Magnetic moment of a massive neutrino due to its pion cloud

R. Tegen[1,2]* and H. G. Miller[1]
[1]*Department of Physics, University of Pretoria, Pretoria 0001, South Africa*
[2]*Institut für Theoretische Physik, JWG-Universität Frankfurt, Robert-Mayer-Strasse 8-10, D-60054 Frankfurt, Germany*
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In this work we have calculated the magnetic moment μ_ν of a massive neutrino, to one-pion loop order. If m_ν varies between ~ 3 meV and ~ 3 eV and m_π between 0 and ~ 140 MeV one obtains values for μ_ν which can compete with the Standard Model result in the chiral limit $m_\pi \to 0$. The photon coupling to the $\pi\pi$-intermediate state leads to the presence of "chiral singularities" [m_π^{-1}- and $\log(m_\pi)$-terms]. There are no such terms for the $W^- W^+$-intermediate state of the Standard Model. As a result μ_ν is particularly sensitive to the mass ratios $(m_\pi/m_\ell, m_{\nu_\ell}/m_\ell)$; this indicates to us that all electromagnetic properties of neutrinos ($\mu_\nu, \langle r^2\rangle$, etc.) are expected to change significantly in a dense or hot medium. Such medium modifications of $\mu_\nu, \langle r^2\rangle$, etc. are entirely due to the $\pi\pi$-intermediate state which acts as a "doorway" state for the photon-neutrino coupling.

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I. INTRODUCTION

There is renewed interest in the electromagnetic properties of massive neutrinos. This interest stems from the recent concrete evidence that neutrinos oscillate with one relatively large mixing angle and one mass difference in the several meV-range [1]. Just like quarks (d', s', b') mix (GIM mechanism) in order to avoid flavor-changing neutral currents (for example $\overline{K^0} \to \mu^- \mu^+$) the reason for neutrino $(\nu_e', \nu_\mu', \nu_\tau')$ mixing might be the need to suppress family changing (B-L) currents (for example $p \to \overline{\nu_e} K^+$) in Grand Unified Theories (GUT), see discussion in [2]. It is important to note that oscillation experiments do not settle the question of neutrino masses. If the neutrino mass spectrum is not hierarchical but nearly degenerate, one absolute mass determination is still necessary. Such experiments are typically 3-body decays; see Ref. [3] for several examples with explicit calculations. Only triton and free-neutron decay would be sensitive enough for a neutrino mass of 1–3 eV. This is the upper limit found by a comprehensive analysis of several experiments, which put the sum of masses of all neutrino species in the range between 0.05 eV and 8.4 eV [4]. Preliminary results on the Majorana versus Dirac question by the Heidelberg-Moscow Collaboration report evidence for neutrinoless double-beta decay of ^{76}Ge and fix the effective neutrino mass in the region [5] 0.05–0.84 eV. If confirmed by an independent experiment this method would be by far the most sensitive direct (Majorana) neutrino mass determination.

In the realm of astrophysics even a small magnetic moment and/or a nonvanishing charge radius $\langle r_E^2 \rangle$ will have an influence on the elastic and inelastic neutrino scattering cross sections, σ, and on the so-called Urca process in neutron stars, $N+N \to N+N+\nu+\overline{\nu}$ [6,7]. Multiplied with the enormous density ϱ prevailing in neutron stars, the resultant mean free path $\lambda = 1/(\varrho\sigma)$ could become significantly altered [8] to have an effect (for example) on the neutrino emissivity of neutron stars. Recently neutrino trapping in supernovae near the proto-neutron star surface has been discussed (see for example Ref. [9]).

In Elementary Particle Physics evidence for nontrivial electromagnetic form factors could help to decide the crucial question "Majorana" versus "Dirac" neutrino. From the SN1987a neutrino flux upper limits have been established [10],

$$\mu_{\nu_e} < 1.5 \times 10^{-10} \mu_B,$$

$$\mu_{\nu_\mu} < 6.8 \times 10^{-10} \mu_B,$$

$$\mu_{\nu_\tau} < 3.9 \times 10^{-7} \mu_B \tag{1}$$

and [15]

$$|\langle r^2 \rangle_{\nu_e}| \leq (10^{-3}\ \text{fm})^2. \tag{2}$$

Here $\mu_B = e/2m_e$ is the Bohr magneton. Both μ_ν and $\langle r^2\rangle_\nu$ must vanish for a purely chiral massless Dirac neutrino. A Majorana neutrino requires $G_{E,M}^{(\nu_l^M)}(q^2) \equiv 0$ (in particular $\mu_{(\nu_M)} = 0$ and $\langle r^2 \rangle_{\nu_M}^{1/2} = 0$), irrespective of the neutrino mass, as in the case of the vanishing of the π^0 ($=\overline{\pi^0}$) and the photon γ ($=\overline{\gamma}$) form factors. In this paper we assume a massive Dirac neutrino and calculate its magnetic moment to one-loop order.

