Probing gravitation, dark energy, and acceleration

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The acceleration of the expansion of the universe arises from unknown physical processes involving either new fields in high energy physics or modifications of gravitation theory. It is crucial for our understanding to characterize the properties of the dark energy or gravity through cosmological observations and compare and distinguish between them. In fact, close consistencies exist between a dark energy equation of state function $w(z)$ and changes to the framework of the Friedmann cosmological equations as well as direct spacetime geometry quantities involving the acceleration, such as ''geometric dark energy'' from the Ricci scalar. We investigate these interrelationships, including for the case of superacceleration or phantom energy where the fate of the universe may be more gentle than the Big Rip.

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I. INTRODUCTION

The acceleration of the expansion of the universe poses a fundamental challenge to the standard models of both particle physics and cosmology. In both cases addition of an unknown physical component, called dark energy, or modification of gravitation, possibly arising from extra dimensions, is required. Most attention has been paid to dark energy as a high energy scalar field, a physical component contributing a presently dominating energy density, characterized by a time varying equation of state. But acceleration is fundamentally linked to gravitation through the Principle of Equivalence and changes to the framework of the Friedmann cosmological equations governing the universal expansion would play a natural role.

Observations from next generation cosmological probes will map the expansion history $a(t)$ at 1% precision, offering the possibility of characterizing the physics responsible for the acceleration. This can be used to test specific models inspired by unified physics involving string theory, supergravity, extra dimensions (e.g. braneworlds), or scalar-tensor gravity, say. Alternately, one can derive general parametrized constraints on the expansion history and propagate these through into quantities such as an effective dark energy equation of state, extra terms in the Friedmann equations, or spacetime geometry characteristics.

Not only the magnitude of the constraints but the interpretation of them is important. We investigate to what extent one can use a common parametrization to describe these very different areas of new physics, and conversely how they can be distinguished. In Sec. II we briefly review dark energy as a scalar field component of the universe. A general modification of the Friedmann equation is analyzed in Sec. III. We examine in Sec. IV the fundamental and general relation between acceleration and spacetime geometry, specifically involving the Ricci scalar, to motivate modifications of gravitation as a possible source of the acceleration—''geometric dark energy.'' In Sec. V we address the issue of superacceleration and whether this leads to a Big Rip. We conclude in Sec. VI, with thoughts on future prospects for understanding how cosmological observations will lead us to specific new physics.

II. PHYSICAL DARK ENERGY

With the discovery of the acceleration of the cosmic expansion [1,2], physicists tended to interpret this in terms of a new physical component of the universe—dark energy—possessing a substantially negative pressure. This is perhaps not surprising since models involving the cosmological constant had been under consideration and the effects of generalized pressure to energy density ratios, or equations of state, on cosmological observations had been worked out, e.g. [3–5]. Yet, as is well known, the cosmological constant can be viewed as belonging to either the right hand, energy-momentum tensor, side of Einstein's field equations or to the left hand, spacetime geometry or gravitation side. Still, in analogy to inflation theory, the observations were treated as a high energy physics scalar field ϕ with a potential $V(\phi)$, often called quintessence.

We here briefly review the essentials so as to later compare and contrast the treatment of gravitation as the source of the acceleration. Dark energy as a physical component possesses an energy density ρ_ϕ and pressure p_ϕ, both generally functions of time t, or equivalently cosmic scale factor a or redshift $z=a^{-1}-1$. The equation of state ratio is defined to be $w_\phi(z)=p_\phi/\rho_\phi$. The cosmological constant is special in possessing $w_\phi=-1$, which ensures that its density and pressure are constant in both time and space.

Like the matter or radiation components of the universe, dark energy is generically globally homogeneous and isotropic. However, in order for dark energy to dominate the energy density of the universe today, but not in the past, in accordance with observations, it must have an effective mass $m\sim\sqrt{V_{,\phi\phi}}\sim H_0\sim 10^{-33}$ eV, where H_0 is the expansion rate today, the Hubble constant. This implies that on scales smaller than the horizon size the dark energy is smooth and unclustered, while on larger scales it possesses inhomogeneities. This clumpiness is important observationally in only restricted circumstances, such as for the growth of matter density perturbations on near horizon scales.

For cosmological observations of the expansion history, e.g. distances and cosmography, and of the growth of matter perturbations on subhorizon scales, the dark energy is simply characterized by its energy density ρ_ϕ [equivalently its fractional contribution to the critical energy density $\Omega_\phi(z) = 8\pi\rho_\phi/3H^2$] and equation of state ratio $w(z)$. The evolution of the energy density follows

$$\rho_\phi(a) = \rho_{\phi,0} e^{3\int_a^1 d\ln a[1+w(a)]}, \tag{1}$$

so only the equation of state ratio and the present density enter. For a spatially flat universe, the present dimensionless dark energy density is related to the matter density by $\Omega_\phi = 1-\Omega_m$.

From the equation of state function one can recreate the high energy physics Lagrangian of the field in terms of its potential and kinetic energies:

$$V(\phi) = \frac{1}{2}\ (1-w)\rho_\phi \tag{2}$$

$$K = \frac{1}{2}\dot{\phi}^2 = \frac{1}{2}(1+w)\rho_\phi \tag{3}$$

$$\phi(a) = \int da \frac{1}{\dot{a}}\dot{\phi} = \int d\ln a H^{-1}\sqrt{2K}, \tag{4}$$

where the last line allows translation from the expansion factor to the value of the scalar field. Thus $w(a)$ really is the central, determining quantity.

Note that the equation of motion of the field ϕ, the Klein-Gordon equation, follows easily from the continuity Friedmann equation: $\dot{\rho}_\phi = -3H(\rho_\phi + p_\phi) = -6HK$. Since $\dot{\rho}_\phi = \dot{K} + \dot{V} = \dot{\phi}\ddot{\phi} + V'\dot{\phi}$, where prime denotes a derivative with respect to the field, we obtain the relevant equation

$$\ddot{\phi} + 3H\dot{\phi} = -V'. \tag{5}$$

It is often convenient to devise a tractable and model independent method of assessing the ability of specific models to reproduce the observations. Parametrization of $w(a)$ in a two dimensional phase space suits this well; there exist many possibilities but one of the simplest,

$$w(a) = w_0 + w_a(1-a), \tag{6}$$

has good success in fitting a variety of scalar field theories, especially those with slow variation (of order the Hubble time) in the equation of state. While there is no requirement that the scalar field partakes of the characteristic time scale of the Hubble expansion, many classes of models do. Furthermore, a reasonable fit to $w(a)$ is only truly needed over the limited redshift range when the dark energy has significant dynamical influence, so Eq. (6) is widely applicable.

For a best fit, w_a is often taken to correspond to the time variation in the equation of state at redshift $z=1$, approximately when dark energy is expected to become significant. That is, $w' = -dw/d\ln a|_{z=1} = w_a/2$. One could also imagine using a different "pivot redshift" to define w_0 and w_a, perhaps that at which the two parameters are decorrelated. However in a coarse sense this is still mathematically equivalent to Eq. (6) and in a fine sense this disrupts the model independence of the parametrization in that the pivot location will depend on the specific model and on the cosmological method of probing it.

