Holography and eternal inflation

David A. Lowe*
Physics Department, Brown University, Providence, Rhode Island 02912, USA

Donald Marolf†
Physics Department, University of California, Santa Barbara, California 93106, USA
(Received 19 March 2004; published 19 July 2004)

We show that eternal inflation is compatible with holography. In particular, we emphasize that if a region is asymptotically de Sitter in the future, holographic arguments by themselves place no bound on the number of past *e*-foldings. We also comment briefly on holographic restrictions on the production of baby universes.

DOI: 10.1103/PhysRevD.70.026001 PACS number(s): 11.25.−w, 04.60.−m, 98.80.Qc

I. INTRODUCTION

New inflation models typically suffer from severe fine-tuning problems associated with the choice of initial state. In the eternal inflation scenario, these problems are avoided because inflating bubbles persist along any time slice, and these inflating regions self-reproduce, leading to a fractal multiverse spacetime structure [1]. From the point of view of a local observer the details of this multiverse structure are irrelevant, except to set up the initial conditions for the observer's own inflating bubble.

In this paper, we examine constraints on the eternal inflation scenario arising from holographic entropy bounds. Historically, the idea of holographic bounds [2,3] and their cousins [4–7] emerged from the study of black hole entropy (but see [8–10]) and some researchers find motivation for such bounds in certain results from string theory. Such bounds are equivalent to the assumption that black holes are maximally entropic objects of a given size; they state that the entropy residing inside the relevant region is bounded by its surface area in Planck units [2,3,11].

On the other hand, for at least one proposed form [12] of the holographic bound, it was argued in [13] that when (i) the number of fields is small, (ii) the matter $T_{\mu\nu}$ is not too anisotropic locally, and (iii) temperatures are below the Planck scale, the bound follows as a consequence of the Einstein equations. Similarly, one generally does not expect holographic bounds to impose additional restrictions on thermodynamics at temperatures below the Planck scale. However, Banks and Fischler argued that holography (of a somewhat different form than that used in [12]), together with certain additional assumptions, requires any late-time observer entering a region dominated by a small value of the cosmological constant to observe a bounded number of *e*-foldings [14]. See [15,16] for subsequent related works.

Here we wish to emphasize one of the additional assumptions of [14]. In particular, [14] considers a scenario where the universe is inflating at early times, passes through a matter-dominated regime, and then becomes asymptotically de Sitter in the future. The assumption of interest is that *the total entropy of the universe in the early-time inflationary region can be computed by local field theory methods even when no observer can directly measure all of this entropy.* In particular, we will see in Sec. II that most of this entropy lies outside the past light cone of any observer.

We are motivated to question this assumption by the observation that a similar assumption in the late-time de Sitter region would already violate any holographic bound on the entropy of the system. This is just the observation that de Sitter space expands to infinite size in the far future, so that any field theory with any finite cutoff contains an infinite number of degrees of freedom. This observation is not new and is well known to proponents of holography who propose that nevertheless de Sitter space is associated only with a finite number of states. The usual resolution (see, e.g., [17]) is to note that this calculation does not contradict the experience of any observer in the spacetime, as such observers have access to only a small part of the entropy—small enough, it turns out, to satisfy a holographic bound. One then supposes that the true entropy of the system is comparable to the maximum entropy measurable by any given observer and that field theory breaks down on scales large enough that it would predict violations of the holographic entropy bounds (see, e.g., [18]).

Our goal here is to show that applying similar reasoning to the system considered in [14] yields a similar conclusion. That is, in contrast to [14] we assume that holographic bounds restrict only the field theory entropy in any past light cone, as field theory may generally acquire holographic corrections on larger scales. In this context we show that holography imposes no restrictions on inflation.[1] In particular, the number of *e*-foldings can be arbitrarily large. To distinguish our assumption from that of [14] we refer to it as "light-cone holography" below.[2] We also comment briefly on claims [21] of holographic restrictions on baby universe formation.