First one notes that a massive Dirac neutrino requires

$$\mu_\nu \sim m_\nu, \varepsilon \tag{3}$$

where ε measures possible deviations from the pure V-A structure of electroweak currents (ε is of order 10^{-3} or smaller [10]). For the standard $SU(2)\times U(1)$ electroweak theory (i.e. $\varepsilon = 0$) extended to include massive neutrinos one finds [11] [the contribution from the $(l^- W^+)$-intermediate state, Fig. 1, has been calculated],

*Electronic address: tegen@th.physik.uni-frankfurt.de

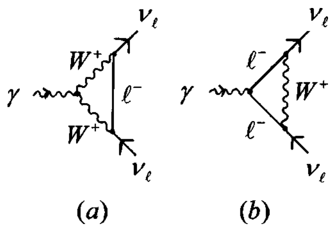

FIG. 1. Neutrino electromagnetic vertex; in the loop a charged (W^+l^-)-pair propagates, and the photon couples to each of them, (a) and (b).

$$\mu_{\nu_e}=\frac{e}{2m_\nu}\left(\frac{3G_F m_W^2}{8\pi^2\sqrt{2}}\right)\frac{m_\nu^2}{m_W^2}=3.2\times10^{-19}(m_\nu/eV)\mu_B \quad (4)$$

much below present upper limits given in Eq. (1). The term in large parentheses can be written as $3g^2/64\pi^2$, where $e=g\sin\theta_W$ and θ_W is the Weinberg-angle. This result [Eq. (4)] is obtained to one-loop order (see Fig. 1) and to leading order in m_e^2/m_W^2. It confirms Eq. (3). This relation deserves more comment. According to Eq. (3) the magnetic moment of a neutral *and* massless particle has to vanish. While the charge distribution of a neutral but extended particle (like the neutron, K^0, etc.) as a function of the distance from the center changes sign (in the case of the more familiar neutron it is a proton-like core surrounded by a negative pion cloud) the mass distribution remains strictly positive. Therefore, in the limit of vanishing mass a particle has to become pointlike. Hence taking the limit $m_\nu\to0$ requires the shrinking of the (W^+l^-) (see Fig. 1) and (π^+l^-) loops (see Fig. 2) in such a way that the magnetic moment vanishes. Furthermore, for $m_\nu\to0$ the neutrino has only one (negative) helicity component, hence $\varepsilon\to0$ in that limit, as indicated in Eq. (3). We will return to this important point further below.

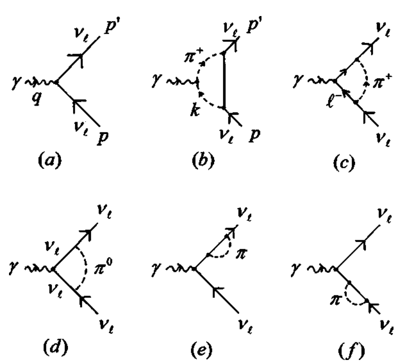

FIG. 2. Electromagnetic coupling of a photon γ with 4-momentum $q=p'-p$ to the neutrino species ν_ℓ ($\ell=e^-,\mu^-,\tau^-$) of 4-momentum p' (final) and p (initial), with a charged $(\pi^+\ell^-)$-pair propagating in the loop. (a) The point-coupling to the neutral neutrino vanishes; (b) the photon couples to the π^+, with the ℓ^- a "spectator" (this diagram contains chiral singularities, see text); (c) the photon couples to the negatively charged lepton ℓ^-, with the π^+ a "spectator" (no chiral singularities, see text); (d) to first order this diagram vanishes [just like (a)]; (e),(f) renormalization of the neutrino mass; it vanishes just like (a) and (d). In the loop a π^+ or a π^0 can propagate in which case a ℓ^- or a neutrino, respectively, would accompany the pion.

Values in Eq. (4) are far below present experimental sensitivities given in Eq. (1) even if one of the neutrinos has a mass as large as 2–3 eV. Diagram (a) and (b) in Fig. 1, however, is not the only contributing intermediate state (to one-loop order). Another, much lighter candidate, apart from supersymmetric particles perhaps, would be the (π^+l^-)-intermediate state (see Fig. 2). There are several reasons which make this pion loop interesting:

(i) The pion mass is much below the W-mass and is known to deviate quite a bit from its vacuum value in a dense and/or hot medium.

(ii) The analogous pion loop contribution to the *neutron* electromagnetic form factors contains chiral singularities [i.e., m_π^{-1} and $\log(m_\pi)$-terms] which dramatically increase the radii for $m_\pi\to0$—we confirm the presence of chiral singularities in the form factors $\Gamma_{1,3}(q^2)$ due to diagram Fig. 2(b).

(iii) Form factor results depend on two masses (m_π,m_e) apart from m_ν; both could be affected by medium effects. Only a full calculation can display the magnitude of this mass dependence.

We have performed a calculation of form factors $G_{E,M}^{(\nu_e)}(q^2)$ to one-pion-loop order (Fig. 2) without any approximations. Here we report our findings for the neutrino magnetic moment $\mu_\nu=(e/2m_\nu)G_M^{(\nu)}(0)$ which is of prime interest due to the peculiar mass dependence [see Eqs. (3), (4)]. Our calculation of all form factors $G_{E,M}^{(\nu_l)}(q^2)$ with their q^2-dependence will be reported elsewhere. Their knowledge in space- and timelike regions of q^2 will be important for accurate calculations of elastic neutrino scattering (via the now possible one-photon exchange) and for the Urca process [using the timelike region for $G_{E,M}^{(\nu_l)}(q^2)$]. The paper is organized as follows: In Sec. II various electromagnetic form factors are defined and related to the magnetic moment and charge. Section III gives details of the gauge and chirally invariant one-loop calculations resulting in form factors $\Gamma_{1,2,3}(q^2)$, which are related to $G_{E,M}^{(\nu_l)}(q^2)$. In Sec. IV we present our numerical results and summarize our findings. Technical details of our calculation are relegated to Appendix A.

