)=\frac{(\Omega_\pi/\omega_\pi)^2-1}{(\Omega_\pi/\omega_\pi)^2+1}.$$

The pion multiplicity distribution is given by

$$P_{n_0,n_+,n_-}=|\langle n_0,n_+,n_-|\psi\rangle|^2$$
$$=|\langle n_0|e^{r_0(a_0^\dagger)^2-r_0^* a_0^2}|0\rangle$$
$$\times\langle n_+,n_-|e^{2r_0(b_0^\dagger c_0^\dagger-b_0c_0)}|0\rangle|^2, \tag{122}$$

defining $S(r_0)$ as the one-mode squeezing operator:

$$S(r_0)=\langle n_0|e^{r_0[(a_0^\dagger)^2-a_0^2]}|0\rangle$$
$$=S_{n_0,0}. \tag{123}$$

$S^{tm}(r_0)$ is then the two-mode squeezing operator

$$\langle n_+,n_-|e^{[r_0(b_0^\dagger c_0^\dagger-b_0c_0)]}|0\rangle=S^{tm}_{n_+,n_-,0}. \tag{124}$$

The neutral and charged pion distributions are

$$P_{n_0,n_c}=\langle S_{n_0,0}\rangle^2\langle S^{\dagger\, m}_{n_+,n_-,0,0}\rangle^2, \tag{125}$$

which is just the product of squeezed distributions for charged and neutral pions and only an even number of pions emerge. Writing $n_+=n_-=n_c$, we get the distribution of charged particles to be

$$P_{n_c}=\sum_{n_0} P_{n_0,n_c}=\frac{[\tanh(r_0)]^{2n_c}}{[\cosh(r_0)]^2} \tag{126}$$

and

$$P_{n_0}=\sum_{n_c} P_{n_0,n_c}=\frac{n_0![\tanh(r_0)]^{n_0}}{[(n_0/2)!]^2\cosh(r_0)2^{n_0}}. \tag{127}$$

The generalized squeezed eigenstate leads to products of two types of squeezed states of pions at zero momentum, the neutral pions being in a one-mode squeezed state and the charged pions being in an $SU(1,1)$ coherent or two-mode squeezed state. Thus the neutral and charged pion distributions are significantly different as the two types of states have different properties. We now illustrate the effect of squeezing in these two distributions. Figures 6 and 7 show the difference in the charged and neutral pion distributions as we vary the squeezing parameter from a low value to a high value.

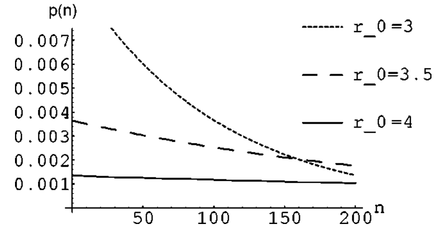

FIG. 6. Variation of P_{nc} with n for the $r_0=3$, 3.5, and 4.

We now illustrate the effect of quenching versus adiabaticity on these two distributions. Figure 8 shows the difference in the charged and neutral pion distributions as we vary the squeezing parameter from the adiabatic limit where the difference is negligible to the quenched limit where the difference is significant.

This behavior seems related to the traditional signal of the DCC—namely, that the probability $P(f)$ of the fraction f of neutral pions to all pions scales as $1/2\sqrt{f}$ whereas the charged pions do not exhibit this behavior. To see this more clearly we examine the "KNO (Koba-Nielsen-Olson) limit" [43] of the probability distributions P_{n0} and P_{nc} (Fig. 9). Define the variable $f=n/\langle n\rangle$. Then, since the neutral pions are always emitted in pairs, the number of neutral pions, $\langle n_0\rangle$, is equal to the number of positive and negative pions, $\langle n_c\rangle$. We may rewrite the distributions (126) and (127) as

$$P_{n_c}(n)=\frac{\langle n\rangle_c^n}{(1+\langle n_c\rangle)^{n+1}} \tag{128}$$

and

$$P_{n_0}(n_0)=\frac{1}{2^2 n}\left(\frac{\langle n_0\rangle}{1+\langle n_0\rangle}\right)^n\frac{2n!}{(1+\langle n_0\rangle)^{1/2}(n!)^2}. \tag{129}$$

Then the KNO asymptotic limit corresponds to the large $\langle n\rangle$ (large squeezing quenched) limit and the charged and neutral pion distributions obey the scaling laws

$$\lim_{n\to\infty,\langle n_c\rangle\to\infty}\langle n_c\rangle P_{n_c}=e^{-f} \tag{130}$$

and

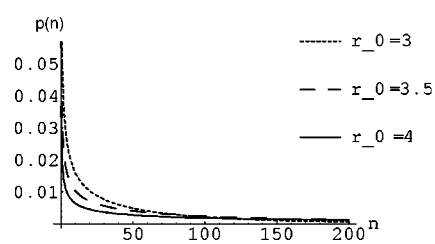

FIG. 7. Variation of P_{nc} with n for the $r_0=3$, 3.5, and 4.