[1]This agrees with results of [19,20] where the quasi–de Sitter case was studied. In particular, in [19] holography plus thermodynamics was shown to imply the linearized Einstein equation for fluctuations around de Sitter.

[2]Bousso's covariant entropy bound [12] is a special case of light-cone holography, but we allow much more general settings here. We emphasize that some form of light-cone holography is essential for *any* consistent holographic description of de Sitter space.

*Electronic address: lowe@brown.edu
†Electronic address: marolf@physics.ucsb.edu

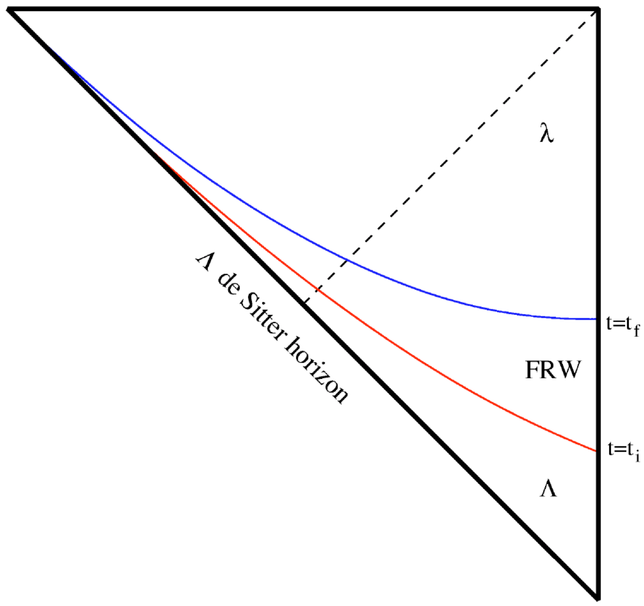

FIG. 1. Spacetime diagram. The dashed line is the past light cone of a late-time observer.

II. BOUND ON *e*-FOLDINGS FROM ENTROPY IN A LIGHT CONE?

In Fig. 1, we show a (rough) conformal diagram of the class of spacetimes considered in [14]. This spacetime region might appear as some portion of an eternally inflating spacetime, which we show in an over-simplified form in Fig. 2. An inflating region with vacuum energy density Λ is patched onto a Friedmann-Robertson-Walker (FRW) phase dominated by some form of matter satisfying $p=\kappa\rho$, which in turn asymptotes to a de Sitter region with small cosmological constant $\lambda \ll \Lambda$. We assume homogeneity, isotropy, and spatial flatness. The latter is at least a good approximation as we are most interested in the case where the Λ region undergoes a large number of *e*-foldings. Following [14], we consider only the region shown in Fig. 1 and do not concern ourselves with holographic restrictions on what occurs to the past of the Λ de Sitter horizon.

The basic idea of our analysis is already clear from this diagram. Consider some observer in the far future. Her past light cone to the future of $t=t_I$ is determined by propagating null rays (dashed line) backwards through the FRW and λ regions. Clearly, then, the spacetime region visible to her but later than time t_i is independent of the number N of *e*-foldings that takes place in the Λ region. Thus light-cone holographic bounds in the FRW region cannot impose any restrictions on N.

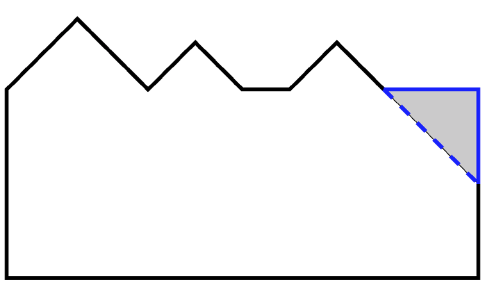

FIG. 2. A representation of the spacetime of eternal inflation. The shaded triangle corresponds to the region shown in Fig. 1.