II. LEPTON ELECTROMAGNETIC FORM FACTORS

Quite generally the electromagnetic current for a pointlike spin-$\frac{1}{2}$ particle is given by

$$e\overline{u(p')}Q\gamma_\mu u(p), \quad (5)$$

where Q is the particle charge in units of the elementary charge $e=\sqrt{4\pi\alpha}$. Applying Eq. (5) to the neutrino ν_l, ($l=e,\mu,\tau$), yields a zero result due to $Q_{\nu_l}=0$ for all l. The presence of the meson cloud due to the weak $\pi l\overline{\nu_l}$-coupling [see Figs. 2(b)–2(f)] complicates the structure of the neutrino electromagnetic current, so that instead of Eq. (5) one has the general form [12]

$$e\overline{u(p')}\left[\frac{p_\mu}{m_\nu}\Gamma_1(q^2)+\frac{p'_\mu}{m_\nu}\Gamma_2(q^2)+\gamma_\mu\Gamma_3(q^2)\right]u(p), \quad (6)$$

where $q_\mu = p'_\mu - p_\mu$. Note that a point-coupling as in Eq. (5) will contribute *only* to $\Gamma_3(q^2)$, see also Eq. (9b) below.

While the spin-1/2 and spin-1 content of the three-field coupling, Eq. (6), gives in general three independent Dirac-scalar form factors $\Gamma_{1,2,3}(q^2)$, the requirement of gauge invariance of the spin-1 electromagnetic current reduces them to two,

$$\Gamma_1(q^2) \equiv \Gamma_2(q^2), \tag{7}$$

but does not restrict $\Gamma_3(q^2)$ due to

$$q^\mu \overline{u(p')} \gamma_\mu u(p) = \overline{u(p')}(m_\nu - m_\nu) u(p) = 0 \tag{8}$$

which follows straight from the free Dirac equation. In Eq. (6) m_l is the mass of the lepton l^- associated with the neutrino ν_l of mass m_ν, within the same lepton family [16]. The two independent form factors $\Gamma_{1,3}(q^2)$ are related to the "Sachs" form factors, $G_{E,M}(q^2)$, and to the "Dirac," $F_1(q^2)$, and "Pauli," $F_2(q^2)$, form factors ($\eta = -q^2/4m_\nu^2 \geqslant 0$)

$$\Gamma_1(q^2) \equiv \Gamma_2(q^2) = \frac{1}{2(1+\eta)}[G_E(q^2) - G_M(q^2)]$$
$$= -\frac{1}{2} F_2(q^2)$$
$$\Gamma_3(q^2) = G_M(q^2) = F_1(q^2) + F_2(q^2). \tag{9}$$

G_E is then given by

$$G_E(q^2) = 2(1+\eta)\Gamma_1(q^2) + \Gamma_3(q^2)$$
$$= F_1(q^2) - \eta F_2(q^2). \tag{10}$$

To first order the purely electromagnetic components [see Figs. 2(a) and 2(d)–2(f)] give vanishing contributions to $G_{E,M}^{(\nu)}$ (which we limit here to Fig. 2).

If Z_l is the bare charge of lepton l^- [appearing in Fig. 2(c)] we find

$$\Gamma_1^{(\nu)}(q^2) = \Gamma_1^{(2b)}(q^2) + Z_l \Gamma_1^{(2c)}(q^2),$$
$$\Gamma_3^{(\nu)}(q^2) = \Gamma_3^{(2b)}(q^2) + Z_l \Gamma_3^{(2c)}(q^2). \tag{11}$$

The normalization is then

$$F_1^{(\nu)}(0) = 0 = 2\Gamma_1^{(\nu)}(0) + \Gamma_3^{(\nu)}(0),$$
$$F_2^{(\nu)}(0) = -2\Gamma_1^{(\nu)}(0) = G_M^{(\nu)}(0) = r_\nu \frac{\mu_\nu}{\mu_B}, \tag{12}$$

with $\mu_B = e/2m_e$, $r_\nu \equiv m_\nu / m_e$, and μ_ν the magnetic moment of the neutrino. Inspection of Eqs. (11) and (12) reveals that Z_l is given by $Z_l = -[2\Gamma_1^{(2b)}(0) + \Gamma_3^{(2b)}(0)]/[2\Gamma_1^{(2c)}(0) + \Gamma_3^{(2c)}(0)]$.

III. ONE-MESON LOOP CONTRIBUTION TO $G_{E,M}(Q^2)$

We proceed now with a calculation of one-meson loop contributions, Figs. 2(a)–2(c) (to avoid cluttering of indices we use subscript ν for neutrino ν_l throughout),

$$\text{Fig. 2(a):}\quad e\overline{u_\nu(p')} Q_\nu \gamma_\mu u_\nu(p) = 0,$$

$$\text{Fig. 2(b):}\quad -\left(-i\frac{G_F F_\pi}{\sqrt{2}}\right)^2 \int \frac{d^4k}{(2\pi)^4} \overline{u_\nu(p')}(\not{k}+\not{q})(1-\kappa\gamma_5)$$
$$\times \frac{i}{\not{p}-\not{k}-m_l+i\varepsilon} \not{k}(1-\kappa\gamma_5) u_\nu(p) \frac{i}{k^2-m_\pi^2+i\varepsilon} e(2k_\mu+q_\mu) \frac{i}{(k+q)^2-m_\pi^2+i\varepsilon},$$

$$\text{Fig. 2(c):}\quad \left(-i\frac{G_F F_\pi}{\sqrt{2}}\right)^2 \int \frac{d^4k}{(2\pi)^4} \overline{u_\nu(p')} \not{k}(1-\kappa\gamma_5)$$
$$\times \frac{i}{\not{p}'-\not{k}-m_l+i\varepsilon}(-e)\gamma_\mu \frac{i}{\not{p}-\not{k}-m_l+i\varepsilon} \not{k}(1-\kappa\gamma_5) u_\nu(p) \frac{i}{k^2-m_\pi^2+i\varepsilon}, \tag{13}$$

where $\kappa \equiv 1+\varepsilon$ is assumed real, $G_F = 1.16637(1) \times 10^{-5}$ GeV^{-2} is the Fermi constant [10], and F_π appears in the invariant amplitude $\mathcal{M}$ describing charged pion decay ($P \equiv p+k$, $l=e$ or μ here),

