# Neutrino mass, dark matter, and inflation

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We show that spontaneous breaking of global $B-L$ symmetry responsible for small neutrino masses via the seesaw mechanism provides a unified picture of hybrid inflation and dark matter if the scale of $B-L$ breaking is close to the GUT scale. The Majoron, which acquires a small mass due to Planck scale breaking of $B-L$, is the dark matter candidate. The coupling of the Majoron field to the neutrino induces a small violation of *CPT* and Lorentz invariance at the present epoch. We discuss some of the phenomenological and cosmological implications of the model.

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## I. INTRODUCTION

While a nonzero neutrino mass is at the moment the only experimental evidence for physics beyond the standard model, as far as particle physics is concerned, in the domain of cosmology, several other questions remain unanswered within the standard model framework and also cry out for new physics. They are: (i) particle physics candidates for dark matter, (ii) a deeper understanding of the mechanism of inflation [1], (iii) the nature of matter-antimatter asymmetry responsible for the observable Universe [2], as well as (iv) an explanation of dark energy. There are many interesting and compelling models of new physics that can explain the different cosmological phenomena listed above individually. However, it is always much more desirable that a single model explain more than one phenomenon (ideally, of course, all of them). In this paper we discuss a model [3,4] originally designed to explain neutrino masses via the seesaw mechanism that seems to provide, after supersymmetrization, an explanation of both inflation and dark matter in a rather novel manner. The model has other desirable features such as gauge coupling unification, stable dark matter, stable proton due to *R*-parity conservation, etc.

We use the supersymmetric singlet Majoron [4] model described in Ref. [3] where the seesaw mechanism [5] for small neutrino masses is implemented by breaking a global $B-L$ symmetry. The idea is to extend the standard model by the addition of one right handed neutrino per family and three standard model singlet superfields $S$, $\Delta$, and $\bar{\Delta}$. Of the new Higgs fields, $S$ has $B-L=0$, $\Delta$ has $B-L=-2$ and $\bar{\Delta}$ has $B-L=+2$. The theoretical rationale for $B-L$ symmetry is rooted in the present neutrino oscillation results, which require that the seesaw scale be much lower than the Planck scale. It is therefore natural to think that it is protected by some symmetry. The simplest symmetry that does this is the $B-L$ symmetry. $B-L$ symmetry could be a global or a local symmetry. If we take it to be a global symmetry, its spontaneous breaking leads to a Nambu-Goldstone boson, the Majoron. Since it is natural to expect that all global symmetries are broken by nonperturbative gravitational (or stringy) effects, we will parameterize these symmetry breaking effects by nonrenormalizable terms in the effective Lagrangian which are suppressed by the Planck scale $M_{P\ell}$. These Planck scale $B-L$ breaking effects then give a tiny mass to the Majoron [6].

In this paper we show that this model leads to (i) a picture of hybrid inflation of the universe that links the neutrino mass (more precisely the seesaw or $B-L$ scale of order $10^{15}$ GeV) to inflation with all the desired features and (ii) Planck scale $B-L$ breaking effects that lead to a mass for the Majoron in the milli-electron-volt range and make it a candidate for dark matter of the Universe in the same way as the familiar axion, even though the parameters of the model are very different. It is, of course, interesting that this model ties the neutrino mass and the scale of inflation to the scale of gauge coupling unification.

An interesting implication of the model is that it leads to a cosmologically induced Lorentz violation for neutrinos. We also find that the superpartners of the Majoron must be in the MeV range in order to be compatible with the successes of big bang nucleosynthesis (BBN).

## II. THE MODEL

As already noted, the supersymmetric singlet Majoron model [3] consists of the following superfields in addition to the well known quark, lepton, Higgs, and gauge fields of the minimal supersymmetric standard model (MSSM): a right handed neutrino field $N_i^c$ ($i$ is the generation index), new Higgs fields $\Delta,\bar{\Delta}$, which carry lepton number $B-L=\pm 2$, and a singlet field $S$, which is $B-L$ neutral. We will show below that this model with an appropriate choice of the superpotential given below leads to F-term inflation as well as to the Majoron as the dark matter candidate. We require the renormalizable part of the theory to be invariant under global

 

$U(1)_{B-L}$ or an effective global symmetry that contains $B-L$ as a subgroup.[1]

