

Light spin- $\frac{1}{2}$ or spin-0 dark matter particles

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We recall and precise how light spin-0 particles could be acceptable dark matter candidates, and extend this analysis to spin- $\frac{1}{2}$ particles. We evaluate the (rather large) annihilation cross sections required, and show how they may be induced by a new light neutral spin-1 boson U . If this one is vectorially coupled to matter particles, the (spin- $\frac{1}{2}$ or spin-0) dark matter annihilation cross section into e^+e^- automatically includes a v_{dm}^2 suppression factor at threshold, as desirable to avoid an excessive production of γ rays from residual dark matter annihilations. We also relate dark matter annihilations with production cross sections in e^+e^- scatterings. Annihilation cross sections of spin- $\frac{1}{2}$ and spin-0 dark matter particles are given by exactly the same expressions. Just as for spin-0, light spin- $\frac{1}{2}$ dark matter particles annihilating into e^+e^- could be responsible for the bright 511 keV γ ray line observed by INTEGRAL from the galactic bulge.

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I. INTRODUCTION

Weakly-interacting massive neutral particles, taken as possible dark matter candidates, should not be too light, otherwise they would not have been able to annihilate sufficiently. Weakly-interacting heavy neutrinos would have had to be heavier than about 2 GeV, for example (to get $\Omega_\nu h^2 \lesssim 1$) [1].

Supersymmetric extensions of the standard model naturally provide such weakly-interacting neutral particles, stable as a result of R -parity conservation (with $R_p = (-1)^{2S} \times (-1)^{(3B+L)}$) [2]. Spin- $\frac{1}{2}$ photinos, or more generally neutralinos, with cross sections roughly of weak-interaction order when the exchanged squarks and sleptons are $\sim m_w$ [3], should be heavier than a few GeV's (for light sfermion masses) at least to annihilate sufficiently, this bound increasing with the exchanged sfermion masses $m_{\tilde{q}, \tilde{l}}$ [4,5]. Given the still unsuccessful hunt for superpartners, in particular at LEP, the lightest neutralino (LSP) of supersymmetric extensions of the standard model is now generally believed to be heavier than about ~ 30 GeV.

Then, how could a light (annihilating) dark matter particle possibly exist? At first it should have *no significant direct coupling to the Z boson*, otherwise it would have been produced in Z decays at LEP. Despite that, it would have to *annihilate sufficiently*—and in fact, much more strongly than through ordinary weak interactions—otherwise its relic energy density would be too high! Can this happen at all, and what could then be the new interactions responsible for light dark matter annihilations?

We have explored in [6] under which conditions a light spin-0 particle could be a viable dark matter candidate. Two different situations have been exhibited, in which the new interactions responsible for the annihilations are due to non-chiral couplings with exchanged heavy fermions such as mirror fermions (case I), in (supersymmetric) theories somewhat reminiscent of $N=2$ extended supersymmetry and/or higher-dimensional theories [7]. Or, such interactions may be mediated by a new neutral spin-1 gauge boson U (case II), similar to the one, light and very weakly coupled, introduced long ago [8]. It is also desirable that the dark matter pair annihilation

cross section into e^+e^- has a v_{dm}^2 suppression factor, so as to avoid an excessive production of γ rays originating from the residual annihilations of dark matter particles (if lighter than ~ 100 MeV) [9]. This is naturally the case when these spin-0 particle annihilations result from the virtual production of a new spin-1 U boson.

We shall show here that, while the first situation (I) is specific of spin-0 particles, the second (II) is not, and could apply to spin- $\frac{1}{2}$ as well as to spin-0 particles. One crucial feature is that the new interactions mediated by the U boson should actually be “not-so-weak” (at lower energies and relatively to weak interactions)—i.e., $\langle \sigma_{ann} v_{rel} / c \rangle \approx$ a few (up to ≈ 10) picobarns—so as to ensure for sufficient annihilations of light dark matter particles, whatever their spin. More precisely, the new U -mediated dark-matter/matter interactions will be *stronger than ordinary weak interactions at lower energies*—but *weaker at higher energies*, at which they are damped by U propagator effects. The smallness of the U couplings to ordinary matter, as compared to e , by several orders of magnitude, then accounts for the fact that these particles have not been observed yet.

A second essential feature is that the annihilation cross sections of such spin- $\frac{1}{2}$ dark matter particles into fermion pairs $f\bar{f}$ through the exchanges of a new neutral spin-1 U boson will, also in this case, have the desired v_{dm}^2 suppression factor at threshold, provided the U boson is *vectorially* coupled to *matter* fermions, as is in any case necessary to avoid a problematic axionlike behavior of its longitudinal polarization state [8].

Indeed, as we shall see, the annihilation, at threshold, of a $C=+$ state (made of two Majorana particles χ , with $J=L=S=0$) into a $f\bar{f}$ final state with $C'=(-)^{(L'+S')}=+$, through a C -violating interaction (axial χ current times vector f current), is forbidden by charge conjugation. This ensures that the annihilation cross section $\sigma_{ann} v_{rel}(\chi\chi \rightarrow e^+e^-)$ has the appropriate $\propto v_{dm}^2$ behavior, automatically suppressing (by a factor $\approx 10^{-5}$) the late annihilations of nonrelativistic relic dark matter particles.

Furthermore, the annihilation cross sections of spin- $\frac{1}{2}$ and spin-0 dark matter particles will, in this case, be given by

exactly the same expressions. Spin- $\frac{1}{2}$ particles then turn out to be acceptable light dark matter (LDM) candidates, as well as spin-0 particles. In particular, their annihilations into e^+e^- pairs could lead to a γ ray signature from the galactic center at low energy, as indicated for spin-0 particles in [6] (before the observations of [10]). Just as the latter, they could be responsible for the bright 511 keV γ ray line recently observed by the INTEGRAL satellite from the galactic bulge [10–12]. Other effects of such light dark matter particles, on nucleosynthesis and energy transfer in stars, were discussed very recently in [13].

II. DARK MATTER DECOUPLING AND RELIC DENSITY

The interactions responsible for the pair annihilations of spin- $\frac{1}{2}$ dark matter particles χ (such as heavy neutrinos or neutralinos, ...) may be written, in the local limit approximation, as effective four-fermion interactions $\mathcal{L} \approx G \bar{\chi} \cdots \chi \bar{f} \cdots f$. The corresponding annihilation cross sections, proportional to G^2 , scale essentially like m_{dm}^2 , m_{dm} being the dark matter particle mass. Such *fermionic* particles annihilating through exchanges of heavy *bosons* of masses $\gtrsim m_W$ cannot be light (in a perturbative theory), since their annihilation cross sections would be too small.

To estimate what annihilation cross sections are actually needed for a correct relic abundance of light dark matter particles (corresponding to $\Omega_{dm} h^2 \approx 0.1$), we express that the annihilation rate $\Gamma = n_{dm} \langle \sigma_{ann} v_{rel} \rangle$ and expansion rate H are approximately equal when the dark matter annihilation reactions freeze out. This occurs at a temperature $T_F = m_{dm}/x_F$, with x_F roughly between ≈ 16 to ≈ 23 for a 1 MeV to 1 GeV particle (cf. Appendix).