The theory of deriving constraints on the dark energy equation of state from a variety of cosmological probes has been well addressed, including aspects of parameter degeneracies and probe complementarity, as well as optimization of observations (e.g. [6–10]). Data from next generation precision cosmology surveys, for example KAOS [11], LSST [12], Planck [13], SNAP [14], etc., should be plentiful and in complementarity capable of determining w_0 and w_a within 1σ uncertainties of roughly 0.05 and 0.15, respectively.

Key clues to the fundamental physics responsible for the acceleration lie in whether w_0 is more negative, more positive, or consistent with the value -1 and whether w_a is negative, positive, or consistent with zero. Measurements consistent with $w_0 = -1$, $w_a = 0$ would provide circumstantial support for a cosmological constant origin, perhaps simply because it is the simplest model, but would also give motivation to look for large scale inhomogeneities in the scalar field since those, possibly in the guise of a sound speed $c_s^2 < 1$, would provide a definitive distinction from the cosmological constant. Of course conversely, values incompatible with the cosmological constant do not rule out its existence, only that its potential energy must be smaller than that of the dominant scalar field.

Even with tightly constrained values of a few characteristics of the equation of state function, such as w_0 and w_a, we will not narrow the field to a specific model. Most potentials have multiple parameters and can cover a swath of such a phase space. What the forthcoming observations will tell us is that certain classes of models are restricted to some parameter range, and other classes are restricted to another parameter range (possibly approaching the limit of a simpler model, such as the cosmological constant). Naturalness and motivation by theory will be needed to winnow the results to a theory of new physics.

But have we been overly narrow in our expectations, by interpreting the observations in terms of a physical component arising from high energy physics? Might the acceleration instead signal new physics from a change in the form of the cosmological expansion equations rather than a change in the ingredients going into them?

III. MODIFICATIONS OF THE FRIEDMANN EQUATIONS

Looking to extensions of general relativity for an explanation of the accelerating expansion has several attractive features. It does not require introduction of hypothetical scalar fields (e.g. quintessence), yet may possess close ties to high energy physics such as string theory or extra dimensions; it does not obviously suffer from fine tuning problems necessarily (e.g. the Ricci scalar naturally evolves; development of density nonlinearities could induce backreaction on the expansion); and it is eminently testable by a number of independent cosmological measurements.

A. General approach

To test the framework of our cosmology theory we should impose prior expectations of the form of a modification as lightly as possible. We have good evidence for the presence of matter density in the universe, from both baryons and dark matter, neither of which can accelerate the expansion, and strong evidence from the cosmic microwave background anisotropy measurements that the universe is consistent with being spatially flat. Taking that as the extent of our knowledge, we can parametrize our ignorance of the physical cause of acceleration with an arbitrary additional term in the Friedmann expansion rate equation:

$$H^2/H_0^2 = \Omega_m(1+z)^3 + \delta H^2/H_0^2. \tag{7}$$

Note that such a phrasing is more general than a parametrization in terms of the matter density exclusively, such as $H^2 = f(\rho)$. While the latter can easily be reduced to the form of Eq. (7) by means of taking $\delta H^2 = f(\rho) - 8\pi\rho/3$, the converse is not true. Indeed, the $f(\rho)$ approach cannot deal with simple time varying dark energy models with nonzero w_a.

Linder and Jenkins [15] showed that the general form Eq. (7) was mathematically equivalent to a time variable dark energy equation of state function

$$w_{\rm DE,eff}(z) \equiv -1 + \frac{1}{3}\frac{d\ln(\delta H^2/H_0^2)}{d\ln(1+z)}, \tag{8}$$

as far as cosmography. That is, observations of the expansion rate and distances alone could not distinguish between these possibilities. This degeneracy might be broken, however, through other information such as the growth rate of matter density perturbations, as discussed below.

In addition to the effective equation of state we can write down other effective "high energy physics" characteristics of the modified gravity theory. The total equation of state of the universe follows immediately from the continuity Friedmann equation, $\dot{\rho} = -3H(\rho+p)$, to give

$$w_{\rm T,eff}(z) \equiv -1 + \frac{1}{3}\frac{d\ln(H^2/H_0^2)}{d\ln(1+z)}. \tag{9}$$

The corresponding potential and kinetic energies of the effective field come from Eqs. (1)–(3):

$$V = \frac{3}{8\pi}\delta H^2 - \frac{H_0^2}{16\pi}\frac{d(\delta H^2/H_0^2)}{d\ln(1+z)} \tag{10}$$

$$K = \frac{H_0^2}{16\pi}\frac{d(\delta H^2/H_0^2)}{d\ln(1+z)}. \tag{11}$$

Note that this is useful as well for treating dark energy models with multiple fields; if there are two components (after all, if we discover that $w \neq -1$ this does not guarantee there is not still a cosmological constant present) then the effective equation of state is a weighted average,

$$w_{\rm DE,eff} = w_1\frac{\delta H_1^2}{\delta H_1^2 + \delta H_2^2} + w_2\frac{\delta H_2^2}{\delta H_1^2 + \delta H_2^2}, \tag{12}$$

where δH_i^2 is the energy density of the ith field.

Various extensions of the Friedmann equation have been considered in the literature. For example [16], consider a term $\delta H^2 \sim H^\alpha$, motivated by infinite scale extra dimensions—a "bulk" encompassing our $4D$ "brane." We initially examine two gravitational source models that lie toward the extremes of present data on the equation of state. The first model is the extra dimensional braneworld "leaking gravity" model [17]. Here the modification to the Friedmann equation arises from a crossover length scale related to the 5-dimensional Planck mass; on larger scales the gravitational force felt in our 4-dimensional brane is reduced. This typically has an effective equation of state more positive than -1 (and corresponds to $\alpha = 1$ above). The second is the vacuum metamorphosis model of [18], originating from a convergent sum of quantum vacuum contributions of a light scalar field coupled to the Ricci scalar curvature. This very elegant approach leads to a rapidly evolving effective equation of state that is more negative than -1. To the extent currently possible these models have some definite physical motivation for their modifications.

We mentioned above that kinematics, i.e. cosmography, would not distinguish between these or other such modifications and dark energy, by virtue of Eq. (8), but that dynamical probes such as the growth of structure might break this degeneracy. Let us investigate this further, both in general terms and with the specific models mentioned.

B. Role of complementary probes

For the growth of structure, it is not only the characteristics of the global, homogeneous and isotropic, universe that enter but the more microphysical properties of the components themselves. Thus sound speed of the dark energy field or interactions with dark matter could give information separate from that contained within the equation of motion governing the cosmic expansion. However, for our present case, we are trying to distinguish modifications of gravity from canonical physical dark energy; if we restrict ourselves to gravitation models obeying the Principle of Equivalence and minimally coupled to matter and nothing else then there are no such microphysical parameters that could break the degeneracy. Then all that enter the perturbation growth are the Hubble drag term depending on $H(z)$ and the dynamical evolution of the matter density, also determined by $H(z)$. For a contrasting view of the braneworld model with a time varying Newton's constant, see [19]. If the dynamics is limited in this way we should expect that if we define a modified Friedmann equation and associated effective equation of state as in Eqs. (7), (8) then we cannot distinguish the gravitational origin from the particularly crafted dark energy model.

However, note that while there is a formal correspondence between a modification δH^2 and an equation of state $w(z)$, one might expect the resulting function to be so complicated that one would be reluctant to ascribe it to a physical dark energy. On the other hand, the modification may be amenable to quite a simple dark energy fit. We examine this for our two test models.