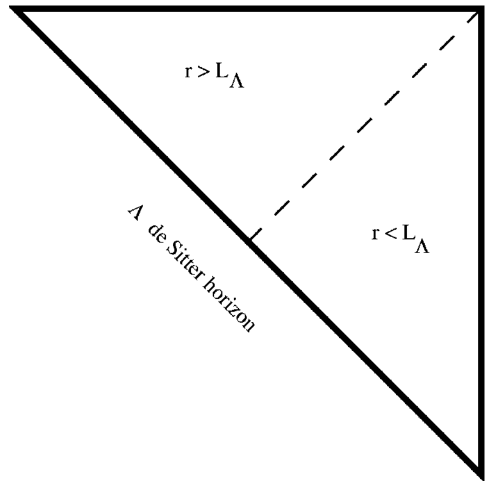

FIG. 3. The inflating patch of Λ de Sitter space.

Now, if we continue to trace the light cone backwards through the Λ region, we will find one of two things to be true. The first possibility is that the light cone at $t=t_i$ is larger than the de Sitter horizon in the Λ region. For this discussion the reader may wish to consult Fig. 3, which depicts the inflating patch of pure Λ de Sitter space with its horizons. We have also indicated in each region the size of the suppressed spheres relative to L_Λ, the Λ-horizon scale. In this first case, standard results from de Sitter space tell us that the light cone *contracts* (when traced backwards) until we reach the de Sitter horizon. Thus, the largest piece of Λ de Sitter spacetime is seen at time $t=t_i$ and, since holographic bounds are most stringent for large regions, light-cone holographic considerations at earlier times can yield only weaker restrictions. In particular, since a bound applied at $t=t_i$ cannot restrict N, it is impossible for a bound at earlier times to do so. More generally, it is clear that $t=t_i$ places the most stringent bound on the entropy.

If on the other hand the light cone is smaller than the Λ de Sitter horizon at $t=t_i$, it will be expanding and will continue to expand when traced further backward. However, it will remain smaller than the horizon size until it crosses the Λ de Sitter horizon. But it is well known that observing a horizon-scale region of de Sitter space does not contradict any holographic bounds. Thus, once again nothing new is learned from the region $t<t_i$.

Thus it is clear that light-cone holographic considerations can place no bound on N. Nonetheless, one might still ask whether they place bounds on other quantities relevant to the scenario above. We now turn to this question and investigate in detail the past light cone of a late-time observer.

Late-time past light cone

There is, however, the issue of just which sort of holographic bound we wish to consider. One popular formulation is Bousso's covariant entropy bound [12]. However, this counts only entropy that flows through *contracting* light sheets and some holographers may desire a tighter bound.[3] Thus, for the rest of this section we will simply assume that holographic considerations restrict the entire entropy visible to any observer on any homogeneous spacelike slice Σ to be less than the area A of the intersection of Σ with the observers past light cone.

Let us also pause to further orient ourselves to the problem at hand. We have already seen that our light-cone holographic bound is satisfied in the Λ region if it is satisfied at $t=t_i$. It is also satisfied in the λ region by the usual arguments for de Sitter space. To address the rest of the FRW region, consider the light cone at $t=t_i$. Because the universe as a whole is homogeneous and expanding, if the light cone were at any point expanding toward the future, it would continue to do so for all time and would not converge at the location of our late-time observer. Thus this light cone must be contracting toward the future at $t=t_i$ and throughout the FRW region. Thus, the part of any constant time slice (at $t>t_i$) visible to our observer is metrically identical to a subset of that at $t=t_i$, but with a lower entropy density (due to the expansion). Thus, if our scenario satisfies the light-cone holographic bound at $t=t_i$, it will do so throughout the entire spacetime.