The superpotential for the model can be written as a sum of three terms:

$$\mathbf{W}=\mathbf{W}_{\mathrm{MSSM}}+\mathbf{W}_0+\mathbf{W}_1. \tag{1}$$

where $\mathbf{W}_{\mathrm{MSSM}}$ is the familiar superpotential for the MSSM; $\mathbf{W}_0$ is the renormalizable part involving the new fields of the model and has the following form:

$$\mathbf{W}_0=h_\nu L H_u N+fNN\Delta+\lambda S(\Delta\overline{\Delta}-v_{BL}^2). \tag{2}$$

We assume that nonperturbative Planck scale effects induce nonrenormalizable terms in the potential and since we expect them to vanish in the limit of vanishing Newton's constant, they should be suppressed by powers of $M_{P\ell}$. We arrange our theory such that the leading Planck scale induced term has the form

$$\mathbf{W}_1=\frac{\lambda_1}{M_{P\ell}^3}(H_u H_d)^2(\Delta)^2. \tag{3}$$

At the nonrenormalizable level, there are many terms that one could write but we find this to provide a good description of physics of interest here and such a form could be guaranteed by additional discrete symmetries. For instance, a symmetry under which $S\to S$; $\Delta\to e^{3i\pi/4}\Delta$; $\overline{\Delta}\to e^{-3i\pi/4}\overline{\Delta}$; $H_u\to H_u$; $H_d\to e^{i\pi/4}H_d$; $\nu^c\to e^{-i\pi/8}\nu^c$; $L\to e^{-i\pi/4}L$; $e^c\to e^c$; $d^c\to e^{-i\pi/4}d^c$ allows all the required terms for MSSM except the $\mu$ term. We include the $\mu$ term in the superpotential since it is a lower dimensional term and it will break the symmetry softly.

This superpotential leads to a potential involving the fields $S$, $\Delta$, and $\overline{\Delta}$ with the following form:

$$V(\Delta,\overline{\Delta},S)=\lambda^2|\Delta\overline{\Delta}-v_{BL}^2|^2+\lambda^2|S|^2(|\Delta|^2+|\overline{\Delta}|^2)+V_{SSB}, \tag{4}$$

where $V_{SSB}$ stands for the supersymmetry breaking terms such as $m_S^2|S|^2$ and $|m_1^2|\Delta|^2+m_2^2|\overline{\Delta}|^2$, etc., where these supersymmetry breaking mass parameters are all in the TeV range. We have ignored the Planck scale $B-L$ breaking terms since they are small and not relevant to the discussion of inflation given in the next section. The minimum of this potential corresponds to $\langle\Delta\rangle=\langle\overline{\Delta}\rangle=v_{BL}$.

[1]For instance one could have a local $B-L$ model like a supersymmetric model based on the gauge group $SU(2)_L\times U(1)_{I_{3R}}\times U(1)_{B-L}$ and have local $B-L$ symmetry be broken by a multiplet $(\Sigma)$ with $B-L\geq 6$. The effective low energy theory in this case has a global U(1) symmetry which behaves like global $B-L$. One can construct a D-term inflation [12] model based on this model with a pair of $\Delta$ and $\Sigma$ fields (and their conjugates), if we choose a form for the superpotential $W=\lambda S(\Delta\overline{\Delta}-M_1^2)+\lambda_1 X\Sigma\overline{\Sigma}$ and a Fayet-Illiopoulos term for the gauged $U(1)_{B-L}$. We do not elaborate on this model here. It has all the properties of the model presented here, with somewhat different parameters.

Once the $B-L$ symmetry is broken, this leads to the seesaw mechanism for small neutrino masses. Typically, a neutrino mass has the form $m_\nu\simeq m_{\nu^D}^2/M_R$, where $m_{\nu^D}$ is of the same order as a typical fermion mass of the standard model and $M_R=fv_{BL}$ with $f\simeq 1$. If we assume that the atmospheric neutrino oscillation is linked to the third generation quarks (as would it be plausible in a quark-lepton symmetric theory), then the Dirac mass would be about 100 GeV and this would give $v_{BL}\simeq 10^{15}$ GeV, which is close to the value preferred by the inflation picture discussed below.

Because the vevs of $\Delta$ and $\overline{\Delta}$ break the global $B-L$ symmetry, in the absence of explicit $B-L$ breaking terms (denoted by $V_{Planck}=0$), we have a massless particle in the theory, the Majoron, given by $\phi\equiv(\chi-\overline{\chi})/\sqrt{2}$, where we have parameterized $\Delta=(1/\sqrt{2})(v_{BL}+\rho)e^{i\chi/v_{BL}}$ and $\overline{\Delta}=(1/\sqrt{2})(v_{BL}+\overline{\rho})e^{i\overline{\chi}/v_{BL}}$. The potential for $\phi$ is flat due to the shift symmetry under which $\phi\to\phi+\alpha$. Once the Planck scale terms are turned on, the potential $V(\phi)$ loses its flatness and can generate a rolling behavior for the $\phi$ field as we see below.