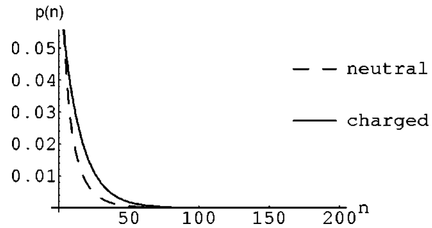

FIG. 8. Variation of P_{n0} (solid line) and P_{nc} (dashed line) with n for the *adiabatic* limit $(r_0=2)$.

$$\lim_{n\to\infty,\langle n_0\rangle\to\infty}\langle n_0\rangle P_{n_0}=\frac{e^{-f}}{2\sqrt{f}}. \tag{131}$$

From these two equations we see that in the large-$\langle n\rangle$ limit—i.e., very high squeezing—the probability distributions of the neutral pions exhibits $1/2\sqrt{f}$ behavior with respect to the charged pions.

We now calculate the correlation function by first calculating the generating function corresponding to the multiplicity distribution P_n. It is given by $Q(\lambda)=\Sigma_\lambda(1-\lambda)^n P_n$. The two-particle zero-momentum correlation function $G^2(0)$ is given by

$$G^2(0)=\frac{\langle n^2\rangle-\langle n\rangle}{\langle n^2\rangle}=\frac{\partial^2 Q/\partial\lambda^2|_{\lambda=1}}{(\partial Q/\partial\lambda)^2|_{\lambda=1}}. \tag{132}$$

The variation of $G^2_{neutral}/G^2_{charged}$ with the squeezing parameter is given in Fig. 10.

Thus we see that in the adiabatic limit $G^2_{neutral}\approx G^2_{charged}$, whereas in the quenched limit $G^2_{neutral}<G^2_{charged}$, giving a very clear indication of the effect of DCC formation on the Bose-Einstein correlations.

To conclude this section, we have shown that the sudden quench approximation in the evolution of the disoriented chiral condensate leads to a substantial amount of squeezing which manifests itself in the dramatic difference between charged and neutral pion distributions. For an adiabatic expansion the difference is much less, so that both the total multiplicity distributions of charged and neutral pions and their second-order correlation functions are dramatic characteristic signals for the DCC and are related directly to the way in which the DCC forms. These are unambiguous; therefore, they must be examined thoroughly in searches for the DCC [16].

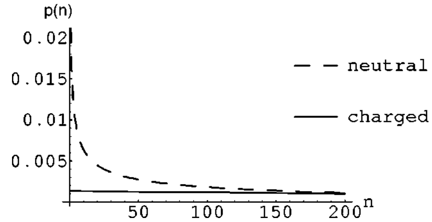

FIG. 9. Variation of P_{n0} (solid line) and P_{nc} (dashed line) with n for the *quenched* limit $(r_0=4)$.

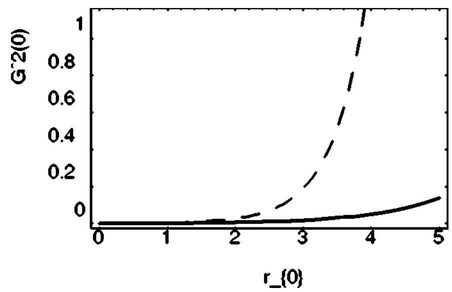

FIG. 10. Difference in correlations of the charged (dashed line) and neutral (solid line) pions as a function of the squeezing parameter r_0.

VI. CONCLUSION

To conclude, in this paper we have constructed effective Hamiltonians for the evolution of the disoriented chiral condensate without and with orientation in isospin space, starting from an $O(4)$ sigma model through the inclusion of second-order quantum fluctuations. We have shown that both Hamiltonians have $SU(1,1)$ symmetries, leading to the presence of squeezed staes in their dynamics. Unlike most earlier studies, our calculations of the effective Hamiltonians are not restricted to zero momentum and take care of back-to-back momentum correlations. The evolution of the wave function in an expanding metric, for one case (without orientation in isospin space), has been considered in detail. The competing effects of the expansion rate and the rolling down time of the system from a state of restored to broken symmetry have been explicitly examined. We find that in the quenched limit (fast expansion) the low-momentum modes are enhanced, signaling DCC formation, whereas in the adiabatic (slow expansion) limit, no such enhancement occurs. This has been shown to be directly related to the value of the squeezing parameter. The manifestation of this difference shows up directly in the total neutral and charged multiplicity distributions at zero momentum and the second-order correlation functions.

Further work is required to incorporate the effects of isospin. This will be done in a subsequent publication [42]. Furthermore, the evolution of the wave function corresponding to the Hamiltonian with orientation in isospin space is also the subject of a forthcoming communication [30]. Within this framework back-to-back momentum correlations and event-by-event analysis of the experimental signals can also be done. This will provide a solid unified picture of the various models of DCC formation, evolution, and decay, together with new experimental signals. Finally, we have achieved our goal of answering Bjorken's troubling questions about DCC's posed at the Trento meeting [18].