To identify the observer's past light cone at $t=t_i$, recall that we assumed homogeneity, isotropy, and spatial flatness. Thus, the metric takes the form

$$ds^2=dt^2-a(t)^2dx_i^2$$

and the Friedmann equation is

$$\left(\frac{\dot{a}}{a}\right)^2=\frac{\rho}{3}+\frac{\lambda}{3}$$

in units where the reduced Planck mass is set to one $[M_{pl}=(8\pi G)^{-1/2}=1]$. For matter satisfying $p=\kappa\rho$, the density satisfies (see, e.g., [22])

$$\rho\propto\frac{s_i^{1+\kappa}}{a^{3(1+\kappa)}}, \tag{1}$$

where s_i is the entropy density in comoving coordinates, at time $t=t_i$. The entropy density s_i is constant during the FRW phase. At the end of inflation ($t=t_i$) the scale factor is $a_i=e^{N_i}\Lambda^{-1/2}$, where N_i is the number of e-foldings during inflation. Note also that inflation ends when ρ_{matter} is of order Λ. The universe then expands in a Friedmann-Robertson-Walker phase dominated by matter with some particular value of κ until the residual cosmological constant λ comes to dominate at $t\approx t_f$. The scale factor at this time is determined by requiring the matter density to decrease to of order λ, so using Eq. (1) we find

$$\frac{a_f}{a_i}\approx\left(\frac{\Lambda}{\lambda}\right)^{1/3(1+\kappa)}. \tag{2}$$

Now consider the past light cone of a late-time observer. In the λ region the light cone at each time encloses a spherical volume whose radius R_f at $t=t_f$ is of the order of the late-time de Sitter radius λ. The requirement that each light ray be null ($ds^2=0$) allows one to propagate the rays back in time and thus to determine the size R_i of the visible region at $t=t_i$. The result is

$$R_i=\frac{a_i}{a_f}R_f+\frac{2}{3\kappa+1}\Lambda^{-1/2}\left[\left(\frac{a_f}{a_i}\right)^{(3\kappa+1)/2}-1\right]. \tag{3}$$

Since $\Lambda\gg\lambda$ and any positive energy condition requires $\kappa\geq-1$, Eqs. (2) and (3) yield

$$R_i\approx\Lambda^{-1/2}\left(\frac{a_f}{a_i}\right)^{(3\kappa+1)/2}. \tag{4}$$

The total entropy in this region is computed using Eq. (1) where, since $\rho=\Lambda$ at $t=t_i$, one finds

$$S_i=R_i^3\Lambda^{1/(1+\kappa)}. \tag{5}$$

Thus, the above version of the holographic bound is equivalent in our context to the requirement

$$S_i\lesssim\frac{1}{\lambda}. \tag{6}$$

Inserting Eqs. (4) and (5) yields

$$\lambda^{-(3\kappa+1)/2(1+\kappa)}\lesssim\lambda^{-1}, \tag{7}$$

which is clearly independent of N. Now, since $\lambda<1$ in Planck units, Eq. (7) is equivalent to

$$\kappa\leq 1$$

and is the same bound that arises from causality considerations. Note that although the region visible to the observer is restricted by Eq. (6), the entropy across the entire initial time slice $t=t_i$ can be arbitrarily large, allowing for an arbitrary number of previous e-foldings. As described earlier, if the light-cone holographic bound is satisfied at $t=t_i$, it will be satisfied at all times. Thus no restriction of any sort arises from light-cone holographic considerations in the spacetime of Fig. 1 and, in particular, light-cone holography does not limit the number of e-foldings.

III. DISCUSSION

We have established that light-cone holography places no bounds on the number of e-foldings to the past of a late-time observer and is thus consistent with the eternal inflation scenario. This conclusion differs from that of [14] because we do not share their assumption that local field theory can correctly compute the entropy of a volume larger than that contained in the past light cone of any observer. Again, we note that assuming local field theory to correctly describe the en-

[3] Though it is not clear to what extent one could hold in general.

tropy of similar large volumes at late times would also contradict holographic bounds. In particular, the corresponding calculation in pure de Sitter space would contradict the idea that asymptotically de Sitter space has a finite number of states, on which the discussion of [14] also rests.[4] Thus, at the current level of holographic understanding, we see no reason to suppose a contradiction between holography and a large number of e-foldings such as would arise in eternal inflation.