$$\mathcal{M}=\frac{G_F F_\pi}{\sqrt{2}} i P^\mu \overline{u_l(p)}\gamma_\mu(1-\kappa\gamma_5)v_{\overline{\nu_l}}(k) \tag{14}$$

so that $F_\pi \equiv \sqrt{2} f_\pi$ is determined by comparison with the measured pion mean-life [10]

$$\begin{aligned}\frac{1}{\tau_{\pi^-}}&=\Gamma_{\pi^-\to l^-\overline{\nu_l}}=\frac{G_F^2F_\pi^2m_\pi^3}{8\pi}t_l^2(1-t_l^2)^2\left[1+\varepsilon\left(1+\frac{2t_\nu}{t_l(1-t_l^2)}\right)+\mathcal{O}(\varepsilon^2,t_\nu^2)\right]\\&=[2.6033(5)\times10^{-8}\ \ \mathrm{s}]^{-1}\end{aligned} \tag{15}$$

for $l=\mu$. One finds $F_\pi=0.92\ m_\pi$. In Eqs. (13), (14), $\varepsilon=\kappa-1$ describes deviations from the pure $(V\text{-}A)$ form of the leptonic current in $\mathcal{M}$, $t_l\equiv m_l/m_\pi\equiv r_\pi^{-1}$, and $t_\nu=m_\nu/m_\pi\equiv r_\nu/r_\pi$ in terms of the parameters r_i, as defined in Appendix A. The loop integrations, Eq. (13), are described in detail in Appendix A. We obtain as the main result $\Gamma_3^{(2b)}(0)$ and $\Gamma_3^{(2c)}(0)$ in Eqs. (A8) and (A11), respectively. $\Gamma_3^{(\nu)}(0)$ is then the combination given in Eq. (11), so that we obtain the magnetic moment in the form

$$\mu_\nu\equiv\frac{e}{2m_\nu}\Gamma_3^{(\nu)}(0)=\frac{e}{2m_\nu}\left(\mu^{(1)}\frac{m_\nu^2}{m_l^2}+\mu^{(2)}\varepsilon\frac{m_\nu}{m_l}+\mu^{(3)}\right) \tag{16}$$

where

$$\begin{aligned}\mu^{(1)}&=\frac{G_F^2F_\pi^2m_l^2}{(4\pi)^2}\,\frac{(1+\kappa)^2+\dfrac{b^2}{a^2}(1-\kappa)^2}{1+b^2/a^2}\int_0^1 dx\int_0^{1-x}dy\\&\times\left[g\left(\frac{c_b}{\lambda^2}\right)\frac{1}{2}[1+(x+y)^2]-3Z_l g\left(\frac{c_c}{\lambda^2}\right)\left(\frac{1}{2}+(x+y)^2-2(x+y)\right)\right],\\\mu^{(2)}&=\frac{G_F^2F_\pi^2m_l^2}{(4\pi)^2}2\int_0^1dx\int_0^{1-x}dy\,g\left(\frac{c_b}{\lambda^2}\right),\\\mu^{(3)}&=\frac{G_F^2F_\pi^2m_l^2}{(4\pi)^2}\,\frac{(1+\kappa)^2+\dfrac{b^2}{a^2}(1-\kappa)^2}{1+b^2/a^2}\int_0^1 dx\int_0^{1-x}dy\\&\times\left\{-\frac{\lambda^2}{2}f\left(\frac{c_b}{\lambda^2}\right)+Z_l\left[\lambda^2 f\left(\frac{c_c}{\lambda^2}\right)+\frac{1}{2}g\left(\frac{c_c}{\lambda^2}\right)\right]\right\}.\end{aligned} \tag{17}$$

Let us consider the limit $m_\nu\to0$ in Eq. (16). The first term $\sim\mu^{(1)}$ in Eq. (16) is $\sim m_\nu$ whereas the second term $\sim\mu^{(2)}$ is $\sim\varepsilon$. Hence both trivially vanish in the limit $m_\nu\to0$. The third term $\sim\mu^{(3)}$ also vanishes for $m_\nu\to0$ but in a nontrivial manner. We recall that for $m_\nu\to0$ the loop has to shrink to a point. This, of course, would be the case for the "trivial" shrinking $\lambda\to0$. There is, however, a more physical shrinking which takes the limit $m_\pi\to\infty$ in the (π^+l^-)-loop. This would mean that $c_b/\lambda^2\to\infty$ and $c_c/\lambda^2\to\infty$, in which case $f(x)\geqslant g(x)$ would vanish, $\lim_{x\to\infty}f(x)=0$ [see the explicit form of f and g after Eq. (A8)].

For a comparison between Eq. (16) and the Standard Model result (4) we notice that only the first term is present in Eq. (4) due to $\varepsilon=0$ and the sensible restriction to the leading order in m_e^2/m_W^2. The large W-mass in Eq. (4) is replaced with m_ℓ in Eq. (16) in compliance with the analogy between Figs. 1(a) and 1(b) and Figs. 2(b) and 2(c). Close inspection of $\mu^{(1)}$ in Eq. (16) and the corresponding term in Eq. (4) reveals that essentially $g=e/\sin\theta_W$ in Eq. (4) is replaced with $(G_F F_\pi m_\ell)$ in the first term of Eq. (16). Unless the double-integral is very large, $\mu^{(1)}$ will lead to generally much smaller results than Eq. (4). The same holds true for the second term (which is limited by the empirical smallness of ε), see our numerical analysis in the next section. The third term in Eq. (16) which has no analogon in the approximated result (4) shows a peculiar m_ν-dependence and is even for the vacuum values of (m_π,m_ℓ) sizeable if m_ν exceeds the meV-range. All three terms display an interesting dynamical enhancement mechanism not present in Eq. (4). The photon coupling to the $\pi\pi$-intermediate state leads to chiral singularities (m_π^{-1} and $\log m_\pi$-terms) not present in the Standard Model result which is derived from diagrams Figs. 1(a) and 1(b), and is free of terms m_W^{-1} or $\log m_W$ for $m_W\to0$. Hence the enhancement is not a direct result of the different