### A. Inflation

This model has the ability to generate inflation in early stages of the Universe. This comes about due to the interplay among the fields $S$ and $\Delta$ and $\overline{\Delta}$. The potential involving them is given by Eq. (2), which has the form required in the hybrid inflation scenario. To see this note that for, $S\gg v_{BL}$, the minimum of the potential corresponds to $\Delta=\overline{\Delta}=0$. We assume that $\lambda(v_{BL}/M_{P\ell})>(m_S/v_{BL})$. The potential is then dominated by the term $V_0=\lambda^2 v_{BL}^4$ and the Universe undergoes an inflationary phase. The $S$ field keeps rolling towards the potential minimum and inflation ends when the field $S$ reaches the value $S\simeq v_{BL}$. However, it is necessary for the potential to have small tilt to drive the field toward its global minimum. This is the hybrid inflation picture [7–10].

One way to generate a slope along the inflationary trajectory (i.e., the S direction) is to include the one loop radiative correction to the tree level potential [8]. This arises because supersymmetry is broken during inflation and there will be a mass splitting between the components in the chiral multiplet. The effective one loop radiative correction is

$$\Delta V=\sum_i \frac{1}{64\pi^2}(-1)^{2J_i}(2J_i+1)M_i^4(S)\ln\left(\frac{M_i^2(S)}{\Lambda_R^2}\right). \tag{5}$$

Where the sum extends over all the spin states $J_i$ with field dependent mass $M_i$ and $\Lambda_R$ is a renormalization scale. The $F_S\equiv\partial W/\partial S\neq 0$ term splits the components of the chiral multiplets $\Delta$ and $\overline{\Delta}$ into a pair of two real scalar fields with masses $m_\pm^2=(\lambda^2/2)(S^2\pm 2v_{BL}^2)$ and a Dirac fermion with mass squared $m_F^2=(\lambda^2/2)S^2$. Since during the inflation $\Delta=\overline{\Delta}=0$, the effective potential reads

$$V_{eff}=\lambda^2\left\{v_{BL}^4+\frac{\lambda^2}{128\pi^2}\left[(S^2-2v_{BL}^2)^2\ln\left(\frac{\lambda^2(S^2-2v_{BL}^2)}{\Lambda^2}\right)+(S^2+2v_{BL}^2)^2\ln\left(\frac{\lambda^2(S^2+2v_{BL}^2)}{\Lambda^2}\right)-2S^4\ln\left(\frac{\lambda^2S^2}{\Lambda^2}\right)\right]\right\}. \tag{6}$$

For $S$ much larger than the $(B-L)$ symmetry breaking scale the above potential becomes

$$V_{eff}\simeq\lambda^2v_{BL}^4\left\{1+\frac{\lambda^2}{16\pi^2}\ln\frac{S}{v_{BL}}\right\}. \tag{7}$$

The log term will provide the driving force for the field $S$ to roll down the potential. In this case the slow roll parameters are given by

$$|\eta|=\frac{1}{2N(S)}\gg\epsilon=\frac{\lambda^2}{32\pi^2}\frac{1}{N(S)}, \tag{8}$$

where $N$ is the number of e-foldings and is given by

$$N(S)=\frac{32\pi^3}{\lambda^2M_{P\ell}^2}S^2. \tag{9}$$

Density perturbations in about $N_{60}=60$ e-foldings before the end of inflation are estimated as

$$\frac{\delta\rho}{\rho}\simeq16\pi\sqrt{\frac{N_{60}}{3}}\left(\frac{v_{BL}}{M_{P\ell}}\right)^2. \tag{10}$$

Fluctuations with amplitude $\sim10^{-5}$ can obtained if $v_{BL}\simeq6\times10^{15}$ GeV.