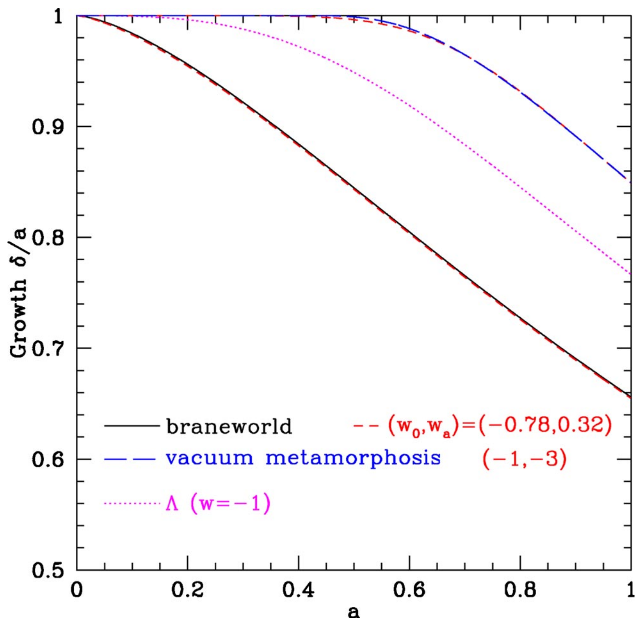

FIG. 1. The growth factor behavior δ/a for two modified gravitation models is compared with that of dark energy models. A clear distinction can be seen relative to the cosmological constant, Λ, model, but simple time varying dark energy models (short dashed, red curves) can be found that reproduce the modified gravity.

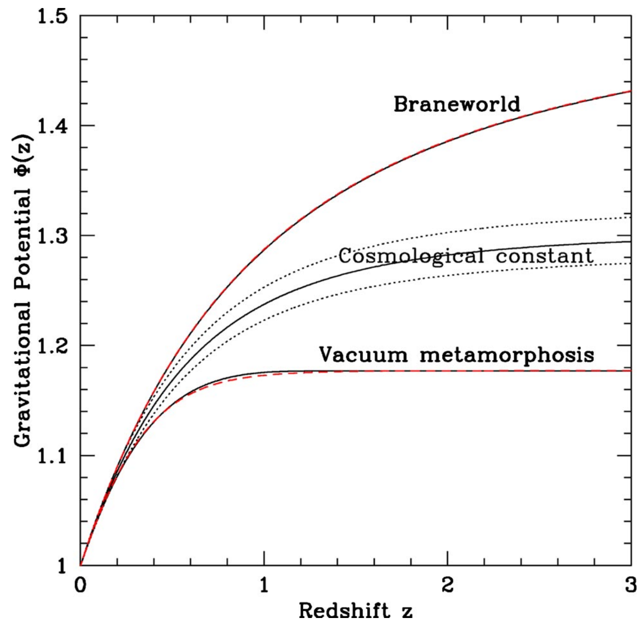

FIG. 2. The gravitational potential $\Phi(z)$ for the same models as Fig. 1 is plotted vs redshift, showing the decay of the potential as the expansion accelerates. Dashed, red curves are for the mimicking (w_0,w_a) models. The dotted outliers to the cosmological constant curve show the deviation expected by a misestimation of the matter density Ω_m by 0.02. The discrimination of modified gravity from a cosmological constant is clear, but from the fit dark energy models is problematic.

For a flat braneworld model, the crossover scale r_c defines an effective energy density $\Omega_{bw}=(1-\Omega_m)^2/4=1/(4H_0^2r_c^2)$ and

$$\delta H^2/H_0^2=2\Omega_{bw}+2\sqrt{\Omega_{bw}}\sqrt{\Omega_m(1+z)^3+\Omega_{bw}}. \quad (13)$$

The cosmography in the form of the supernova magnitude-redshift relation is excellently fit[1] by the simple dark energy model of $(w_0,w_a)=(-0.78,0.32)$. We take both models to have the same matter density, $\Omega_m=0.28$.

For the vacuum metamorphosis model, the cosmic expansion causes the quantum vacuum to undergo a phase transition at a redshift z_j away from the matter dominated behavior. So the modification to the Friedmann equation goes from zero at high redshift to

$$\delta H^2/H_0^2=(1-m^2/12)(1+z)^4+m^2/12-\Omega_m(1+z)^3, \quad (14)$$

for $z<z_j$, where $z_j=[m^2/(3\Omega_m)]^{1/3}-1$ and $m^2=3\Omega_m[(4/m^2)-(1/3)]^{-3/4}$. For $\Omega_m=0.28$, $m^2=10.93$ and $z_j=1.35$. Despite the rapid evolution in the effective equation of state, the magnitude-redshift relation is excellently fit by $(w_0,w_a)=(-1,-3)$. Note that this is a physical model for an effective phantom energy, i.e. where $w<-1$.

Does the dynamical probe of the growth of matter density perturbations preserve the degeneracy between the gravitational source and the high energy physics (dark energy) source for accelerating expansion? Figure 1 emphatically affirms this. While the growth evolution of either of the models is readily distinguishable from a cosmological constant universe, the models cannot be separated from their dark energy counterparts. (One could equally well have first fit the growth history and then looked for deviations in the magnitude-redshift curves.)

Note that the braneworld scenario, with its more positive equation of state, shuts off growth earlier since its influence on the expansion was greater at early times, while the vacuum metamorphosis model shows increased growth even compared to the cosmological constant case, as generically expected for $w<-1$ models. Recall that the linear mass power spectrum is proportional to the square of the growth factor, so the models differ $\sim 25\%$ in power amplitude from the cosmological constant.

If we normalize to the present amplitude of structure (this would roughly correspond to normalizing the power spectra of the different models by the present mass variance σ_8 rather than to the high redshift CMB power) the situation does not change. Figure 2 plots this in the form of the gravitational potential of the mass perturbations. Again the gravity and dark energy models lie virtually on top of each other. To indicate a measure of the ability of cosmological observations to distinguish models, for the cosmological constant case we show the effect of variation in Ω_m by ± 0.02 (dotted lines).

The parametrization in terms of dark energy variables w_0, w_a is nearly the simplest possible, but it is highly successful in mimicking the more complicated gravitational modifica-

[1]Here and in the rest of the paper we mean specifically that the dark energy model reproduces the modified gravity results to within 0.01 mag over the redshift range $z=0-2$.

tion. The possibility of discriminating between dark energy and gravity would be even worse for either a more complicated dark energy ansatz or a nonparametric analysis in terms of the expansion history $a(t)$ or density history $\rho(t)$ directly. Correlations between cosmological quantities tend to dilute the precision of the nonparametric approach relative to the equation of state fit by roughly a factor of 2, e.g. 0.02 mag or 1% distance measurements reconstruct the expansion history to only 2% precision [20].

Another possible cosmological probe is the CMB temperature power spectrum. This is primarily dependent on dark energy or low redshift modifications of the Friedmann equation through the geometric quantity of the distance to the last scattering surface. However it is generally not nearly as sensitive to the equation of state as the supernova magnitude-redshift data. In any case, the distances to the last scattering surface agree between each gravity model considered and its corresponding dark energy version to 0.1%, below what Planck will be able to achieve.