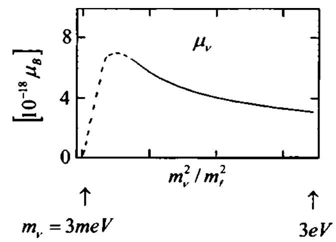

FIG. 3. Magnetic moment as a function of neutrino mass, in units of $10^{-18}\mu_B$. $\Lambda_E=1$ GeV, $m_\pi=140$ MeV. The dotted line indicates the limiting procedure $m_\pi\to\infty$ as $m_\nu\to 0$, see text. The Standard Model result (4) is (in units of $10^{-18}\mu_B$) ≈ 1 for $m_\nu=3$ eV and $\approx 10^{-3}$ for $m_\nu=3$ meV.

boson propagators in the loops (Figs. 1 and 2) but a result of the peculiar photon-boson and boson-fermion couplings present in Figs. 1(a) and 1(b) and Figs. 2(b) and 2(c), respectively. For $m_\pi\to 0$ the underlying chiral symmetry is exact and the photon-nucleon [12], photon-pion [13], and photon-neutrino (this work) coupling goes preferably via the $\pi\pi$-intermediate (''doorway'') state. Particle properties in vacuum are not affected as the empirical pion mass is far from zero. Here is where medium effects come in. Those are known to affect masses; relatively large effects have been reported for the pion and nucleon masses. Medium effects on m_W are generally expected to be much smaller than those for m_π. Hence medium effects should selectively modify electromagnetic properties of neutrinos (like $\mu_\nu,\langle r^2\rangle$, etc.) due to diagrams Figs. 1(a) and 1(b) (small effect is expected) and diagrams Figs. 2(b) and 2(c) (drastic effects possible).

In the next section we present our numerical results for various values of $(\lambda, m_\pi, m_l, m_\nu)$.

IV. RESULTS AND DISCUSSION

The loop integration requires a cutoff $\Lambda_E=\lambda m_\ell$. In compliance with the analogous one-loop calculation [12] for the neutron form factor we choose the same cutoff $\Lambda_E=1$ GeV. Next we observe in Eqs. (16) and (17) that $\mu^{(1)}$ and $\mu^{(3)}$ also contain ε-terms, $(1+\kappa)^2=(2+\varepsilon)^2=4+4\varepsilon+O(\varepsilon^2),(1-\kappa)^2=\varepsilon^2$. To order ε ($\varepsilon\leqslant 10^{-3}$) there are no $(b^2/a^2)(1-\kappa)^2$-terms in $\mu^{(1,3)}$. In the first term $\sim\mu^{(1)}$ in Eq. (16) the term $\varepsilon(m_\nu^2/m^2\ell)$ is negligible compared with the leading term. In $\mu^{(3)}$, $\varepsilon\ll 1$ can be neglected. The only numerically noticeable dependence on b/a stems from the normalization $1/(1+b^2/a^2)$. Remarkably, $\mu^{(2)}$ does not depend on b/a. In our numerical results below we will assume $b\ll a$ throughout. Other values can easily be implemented using Eq. (17). The actual values b/a have to be supplied by measurements of the helicity ''profile'' of a given ''beam'' of neutrinos. For $m_\nu\ll m_\ell$, $m_\pi=140$ MeV (i.e., the vacuum values) it is easy to see that $\mu^{(3)}$ dominates μ_ν. In Fig. 3 we display the neutrino mass dependence of μ_ν for $m_\nu\in[3\text{ meV},3\text{ eV}]$. μ_ν (solid curve) is dominated by the third term in Eq. (16) which is positive; in units of μ_B the first term is negative and of order $10^{-25}-10^{-28}$; the second term is negative as well and of order 10^{-23} over the neutrino mass range displayed here. For $m_\nu\leqslant 3$ eV our (positive) result is comparable with the Standard Model result (4). If m_ν turns out to be in the meV-range the result depends on unknown details of $m_\pi\to\infty$ (shrinking of loop when the neutrino becomes pointlike for $m_\nu=0$, see dashed curve in Fig. 3).

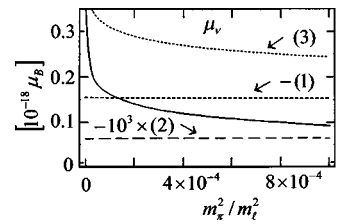

FIG. 4. Magnetic moment as a function of the pion mass, in units of $10^{-18}\mu_B$. In the vacuum $m_\pi^2/m_\ell^2\approx 10^5$ and outside the displayed range. $\Lambda_E=1$ GeV as in Fig. 3, but now $m_\nu/m_\ell=1$ due to medium effects. For $m_\pi=0$ (''chiral restoration in the medium''), μ_ν diverges due to the presence of chiral singularities m_π^{-1} and $\log(m_\pi)$, see text. Solid line: sum of all three contributions, Eq. (16); dashed curve: $-\mu^{(1)}m_\nu/m_\ell$; long-dashed curve: $-10^3\mu^{(2)}\varepsilon$ ($\varepsilon=10^{-3}$ assumed); dotted curve: $\mu^{(3)}/r_\nu$. Both the second and third contribution in Eq. (16) are negative. Note that the second contribution is enhanced by 10^3.