Another way to generate a slope to the potential is to add soft supersymmetry breaking mass $m_S$ to the inflaton field [9]. In this case the slow roll parameters are given by

$$\epsilon=4\pi\frac{M_{P\ell}^2m_S^2}{V_0}\left(\frac{m_S^2S^2}{V_0}\right),$$

$$\eta=8\pi\frac{M_{P\ell}^2m_S^2}{V_0}\gg\epsilon. \tag{11}$$

The requirement that $\eta\ll1$ (say 0.01) gives

$$\lambda\simeq10^{3/2}\left(\frac{M_{P\ell}}{v_{BL}}\right)\left(\frac{m_S}{v_{BL}}\right). \tag{12}$$

In order for the soft supersymmetry breaking to dominate over the one loop radiative correction the parameter $\lambda$ must be smaller than $3\times10^{-5}$. The value of $S$ after suffering $N_{60}\simeq60$ e-foldings between the horizon exit and the end of inflation is given by

$$S_{60}=v_{BL}\exp(60\eta). \tag{13}$$

The $\Delta$ field plays the role of the inflaton field, which then oscillates and reheats the Universe via the production of neutrinos due to $NN\Delta$ coupling. The density fluctuations lead to strong constraints on the parameters of the model and are given by

$$\frac{\delta\rho}{\rho}\simeq\lambda^3\sqrt{8\pi}\left(\frac{v_{BL}}{M_{P\ell}}\right)^3\left(\frac{v_{BL}}{m_S}\right)^2\exp(-60\eta). \tag{14}$$

For $v_{BL}\simeq2\times10^{14}$ GeV, $m_S\simeq2$ TeV, and $\lambda\sim10^{-4.5}$, we get $\delta\rho/\rho\simeq10^{-5}$. It is interesting that the value for $v_{BL}$ required for understanding neutrino masses also gives the correct order of magnitude for density fluctuations generated by inflation.

Let us now discuss the consistency of our model with the WMAP observations [11]. According to their Table 3 (row $D$), which gives the best fit values for different inflation parameters, the value of $\epsilon$ can be extremely small, which is therefore consistent with the prediction of this model.

Finally, we discuss the question of reheating after inflation ends. The reheating can be assumed to be caused by the decay of the field $S$ when it starts oscillating around the minimum. The reheating temperature is given by the approximate formula $T_R\sim\sqrt{M_{P\ell}\Gamma_S}$ where $\Gamma_S$ is the decay width of the inflaton. To obtain the decay width of $S$, we have to isolate the decay modes. We expect that the $\lambda$ coupling in the renormalizable part of the superpotential will give the dominant contribution to the decay width.

To see this in detail, note that if both $\Delta$ and $\bar{\Delta}$ have equal vacuum expectation values (VEVs),[2] the combination $\psi\equiv(1/\sqrt{2})(\psi_\Delta-\psi_{\bar{\Delta}})$ is the fermionic partner of the Majoron field (denoted by $\phi$) and is therefore light, whereas $(1/\sqrt{2})\times(\psi_\Delta+\psi_{\bar{\Delta}})$ is the fermionic partner of the superheavy supermultiplet and therefore has same mass as $S$ field. However one can see from the superpotential that $S$ has a coupling to $\psi\psi$ and can therefore decay to these states. The decay width is then given by $\Gamma_S\sim(\lambda^2/4\pi)m_S\simeq10^{-14.5}v_{BL}$. Note that the inflaton gets its dominant mass from the $B-L$ breaking VEV $v_{BL}$ and a smaller mass from supersymmetry breaking. Therefore $M_S\simeq v_{BL}$. This leads to a reheating temperature of $T_R\simeq10^9$ GeV. In this case a gravitino with mass $m_{3/2}\simeq100$ GeV does not spoil the success of big bang nucleosynthesis.

It is also worth pointing out that the supergravity (SUGRA) embedding of this model is free of the $\eta$ problem that is generic to F-term SUGRA models [12].

### B. Nonrenormalizable terms, Majoron mass, and Majoron as the ultralight dark matter

In the presence of the nonrenormalizable terms, the Majoron picks up a nonzero mass. If we choose the leading order nonrenormalizable term to be $(H_uH_d)^2(\Delta^2)/M_{P\ell}^3$ (which can be done by a judicious choice of discrete sym-

[2]These VEVs will differ by a small amount proportional to the supersymmetry breaking, which can therefore be neglected for large values of $v_{BL}$ that we are interested in.

metries as discussed above), then we get the Majoron mass $m_\phi^2 \simeq \mu v_{wk}^4 / M_{P\ell}^3 \simeq (10^{-15}\ \mathrm{eV})^2$.[3]

In order to discuss how massive Majorons constitute the dark matter of the Universe, let us discuss the evolution of the Majoron field $\phi$ as the Universe evolves. The Majoron potential, which arises solely from the $B-L$ breaking terms, can be written using Eq. (3) in the form

$$V(\phi) = \lambda_1 \Lambda^4 \left( 1 + \cos \frac{\phi}{v_{BL}} \right), \quad \lambda_1 \simeq 1 \tag{15}$$

where $\lambda_1 \Lambda^4 = m_\phi^2 v_{BL}^2 \simeq (1.7 \times 10^{-3}\ \mathrm{GeV})^4$. We have added a constant term, akin to a cosmological constant, so that after the amplitude for the $\phi$ field oscillation damps to its minimum at $\phi = \pi v_{BL}$ the value of the potential is zero. The field $\phi$ and the radiation energy density $\rho$ satisfy the coupled scalar field–Einstein-Friedman equations