For eternal inflation to be self-reproducing, the inflaton must be able to fluctuate up its potential with some finite probability, giving rise to inflating regions with an increasing rate of expansion. One may also ask if there are holographic constraints on this process. Discussions of related issues have appeared in [25].

Let us begin with a clear example that illustrates how this mechanism can be compatible with holography. Consider a region of spacetime with some effective Hubble parameter H, homogeneous over many horizon volumes. Suppose a bubble with effective Hubble parameter $H' > H$ is nucleated inside this region with a size larger than the horizon size set by H. This process occurs with finite probability in the eternal inflation scenario [1]. Applying the holographic bound to this situation [11] one finds that the generalized second law yields no constraint on the evolution of this super-horizon size bubble, as it is unable to collapse. The bubble is then free to expand in a manner compatible with holography, and no contradiction is later reached when inflation ends in this bubble and a vast amount of entropy is produced, despite the fact the bubble started out at a scale associated with some grand unified theory (GUT), with low entropy. From a quantum mechanical viewpoint, the system starts in a special state of low entropy, but as time evolves the state explores a larger subspace of the full Hilbert space of states, corresponding to the H' bubble expanding into the ambient H region. Clearly we have ignored inhomogeneities, but we believe this example suffices to illustrate the essential compatibility of the seeding mechanism of eternal inflation with holography.

Another oft-discussed situation occurs when the initial radius of the bubble H' is smaller than the ambient spacetime's inverse Hubble scale H^{-1}. For $H' > H$ this bubble will collapse and can form a black hole whose interior becomes a baby universe that undergoes inflation. For uncharged bubbles, Farhi and Guth [26,27] concluded the initial conditions for the formation of such a bubble are always singular. This may prevent the formation of such bubbles in the first place. Even if they are formed, a curvature singularity separates any external observer from the inflating interior of the bubble, so the application of holography is not entirely clear. The case of charged bubbles was analyzed in [28]. There it was found that these problems can be avoided, but a new difficulty appears because the inflating region lies inside a Cauchy horizon, which is unstable [29–34].

Let us nonetheless assume that such problems are somehow solvable and that sub-horizon scale bubbles do play a role in seeding eternal inflation. In this scenario, we wish to analyze possible constraints of holography. If semi-classical physics is valid in the appropriate regions of spacetime, the bubbles will collapse and form horizons. Banks has argued that the entropy of such bubbles should be bounded by the Bekenstein-Hawking entropy of the resulting horizon [21]. In particular, the argument is that universes such as ours today have $S \approx 10^{85}$, which requires an event horizon of radius $10^8 m$, somewhat larger than the radius of the Earth. The probability of nucleating such a large bubble in the early universe is extremely small.

However, there are a number of subtleties in the above argument. Let us at least enumerate some of the possibilities, assuming we start with a GUT scale bubble that collapses to form a black hole. Such a GUT bubble might have formed during quantum fluctuations in the eternal inflation foam or perhaps through interactions in a very high energy particle accelerator.