The striking analogy with the one-loop contributions to the neutron form factors $G_{E,M}^n(q^2)$ suggests to first take the limit $m_\ell\to m_\nu$ and then consider the limit $m_\pi\to 0$. The anticipated terms m_π^{-1} and $\log(m_\pi)$ do indeed appear for $m_\nu/m_\ell\to 1$ for m_π near zero. This is displayed in Fig. 4. Note that there are no terms m_W^{-1} or $\log m_W$ (for $m_W\to 0$) in the Standard Model result for μ_ν. This is a peculiarity of the $\pi\pi$-intermediate state and its coupling to the photon (a consequence of the underlying chiral symmetry). We emphasize that enhancement effects (over and above the vacuum value for μ_ν) appear only once the mass ratios m_π/m_ℓ and m_ν/m_ℓ are drastically modified due to the pions interaction with a dense or hot medium.

There is some controversy over the medium corrections on m_π. Our results indicate that, should m_π *increase* due to medium effects, all electromagnetic effects $G_{E,M}^{(\nu)}(q^2)$ would become much weaker than the Standard Model contributions given in Eq. (4). There is, however, the possibility that some medium effects (dense and/or hot) conspire to *lower* m_π (NJL-type models consistently *lower* m_π, see [14]). In that case neutrino *electromagnetic* processes compete with weak processes, Eq. (4), and have to be taken into account when neutrino elastic (and inelastic) scattering is considered in neutron stars, etc., resulting in a much shorter mean free path in layers where the medium-corrected pion mass approaches the chiral limit.

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APPENDIX: EVALUATION OF LOOP INTEGRALS

First we calculate the matrix element in Fig. (2b), Eq. (13),

$$\begin{aligned}&\overline{u_\nu(p')}(\not k+\not q)(1-\kappa\gamma_5)(\not p-\not k+m_l)\not k(1-\kappa\gamma_5)u_\nu(p)\\&=(1+\kappa^2)\overline{u_\nu(p')}(\not k+\not q)(\not p-\not k)\not k\left(1-\frac{2\kappa}{1+\kappa^2}\gamma_5\right)u_\nu(p)\\&+(1-\kappa^2)m_l\overline{u_\nu(p')}(\not k+\not q)\not k u_\nu(p).\end{aligned}\tag{A1}$$

In general $u_\nu(p)$ is a superposition of both helicity states $u_\pm$, for which $\gamma_5 u_\pm=\pm u_\pm$,

$$u_\nu=\frac{1}{\sqrt{a^2+b^2}}(au_-+bu_+).\tag{A2}$$

It is easy to see that "crossed" terms $\overline{u}_-\cdots u_+$ and $\overline{u}_+\cdots u_-$ in Eq. (A1) vanish. For diagonal terms one finds

$$\begin{aligned}\text{Eq. (A1)}=&(a^2\{\overline{u}_-\not k u_-[(1+\kappa)^2(2k\cdot p-k^2-2m_\nu^2)+2m_lm_\nu(1-\kappa^2)]+\overline{u}_-u_-m_l(k^2-2k\cdot p)[(1-\kappa^2)-r_\nu(1+\kappa)^2]\}\\&+b^2\{\overline{u}_+\not k u_+[(1-\kappa)^2(2k\cdot p-k^2-2m_\nu^2)+2m_lm_\nu(1-\kappa^2)]+\overline{u}_+u_+m_l(k^2-2k\cdot p)\\&\times[(1-\kappa^2)-r_\nu(1-\kappa)^2]\})/(a^2+b^2)\end{aligned}\tag{A3}$$

and $r_\nu=m_\nu/m_l$, $1+\kappa\equiv 2+\varepsilon, 1-\kappa^2=-2\varepsilon+\mathcal{O}(\varepsilon^2)$.

Using the Feynman parametrization for products in denominators [see for example Eq. (B1) of Ref. [12]] one finds

$$\begin{aligned}(13\mathrm{b})=&-ieG_F^2F_\pi^2\int_0^1dx\int_0^{1-x}dy\int\frac{d^4k}{(2\pi)^4}\frac{2k_\mu+q_\mu}{[(k+l)^2-m_l^2c_b(x,y,\eta)+i\varepsilon]^3}[a^2\{\overline{u}_-\not k u_-P_1(k,p,\kappa,m_\nu)\\&+\overline{u}_-u_-m_lP_2(k,p,\kappa,m_\nu)\}+b^2\{\overline{u}_+\not k u_+P_1(k,p,-\kappa,m_\nu)+\overline{u}_+u_+m_lP_2(k,p,-\kappa,m_\nu)\}]/(a^2+b^2)\end{aligned}\tag{A4}$$

where $P_1\equiv(1+\kappa)^2(2k\cdot p-k^2-2m_\nu^2)+2m_lm_\nu(1-\kappa^2)$, $P_2\equiv(k^2-2k\cdot p)[(1-\kappa^2)-r_\nu(1+\kappa)^2]$, $l_\mu\equiv q_\mu y-p_\mu(1-x-y)$ $=p_\mu(x-1)+yp'_\mu$, $c_b\equiv r_\nu^2(1-x-y)^2+(1-r_\nu^2)(1-x-y)+4\eta xyr_\nu^2+r_\pi^2(x+y)$, and $r_\pi=m_\pi/m_l$.