C. Discrimination from Λ

While the degeneracies exhibited between the two gravity models and their dark energy matches are quite interesting, data favors an effective equation of state closer to $w=-1$. However, the braneworld model can only supply this for matter densities $\Omega_m \ll 1$. For $\Omega_m = 0.28$ its rough, averaged equation of state is $\overline{w} \approx -0.7$ while that for vacuum metamorphosis is $\overline{w} \approx -1.3$. Suppose future data continues to narrow in around the value $w=-1$; are there gravitational modifications that may be confused with a cosmological constant fit?

We devise additional terms δH^2 such that they mimic dark energy near the cosmological constant value. These modifications to the Friedmann expansion equation are essentially *ad hoc*, though they bear some functional resemblance to physics models such as braneworld and *k*-essence tachyon field scenarios (cf. [21,22]):

$$\text{Case 1:} \quad H^2 = (8\pi/3)\rho + \sqrt{A' + B'/\rho} \tag{15}$$

$$\text{Case 2:} \quad H^2 = (8\pi/3)\rho + \sqrt{A' + B'\rho} \tag{16}$$

$$\text{Case 3:} \quad H^2 = (8\pi/3)\rho + \sqrt{A'\rho + B'/\rho}. \tag{17}$$

These are universes with matter density as the only physical component dynamically important today, but with modifications to the Friedmann expansion equation. By evaluating these expressions at the present, one derives an expression for the constant A' in terms of B' and Ω_m, so there are only two free parameters. It is convenient to define a dimensionless quantity, $B = B'(8\pi/3H_0^6)$ in cases 1 and 3 or $B = B'(3/8\pi H_0^2)$ in case 2.

Case 1 has the property that the effective dark energy equation of state ranges between $w \in [-3/2, -1]$. At high redshifts, $w \to -1$ and today $w(0) = -1 - B/[2\Omega_m(1-\Omega_m)^2] \approx -1 - 3.4\,B$. For case 2 the range is $w \in [-1, -1/2]$, with the value evolving from $w = -1/2$ at $z \gg 1$ to $-1 + 0.27B$ today. The relatively large value of w at early times is likely to interfere with structure formation. Very roughly, cases 1 and 2 are milder versions of the vacuum metamorphosis and braneworld scenarios, respectively.

Case 3 is intriguing in that $w \in [-3/2, -1/2]$, crossing the cosmological constant value of -1. Thus one might imagine that this model could mimic on average the cosmological constant at recent times—and is worth studying in detail. Unfortunately, the transition between its asymptotic values is quite sharp owing to the difference of six powers of the scale factor in the two terms in the square root. One could adopt a wholly *ad hoc* model containing $\sqrt{A'\rho^{\alpha} + B'\rho^{-\beta}}$ but we would likely learn little physics motivation. Instead we keep case 3 as is and use it as an interesting, if extreme, test case to investigate model degeneracy. Because of its rapid transition, if this model can be well fit by a simple dark energy model then much less radical forms likely will be as well.

In this phenomenology we walk a fine line: if $w > -1$ by too much, the model will be uninteresting since it is easily ruled out by observations, but if $w \approx -1$ then the modification is too strongly degenerate with simple physical dark energy models to probe physics well. Observations are less stringent on ruling out models with $w < -1$, so these are useful to explore further, and if they cross through the interesting $w = -1$ value then their time averaged equation of state may well satisfy future constraints. Thus case 3 allows investigation of the extent to which distance and growth probes can break degeneracies between classes of physics responsible for the acceleration.

We first consider for which values of the parameter B we can fit the data for the least sensitive dark energy probe: the CMB measurement of the distance to the last scattering surface. If we require the distance to match the distance in the cosmological constant case to a certain precision, then we obtain upper limits to B in cases 1 and 2, and a range in case 3. This is because in cases 1 and 2 the value $B=0$ corresponds exactly to the cosmological constant, so these cases will never be fully ruled out under our assumption that the true model is that of the cosmological constant Λ. However case 3 is distinct from a Λ model throughout its parameter space.

Figure 3 illustrates the allowed parameter space for the case of WMAP precision: the last scattering surface distance d_{lss} known to 3.3% (1σ). For case 1, the area between the long dashed curve and the dotted curve at $w=-1$ is allowed, corresponding to $B < 0.427$. For case 2, the area between the short dashed curve and the dotted curve at $w=-1$ is allowed, corresponding to $B < 0.144$. Note that in both cases the allowed effective equations of state are fairly slowing varying functions of redshift, so we expect ease in fitting them to a (w_0, w_a) dark energy model and difficulty in discrimination from the cosmological constant with whatever probe for $B \ll 1$. So we will not consider them further. In case 3, the CMB data would restrict the model to have $0.099 < B < 0.145$, with a perfect match of the distance for $B = 0.131$. Nevertheless, the equations of state clearly do not resemble that of the cosmological constant, and have a strong time dependence. (Note that Planck precision of 0.7% would limit B to between 0.126 and 0.135.) The CMB distance to last scattering, normally thought fairly insensitive to

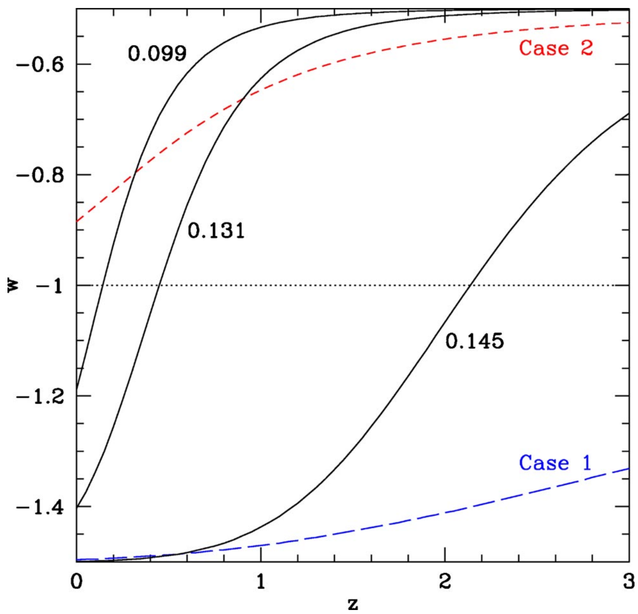

FIG. 3. The effective equations of state corresponding to the modified Friedmann equations (15)–(17) are plotted vs redshift. The parameter space allowed under CMB constraints for cases 1 and 2 lie between the respective curves shown and the $w=-1$ line, i.e. they can mimic a cosmological constant arbitrarily closely. Case 3 curves (labeled by value of B) can fit the CMB distance of the Λ model with much more strongly varying equations of state, lying between the left and right solid curves, with a perfect fit given by the middle solid curve.

time variation, can put tension on regions of parameter space for these time varying models.

Next we apply the supernova magnitude-redshift and growth tests to the models given by case 3 and see to what extent these can distinguish the gravitational model from the cosmological constant, or from the best fitting effective dark energy model. All of these gravity models can be distinguished from the cosmological constant, Λ model through the magnitude-redshift probe; the magnitude differences range from 0.1–0.2. It is not easy to mimic the cosmological constant behavior with a modification δH^2 except as $\delta H^2 \to \Lambda$.

However this is a separate issue from whether the modification matches *some* dark energy model. In general the degeneracy between a gravitational source and effective dark energy model remains. We find excellent fits by the simple (w_0,w_a) parametrization as follows: $B=0.099$ corresponds to $(w_0,w_a)=(-1.12,1.2)$, $B=0.131$ to $(-1.49,1.64)$, and $B=0.145$ to $(-1.52,0.2)$. Recall that $w' \approx w_a/2$. The model that exactly reproduces the d_{lss} for the Λ $(w'=0)$ model has a $w' \approx 0.8$!