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi) \tag{16}$$

and

$$H^2 = \frac{8\pi G_N}{3} \left[ \rho + \frac{1}{2}\dot{\phi}^2 + V(\phi) \right]. \tag{17}$$

In the early Universe, clearly the $V'$ term is negligible. The Hubble parameter $H$ is then dominated either by the kinetic energy of $\phi$ or the relativistic energy density in radiation. Solving Eq. (16), one finds that $\dot{\phi} \propto R^{-3}$, where $R$ is the scale factor. So regardless of whatever initial $\dot{\phi}$ the field starts out with, it completely damps down to a very small value as the Universe expands. Thus the value of $\phi$ freezes over for most of the early Universe at some random value. When $m_\phi \simeq 3H$, the potential term will dominate Eq. (16) and the field will oscillate around its minimum value of $\phi = \pi v_{BL}$. These oscillations, which do not damp, behave like matter [14] and contribute to the dark matter density. Below, we calculate this contribution and find that for the range of parameters of interest, it gives the right order of magnitude for $\Omega_{matter}$.

As noted already, the value of the $\phi$ field remains frozen at its initial value (taken to be $v_{BL}$) until the epoch when $m_\phi \simeq 3H \simeq 3T_i^2 / M_{P\ell}$ at which time the $\phi$ field starts to oscillate. The temperature at which it does this is given by $T_i \simeq \sqrt{M_{P\ell} m_\phi / 3} \simeq 10^{-3}$ GeV. As it oscillates, it is easy to show that it acts like a pressureless gas and thus can be treated as an ensemble of nonrelativistic particles [14]. The contribution of the Majoron to the energy density now is given by

[3]If this was the only term present then there would be domain walls in the theory since this nonrenormalizable term breaks the symmetry down to a $Z_2$. They would have cosmological implications [13]. However, consistent with our softly broken discrete symmetry we are allowed to write down other terms that break this symmetry completely and we do not have domain walls in the theory.

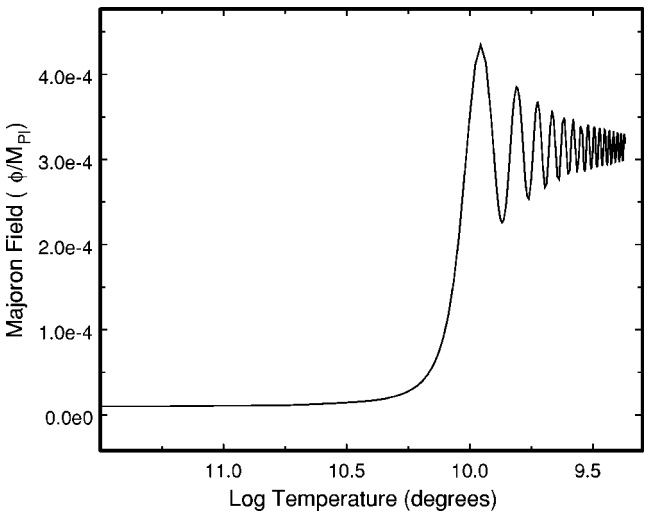


FIG. 1. The evolution of the Majoron field (in units of $M_{Pl}$) as a function of temperature for the initial value of the field $\phi(0) = 10^{-5}$. The field settles down to the value $\phi = \pi v_{BL}$; $v_{BL} = 10^{-4} M_{Pl}$, driven by the first minimum of the potential. The logarithm is with respect to base 10 and $e^{-4}$ in the ordinate stands for $10^{-4}$.

$$\Omega_\phi \simeq \frac{m_\phi^2 v_{BL}^2}{H_0^2 M_{P\ell}^2} \left( \frac{T_0}{T_i} \right)^3 \simeq 0.1. \tag{18}$$

The ratio of the Majoron (dark matter) to the radiation energy density at their equipartition temperature $T_{EQ} \sim 1$ eV is given by

$$\frac{\rho_\phi}{\rho_R}(T_{EQ}) \simeq \frac{\Lambda^4 T_0^4}{\rho_R^{(0)} T_{EQ} T_i^3}, \tag{19}$$

where $T_0 \simeq 2.4 \times 10^{-4}$ eV and $\rho_R^{(0)} \sim 2 \times 10^{-15}\ \mathrm{eV}^4$ are the temperature and the energy density of the photons today. For $\lambda_1 \Lambda^4 \simeq 1\ \mathrm{MeV}^4$ and $T_i \sim 1$ MeV one obtains $(\rho_\phi / \rho_R)(T_{EQ}) \sim 1$. Hence the Universe remains radiation dominated until the temperature drops below $\simeq 1$ eV.