Next we perform a shift of the origin $k\to\tilde{k}=k+l$ and use the fact that under symmetric integration $\tilde{k}_\mu\tilde{\not k}\tilde{k}_\lambda f(\tilde{k}^2)$ $\to 0, 2\tilde{k}_\mu\tilde{\not k}\tilde{k}^2f(\tilde{k}^2)\to(1/2)\tilde{k}^4f(\tilde{k}^2)\gamma_\mu, \tilde{k}_\mu f(\tilde{k}^2)\to 0$ for any function f, with the result

$$\begin{aligned}\text{Eq. (13b)}=&-ieG_F^2F_\pi^2\int_0^1dx\int_0^{1-x}dy\int\frac{d^4\tilde{k}}{(2\pi)^4}\frac{1}{[\tilde{k}^2-m_l^2c_b(x,y,\eta)+i\varepsilon]^3}\Bigg\{a^2\Bigg[\overline{u}_-\gamma_\mu u_-P_3(x,y,\tilde{k}^2,\kappa)+\overline{u}_-u_-\\&\times\left(\frac{p'_\mu}{m_\nu}P_4(x,y,\tilde{k}^2,\kappa)+\frac{p_\mu}{m_\nu}P_5(x,y,\tilde{k}^2,\kappa)\right)\Bigg]+b^2\Bigg[\overline{u}_+\gamma_\mu u_+P_3(x,y,\tilde{k}^2,-\kappa)+\overline{u}_+u_+\\&\times\left(\frac{p'_\mu}{m_l}P_4(x,y,\tilde{k}^2,-\kappa)+\frac{p_\mu}{m_l}P_5(x,y,\tilde{k}^2,-\kappa)\right)\Bigg]\Bigg\}\Bigg/(a^2+b^2)\end{aligned}\tag{A5}$$

where $P_3(x,y,\tilde{k}^2,\kappa)=\frac{1}{2}(1+\kappa)^2\tilde{k}^2[m_l^2R_1(x,y,\eta)-\tilde{k}^2]$, $P_4(x,y,\tilde{k}^2)=m_l^2\tilde{k}^2f_1(x,y)+m_l^4f_2(x,y)$, $P_5(x,y,\tilde{k}^2)=P_4(y,x,\tilde{k}^2)$, $R_1(x,y,\eta)=-4\eta xyr_\nu^2-r_\nu^2[1+(x+y)^2]+\mathcal{O}(\varepsilon r_\nu)$, $f_1(x,y)=2r_\nu(x+y)(4y-1)+\mathcal{O}(\varepsilon)$, $f_2(x,y)=4r_\nu(1-2y)[(x+y)R_1+2r_\nu]+\mathcal{O}(\varepsilon)$. The $d^4\tilde{k}$-integration can now be done using

$$I_n(\tilde{m}^2)\equiv\int\frac{d^4l}{(2\pi)^4}\frac{1}{[l^2-\tilde{m}^2+i\varepsilon]^n}.$$

It is well known that I_n converges for $n\geqslant 3$,

$$I_n(\tilde{m}^2)=(-1)^n\frac{i}{16(n-1)(n-2)\pi^2}\frac{1}{(\tilde{m}^2)^{n-2}}.\tag{A6}$$

Equation (A5) contains $I_{1,2}$ as well, which both have to be regularized. For consistency we regularize *all* $I_n(n=1,2,3)$ in Eq. (A5),

$$I_1^{co}(m_l^2 c_b) = -\frac{i}{(4\pi)^2} m_l^2\left[\lambda^2 - c_b \log\left(1+\frac{\lambda^2}{c_b}\right)\right],$$

$$I_2^{co}(m_l^2 c_b) = \frac{i}{(4\pi)^2}\left[\log\left(1+\frac{\lambda^2}{c_b}\right) - \frac{\lambda^2}{\lambda^2+c_b}\right],$$

$$I_3^{co}(m_l^2 c_b) = -\frac{i}{(4\pi)^2}\frac{1}{2m_l^2 c_b}\left(\frac{\lambda^2}{\lambda^2+c_b}\right)^2. \tag{A7}$$

The superscript "co" refers to a Euclidean cutoff $\Lambda_E \equiv \lambda m_l$; as discussed in Ref. [12] this procedure is numerically close to the Pauli-Villars regularization (which is—with the exception of dimensional regularization—the only scheme consistent with gauge- and chiral-invariance).

The comparison of Eq. (A5) with Eq. (6) confirms separate gauge invariance for this diagram, Fig. 2(b), $\Gamma_1^{(2b)}(q^2) \equiv \Gamma_2^{(2b)}(q^2)$, and gives [17] [note for $\eta=0$, $c_b = r_\nu^2(1-x-y)^2 + (1-r_\nu^2)(1-x-y) + r_\pi^2(x+y)$],

$$\Gamma_3^{(2b)}(0) = \frac{G_F^2 F_\pi^2 m_l^2}{(4\pi)^2}\int_0^1 dx \int_0^{1-x} dy \left[g\left(\frac{c_b}{\lambda^2}\right)\left(\frac{r_\nu^2}{2}[1+(x+y)^2]\frac{(1+\kappa)^2 + \frac{b^2}{a^2}(1-\kappa)^2}{1+b^2/a^2} - r_\nu(1-\kappa^2)\right) - \frac{\lambda^2}{2} f\left(\frac{c_b}{\lambda^2}\right)\frac{(1+\kappa)^2+\frac{b^2}{a^2}(1-\kappa)^2}{1+b^2/a^2}\right] \tag{A8}$$

where $f(x) \equiv 3x\log(1+1/x) - 1 - 2x/(1+x) - x/[2(1+x)^2]$ is (up to a constant factor) the combination $(I_1 + 2m_l^2 c_b I_2 + m_l^4 c_b^2 I_3)$, and $g(x) \equiv -\log(1+1/x) + 1/(1+x) + 1/[2(1+x)^2]$ is (up to a constant factor) the combination $(I_2 + m_l^2 c_b I_3)$.