Again, the growth of matter perturbations does not break the degeneracy, as seen in Fig. 4. Gravity models can be distinguished from each other, and dark energy models from each other, but the mapping to the effective equation of state holds firm. Note that the growth behavior of the models from cases 1 and 2 with the largest B values allowed by the CMB data roughly agree with the extremes plotted for case 3. This implies that if CMB data is consistent with the cosmological

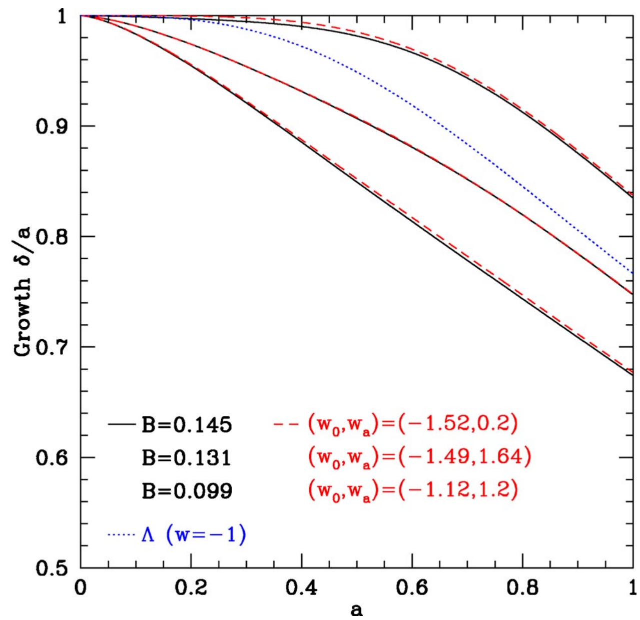

FIG. 4. As Fig. 1 but for case 3 modified gravity. A fairly clear distinction in growth behavior exists relative to the cosmological constant model, but not with respect to each corresponding, simple, time varying dark energy (dashed, red curves). These were chosen to match the magnitude-redshift relation, so neither expansion history nor growth history here distinguishes between a gravitational and dark energy explanation for the acceleration of the universe.

constant then the growth behavior should lie in the region between the upper and lower growth curves (at least for the three case forms considered).

Differences between the $B=0.131$ (exact match in d_{lss}) model and the cosmological constant amount to less than 8% in the power spectrum, so the magnitude-redshift data would

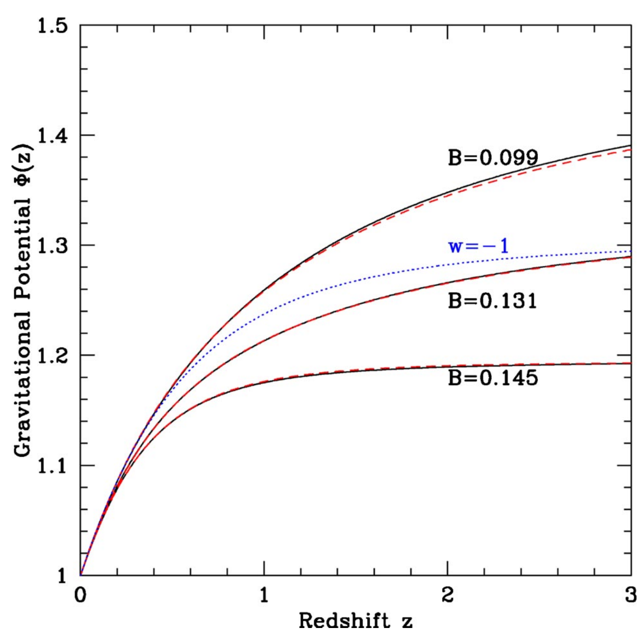

FIG. 5. The gravitational potential behavior as in Fig. 2, but for the case 3 modified models (black solid curves) and dark energy models (red dashed curves, blue dotted curve for $w=-1$) in Fig. 4.

be the most incisive probe. In Fig. 5 we again normalize to the present matter power spectrum and plot the gravitational potential decay behavior. Even such an extreme modified gravity model as the rapidly varying case 3 cannot be distinguished from a dark energy parametrized by (w_0, w_a). [Note that in fitting (w_0, w_a) models we impose $w(z) \leq -0.5$ in the growth equation to match the allowed equation of state range of the B models, but this in fact does not affect the results very much.]

IV. ACCELERATION DIRECTLY

Through the Principle of Equivalence, acceleration has a very direct relation to the nature of gravitation and to the spacetime geometry. In turn, mapping the expansion history and observations of cosmological distance relations, or cosmography, has a clear connection to the spacetime geometry. This allows future data to directly constrain modifications of general relativity, testing the framework of the gravitation theory not merely the ingredients of the universe. It seems useful to try to make this connection between the measurements and theory as explicit as possible, especially in the hope of distinguishing a gravitational origin for the acceleration of the expansion of the universe from a physical dark energy origin.

A. Principles

Starting with Robertson-Walker metric for a homogeneous and isotropic universe, and imposing spatial flatness, leads to the relation between the expansion factor $a(t)$ and the spacetime geometry quantity of the Ricci scalar curvature R:

$$R = 6\left(\frac{\ddot{a}}{a} + H^2\right), \tag{18}$$

where $H = \dot{a}/a$ is the Hubble parameter. No dynamics, i.e. specific $a(t)$ relation or physical theory, is assumed. Cosmography directly probes a and its derivatives, and hence the quantities R and H. It is possible that these are not the whole story, that the gravitational action contains other terms and so the interpretation of the observations in terms of the theory of gravity is more complicated, but so long as the metric holds, then the relation (18) is still good. (See [23] for the case of R^{-n} terms in the action.)

Observations of acceleration, $\ddot{a} > 0$, then inform us about the Ricci scalar. In particular, acceleration imposes the condition

$$R > 6H^2. \tag{19}$$

Again, this is wholly equivalent at this level to an effective total equation of state parameter for the universe,

$$w_{\rm T,eff} \equiv \frac{1}{3}\left(1 - \frac{R}{3H^2}\right). \tag{20}$$

Indeed we see that $R > 6H^2$ corresponds to the usual condition $w < -1/3$. The use of w is purely a symbolic definition and does not rely on a physical link that would come from, e.g., employing the relation $R = 8\pi T$ between the Ricci scalar and the trace of the energy-momentum tensor that general relativity provides.

Note that consistency holds between the two approaches of this section and Sec. III. In some sense we have modified the acceleration ($\ddot{a}$) Friedmann equation here and the velocity ($\dot{a}^2$) Friedmann equation in the previous section. To demonstrate consistency, start with Eq. (8) and substitute in Eq. (7). Using the identity

$$(H^2)^{\dot{}} = 4H^3[R/(12H^2) - 1], \tag{21}$$

one obtains

$$w_{\rm DE,eff}(z) = \frac{1}{3}\,\frac{H^2}{\delta H^2}\left(1 - \frac{R}{3H^2}\right). \tag{22}$$

Finally, since the total equation of state of the universe is related to the effective dark energy, or "parametrized ignorance," equation of state by $w_T(z) = w_{\rm DE,eff}(z)\Omega_{\rm DE,eff}(z)$, we find

$$w_T = w_{DE}(\delta H^2/H^2) = \frac{1}{3}\left(1 - \frac{R}{3H^2}\right), \tag{23}$$

as in Eq. (20). One can only go from Eq. (20) to Eq. (8), however, if one defines an appropriate split between knowledge and ignorance, i.e. the Ω_m and δH^2 terms.