Thus we see that Majoron oscillations can act as a nonrelativistic dark matter with the right order of magnitude for $\Omega_{matter}$. We therefore conclude that the same range of parameters that gave the neutrino mass values in the right order and also the density fluctuations required by cosmic microwave background (CMB) measurements also seems enable the Majoron to play the role of dark matter.

To make these ideas concrete, we have solved numerically the coupled equations (16) and (17) in the regime prior to and after the Majoron field begins oscillating. That is the region in which the Hubble parameter, which includes the sum of radiation, matter and the Majoron field energies, in the second term in the right hand side of Eq. (16), has decreased to the point that the right hand side has become appreciable in comparison with that term. For natural choices of the parameters discussed above ($v_{BL}$ and $v_{WK}$ $10^{-4} M_{P\ell}$ and 100 GeV, respectively) the coefficient $\Lambda^4$ in the Eq. (15) becomes $(10^{-4}\ \mathrm{GeV})^4$. In Figs. 1 and 2, we plot the solution for the Majoron field $\phi$ for two initial $\phi$ values: $\phi / v_{BL}$

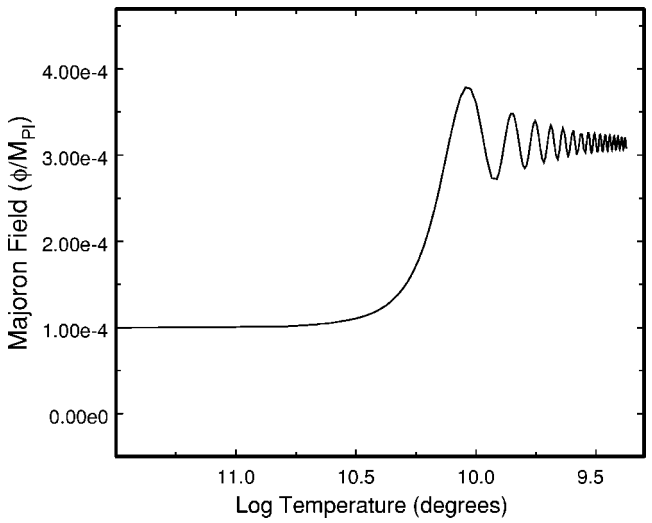


FIG. 2. The evolution of the Majoron field (in units of $M_{Pl}$) as a function of temperature for the initial value of the field $\phi(0) = 10^{-4}$. In this case also, the field settles down to the same value $\phi = \pi v_{BL}$; $v_{BL} = 10^{-4} M_{Pl}$, driven by the first minimum of the potential. The notation is same as in Fig. 1. Note that the amplitude of oscillation is smaller than that in Fig. 1 by about a factor of half.

$= 0.1\pi$ and $\pi/3$. We note that the amplitude of the final field oscillation is less in the second case. For still larger initial values of $\phi$, the amplitude falls off more sharply. On the other hand, both for smaller initial $\phi$ values and over a wide range of $\dot{\phi}$ values, the final result for the Majoron oscillation amplitudes are the same.

In Figs. 3 and 4 we plot the resulting ratio $\rho_\phi/\rho_\gamma$ for the same choice of parameters as in Figs. 1 and 2, respectively, over a broad region in which the oscillatory frequency is not too large compared to the expansion rate so that numerical computations with a reasonable number of steps can be carried out and are reliable. One sees that this ratio increases