(i) For **10 MeV $\lesssim m_{dm} \lesssim 1$ GeV**, the freeze-out occurs at T_F (with, roughly, $0.6 \text{ MeV} \lesssim T_F \lesssim 50 \text{ MeV}$) after muons have annihilated (most of them at least), but not electrons yet. The effective number of degrees of freedom is then $g_* \approx \frac{43}{4}$. The surviving particles get diluted by the expansion of the Universe, proportionally to T^3 , with an extra factor $\frac{4}{11}$ corresponding to the subsequent annihilation of e^+e^- pairs into photons, so that the relic density of dark matter particles may now be expressed as [14]

$$n_{\odot dm} = \frac{4}{11} \frac{T_{\odot \gamma}^3}{T_F^3} n_{dm}, \quad (1)$$

$T_{\odot \gamma} \approx 2.725 \text{ K} \approx 2.35 \times 10^{-13} \text{ GeV} (\approx 11.9 \text{ cm}^{-1})$ being the present photon temperature. We shall denote by $N_{\odot dm} = (2)n_{\odot dm}$ the total density of dark matter (particles + anti-particles), with the factor 2 present only in the case of non-self-conjugate dark matter particles.

The resulting freeze-out equation $\Gamma \approx H$, i.e.,

$$n_{\odot dm} \frac{11}{4} \frac{T_F^3}{T_{\odot \gamma}^3} \langle \sigma_{ann} v_{rel} \rangle \approx 1.66 \sqrt{g_*} = \frac{43}{4} \frac{T_F^2}{m_{Pl}}, \quad (2)$$

sufficient as a first approximation, fixes the relic energy density

$$\begin{aligned} \rho_{dm} &\approx (2)n_{\odot dm} m_{dm} \approx (2)x_F n_{\odot dm} T_F \\ &\approx (2) \frac{4}{11} 1.66 \sqrt{g_*} = \frac{43}{4} x_F \frac{T_{\odot \gamma}^3}{m_{Pl}} \frac{1}{\langle \sigma_{ann} v_{rel} \rangle} \\ &\approx (2) \frac{x_F}{20} \frac{4.2 \times 10^{-56} \text{ GeV}^2}{\langle \sigma_{ann} v_{rel} \rangle}. \end{aligned} \quad (3)$$

Dividing by the critical density $\rho_c/h^2 \approx 1.054 \times 10^{-5} \text{ GeV/cm}^3$ (times $\hbar \approx 6.58 \times 10^{-25} \text{ GeV}\cdot\text{s}$) to get the density ratio $\Omega_{dm} h^2$, we find

$$\begin{aligned} \frac{\Omega_{dm} h^2}{0.1} &\approx \frac{x_F}{20} (2) \frac{6 \times 10^{-26} \text{ cm}^3/\text{s}}{\langle \sigma_{ann} v_{rel} \rangle} \\ &\approx (2) \frac{x_F}{20} \frac{2 \times 10^{-36} \text{ cm}^2}{\langle \sigma_{ann} v_{rel}/c \rangle}, \end{aligned} \quad (4)$$

with the extra factor 2 present in the case of non-self-conjugate particles.

More precisely, there is also, from an approximate solution of the Boltzmann equation, an expected increase of the required cross section by a factor ≈ 2 , for $\langle \sigma_{ann} v_{rel} \rangle$ behaving at threshold like v_{dm}^2 , as compared to a constant [15,16].

Indeed the later annihilations that would still occur below the temperature T_F given by Eq. (2) are further inhibited by this v_{dm}^2 factor, preventing the dark matter density from reaching the equilibrium value corresponding to this T_F , as it would be given by Eqs. (2)–(4). Altogether, obtaining the right amount of dark matter ($\Omega_{dm} h^2 \approx 0.1$) requires typically

$$\langle \sigma_{ann} v_{rel}/c \rangle \approx (2) (4 \text{ or } 2) \text{ pb}, \quad (5)$$

depending whether $\langle \sigma_{ann} v_{rel}/c \rangle$ behaves like v_{dm}^2 ($\approx (2) 4 \text{ pb}$, the most interesting case for us here), or as a constant ($\approx (2) 2 \text{ pb}$) [17], the factor 2 being associated with non-self-conjugate dark matter particles.

(ii) Let us now consider *lighter dark matter particles* (say **$m_{dm} \lesssim 10 \text{ MeV}$**), actually the most interesting situation. At first, for particles lighter than about 2 to 3 MeV, that would decouple (at $T_F = m_{dm}/x_F \lesssim 0.15 \text{ MeV}$) after most electrons have annihilated, the dilution factor of $\frac{4}{11}$ is no longer present in Eqs. (1)–(3). In addition, the g_* at dark matter freeze-out is no longer $\frac{43}{4}$ when electrons have disappeared. It may in fact be expressed in terms of the neutrino temperature as

$$g_* \approx 2 + \frac{7}{8} (2 \times 3) \left(\frac{T_\nu^4}{T_F^4} \right), \quad (6)$$

which would be ≈ 3.36 according to the standard model, where $T_\nu/T \approx (\frac{4}{11})^{1/3}$ as an effect of electron annihilations.

The neutrino contribution to g_* , however, is then no longer the same as in the standard model. Indeed dark matter particles annihilating after neutrino decouple, at $T \approx 3.5 \text{ MeV}$ for ν_μ, ν_τ or $\approx 2 \text{ MeV}$ for ν_e (assuming neutrino interactions with dark matter do not keep them longer in thermal equilibrium with photons, as discussed in [13])

TABLE I. Estimates of the annihilation cross sections $\langle \sigma_{ann} v_{rel}/c \rangle$ at freeze-out required for a correct relic abundance ($\Omega_{dm} h^2 \approx 0.1$).

Spin- $\frac{1}{2}$ Majorana (χ)	Spin- $\frac{1}{2}$ Dirac (ψ)	Spin-0 (φ complex)
4–5 pb	8–10 pb	8–10 pb

would also heat up the photon gas as compared to neutrinos, so that the resulting neutrino temperature would be less than the usual $(\frac{4}{11})^{1/3} T$, resulting in a lower contribution of neutrinos to the g_* at T_F than in the standard model (down to 0.71 to 0.94 instead of 1.36), due to dark matter annihilations themselves [18]!

Such a phenomenon, if a significant fraction of dark matter annihilations were to occur after neutrino decoupling but before the neutron/proton ratio freezes out, could have important implications, potentially allowing for less primordial helium than in the standard model (or conversely allowing for new species, e.g., additional light inos, etc., to contribute to the expansion rate in a way which would otherwise have been forbidden). This effect, that we found qualitatively, is discussed in detail in [13], as well as, more generally, the effects of light dark matter particles on the big bang nucleosynthesis, with the conclusion that light masses $m_{dm} \lesssim 2$ MeV are disfavored as they would severely disturb the BBN concordance.