The generality of the link of the total equation of state with the spacetime geometry and the dynamical Eq. (21) has an exciting implication. The equations point up the centrality of the variable $\mathcal{R} \equiv [R/(12H^2)](z)$, since both the equation of state and the Hubble expansion parameter can be defined in terms of it. That is, through Eq. (21) H is determined by

$$\frac{H^2}{H_0^2} = e^{4\int_0^{\ln 1+z} d\ln y(1-\mathcal{R})}. \tag{24}$$

Knowledge of the spacetime quantity $\mathcal{R}$ therefore allows us to solve for H, R, the comoving distance $r(z) = \int dz/H$ and others, the magnitude-redshift relation $m(z) = 5\log[(1+z)r]$, etc. This is a powerful simplification.

Furthermore, we will see in Sec. V that $\mathcal{R} = 1$ is a critical value, corresponding to $w_T = -1$ (a de Sitter state) and a universe on the cusp between ordinary acceleration and superacceleration.

B. Parametrization

As with the equation of state $w(z)$, forthcoming observational data will not be strong enough to reconstruct directly the entire function, here $\mathcal{R}(z)$. Instead we must learn about the physics encoded in it, whether gravitational or high energy, in smaller steps. Following the equation of state we might try to parametrize $\mathcal{R}$ in different models by a fitting form containing a few parameters. Suppose in analogy to Eq. (6) we write

$$\mathcal{R} = r_0 + r_1(1-a), \tag{25}$$

with r_0 representing the present value and r_1 giving a measure of its time variation. This seems a reasonable minimal parametrization for the same reasons as with $w(a)$; one might expect that the spacetime geometry should be slowly varying with the expansion.

In this ansatz, the Hubble parameter is

$$H = H_0 a^{2(r_0+r_1-1)} e^{2r_1(1-a)}. \tag{26}$$

If we want to ensure a matter dominated epoch at high redshifts ($a \ll 1$), then we require H to asymptotically vary as $a^{-3/2}$, thus

$$r_0 + r_1 = 1/4. \tag{27}$$

This leaves us with only a one parameter family and so we could elaborate the fitting form (25) to allow a second parameter. However, we find that for redshifts $z \lesssim 2$, where most of the cosmological probe data will lie, the linear fit is a superb approximation to a wide variety of physical dark energy models—as long as the constraint condition Eq. (27), unnecessary at these redshifts, is not imposed. However one could certainly be fancier and attempt a fit that both satisfies the moderate redshift fitting and the asymptotic constraint, such as $\mathcal{R} = 1/4 + r_0 a \tanh(r_1 a)$ or $\mathcal{R} = 1/4 + r_2 a^2 + r_3 a^3$ (i.e. a cubic polynomial with the zeroth order term fixed by the matter domination asymptote and the first order term fixed by the smooth approach to this asymptote; thus we are left with two free parameters). But the linear fit suffices, matched smoothly to a matter dominated asymptote for high redshift calculations.

Finally, if we have a specific function $\mathcal{R}$ then we can derive the corresponding dark energy model, or its effective equivalent, upon imposing a split between matter and dark energy, i.e. choosing Ω_m. The effective dark energy equation of state is then

$$w_{\rm DE,eff}(a) = \frac{1}{3}(1 - 4\mathcal{R})[1 - \Omega_m e^{-\int d\ln y(1-4\mathcal{R})}]^{-1}, \tag{28}$$

and the scalar field potential and kinetic energies follow from Eqs. (1)–(4) as before. Explicitly,

$$V = (1 + 2\mathcal{R})\frac{H^2}{8\pi} - \frac{3H_0^2}{16\pi}\Omega_m a^{-3} \tag{29}$$

$$K = (1 - \mathcal{R})\frac{H^2}{4\pi} - \frac{3H_0^2}{16\pi}\Omega_m a^{-3}. \tag{30}$$

C. Distinction from dark energy

As we carried out previously for the Friedmann modifications δH^2, we can investigate the discrimination between this direct acceleration, or "geometric dark energy," model and physical dark energy for various cosmological probes. Once again the straightforward parametrization of dark energy in terms of (w_0, w_a) provides an excellent fit to the geometry model [note that this is *not* a consequence of the similar forms of Eqs. (25) and (6), due to the presence of matter; furthermore, the fit is similarly successful when using $\mathcal{R} = 1/4 + r_2 a^2 + r_3 a^3$].

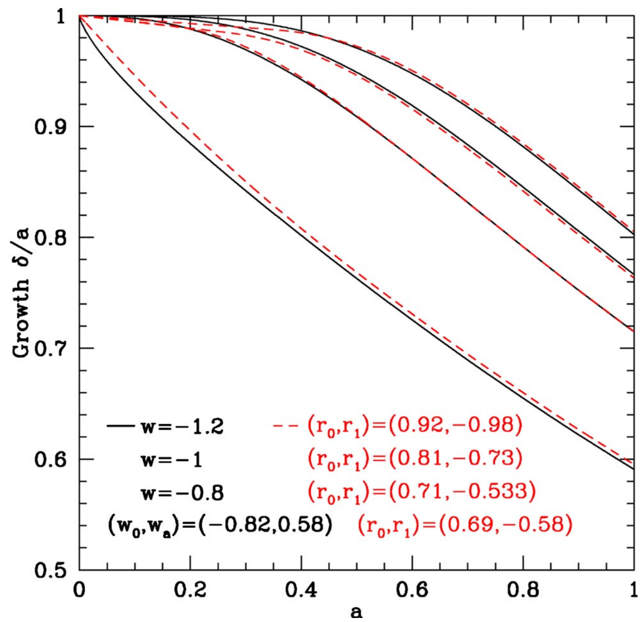

FIG. 6. Growth factor as in Fig. 4, but for the Ricci geometric dark energy models (red, dashed curves). Simple parametrizations of these models can match the behavior of dark energy models (solid, black curves), including the cosmological constant. Slight deviations occur at higher redshifts.

We examine four dark energy–Ricci geometry pairs. For the cosmological constant, the fit is provided by $(r_0, r_1) = (0.81, -0.73)$, for the time varying equation of state SUGRA model with $(w_0, w_a) = (-0.82, 0.58)$ the analog is $(r_0, r_1) = (0.69, -0.58)$, and for $w = -0.8$ and $w = -1.2$ they are $(0.71, -0.533)$ and $(0.92, -0.98)$ respectively. Each pair possesses magnitude-redshift diagrams agreeing within 0.01 mag out to $z = 2$.

Dynamical aspects within the matter density perturbation growth equation still contain no leverage to break the degeneracy in any substantial way. For reference we write the growth equation of a linear matter density perturbation $\delta = \delta\rho/\rho$:

$$G'' + (3 + 2\mathcal{R})a^{-1}G' + [1 + 2\mathcal{R} - (3/2)\Omega_m a^{-3}/(H^2/H_0^2)]a^{-2}G = 0, \tag{31}$$

where $G = \delta/a$ is the normalized growth and prime denotes a derivative with respect to scale factor a. (The growth equation given a modification δH^2 is written in [15].)