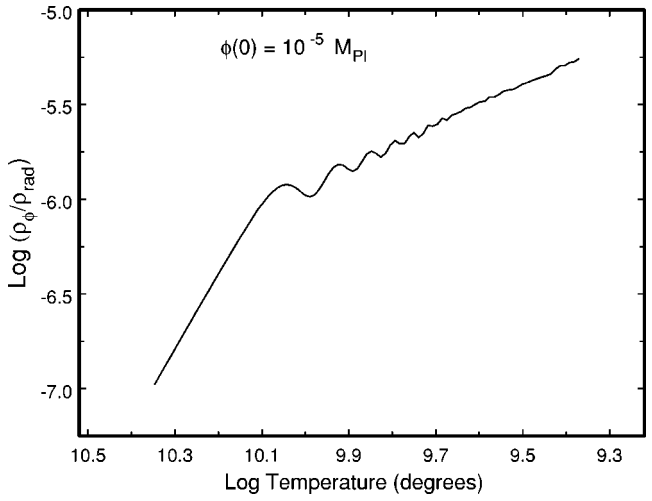


FIG. 3. The ratio of energy density in the field $\phi$, $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$ over the radiation energy density $\rho_{rad}$ as a function of the temperature of the Universe for initial value of $\phi(0) = 10^{-5} M_{Pl}$. After the field begins oscillating it behaves just like matter, as it can be seen from the above ratio which increases like $T$.

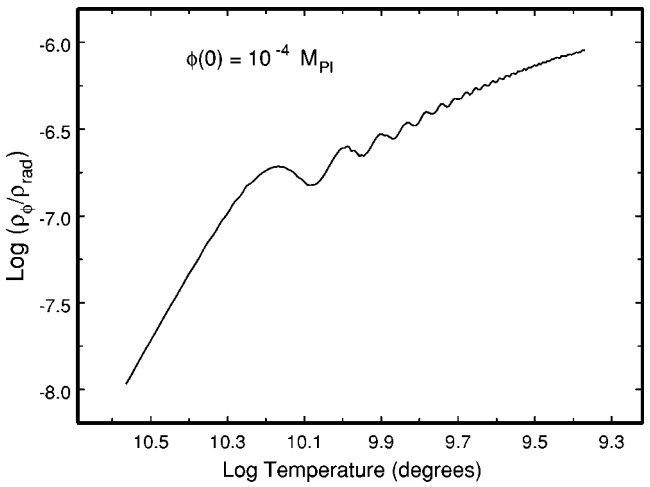


FIG. 4. The ratio of energy density in the field $\phi$, $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$ over the radiation energy density $\rho_{rad}$ as a function of the temperature of the Universe for initial value of $\phi(0) = 10^{-4} M_{Pl}$. After the field begins oscillating it behaves just like matter, as it can be seen from the above ratio which increases like $T$.

linearly with decreasing temperature, as is appropriate for pressureless matter. One can find today's value for $\rho_\phi/\rho_\gamma$ simply by multiplying the values read-off the figure by $T/T_0$. These numerical computations confirm that our parameter choice leads to the correct order of magnitude for today's ratio (that is, oscillating $\phi$ field giving $\Omega_{DM} \sim 0.3$).

Finally we note that the frequency of the Majoron field oscillation is given by $\omega = (\lambda_1 \Lambda^4 / v_{BL}^2)^{1/2} \simeq 0.2\ \mathrm{sec}^{-1}$. The oscillations begin around the epoch of nucleosynthesis. The energy in these oscillations around this epoch is very small compared to that in $\rho_\gamma$ so that it does not affect the BBN considerations. Furthermore it is worth noting that the frequency $\omega$ is independent of the value of $v_{BL}$.

## III. OTHER CONSEQUENCES OF THE MODEL

Some consequences of the superlight Majoron for cosmology has recently been discussed in Ref. [15]. We consider other implications that particularly relate to the model in this paper in this section.

### A. Cosmologically induced Lorentz violation for neutrinos

One important implication of our proposal is that it leads to a cosmologically induced violation of Lorentz invariance. This comes about because in our model the Majoron field has a derivative coupling of the form $(1/v_{BL})\bar{\nu}\gamma_\mu \nu \partial^\mu \phi$. In the late Universe when $\dot{\phi} \neq 0$, this leads to an effective Lorentz violating term of the form $(\dot{\phi}/v_{BL})\nu^\dagger \nu$ in the effective low energy Lagrangian. This effect is Lorentz violating and will manifest itself in the neutrino oscillation process [16]. The maximum value of the parameter $\dot{\phi}/v_{BL}$ in our model is $m_\phi \simeq 10^{-23}$–$10^{-24}$ GeV. However, since the Lorentz violating term is family universal, it will most likely manifest itself in a transition from $\nu_\alpha$ to $\bar{\nu}_\beta$. So the only way to detect this will be to measure $P(\nu_\alpha \to \bar{\nu}_\beta) - P(\bar{\nu}_\beta \to \nu_\alpha)$. In the Lorentz

invariant case, such effects are generally suppressed by the mass of the neutrino [17] and are therefore likely to be very small. The detailed experimental implications of such Lorentz violating effects are currently under investigation.