Given INTEGRAL results, we tend to favor light dark matter masses just above this value, so as to maximize the number of e^+ produced for a given dark matter energy density; and, also, to avoid these e^+ from dark matter annihilations (with their associated e^- , produced with an energy close to m_{dm}) having too much energy dissipated in γ rays (as it would happen for a not-so-small m_{dm}), before they can form positronium and annihilate, leading to the bright 511 keV γ ray line.

Ignoring for a moment this mass restriction as we discuss relic abundances, for a particle in the $\approx \frac{1}{2}$ –2 MeV mass range, the required cross section gets increased (from the absence of the $\frac{4}{11}$ dilution factor and the lower value of g_* , and x_f) by a factor $\frac{11}{4} \sqrt{g_*/(\frac{43}{4})} (x_F/20 \approx 0.85) \approx 1.2$, as compared to a ≈ 100 MeV particle, a rather moderate increase of about 20%. No spectacular difference is then expected when the mass grows from 2 to 10 MeV, the effects of the $\frac{4}{11}$ dilution factor and of the larger g_* approaching $\frac{43}{4}$ getting progressively reestablished.

(iii) Altogether for cross sections behaving like v_{dm}^2 , the required annihilation cross sections at freeze-out $\langle \sigma_{ann} v_{rel}/c \rangle$ are of the order of 4 to 5 picobarns for a self-conjugate (Majorana) dark matter particle, or 8 to 10 picobarns for a non-self-conjugate one, e.g., a complex scalar, as summarized above in Table I. (We do not consider real self-conjugate spin-0 particles, as Bose statistics does not allow for the desired P -wave annihilation.)

This corresponds roughly, for present annihilations of residual dark matter particles having a velocity $v_{dm} \approx 3$

$\times 10^{-3}$ the velocity at freeze-out in the primordial Universe (≈ 0.4 c), to annihilation cross sections

$$\langle \sigma_{ann} v_{rel}/c \rangle \approx (4 \text{ to } 10) \cdot 10^{-5} \text{ pb}, \quad (7)$$

respectively. Although large, this is indeed the right order of magnitude for light dark matter particle (in the $\approx$ MeV range) annihilations to be at the origin of the 511 keV γ ray signal observed by INTEGRAL from the galactic bulge [10,11,19].

In any case, since $G_F^2 (1 \text{ GeV})^2 / 2\pi \approx 0.8 \times 10^{-38} \text{ cm}^2$, cross sections (at freeze-out) of weak interaction order are, for light masses $m_{dm} \ll \text{GeV}$, by far too small for a correct relic abundance. Significantly larger annihilation cross sections are needed, requiring new types of interactions, as discussed in [6].

III. ANNIHILATIONS THROUGH HEAVY FERMION EXCHANGES

In case (I) (cf. Introduction), one arranges, for spin-0 dark matter particles, to have annihilation cross sections behaving as the inverse of the (large) squared masses of exchanged fermions (rather than the fourth power of exchanged boson masses). Spin-0 dark matter particles (φ) are taken to have Yukawa interactions coupling ordinary quarks and leptons f to heavy fermions F such as *mirror fermions* [20]. The low-energy effective Lagrangian density responsible for their pair-annihilation into $f\bar{f}$ may be written as

$$\mathcal{L} \approx \frac{C_l C_r}{m_F} \varphi^* \varphi \bar{f}_R f_L + \text{h.c.}, \quad (8)$$

where C_l and C_r denote the Yukawa couplings of the spin-0 dark matter particles to the left-handed and right-handed fermion fields, respectively. The resulting annihilation cross section at threshold is of the type

$$\sigma_{ann} v_{rel} \approx \frac{C_l^2 C_r^2}{\pi m_F^2}, \quad (9)$$

in the case of *nonchiral* couplings (i.e., for $C_l C_r \neq 0$). This cross section, largely independent of the dark matter mass, can be quite significant even for light spin-0 dark matter particles [21].

However, in the absence of a P -wave suppression factor proportional to v_{dm}^2 (since the couplings (8) involve fermion fields of both chiralities, allowing for nonvanishing S -wave annihilations into $f\bar{f}$) [22], one runs the risk, at least for lighter dark matter particles ($\lesssim 100 \text{ MeV}/c^2$), of too much γ ray production due to residual annihilations of dark matter particles [9] (unless there is an asymmetry between dark matter particles and antiparticles).

It is thus preferable to consider annihilations induced through the virtual production of a new light neutral spin-1 gauge boson U (case II). We can then get both an appropriate relic abundance, together with the desired v_{dm}^2 suppression factor in the annihilation cross sections. We shall point out later, in Sec. VI, that these two features are not specific to light spin-0 dark matter particles, but may apply as well to spin- $\frac{1}{2}$ particles.

IV. SPIN-0 DARK MATTER ANNIHILATIONS THROUGH U EXCHANGES

A. Spin-0 annihilation cross sections

In the local limit approximation (valid, in the annihilation case, for $2E \ll m_U$), dark matter interactions may be described by an effective Lagrangian density involving the product of the dark matter (φ) and quark and lepton (f) contributions to the U current, i.e.,

$$\mathcal{L} = \frac{C_U}{m_U^2} \varphi^* i \overleftrightarrow{\partial}_\mu \varphi (f_V \bar{f} \gamma^\mu f + f_A \bar{f} \gamma^\mu \gamma_5 f), \quad (10)$$

with “Fermi-like” coupling constants $G_V \approx C_U f_V / m_U^2$, $G_A \approx C_U f_A / m_U^2$. C_U and f_V and/or f_A denote the couplings of the new gauge boson U to the spin-0 dark matter field φ and the matter fermion field f considered, respectively. (If the local limit approximation is not valid, U propagator effects may be taken into account by replacing $-m_U^2$ by $s - m_U^2 = 4E^2 - m_U^2$.) The contributions of the vector and axial f currents do not interfere for unpolarized cross sections (an interference term would have to involve the totally antisymmetric ϵ tensor and must therefore vanish), and may be considered separately. The f_V coupling (i.e., in fact the product $C_U f_V$) is invariant under charge conjugation, while f_A (which is likely to be absent, otherwise we would generally have an unwanted axionlike behavior of the new light gauge boson U [23]) is C -violating.

1. No S -wave annihilation for spin-0 dark matter particles

The threshold behavior of the annihilation cross section $\sigma_{ann}(\varphi\bar{\varphi} \rightarrow f\bar{f})$ may be understood easily from simple arguments based on charge conjugation. The initial $\varphi\bar{\varphi}$ state has $C = (-)^L = +$ in an S wave ($L=0$). The final $f\bar{f}$ state then also has $C' = (-)^{(L'+S')} = +$ (since angular momentum conservation requires $J' = J = 0$).

In the case of an *axial* coupling (f_A) to the fermion field f , the relevant terms in the Lagrangian density (10), being C -violating, cannot induce the decay $\varphi\bar{\varphi}_{S\text{-wave}} \rightarrow f\bar{f}$.