(i) Question (a): Are coherent states the right quantum DCC description, or should one go beyond to squeezed states, etc.? Answer (a): Yes, one is naturally lead to squeezed states from a quantum field theoretic perspective.

(ii) Question (b): How does one link DCC thinking to quantum effects in the data, especially the Bose-Einstein correlations. Is the DCC just another way of talking about the same thing? Answer (b): DCC formation leads to a significant difference between the charged and neutral pion Bose-

Einstein correlations corresponding to large squeezing. So it is not another way of talking about the same thing.

(iii) Question (c): How does one enforce quantum number conservation, especially charge? Answer (c): Charge conservation is automatically guaranteed by the isospin analysis shown in Ref. [42,56,58–61]. A word about other quantum numbers such as strangeness is in place here [44]. Schaffner-Bielich and Randrup [45] have examined the inclusion of strangeness in effective models of the DCC by considering the $SU(3)$ extension of the linear sigma model. They have made the observation that the nonequilibrium dynamics results in the enhancement of neutral kaons but to a lesser degree than the pions. Furthermore, they state that the kaons emitted form a DCC have a flat distribution much like the charged pions in the quenched limit that we have shown in this paper. The examination of this result in the context of our formalism is warranted, but requires a generalization of our model to the $SU(3)$ sigma model and will be reported in a later communication. In particular the difference between the neutral kaon distributions in a quenched versus adiabatic limit would be interesting to study and give another good signal for DCC formation.

(iv) Question (d): DCC production may imply anomalous bremsstrahlung, due to the large quantum fluctuations in charge. Can this be calculated from first principles? Answer (d): This can be easily incorporated in our model and will be shown in a later communication.

(v) Question (e): Can one really set up the problem at the quantum field theory level? Answer (e): An emphatic yes. Albeit, the full quantum theoretical effective Hamiltonian with the contribution of the quarks in this framework has yet to be considered. This will provide a new approach to supplement the work done in Refs. [46–50,55]. Some of these studies have examined the effect of coupling of the quark field with the condensate and have shown that the growth and decay of the condensate are governed entirely by meson fluctuations. However, in Ref. [50], a statement has been made that in a rapidly expanding plasma pair production of quarks may be as important as pion production. To make a quantitative judgement on this statement one would have to extend our work to the linear sigma model with quarks, as expansion is included in our model. This would perhaps lead to new and improved signals of the DCC.

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APPENDIX: RELATIONSHIP BETWEEN THE TRANSMISSION COEFFICIENTS AND SQUEEZING PARAMETER FOR A TIME-DEPENDENT HARMONIC OSCILLATOR

The process of particle creation and excitation of parametric oscillators can be related to the transmission and reflection of Schrödinger waves over a potential barrier. The method we describe for the relation is closely related to that of Ref. [51]. Let us consider the evolution of the wave function of the equation

$$\xi''_{i,k}+[k^2-V_{i,k}(\eta)]\xi_{i,k}=0. \tag{A1}$$

If we define $k^2-V_{i,k}(\eta)=\omega(\eta)^2$, then the above equation is a time-dependent harmonic oscillator $\xi''(t)+\omega^2(\eta)\xi(t)=0$. In the event of a change in frequency of the oscillator from ω_- to ω_+ the asymptotic form of the real solution at $\eta\to\pm\infty$ is

$$\xi_{\pm}(t)=\frac{1}{2}(a_{\pm}e^{i\omega_{\pm}t}+a^*_{\pm}e^{-\omega_{\pm}t}). \tag{A2}$$

We compare this with the complex solution of a 1D oscillator treated as a reflection over a barrier $k^2-V_{i,k}$ with η corresponding to the spatial variable of a Schrödinger-type equation:

$$\xi_{c-}(t)=e^{i\omega_- t}+Re^{-i\omega_- t},$$

$$\xi_{c+}(t)=Te^{i\omega_+ t}, \tag{A3}$$

to identify $a_-=1+R^*$ and $a_+=T$.

Now consider the particle creation problem modeled by a time-dependent harmonic oscillator [32]. The solution can be represented by

$$\xi_{r-}=\frac{e^{-i\omega_i t}}{\sqrt{2\omega_i}},$$

$$\xi_{r+}(t)=\alpha\frac{e^{-i\omega_f t}}{\sqrt{2\omega_f t}}+\frac{\beta e^{i\omega_f t}}{\sqrt{2\omega_f t}}. \tag{A4}$$

Now the definition of the transmission coefficient is

$$T=\frac{|\xi_{out}|^2 v_{out}}{|\xi_{in}|^2 v_i n}. \tag{A5}$$

If we regard t as a spatial variable, then these represent reflection and transmission over a 1D barrier. In particle creation problems the potential barrier reflection and the transmission coefficient can be related to the squeezing parameter in a particle creation problem by the relation $\sinh^2 r=R/T=\langle n\rangle=\beta\beta^*$ [52–61].

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