(i) One interpretation of the Bekenstein-Hawking entropy S_{BH} is that $e^{S_{BH}}$ bounds the dimension of the Hilbert space of states associated with the region inside the horizon. Let us denote this Hilbert space by $\mathcal{H}$. This interpretation is supported by string theory calculations of black hole entropy via D-brane methods. At first glance, the idea that a black hole with initial Bekenstein-Hawking entropy of order $S_{BH} \approx 10^6$ expands to give a large universe appears to conflict with this idea. In particular, suppose one assumes that time evolution maintains a sharp distinction between those states in $\mathcal{H}$ and those orthogonal to $\mathcal{H}$. By this we mean both (a) that $\mathcal{H}$ and its complement do not significantly mix under time evolution and (b) that the two classes of states appear quite different to local observers which experience them. In this case, a local observer inside the bubble can estimate the dimension of the Hilbert space of states similar to what she observes and compare this with $\mathcal{H}$ for an initial GUT size bubble. While it has been argued [35] that precise measurements are impossible for this observer, it is clear that there are certain classes of scattering observables that can be measured with very high precision. The observer in our present universe would then be able to conclude with a high degree of certainty that her Hilbert space is larger than that inherited from a GUT scale bubble and rule out creation of her universe via such a black hole.

(ii) However, it is not clear to what extent the assumption in (i) above is physically justified. In particular, con-

[4]The authors of [14] express their skepticism of the existence of a consistent theory which approximates local field theory in the inflating Λ region and leads to similar predictions for the cosmic microwave background (CMB), but yields a smaller total entropy in the inflating regime. We have no such example to offer, but see no reason why creating such a model is fundamentally more difficult than achieving the same goal for de Sitter space itself, a task not yet completed for which the authors of [14] expect success. See [23,24] for some steps toward a model for de Sitter with a finite dimensional Hilbert space. Unfortunately, until a model exists for the perhaps simpler pure de Sitter case, there will be few solid grounds on which to resolve this difference of opinion.

sider assumption (ia), that the bubble remains in the space $\mathcal{H}$ under time evolution. Certainly the original black hole Hawking radiates and may well disappear in the distant future. Black hole complementarity suggests that the state of the Hawking particles is actually equivalent to a state inside the black hole and in particular to any baby universe so created. Since these Hawking particles explore a much larger Hilbert space, it is conceivable then that the entropy of the bubble is not constrained by holographic bounds. From the external point of view, the late time de Sitter phase with large entropy could have a complementary description as a high entropy state in the Hilbert space $\mathcal{H}\times\mathcal{O}$, where $\mathcal{O}$ is the Hilbert space of states outside the horizon.

(iii) Let us also consider assumption (ib), that $\mathcal{H}$ and its compliment appear quite different (for all time) to local observers which experience them. Such observers are unable to measure the exact late-time state of the full system and so end up measuring the entropy of a locally accessible subsystem. It is not clear to us whether observations of such subsystems can indeed distinguish between universes produced via black holes and those which arose from other initial conditions. Let us suppose now that they cannot. Let us also note that the von Neumann ($\mathrm{Tr}\,\rho\ln\rho$) entropy of such a subsystem may well *exceed* the entropy of the full system, because the observer is unable to measure correlations with causally disconnected regions of the asymptotically de Sitter space. Indeed the exact von Neumann entropy will vanish if the system is in a pure state. It is thus conceivable that a late time observer could see a vastly larger entropy than the Bekenstein-Hawking entropy associated with the horizon of an initial black hole from which our universe somehow emerged.

We see that in order to arrive at a contradiction, one would need to prove the existence of more than $e^{S_{BH}}$ states which (a) are macroscopically indistinguishable from our universe and (b) could have been formed from a GUT-scale black hole. We conclude that successful production of a de Sitter region with large apparent entropy must produce some fine-tuning of the universe, but not that it is otherwise ruled out without additional assumptions. Such a fine-tuning is not a surprise, as the instability of the charged black hole's Cauchy horizon and the resulting singularity already indicate that successful production of a universe via a black hole is either far from generic or is dependent on high energy effects not currently understood and associated with the singularity. If one believes that the process is possible at all, it is plausible that any fine-tunings required by holography are a natural result.