In the case of a *vector* coupling (f_V), the relevant terms in (10) are indeed C -conserving, but the $f\bar{f}$ final state, being vectorially produced through the virtual production of a U boson (as if it were through a one-photon exchange), must have $C' = -$ (while $C' = +$ from angular momentum conservation).

In both cases of vector and axial couplings (or for a linear combination of them), there can be no S -wave term in the annihilation cross section. The dominant (P -wave) terms in $\sigma_{ann} v_{rel}$ are then proportional to the square of the dark matter particle velocity in the initial state, i.e.,

$$\sigma_{ann} v_{rel}(\varphi\bar{\varphi} \rightarrow f\bar{f}) \propto v_{dm}^2 \quad (11)$$

at threshold.

Stated in other terms, the total U charge-density and current of a $\varphi\bar{\varphi}$ pair must vanish at threshold. The annihilation amplitudes, proportional to $i(p_1^\mu - p_2^\mu)$, vanish proportion-

ally to the rest-frame momenta p_{dm} of the initial particles, or to v_{dm} , as a result of the derivative nature of the U coupling to scalar particles.

Let us now evaluate explicitly these annihilation cross sections (as given in [6] at threshold, for $E \approx m_{dm}$). Still another way to obtain them, without calculation, from the corresponding production cross sections in e^+e^- annihilations, will be given in Sec. V.

2. Vector coupling to fermions

The following factor in the “squared amplitude” is easily evaluated, in the center of mass reference frame:

$$\begin{aligned} & -(p_1 - p_2)_\mu (p_1 - p_2)_\nu \text{Tr}[(\not{p}_3 + m_f) \gamma^\mu (-\not{p}_4 + m_f) \gamma^\nu] \\ &= -4(p_1 - p_2)_\mu (p_1 - p_2)_\nu (-p_3^\mu p_4^\nu - p_3^\nu p_4^\mu \\ & \quad + g^{\mu\nu} (p_3 \cdot p_4 + m_f^2)) \\ &= -16p_{1i} p_{1j} (2p_3^i p_3^j + 2g^{ij} E^2) \\ &= 32p_{dm}^2 E^2 (1 - \beta_f^2 \cos^2 \theta). \end{aligned} \quad (12)$$

Averaging over angles, and multiplying by $C_U^2 f_V^2 / (m_U^2 - 4E^2)^2$, and by $[1/(2\pi)^2][1/(2E)^4] 4\pi p_f E_f / 2 = (1/32\pi) \times (1/E^2) \beta_f$ for the phase space integration, we get

$$\begin{aligned} \sigma_{ann} v_{rel} &= \frac{2}{3\pi} v_{dm}^2 \frac{C_U^2 f_V^2}{(m_U^2 - 4E^2)^2} E^2 \left(\frac{3}{2} \beta_f - \frac{1}{2} \beta_f^3 \right) \\ &= \frac{2}{3\pi} v_{dm}^2 \frac{C_U^2 f_V^2}{(m_U^2 - 4E^2)^2} \sqrt{1 - \frac{m_f^2}{E^2}} \left(E^2 + \frac{m_f^2}{2} \right). \end{aligned} \quad (13)$$

This reduces, at threshold ($s = 4E^2 \approx 4m_{dm}^2$), to the same expression as in [6] (choosing $f_{U_l} = f_{U_r} = f_V$). We recognize, in terms of the velocity parameter $\beta_f = v_f/c = (1 - m_f^2/E^2)^{1/2}$, the usual kinematic factor relative to the vectorial production of a pair of spin- $\frac{1}{2}$ Dirac fermions,

$$\frac{3}{2} \beta_f - \frac{1}{2} \beta_f^3 = \sqrt{1 - \frac{m_f^2}{E^2}} \left(E^2 + \frac{m_f^2}{2} \right). \quad (14)$$

3. Axial coupling to fermions

We can still use the previous calculation, replacing f_V by f_A , and changing m_f^2 into $-m_f^2$ within the expression of the “squared amplitude” $|A|^2$. $p_3 \cdot p_4 + m_f^2 = 2E_f^2$ is replaced by $p_3 \cdot p_4 - m_f^2 = 2p_f^2$, and $(1 - \beta_f^2 \cos^2 \theta)$ by $(\beta_f^2 - \beta_f^2 \cos^2 \theta) = \beta_f^2 \sin^2 \theta$. The kinematic factor (14) which appears for the vectorial production of the $f\bar{f}$ pair gets simply replaced by the corresponding factor β_f^3 appropriate to the axial production of spin- $\frac{1}{2}$ particles. Expressing the latter in terms of m_f and the dark matter particle energy $E_{dm} = E_f = E$, we get

$$\sigma_{ann}v_{rel} = \frac{2}{3\pi} v_{dm}^2 \frac{C_U^2 f_A^2}{(m_U^2 - 4E^2)^2} E^2 \left(1 - \frac{m_f^2}{E^2}\right)^{3/2}. \quad (15)$$

Again this reduces, at threshold ($E \approx m_{dm}$), to the same expression as obtained from [6] (choosing $f_{U_l} = -f_{U_r} = f_A$).

If the U coupling to the fermion field f includes both vector and axial contributions, the annihilation cross section is the sum of the two contributions (13) and (15).

B. Constraints on the U couplings

Numerically, and to get an idea of the size of the couplings (depending also on the masses m_U and m_{dm}) required to get appropriate values of the annihilation cross sections at decoupling (i.e., about 8 to 10 picobarns, cf. Sec. II), we can write the above expressions (13) and (15) as

$$\sigma_{ann}v_{rel} \approx \frac{v_{dm}^2}{0.16} \left(\frac{C_U f_{V,A}}{10^{-6}} \right)^2 \left(\frac{m_{dm} \times 3.6 \text{ MeV}}{m_U^2 - 4m_{dm}^2} \right)^2 \text{ pb}, \quad (16)$$

still to be multiplied by the appropriate kinematic factor (<1) relative to the vectorial ($\frac{3}{2}\beta_f - \frac{1}{2}\beta_f^3$) or axial (β_f^3) production of spin- $\frac{1}{2}$ particles [24]. We shall in fact consider mostly vectorial couplings of the U to ordinary matter (f_V), with values much smaller than the electric charge ($e \approx 0.3$) by several orders of magnitude. The resulting U boson effects on ordinary particle physics processes, charged lepton $g-2$, etc., then appear sufficiently small [6].