Figure 6 shows the growth curves for these four pairs of models. At higher redshift the geometric models do have a deviation in growth behavior relative to the dark energy models, but this is small. Note that we enforce matter domination asymptotically, matching the (r_0, r_1) parametrization onto $\mathcal{R} = 1/4$ at high redshift, but this is unlikely to be responsible for the deviation as the effect goes in the opposite

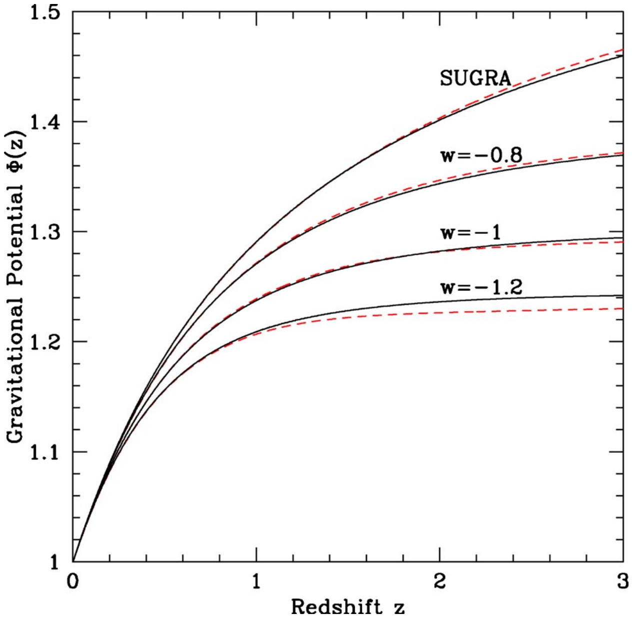

FIG. 7. The gravitational potential $\Phi(z)$ corresponding to the models of Fig. 6. Slight deviations at higher redshifts occur between the Ricci models (dashed red curves) and their corresponding dark energy partners (solid black curves). Deviations in *slope* focus on the behavior at $z \approx 1-2$.

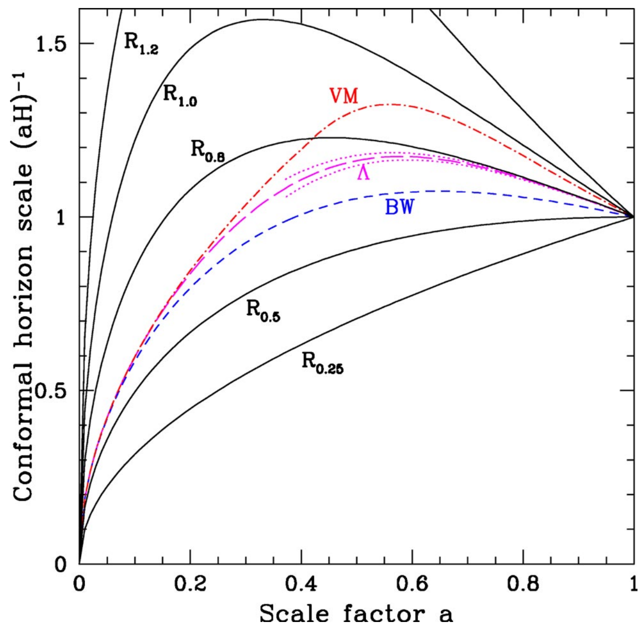

FIG. 8. The expansion history is plotted in terms of conformal horizon scale vs scale factor for various modified gravity and spacetime geometry models. The Ricci geometric dark energy models (solid, black curves) are subscripted with the present value r_0, and have the form $\mathcal{R}=r_0+(1/4-r_0)(1-a)$. All models are matter dominated in the past. Negative slopes indicate an accelerating epoch while slopes more steeply negative than a critical value $(-1$ at the present) indicate superacceleration.

direction, increasing the growth, and would enter at a different redshift than seen.

The deviation can be seen more clearly in the gravitational potential decay behavior of Fig. 7. Especially for the $w=-1.2$ case a distinction between the Ricci geometry and dark energy models can be seen, but this amounts to less than 1% difference out to $z=3$. So for both cosmography and growth of structure, interpretation in terms of an effective equation of state remains a robust path, though not one that allows us to probe all the details of the fundamental physics responsible.

Studying the behavior of the gravitational potential in Fig. 7 does offer one possible hope for elucidating the physics model in more detail. At high redshift, $z \gg 1$, we expect that all models approach the matter dominated behavior where the gravitational potential is constant. This corresponds to the linear perturbation growth $\delta \sim a$. Such behavior, of the development of structure through gravitational instability of adiabatic density perturbations, has been broadly successful in explaining the appearance of large scale structure in our universe. In such a decelerating phase of the expansion, the origin of the accelerating physics should be largely moot.

At low redshift, $z \ll 1$, all the models within the region of parameter space our universe seems to inhabit show a similar behavior, all the curves of $\Phi(z)$ possessing nearly the same slope and so overlapping. This does not arise from any fundamental requirement but is a coincidence for models with behavior not too different from a cosmological constant and for our universe at the present time, not too long after the acceleration began. (A similar coincidence makes the contours of constant age of the universe lie parallel to those of angular size corresponding to the first acoustic peak of the CMB, allowing for tight constraints on the age via CMB measurements [24].)

Since the slope of the gravitational potential-redshift relation is therefore fixed at the two ends (roughly 1/2 at $z=0$ and 0 at $z \gg 1$), there will be some intermediate redshift where the deviation in slope $d\Phi/dz$ between models is maximal. This in fact occurs when the dark energy or other accelerating mechanism begins to be dynamically significant, and the changing slope or curvature offers clues to the underlying physics, localized to this redshift.

Certain cosmological observations relevant to the key redshift range of $z \approx 1-2$ in fact are sensitive to this effect. One is the integrated Sachs-Wolfe effect (ISW), where the CMB photon interaction with the time varying gravitational potential of large scale structure in the process of formation leads to CMB anisotropies on large angles or low multipoles. This involves $d\Phi/d\eta = H d\Phi/dz$ (see, for example, [25]). Another prospective probe is the CMB bispectrum, related to the three point correlation function of temperature anisotropies arising from nongaussianities induced by weak gravitational lensing of the CMB by large scale structure (see [26]). This involves $\Phi(d\Phi/dz)$ and has been recognized to allow CMB measurements to have some sensitivity to the time variation of the dark energy equation of state [27], similarly localized to $z \approx 1-2$. Both these methods may be able to play a role in breaking the degeneracy between the physics of the spacetime geometry Ricci term and a physical dark energy.

V. SUPERACCELERATION AND THE BIG RIP

As alluded to in Sec. IV A, the value of the normalized Ricci scalar curvature $\mathcal{R}=R/(12H^2)=1$ has a special role. The condition for superacceleration, where the acceleration increases with time, is $\mathcal{R}>1$, which could be written $w_{\rm eff}<-1$. For the case of a physical dark energy component this implies that its energy density increases with expansion. An important point regarding superacceleration is that it corresponds to $(\ddot{a}/a)^{\dot{}}>0$ and not $(\ddot{a})^{\dot{}}>0$. That is, the conformal acceleration is the relevant quantity.