Next we calculate the contribution of the vertex correction, Fig. 2(c). The corresponding matrix element in Eq. (13c) is now [first for left-handed neutrinos, i.e., $b=0$ in Eq. (A2)]

$$\begin{aligned}
&\overline{u_\nu(p')}\not{k}(1-\kappa\gamma_5)(\not{p}'-\not{k}+m_l)\gamma_\mu(\not{p}-\not{k}+m_l)\not{k}(1-\kappa\gamma_5)u_\nu(p)\\
&\quad= \overline{u_\nu(p')}\gamma_\mu u_\nu(p)\big((1+\kappa)^2\{4k\cdot p' k\cdot p + k^2[k^2 - m_\nu^2 - m_l^2 - 2k\cdot(p+p')]\} + 2m_l m_\nu(1-\kappa^2)k^2\big)\\
&\qquad + \overline{u_\nu(p')}\not{k}u_\nu(p)2k_\mu[(1+\kappa)^2(m_\nu^2 + m_\ell^2) - (1-\kappa^2)2m_l m_\nu] + \overline{u_\nu(p')}u_\nu(p)2k_\mu(k^2 - 2k\cdot p')\\
&\qquad \times[m_\nu(1+\kappa)^2 - m_\ell(1-\kappa^2)] + \overline{u_\nu(p')}\not{k}\gamma_\mu u_\nu(p)2k\cdot q[m_\nu(1+\kappa)^2 - m_\ell(1-\kappa^2)].
\end{aligned} \tag{A9}$$

As before, see Eq. (A3), the result for right-handed neutrinos is obtained by replacing κ with $-\kappa$ in Eq. (A9) throughout. The next steps follow those after Eq. (A3),

$$\begin{aligned}
\text{Eq. (13c)} = &-ieG_F^2F_\pi^2\int_0^1 dx\int_0^{1-x}dz\int\frac{d^4k}{(2\pi)^4}\frac{m_\ell^2}{[(k+l)^2 - m_l^2 c_c(x,z,\eta)+i\varepsilon]^3}\Bigg[a^2\Bigg(\bar{u}_-\gamma_\mu u_- Q_1(k,p,\kappa,m_\nu)\\
&+\bar{u}_-u_-\frac{2k_\mu}{m_l}Q_2(k,p,\kappa,m_\nu) + \bar{u}_-\not{k}u_-\frac{2k_\mu}{m_l^2}Q_3(k,p,\kappa,m_\nu) + \bar{u}_-\not{k}\gamma_\mu u_-\frac{2k\cdot q}{m_l^3}Q_4(k,p,\kappa,m_\nu)\Bigg)\\
&+b^2\Bigg(\bar{u}_+\gamma_\mu u_+ Q_1(k,p,-\kappa,m_\nu) + \bar{u}_+u_+\frac{2k_\mu}{m_l}Q_2(k,p,-\kappa,m_\nu) + \bar{u}_+\not{k}u_+\frac{2k_\mu}{m_l^2}Q_3(k,p,-\kappa,m_\nu)\\
&+\bar{u}_+\not{k}\gamma_\mu u_+\frac{2k\cdot q}{m_l^3}Q_4(k,p,-\kappa,m_\nu)\Bigg)\Bigg]\Bigg/(a^2+b^2)
\end{aligned} \tag{A10}$$

where now $c_c \equiv r_\nu^2(x+z)^2 + (1-r_\nu^2)(x+z) + 4\eta xz r_\nu^2 + r_\pi^2(1-x-z)$,

$$Q_1=(1+\lambda)^2\left[\frac{4k\cdot p'}{m^2\ell}k\cdot p+k^2\left(\frac{k^2}{m^2\ell}-\frac{2k\cdot(p+p')}{m^2\ell}-r_\nu^2-1\right)\right]+(1-\lambda^2)2k^2r_\nu,$$

$Q_2=(k^2-2k\cdot p')Q_4/m^2\ell$, $Q_3/m^2\ell=(1+\lambda)^2(1+r_\nu^2)-2r_\nu(1-\lambda^2)$, $Q_4/m^2\ell=(1+\lambda)^2r_\nu-(1-\lambda^2)$ and $l_\mu=-xp_\mu-zp'_\mu$, so that $\ell^2=m_\nu^2(x+z)^2+4m_\nu^2\eta xz$ and $-l\cdot(p+p')=2(x+z)m_\nu^2(1+\eta)$.

Again we find $\Gamma_1^{(2c)}(q^2)\equiv\Gamma_2^{(2c)}(q^2)$, i.e., separate gauge invariance of diagram, Fig. 2(c). For the magnetic contribution we finally get

$$\Gamma_3^{(2c)}(0)=\frac{G_F^2F_\pi^2m_l^2}{(4\pi)^2}\,\frac{(1+\kappa)^2+\dfrac{b^2}{a^2}(1-\kappa)^2}{1+b^2/a^2}\int_0^1dx\int_0^{1-x}dy$$
$$\times\left\{-g\left(\frac{c_c}{\lambda^2}\right)3r_\nu^2\left(\frac{1}{2}+(x+y)^2-2(x+y)\right)+\left[\lambda^2f\left(\frac{c_c}{\lambda^2}\right)+\frac{1}{2}g\left(\frac{c_c}{\lambda^2}\right)\right]\right\}. \qquad \text{(A11)}$$

Note there is no term $\sim r_\nu(1-\kappa^2)$ in $\Gamma_3^{(2c)}(0)$.

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[15] Note that both positive and negative values for $\langle r^2\rangle_{\nu_e}$ have been reported [10], see "nonstandard contributions to neutrino scattering" in the full listing.
[16] At this point we are not considering neutrino oscillations, which are expected to become relevant only at much larger length scales.
[17] The explicit form of $\Gamma_{1,2,3}^{(2b)}(q^2)$ as a function of q^2 is not of interest here and will be reported elsewhere.