In particular, for a vectorially coupled U boson somewhat heavier than the electron but lighter than the muon, the comparison between the additional U contributions to the muon and electron anomalies and the possible difference between the experimental and standard model values indicates that [6,25]

$$\begin{aligned} \delta a_\mu &\approx \frac{f_{V\mu}^2}{8\pi^2} \approx (2 \pm 2) 10^{-9}, \\ \delta a_e &\approx \frac{f_{Ve}^2}{12\pi^2} \frac{m_e^2}{m_U^2} \approx (4 \pm 3) 10^{-11}, \end{aligned} \quad (17)$$

so that

$$f_{V\mu} \lesssim 6 \times 10^{-4}, \quad f_{Ve} \lesssim 2 \times 10^{-4} m_U (\text{MeV}). \quad (18)$$

One should also have, for a U mass larger than a few MeV's,

$$|f_{V\nu} f_{Ve}| \lesssim G_F m_U^2 \approx 10^{-11} (m_U (\text{MeV}))^2 \quad (19)$$

so that U exchanges do not modify excessively neutrino-electron low-energy elastic scattering cross sections, in good agreement with standard model values [26]. This requires that the U couplings to neutrinos, at least, be sufficiently small. This also requires, conversely, that the U coupling C_U to dark matter be not too small, so as to get, from Eq. (16), appropriate values of the annihilation cross section $\langle \sigma_{ann} v_{rel} \rangle$ [27].

The same cross section formulas (with vector couplings f_V), etc., may also be used, as we shall see in Sec. VI, in the case of spin- $\frac{1}{2}$ dark matter particles, as well as for spin-0 particles.

V. RELATING PRODUCTION AND ANNIHILATION CROSS SECTIONS

The production and annihilation cross sections of dark matter particles may be easily related, as the corresponding amplitudes are related by CPT (or simply T , when CP invariance holds). When computing cross sections σv_{rel} we sum on final state polarizations while averaging over initial ones. The integration of the squared modulus of the transition amplitudes ($|A|^2$)—identical when one appropriately exchanges the initial and final states—over the final particle momenta, makes the velocity of the latter particles ($\beta_f = v_f$, or $\beta_{dm} = v_{dm}$, for $f, \bar{f}$ or dark matter particles) appear. The production (in e^+e^- scatterings) and annihilation cross sections are then related by

$$\sigma_{prod} v_e (e^+ e^- \rightarrow \varphi \bar{\varphi}) / v_{dm} \equiv \frac{1}{4} \sigma_{ann} v_{dm} (\varphi \bar{\varphi} \rightarrow e^+ e^-) / v_e. \quad (20)$$

From the usual electromagnetic pair production cross section of charged spin-0 particles in e^+e^- annihilations (neglecting m_e),

$$\sigma_{prod}^{(\gamma)} = \frac{4\pi\alpha^2}{3s} \frac{1}{4} \beta_{dm}^3 = \frac{e^4}{48\pi s} \beta_{dm}^3, \quad (21)$$

we immediately get (replacing e^4/s^2 by $C_U^2 f_V^2 / (s - m_U^2)^2$) the production cross section, through U exchanges, of neutral spin-0 dark matter particles,

$$\sigma_{prod}(e^+ e^- \rightarrow \varphi \bar{\varphi}) = \frac{1}{12\pi} \frac{C_U^2 f_V^2}{(4E^2 - m_U^2)^2} E^2 \beta_{dm}^3. \quad (22)$$

Multiplying it by $v_{rel} \approx 2$, by the spin factor 4 and the velocity ratio ($\beta_e \approx 1$)/($\beta_{dm} = v_{dm}$) appearing in (20), we get the corresponding annihilation cross section,

$$\sigma_{ann} v_{rel} (\varphi \bar{\varphi} \rightarrow e^+ e^-) = \frac{2}{3\pi} v_{dm}^2 \frac{C_U^2 f_V^2}{(m_U^2 - 4E^2)^2} E^2, \quad (23)$$

which, once the kinematic factor $\frac{3}{2}\beta_f - \frac{1}{2}\beta_f^3$ (taking into account the effect of nonvanishing m_e) is reintroduced, coincides precisely with Eq. (13). In a similar way replacing f_V by f_A , and reintroducing the appropriate kinematic factor β_f^3 , we recover Eq. (15) for the annihilation cross section $\sigma_{ann} v_{rel}$ through an axial coupling to the matter fermion field f .

The v_{dm}^2 suppression factor in the annihilation cross section $\sigma_{ann} v_{rel}$ of spin-0 particles φ appears simply as a *reflection by CPT of the well-known β^3 factor* for the pair production of spin-0 particles in e^+e^- annihilations (with, in both cases, a P wave for the $\varphi\bar{\varphi}$ state).

Equation (22) may be used to discuss the pair production of spin-0 dark matter particles in e^+e^- annihilations. When the U and φ particles are light it may be written under any of the equivalent forms:

$$\begin{aligned}\sigma_{prod}(e^+e^- \rightarrow \varphi\bar{\varphi}) &\simeq \frac{C_U^2 f_V^2}{48\pi s} \simeq \frac{C_U^2 f_V^2}{192\pi E_e^2} \simeq \frac{\alpha_U \alpha_V}{\alpha^2} \frac{21.7\text{nb}}{s(\text{GeV}^2)} \\ &\simeq C_U^2 f_V^2 \frac{2.58\mu\text{b}}{s(\text{GeV}^2)} \\ &\simeq \left(\frac{C_U f_V}{10^{-6}}\right)^2 \frac{2.6 \times 10^{-42} \text{ cm}^2}{(\sqrt{s}(\text{GeV}))^2}. \quad (24)\end{aligned}$$

For the relevant values of $C_U f_V$ (or conceivably $C_U f_A$) considered, these production cross sections get very small at high energies, much below neutrino production cross sections, so that the direct production of such dark matter particles is in general not expected to lead to easily observable signals in e^+e^- annihilations [6].

Let us now turn to spin- $\frac{1}{2}$ particles. Their pair production through an axial coupling involves a β_{dm}^3 factor [28]. It reflects precisely, as we saw, in a v_{dm}^2 suppression factor for the corresponding annihilation cross section at threshold (at least as long as the masses m_f of the produced fermions are neglected, a point to which we shall return in Sec. VI).

Since the *production* cross sections of spin-0 and (axially coupled) spin- $\frac{1}{2}$ particles in e^+e^- annihilations are given by similar formulas we expect that the corresponding *annihilation* cross sections into $f\bar{f}$ pairs be given, also, by similar formulas. Still it is essential to clarify under which circumstances these annihilation cross sections will continue to behave at threshold like v_{dm}^2 , when nonvanishing fermion masses m_f are taken into account.

VI. SPIN- $\frac{1}{2}$ DARK MATTER PARTICLES

We now consider spin- $\frac{1}{2}$ Majorana fermions χ , which can only have an axial coupling to the U boson. The analysis, in fact, applies as well to Dirac fermions (ψ), provided they are also axially coupled to the U .

Again the vector and axial couplings of the U boson to the fermions f may be considered independently. The effective Lagrangian density (similar to the one responsible for the effective interactions of photinos with matter through $\tilde{q}$ or $\tilde{l}$ exchanges) may now be written as

$$\mathcal{L} = \frac{C_U}{2m_U^2} \bar{\chi} \gamma_\mu \gamma_5 \chi (f_V \bar{f} \gamma^\mu f + f_A \bar{f} \gamma^\mu \gamma_5 f). \quad (25)$$

In contrast to the previous case of a spin-0 field φ , the coupling f_V is now C -violating while f_A (still normally presumed to be absent as it would be related with an unwanted axionlike behavior of the U boson) is C -conserving.