This is analogous to the condition for acceleration, or inflation, where $(aH)^{\dot{}}>0$, meaning the conformal horizon $(aH)^{-1}$ shrinks with time. Indeed such an acceleration condition is equivalent to $\dot{H}>-H^2$ while superacceleration relies on $\dot{H}>0$, equivalent to $(\ddot{a}/a)^{\dot{}}>0$. More explicitly, if $R<12H^2$ then $(\ddot{a}/a)<H^2$. If this holds for all future times then $(\ddot{a}/a)^{\dot{}}<(H^2)^{\dot{}}=2H[(R/6)-2H^2]<0$. Thus superacceleration is $(\ddot{a}/a)^{\dot{}}>0$ and not $(\ddot{a})^{\dot{}}>0$. The latter condition would be satisfied by a dark energy equation of state ratio $w<-2/3$, while $(\ddot{a}/a)^{\dot{}}>0$ corresponds to $w<-1$.

Figure 8 illustrates the behavior of the conformal horizon in various cases, including those of Ricci geometric dark energy models listed by their present value of $\mathcal{R}$. Those shown follow Eq. (25) with constraint Eq. (27). Any model with a region of negative slope is accelerating during such an epoch; e.g. the $r_0=0.5$ model is just starting to accelerate today, corresponding to $w_T=-1/3$. The cosmological constant model has nearly the same acceleration today as for $r_0=0.8$, and $r_0=0.25$ is the (decelerating) Einstein–de Sitter cosmology. Superacceleration requires a slope more steeply negative than $-(a^2H)^{-1}$, i.e. -1 today. This condition for superacceleration can be rewritten in terms of the logarithmic slope of the conformal diagram as

$$\frac{d\ln(aH)^{-1}}{d\ln a}<-1. \tag{32}$$

It occurs for models steeper today than $r_0=1$, or more generally $\mathcal{R}>1$ or $w_T<-1$.

From this diagram one can read off that a model such as vacuum metamorphosis is accelerating today but not superaccelerating. Although it acts as a component with $w<-1$, the total equation of state of the universe, including matter, is $w_T>-1$. Even a currently superaccelerating model like $r_0=1$ only began accelerating at $z=2$, so we see that there is a relatively narrow range of redshifts—not ‘‘fine tuned’’—when this extraordinary property of the universe will be evident.

Note that such increasing conformal acceleration implies the existence of a Rindler horizon in the spacetime. That is, points at a distance $r>1/g$ from an observer, where g is the conformal acceleration, recede at greater than the speed of light and so are hidden behind a horizon [28,29]. Generically such a horizon radiates particles at a temperature $T=g/(2\pi)$, analogous to Hawking radiation from a black hole horizon.

Now we have seen that a component with $w<-1$, so-called phantom energy, leads to superacceleration. This implies a Big Rip scenario for the fate of the universe, according to [30], where the increasing acceleration overcomes all other attractive forces. However we conjecture that the particle creation from the Rindler horizon gives an energy density in radiation that grows faster than the phantom energy. Illustratively, $\rho_R\sim T^4\sim(\ddot{a}/a)^4\sim\rho_{ph}^4$ while phantom energy dominates the universe. So the ratio $\rho_R/\rho_{ph}\sim\rho_{ph}^3$ and this grows with time since $w<-1$. Therefore at some point the radiation energy density will overtake the phantom energy density, shutting off the superacceleration. Without superacceleration the particle creation declines, the radiation energy redshifts away, and the phantom energy can again dominate. Depending on the details, this may lead either to an attractor at $w=-1$ or a cycle of superacceleration and hot, radiation (and matter) dominated phases of the universe.

VI. CONCLUSION

To face the challenge of determining the fundamental physics responsible for the acceleration of the universe, we need to bring to bear next generation observations of the expansion history and possibly its dependent growth history. The precision and accuracy of these future observations will guide us a long way to identifying new physics. We see that at the heart of the next step lies a single function—the effective equation of state $w(z)$. Mapping this describes the cosmology; models with the same function, or equivalently same expansion history, will agree on the cosmological tests, whether distance-redshift, growth of structure, etc. Furthermore the simple parametrization in terms of the present value, w_0, and a measure of the time variation, w_a, proves extraordinarily robust regardless of the exact reason for elaborating on the matter density term in the Friedmann equation.

This is not to say there is no complementarity between cosmological probes; indeed that is a crucial ingredient in constraining the *values* of the equation of state parameters. And next generation experiments will be superb at achieving this. The simplicity of a two parameter functional form means we cannot easily appeal to ‘‘naturalness’’ to decide which physics model—dark energy or modified gravity, say—is a most likely explanation. Despite the models considered here, though, there is no guarantee that an arbitrary modification δH^2 can be fit in terms of w_0, w_a. Regardless, the function $w(z)$ encodes all the standard, ‘‘smooth’’ information regardless of origin.

We have illustrated this for several classes of physics including scalar field dark energy, modifications of general relativity in the Friedmann equation, and direct acceleration through Ricci ‘‘geometric dark energy,’’ both in general and for specific models. Explicit examples of the fits were given for probes such as magnitude-redshift, growth factor or gravitational potential, and distance to the CMB last scattering surface. This held even for models with quite large time variation of the effective equation of state.

One possible breakdown of the simple dark energy mimic ability might occur through the curvature of the gravitational potential decay behavior; the slope is remarkably model independent at low redshifts and asymptotically matter dominated at high redshift, but the localized deviation in between might provide a clue to the accelerating physics. Precision observations of the integrated Sachs-Wolfe effect or the lensing induced CMB bispectrum, yet untested, might be useful probes for this.

We considered the implications of acceleration in general, regardless of origin, through the Ricci scalar curvature. This is pleasingly directly related to the expansion and fate of the universe. In a conformal horizon history diagram (Fig. 8) we illustrate conditions for both acceleration and superacceleration, and briefly discuss the role of superacceleration in particle production that could nullify the Big Rip and indeed possibly provide an attractor for the universe to an apparent cosmological constant state.

The picture of an achievable and wide ranging goal in measuring $w(z)$ is attractive. In our quest for understanding fundamental physics, though, we always want to push deeper. The virtues of simplicity and broad applicability contest with lack of leverage in separating the root causes. But it is only in the absence of new dynamics, new equations of motion, that the equation of state $w(z)$ or the expansion history $a(t)$ rules all. New terms—interactions or graininess—lead to complexity but a grip on deeper details of the new physics. This graininess could come from an observable consequence of dark energy perturbations or a noncanonical sound speed, separating it from a ''smooth'' gravity law (though it is only useful if it occurs within a realm accessible to precision observations). Conversely, couplings in the gravitational sector, going beyond the Ricci spacetime geometry approach analyzed here, could distinguish a gravitational origin from one of dark energy. This could arise in scalar-tensor theories, or metric perturbation terms $\dot{h}$ in the growth equation, or local curvature dependent effects δR, e.g. back reaction from structure formation.

This is rather analogous to the situation in early universe acceleration—inflation theory. The incredible simplicity and generic power of it in solving cosmological and high energy physics conundra is immensely attractive, and we should not lose sight of it, just as we should not lose sight of the crucial role of $w(z)$. But acceleration, then and now, is very much more than just a de Sitter state. We *want* complexity in the form of perturbations, tilt, gravitational waves to learn about the details of the fundamental physics. For the CMB, measuring $\delta T/T$, or the power spectrum, is a stunning experimental accomplishment, just as $w(z)$ will be, but we want to explore further through non-Gaussianities, polarization, etc. So too we look forward to probing gravity, dark energy, and acceleration.

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