In summary, we see that holography appears quite compatible with eternal inflation. In particular, a late time observer sees no bound on the number of e-foldings or on any other parameters in the model of Fig. 1. Furthermore, the mechanisms of self-reproduction in eternal inflation survive holographic constraints. Holography may place strong constraints on branches of the eternal inflation spacetime that somehow emerge from black hole interiors, but even here such a conclusion follows only if one introduces additional assumptions. Because quantum gravitational processes are necessarily involved in the production of such regions, any such assumptions are necessarily difficult to test and must remain inherently speculative.

ACKNOWLEDGMENTS

D.L. thanks T. Hertog and D. Lyth for helpful discussions and KITP for hospitality. D.L. is supported in part by DOE grant DE-FE0291ER40688-Task A and NSF grant PHY99-07949. D.M. was supported in part by NSF grants PHY00-98747, PHY99-0747, and PHY03-42749.

[1] A.D. Linde, D.A. Linde, and A. Mezhlumian, Phys. Rev. D **49**, 1783 (1994).
[2] G. 't Hooft, gr-qc/9310026.
[3] L. Susskind, J. Math. Phys. **36**, 6377 (1995).
[4] J.D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).
[5] J.D. Bekenstein, gr-qc/0007062.
[6] J.D. Bekenstein, Phys. Lett. B **481**, 339 (2000).
[7] R. Brustein and G. Veneziano, Phys. Rev. Lett. **84**, 5695 (2000).
[8] D. Marolf and R. Sorkin, Phys. Rev. D **66**, 104004 (2002).
[9] D. Marolf and R. Sorkin, Phys. Rev. D **69**, 024014 (2004).
[10] D. Marolf, D. Minic, and S. Ross, Phys. Rev. D **69**, 064006 (2004).
[11] R. Easther and D.A. Lowe, Phys. Rev. Lett. **82**, 4967 (1999).
[12] R. Bousso, J. High Energy Phys. **07**, 004 (1999).
[13] E.E. Flanagan, D. Marolf, and R.M. Wald, Phys. Rev. D **62**, 084035 (2000).
[14] T. Banks and W. Fischler, astro-ph/0307459.
[15] B. Wang and E. Abdalla, Phys. Rev. D **69**, 104014 (2004).
[16] R.-G. Cai, JCAP **02**, 007 (2004).
[17] T. Banks and W. Fischler, hep-th/0102077.
[18] A.G. Cohen, D.B. Kaplan, and A.E. Nelson, Phys. Rev. Lett. **82**, 4971 (1999).
[19] A. Frolov and L. Kofman, JCAP **05**, 009 (2003).
[20] U.H. Danielsson, JCAP **03**, 002 (2003).
[21] T. Banks, hep-th/0211160.
[22] W. Fischler, A. Loewy, and S. Paban, J. High Energy Phys. **09**, 024 (2003).
[23] A. Guijosa and D.A. Lowe, Phys. Rev. D **69**, 106008 (2004).
[24] M.K. Parikh and E. Verlinde, hep-th/0403140.
[25] D. Coule, Int. J. Mod. Phys. D **12**, 963 (2003).
[26] E. Farhi and A.H. Guth, Phys. Lett. B **183**, 149 (1987).
[27] E. Farhi, A.H. Guth, and J. Guven, Nucl. Phys. **B339**, 417 (1990).
[28] G.L. Alberghi, D.A. Lowe, and M. Trodden, J. High Energy Phys. **07**, 020 (1999).
[29] M. Simpson and R. Penrose, Int. J. Theor. Phys. **7**, 183 (1973).

[30] S. Chandrasekhar and J. Hartle, Proc. R. Soc. London **A384**, 301 (1982).
[31] E. Poisson and W. Israel, Phys. Rev. D **41**, 1796 (1990).
[32] L.M. Burko and A. Ori, Phys. Rev. D **57**, 7084 (1998).
[33] M. Dafermos, gr-qc/0307013.
[34] M. Dafermos, gr-qc/0401121.
[35] T. Banks, W. Fischler, and S. Paban, J. High Energy Phys. **12**, 062 (2002).