A. σ_{ann} vanishes at threshold, for vector couplings—but not for axial ones

At threshold the antisymmetry of our 2-Majorana $\chi\chi$ state (in an S wave) imposes that the total spin be $J=S=0$, so that the production (indifferently through vector and/or axial couplings) of massless fermion pairs $f\bar{f}$ with total angular momentum $\lambda = \pm 1$ along their direction of propagation is forbidden. But the (S -wave) annihilation cross section $\sigma_{ann} v_{rel}$ could in principle include, at threshold, nonvanishing contributions proportional to m_f^2 .

Our initial 2-Majorana $\chi\chi$ state has $C=+$. If it is in an S wave (so that $J=0$), the final $f\bar{f}$ state must have $C' = (-)^{(L'+S')} = +$ (with $L'=S'$ from angular momentum conservation). It follows immediately that the (C -violating) vectorial coupling f_V cannot contribute to the S -wave $\varphi\varphi \rightarrow f\bar{f}$ annihilation amplitude.

A second reason is that a U boson with a vectorial coupling to the f can only produce a $f\bar{f}$ pair with $C' = -$, while it must have $C' = +$ from angular momentum conservation. For either reason, the f_V contribution to the S -wave annihilation cross section must vanish, so that

$$\sigma_{ann} v_{rel}(\chi\chi \rightarrow f\bar{f}) \propto v_{dm}^2 \text{ at threshold.} \quad (26)$$

The situation would clearly be different for the production of massive fermion pairs $f\bar{f}$ through an *axial* current (with $C=+$), rather than a *vector* current (with $C=-$). The f_A contribution to the S -wave annihilation cross section has now no reason to vanish, as soon $m_f \neq 0$, since

(i) the corresponding C -conserving operator in (25) can indeed induce a $\chi\chi \rightarrow f\bar{f}$ transition from a $C=+$ to a $C' = +$ state; and

(ii) the axial fermionic current $\bar{f} \gamma_\mu \gamma_5 f$, being C -even, is capable of creating a $f\bar{f}$ pair in a $C' = +$ state.

Constant terms proportional to m_f^2 (undesirable for us here, at least for light dark matter particles ≤ 100 MeV) do then appear in the annihilation cross section $\sigma_{ann} v_{rel}$.

It is remarkable that the constraint of *vector* couplings of the light U to the matter fermions f , obtained here from the requirement of annihilation cross sections behaving like v_{dm}^2 , is essentially the same as already necessitated from the fact that such a light U boson (given the masses and couplings considered) would have an unacceptable axionlike behavior if it had sizeable axial couplings f_A to the matter fermions f [8].

B. From spin-0 to spin- $\frac{1}{2}$ dark matter annihilation cross sections

We now intend to compare the pair *annihilation* cross section for (Majorana) spin- $\frac{1}{2}$ particles χ (with axial coupling $C_U/2$), and for spin-0 particles of U -charge C_U , by relating them to the corresponding *production* cross sections in e^+e^- annihilations.

In the limit of vanishing m_e the production cross section of a pair of Dirac particles ($\psi\bar{\psi}$) through an axial coupling ($C_U \bar{\psi} \gamma_\mu \gamma_5 \psi$) to the U is proportional to β_{dm}^3 . It is related to the production cross section for a pair of spin-0 particles ($\varphi\bar{\varphi}$), proportional to $\frac{1}{4}\beta_{dm}^3$ (with in both cases a β_{dm}^3 factor

associated with a P -wave production of these particles in the final state), by

$$\sigma_{prod}(e^+e^- \rightarrow \psi\bar{\psi}) \equiv 4\sigma_{prod}(e^+e^- \rightarrow \varphi\bar{\varphi}). \quad (27)$$

By using the relations of Sec. V between annihilation and production cross sections (as expressed by (20) for spin-0 particles), we get from Eq. (27) the following relation between the (Dirac) spin- $\frac{1}{2}$ and spin-0 annihilation cross sections:

$$\sigma_{ann}(\psi\bar{\psi} \rightarrow e^+e^-) = \sigma_{ann}(\varphi\bar{\varphi} \rightarrow e^+e^-). \quad (28)$$

To relate the annihilation cross sections of Dirac and Majorana particles we can use the following trick: by writing the decomposition $\psi = (\chi - i\chi')/\sqrt{2}$ of the Dirac spinor field ψ , so that

$$\bar{\psi}\gamma_\mu\gamma_5\psi = \frac{1}{2}\bar{\chi}\gamma_\mu\gamma_5\chi + \frac{1}{2}\bar{\chi}'\gamma_\mu\gamma_5\chi', \quad (29)$$

and considering an initial state in which each of the two annihilating particles is either a ψ or a $\bar{\psi}$ (i.e., just as well, equivalently, either a χ or a χ'), we see that the pair annihilation cross section of Dirac particles (ψ , with axial coupling C_U) is the same as for Majorana particles (χ , with axial coupling $C_U/2$). It follows that

$$\begin{aligned} \sigma_{ann}(\chi\chi \rightarrow e^+e^-) &= \sigma_{ann}(\psi\bar{\psi} \rightarrow e^+e^-) \\ &= \sigma_{ann}(\varphi\bar{\varphi} \rightarrow e^+e^-), \end{aligned} \quad (30)$$

the latter being given by Eq. (13) or (23). Altogether we get

$$\sigma_{ann}v_{rel}(\chi\chi \rightarrow e^+e^-) = \frac{2}{3\pi}v_{dm}^2 \frac{C_U^2 f_V^2}{(m_U^2 - 4E^2)^2} E^2. \quad (31)$$

We can now take into account explicitly the effect of the electron mass in the final state. Our previous arguments showed that no S -wave annihilation cross section may be induced from a nonvanishing m_e , in the case of a vectorial coupling f_V (in contrast with f_A). The only expected effect of a nonvanishing m_e , or more generally of fermion masses

m_f , will simply be a multiplication of (31) by the usual kinematic factor for the vectorial production of a $f\bar{f}$ pair, $\frac{3}{2}\beta_f - \frac{1}{2}\beta_f^3$. This yields

$$\begin{aligned} \sigma_{ann}v_{rel}(\chi\chi \rightarrow e^+e^-) &= \frac{2}{3\pi}v_{dm}^2 \frac{C_U^2 f_V^2}{(m_U^2 - 4E^2)^2} E^2 \left(\frac{3}{2}\beta_f - \frac{1}{2}\beta_f^3 \right) \\ &= \frac{2}{3\pi}v_{dm}^2 \frac{C_U^2 f_V^2}{(m_U^2 - 4E^2)^2} \sqrt{1 - \frac{m_f^2}{E^2}} \left(E^2 + \frac{m_f^2}{2} \right). \end{aligned} \quad (32)$$

Remarkably enough, this cross section is actually identical to the cross section (13) for the pair annihilation of spin-0 dark matter candidates! In particular, we get in both cases the same v_{dm}^2 suppression factor of the annihilation cross sections, as desirable to avoid an excessive production of gamma rays originating from residual light dark matter annihilations. The (collisional and free-streaming) damping effects [6,29] associated with such particles are also, in both cases, sufficiently small.