### B. BBN constraints on Majoron superpartners

A second phenomenological implication is that if we ignore the supersymmetry breaking effects, then Majoron belongs to a supermultiplet together with a scalar partner (to be called sMajoron) and a fermion, Majorino, both of which are massless. In the presence of supersymmetry breaking effects, the sMajoron ($\sigma$) and Majorino ($\psi$) pick up mass in the MeV to GeV range. The precise values of these masses are however constrained by cosmological consideration for the model to be viable. We discuss them in this section.

This question was discussed in Ref. [3] for the case where the $B-L$ breaking scale is in the TeV range. It was found that in that case the masses of $\sigma$ and $\psi$ can be in the TeV range. This holds as long as $v_{BL} \leqslant 10^8$ GeV. In our case, however, $v_{BL} \simeq 10^{15}$ GeV. We will therefore end up with the sMajoron ($\sigma$) and Majorino ($\psi$) masses in the 10 MeV or lower range.

The primary goal is to make sure that the new particles do not affect big bang nucleosynthesis (BBN). Since the couplings of the $\phi$, $\sigma$, and $\psi$ to standard model particles are all suppressed by $v_{BL}$, the lifetimes of $\sigma$ and $\psi$ are much longer than one second, the BBN epoch. Therefore if they are heavy and their abundances at the BBN epoch are not suppressed, then they will have adverse effect on nucleosynthesis. We therefore have to calculate their abundance at the BBN epoch.

Due to the suppressed couplings of $\phi$, $\sigma$, and $\psi$, they will decouple in the very early stage of the Universe. To calculate the decoupling temperature, we note that the typical annihilation rates are for $\phi$, $\sigma$, and $\psi$ are given by $R \simeq T^5/v_{BL}^4$. Using the decoupling condition $R(T_D) < H(T_D)$, we get

$$T_D^3 \simeq g_*^{1/2}(T_D) \frac{v_{BL}^4}{M_{P\ell}}, \tag{20}$$

which leads to $T_D \simeq 10^{-1} v_{BL}$, which is of order $10^{14}$ GeV. After decoupling the $\phi$, $\sigma$, and $\psi$ densities simply dilute due to the expansion of the Universe. Their contribution to the energy density of the Universe at the BBN epoch is given by

$$\frac{\rho_{\phi,\sigma,\psi}}{\rho_\gamma} \simeq \frac{n_{\phi,\sigma,\psi}}{n_\gamma} \frac{m_{\phi,\sigma,\psi}}{T_{BBN}} \simeq \frac{g_*(T_{BBN})}{g_*(T_D)} \frac{m_{\phi,\sigma,\psi}}{T_{BBN}}. \tag{21}$$

Success of BBN requires that $\rho_{\phi,\sigma,\psi}/\rho_\gamma$ should be much less than one. Since $g_*(T_{BBN})/g_*(T_D) \simeq 100$, we therefore must have $m_{\sigma,\psi} \ll 100$ MeV.

We further note that since these particles $\phi, \sigma, \psi$ are singlets under the standard model, they are not in conflict with any known low energy observations despite their small mass.

### C. Implications for leptogenesis

Finally, we wish to comment that in this model one can employ the mechanism of leptogenesis [18] to understand the origin of matter in the Universe. One can have a right handed neutrino [19] at the intermediate scale range of $10^9$ GeV range (or even a pair of them nearly degenerate [20]), whose decay would produce a lepton asymmetry, which via the sphaleron interactions would get converted to baryons. The only new aspect of our model is the presence of the Majoron at very low energies. In principle its interactions can erase the baryon asymmetry since its interactions violate lepton number. However, in our case since the scale $v_{BL}$ is very high, as noted already, the Majorons decouple around $10^{14}$ GeV and are therefore ''impotent'' as far as their effect on lepton asymmetry is concerned. If for instance the $v_{BL}$ scale was in the range below $10^9$ GeV, we would have no leftover lepton asymmetry at the weak scale to be converted to baryons. It is therefore interesting that the high scale of $v_{BL}$ is required from various considerations.

In conclusion, we have presented a simple model for neutrinos using the supersymmetric extension of the singlet Majoron model that provides a unified framework for understanding inflation and dark matter for the same set of parameters required by neutrino masses. We find that the $v_{BL}$ scale is constrained from various considerations to be around the conventional grand unification scale $\sim 10^{15}$ GeV. We have checked our results using a numerical solution of the evolution equation for the Majoron field. The high $v_{BL}$ scale makes the Majoron and its superpartners highly invisible in collider experiments.

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