The effect of m_f in the case of an axial coupling to fermions, however, will now be much more drastic than a simple multiplication by β_f^3 , since new terms not behaving like v_{dm}^2 (and proportional to m_f^2) will appear in the annihilation cross section.

C. Direct evaluation of spin- $\frac{1}{2}$ cross sections

Now that we know the result, at least in the case of a vector coupling f_V , without performing any explicit calculation, we can verify it explicitly. Since the annihilation cross section for a pair of *Majorana* particles (χ , with an axial coupling $C_U \frac{1}{2} \gamma^\mu \gamma_5$ to the U boson) is the same as for *Dirac* fermions (ψ , with an axial coupling $C_U \gamma^\mu \gamma_5$ to the U), we can evaluate the “squared amplitudes,” and resulting cross sections, as if we were dealing with such Dirac fermions. With an overall $\frac{1}{4}$ factor from the average on the incoming spin states, a first trace factor corresponding to the pair annihilation of dark matter particles through an axial coupling, and a second one to the $f\bar{f}$ pair production through a vector coupling, we evaluate

$$\begin{aligned} &\frac{1}{4} \text{Tr}[(\not{p}_1 + m_{dm}) \gamma_\mu \gamma_5 (-\not{p}_2 + m_{dm}) \gamma_\nu \gamma_5] \text{Tr}[(\not{p}_3 + m_f) \gamma^\mu (-\not{p}_4 + m_f) \gamma^\nu] \\ &= 4(-p_{1\mu}p_{2\nu} - p_{1\nu}p_{2\mu} + g_{\mu\nu}(p_1 \cdot p_2 - m_{dm}^2))(-p_3^\mu p_4^\nu - p_3^\nu p_4^\mu + g^{\mu\nu}(p_3 \cdot p_4 + m_f^2)) \\ &= 4[2p_1 \cdot p_3 p_2 \cdot p_4 + 2p_1 \cdot p_4 p_2 \cdot p_3 - 2p_3 \cdot p_4 m_{dm}^2 + 2p_1 \cdot p_2 m_f^2 - 4m_{dm}^2 m_f^2] \\ &= 4[2(E^2 - p_{dm} p_f \cos\theta)^2 + 2(E^2 + p_{dm} p_f \cos\theta)^2 - 2(2p_f^2 + m_f^2)m_{dm}^2 + 2(2p_{dm}^2 + m_{dm}^2)m_f^2 - 4m_{dm}^2 m_f^2] \\ &= 4[4p_{dm}^2 p_f^2 (1 + \cos^2\theta) + 8p_{dm}^2 m_f^2] \rightarrow 16\beta_{dm}^2 E^2 \left(\frac{4}{3}p_f^2 + 2m_f^2 \right) = 32\frac{2}{3}v_{dm}^2 E^2 \left(E^2 + \frac{m_f^2}{2} \right). \end{aligned} \quad (33)$$

Averaging over angles as done above, reintroducing the factor $C_U^2 f_V^2 / (m_U^2 - 4E^2)^2$, and multiplying (as for a spin-0 particle in Sec. IV A) by $(1/32\pi)(1/E^2)\beta_f$ for the phase space integration, we recover precisely the previous expression (32) for the annihilation cross section of spin- $\frac{1}{2}$ dark matter particles, a result also identical to the one obtained in (13) for spin-0 particle annihilations.

Let us also give for completeness the corresponding spin- $\frac{1}{2}$ annihilation cross section in the case of an axial matter fermion coupling f_A . By changing $m_f^2 \rightarrow -m_f^2$ at appropriate places in the calculation of Eq. (33), this expression of the squared amplitudes gets replaced by the now symmetric one

$$\begin{aligned} & 4[2p_1 \cdot p_3 p_2 \cdot p_4 + 2p_1 \cdot p_4 p_2 \cdot p_3 \\ & - 2p_3 \cdot p_4 m_{dm}^2 - 2p_1 \cdot p_2 m_f^2 + 4m_{dm}^2 m_f^2] \\ & = 16[p_{dm}^2 p_f^2 (1 + \cos^2 \theta) + m_{dm}^2 m_f^2], \end{aligned} \quad (34)$$

which leads to

$$\begin{aligned} \sigma_{ann} v_{rel} &= \frac{1}{2\pi} \frac{C_U^2 f_A^2}{(m_U^2 - 4E^2)^2} \sqrt{1 - \frac{m_f^2}{E^2}} \\ & \times \left[\frac{4}{3} (E^2 - m_f^2) v_{dm}^2 + \frac{m_{dm}^2}{E^2} m_f^2 \right]. \end{aligned} \quad (35)$$

It does coincide with (32) (replacing f_A by f_V) in the limit of vanishing fermion masses m_f , for which there is no physical distinction between vector and axial matter fermion couplings, so that we get in both cases a v_{dm}^2 suppression factor in the annihilation cross section. But, as anticipated earlier, this overall v_{dm}^2 factor no longer subsists for nonvanishing fermion masses m_f , for which one recovers a nonvanishing S-wave term in the annihilation cross section (35), proportional to m_f^2 .

D. Final remarks

Altogether, in the case of a vector coupling f_V of the U boson to quark and lepton fields f , spin- $\frac{1}{2}$ dark matter particles have the required characteristics for light dark matter (LDM) particles annihilating into e^+e^- pairs, just as well as spin-0 particles. In both cases, U -induced dark-matter/electron interactions should be significantly stronger than ordinary weak interactions at low energy (but weaker at high energies), which requires the U to be more strongly coupled to dark matter than to ordinary matter—also resulting in significant U -induced dark matter self-interactions. Finally, light spin- $\frac{1}{2}$ dark matter particles appear more attractive than spin-0 ones, as the smallness of their mass is easier to understand, and provide valuable alternative scenarios to be discussed and confronted with the standard ones.

APPENDIX: EVALUATING x_F

For a dark matter particle of mass m_{dm} with g degrees of freedom (including antiparticles), freezing out in the nonrelativistic regime at a temperature $T_F = m_{dm}/x_F$, the residual number density (evaluated as if in equilibrium) at T_F is

$$N_{dm} \simeq g \left(\frac{m T_F}{2\pi} \right)^{3/2} e^{-m/T_F} \simeq g T_F^3 \left(\frac{x_F}{2\pi} \right)^{3/2} e^{-x_F}. \quad (A1)$$

We work here in the naive approximation in which the residual abundance of dark matter particles is taken to be determined from its equilibrium value at the freeze-out temperature T_F . If this occurs after e^+e^- annihilations, dark matter particles get diluted by the expansion of the Universe, as for photons, so that their present density reads

$$N_{\circ dm} = g T_{\circ\gamma}^3 \left(\frac{x_F}{2\pi} \right)^{3/2} e^{-x_F}, \quad (A2)$$

with $T_{\circ\gamma} \simeq 1685 \text{ cm}^{-3}$. For particles in the $\approx 1 \text{ MeV}$ mass range decoupling after e^+e^- annihilations, we get

$$\frac{\Omega_{dm} h^2}{0.1} \simeq \frac{N_{\circ dm} m_{dm}}{(\rho_c/h^2) \times 0.1} \simeq \frac{g}{2} \frac{m_{dm}}{\text{MeV}} 2.03 \times 10^5 (x_F)^{3/2} e^{-x_F}, \quad (A3)$$

which determines (e.g., by taking $\ln$) x_F as a function of m_{dm} (and g), e.g., for 1 MeV $x_F \simeq 16.4$ for $g=2$ (or 17.2 for $g=4$).

For heavier particles decoupling after $\mu^+\mu^-$ but before e^+e^- annihilations there is a further $\frac{4}{11}$ reduction factor in the relic density as compared to (A2), so that

$$\frac{\Omega_{dm} h^2}{0.1} \simeq \frac{g}{2} \frac{m_{dm}}{\text{MeV}} 7.4 \times 10^4 (x_F)^{3/2} e^{-x_F}. \quad (A4)$$

This gives approximately, for $m_{dm} = 10 \text{ MeV}$, 100 MeV or 1 GeV , $x_F \simeq 17.8$, 20.3 or 22.8 , respectively, for a complex spin-0 or a Majorana particle ($g=2$). For a Dirac particle with $g=4$, these values are increased by ≈ 0.8 , to about 18.6 , 21.1 or 23.6 , respectively.

These estimates for x_F , although naive, are sufficient for a first estimate of the required annihilation cross sections at freeze out. Since we now demand a fixed $\Omega_{dm} h^2 \simeq 0.1$ for any given m_{dm} (rather than estimating an unknown $\Omega_{dm} h^2$ as a function of m_{dm} and $\langle \sigma_{ann} v_{rel} \rangle$), the corresponding x_F as evaluated above is directly fixed by m_{dm} (and g) through (A3) or (A4), without any direct reference here to $\langle \sigma_{ann} v_{rel} \rangle$ (itself a function of m_{dm} and x_F).

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$$\rho_{dm} \approx (2) 1.09 \times 10^4 \frac{x_F}{g'_*/\sqrt{g_*}} \frac{1}{m_{Pl} \langle \sigma v \rangle} \text{cm}^{-3},$$

and, dividing by ρ_c/h^2 ,

$$\Omega_{dm} h^2 \approx (2) \frac{x_F}{g'_*/\sqrt{g_*}} \frac{1.04 \times 10^9}{m_{Pl} \times (1 \text{ GeV}) \langle \sigma v \rangle},$$

as in [15], for a velocity-independent $\langle \sigma_{ann} v_{rel} \rangle$.

- [17] These formulas, which apply directly for m_{dm} in the 10 MeV to 1 GeV range, can be immediately extended to heavier particles by including the extra factor $\sqrt{g_*} \times \frac{43}{4}/g'_*$ (usually the same as $\sqrt{(\frac{43}{4})/g_*}$), and using the appropriate (slightly larger) value of x_F . For comparison, in the familiar case of heavy dark matter particles, the significantly larger value of g_* brings the required cross sections $\langle \sigma_{ann} v_{rel} \rangle$ close to about (2) $3 \times 10^{-26} \text{ cm}^3/\text{s}$ [as compared to (4)], in the case of v_{dm} -independent cross sections [resp. (2) $6 \times 10^{-26} \text{ cm}^3/\text{s}$ when they are $\propto v_{dm}^2$].
 [18] Once dark matter particles and electrons have annihilated, the neutrino temperature gets related to the photon temperature by

$$\frac{T_\nu^3}{T^3} \approx \frac{g_{*\gamma}=2}{\left(g_{*e+\gamma}=\frac{11}{2}\right)+g_{*dm}} \approx \frac{8}{29, 36 \text{ or } 30},$$

with $g_{*dm} = \frac{7}{4}$, $\frac{7}{2}$ or 2 for a Majorana, Dirac or complex spin-0

particle, respectively. This leads to a total $g_{*F} \approx 2.94$, 2.71 or 2.90, respectively, instead of 3.36 in the standard model.

- [19] With a measured flux of about $10^{-3} \text{ photon cm}^{-2} \text{ s}^{-1}$, the required cross section may be estimated $\propto 10^{-4} \text{ pb}$, times $(m_{dm}/1 \text{ MeV})^2$, depending on the density distribution within the galactic bulge. Note also that the rate of annihilation events is proportional to $\frac{1}{2} N_{dm}^2 \langle \sigma_{ann} v_{rel} \rangle$ for self-conjugate dark matter particles, and to $(N_{dm}/2)^2 \langle \sigma_{ann} v_{rel} \rangle$, for non-self-conjugate ones (the number densities of particles and antiparticles being both $N_{dm}/2$). The required cross sections are thus, also in this case, twice as large for non-self-conjugate particles as for self-conjugate ones.
 [20] Furthermore, in theories inspired from $N=2$ extended supersymmetry, or higher-dimensional theories [7], the spin-0 field φ may be a spin-0 photon field, or companion of the weak hypercharge gauge field B^μ , possibly described by the extra fifth and sixth components of the corresponding 6d-gauge fields, A^μ or $B^\mu \dots$.
 [21] For example, $\alpha^2/(100 \text{ GeV})^2 \approx 2 \times 10^{-36} \text{ cm}^2$, which shows that appropriate values of $\langle \sigma_{ann} v_{rel}/c \rangle \approx (2 \text{ to } 5) 10^{-36} \text{ cm}^2$ may easily be obtained, from the exchanges of heavy fermions F in the $\sim 100\text{--}1000 \text{ GeV}$ mass range, with Yukawa couplings C_l and C_r remaining in the perturbative regime.
 [22] The effective Lagrangian density (8) is invariant under charge conjugation, allowing for a nonvanishing annihilation cross section at threshold of a $\varphi\bar{\varphi}$ initial state with $C=(-)^L = +$ into a $f\bar{f}$ state with $C'=(-)^{(L'+S')} = +$.
 [23] In the presence of an axial contribution (f_A) to the U matter current, the longitudinal polarization state of the U would behave very much as an (unwanted) spin-0 axionlike particle, having pseudoscalar couplings to quarks and leptons $\approx f_A m_{q,l}/m_U$ [8].
 [24] $v_{dm}^2 \approx 0.16 c^2$ is associated with a kinetic energy $\approx 0.09 m_{dm}$, which would correspond roughly, to fix ideas, to $(\frac{3}{2}) T_F = (3/2 x_F) m_{dm}$ with $x_F \approx 16$ or 17.
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