Complete analysis of baryon magnetic moments in the $1/N_c$ expansion

Richard F. Lebed* and Daniel R. Martin†
Department of Physics and Astronomy, Arizona State University, Tempe, Arizona 85287-1504, USA
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We generate a complete basis of magnetic moment operators for the $N_c=3$ ground-state baryons in the $1/N_c$ expansion, and compute and tabulate all associated matrix elements. We then compare to previous results derived in the literature and predict additional relations among baryon magnetic moments holding to subleading order in $1/N_c$ and flavor SU(3) breaking. Finally, we predict all unknown diagonal and transition magnetic moments to $\leqslant 0.15\mu_N$ accuracy, and suggest possible experimental measurements to improve the analysis even further.

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I. INTRODUCTION

The generalization of quantum chromodynamics from 3 to $N_c>3$ color charges, called large N_c QCD, has opened a path to substantial progress in understanding strong interactions at both the formal and phenomenological levels. Formal successes spring from the fact that large N_c QCD exhibits a well-defined limit, meaning that the renormalization group equations remain finite and nontrivial as $N_c\to\infty$. Furthermore, the counting of explicit N_c factors organizes QCD Feynman diagrams into topological classes of decreasing significance with increasing powers of $1/N_c$, which defines the $1/N_c$ expansion. Phenomenological successes build on these formal $1/N_c$ power-counting results, but add one crucial extra ingredient: Observables calculated to appear at $O(1/N_c)$ or $O(1/N_c^2)$ are empirically found to be a factor 3 or 9, respectively, smaller than corresponding quantities calculated to be $O(N_c^0)$; this means that even QCD with N_c as small as 3 belongs to the class of theories for which the $1/N_c$ expansion is meaningful. We note only that the literature to date that provides evidence substantiating these statements has become so extensive, that nothing short of a review article [1] can do it justice.

Nevertheless, a multitude of problems utilizing the $1/N_c$ expansion, even for well-known observables, remain unsolved. In this paper we focus on one very specific such set, the magnetic moments of the u,d,s baryons in the ground-state multiplet. In the case of large N_c, this multiplet consists of a tower of states [2] completely symmetric under combined spin and flavor transformations, thus providing justification for the group-theoretical aspects of the old three-flavor SU(6) classification for baryons. The nonstrange members of the multiplets in this tower carry the (I,J) quantum numbers $(\frac{1}{2},\frac{1}{2}),(\frac{3}{2},\frac{3}{2}),\ldots,(N_c/2,N_c/2)$. The first (I,J) multiplet represents nucleons, which reside in an SU(3) multiplet that is an octet for $N_c=3$; the second represents Δ resonances in an SU(3) multiplet that is a decuplet for $N_c=3$. Here we continue to use the SU(3) labels **8** and **10**, despite the fact that the corresponding SU(3) representations are much larger for $N_c>3$ [3]. Of course, for $N_c=3$ this tower truncates after the Δ's. While the mass of each baryon is $O(N_c^1)$, mass splittings between two low-lying states in the tower [i.e., $I=J=O(N_c^0)$] is $O(1/N_c)$ [4], supporting the notion of a true degenerate spin-flavor multiplet. In fact, it is only because our universe is somewhat closer to the chiral limit than the large N_c limit that Δ and its partners in the SU(3) decuplet are unstable under strong decays: $m_\pi=O(m_{u,d})=O(N_c^0)<m_\Delta-m_N=O(1/N_c)$.

In a complete analysis organized according to $1/N_c$, the whole set of states in the spin-$\frac{1}{2}$ **8** and spin-$\frac{3}{2}$ **10** [and SU(3) multiplets associated with spin $\frac{5}{2},\frac{7}{2},\ldots,N_c/2$, which decouple for $N_c=3$], must be considered together as a single, completely symmetric spin-flavor multiplet with N_c fundamental representation (quark) indices; we continue to denote this multiplet by the old SU(6) label **56**, although again for $N_c>3$ the dimension of this representation is much greater. The instability of spin-3/2 baryons is taken into account simply by maintaining finite values for m_π and $1/N_c$ in the full Hamiltonian.

We hasten to add that magnetic moments for baryons in the **56** have been considered in the $1/N_c$ expansion in the past—in fact, in papers dating back a decade or more. There are three papers in particular that have examined these magnetic moments in the $1/N_c$ expansion: Jenkins and Manohar (JM) [5], Luty, March-Russell, and White (LMRW) [6], and Dai, Dashen, Jenkins, and Manohar (DDJM) [7]. Each of these papers contains a scheme for including a particular set of operators that contribute to magnetic moments, and each is discussed in detail below, once we establish a notation to describe the formalism.

In short, however, the essential improvement of the current work over previous papers is completeness. Once all relevant baryon states are assumed to fill a complete spin-flavor multiplet—in this case, the **56**—then only a finite number of operators exist with distinct spin-flavor transformation properties that can generate nonvanishing baryon bilinears in a Hamiltonian. This number precisely equals the number of distinct observables associated with the given quantum numbers. For example, the **56** allows precisely 19 linearly independent mass operators (those with $\Delta J=0,\Delta J^3=0,\Delta Y=0,\Delta I^3=0$, T even) with distinct spin-flavor properties, corresponding to the masses of the eight **8**

*Electronic address: Richard.Lebed@asu.edu
†Electronic address: daniel.martin@asu.edu

baryons, the ten **10** baryons, and the spin-singlet $\Sigma^0\Lambda$ mixing. In the magnetic moment case ($\Delta J=1, \Delta J^3=0, \Delta Y=0, \Delta I^3=0$, T odd), one finds 27 linearly independent operators, corresponding to the eight **8** baryons, ten **10** baryons, the $\Delta J=1$ $\Sigma^0\Lambda$ mixing, and eight SU(3)-breaking $\Delta J=1$ mixings between states of the same I^3 and Y in the **8** and **10**, such as $\Delta^+ p$. Quite simply, these descriptions represent two complete bases of a vector space corresponding to a particular class of observable: One basis is organized in such a way as to give one basis vector for each observable for a given state, and the other basis is organized according to quantum numbers of the spin-flavor symmetry. In such an analysis, an arbitrary amount of symmetry breaking can be accommodated.

This approach was used to classify all static observables of the literal SU(6) **56** (i.e., using only $N_c=3$) in Ref. [8], with a deeper study of quadrupole moments in Ref. [9]. It has been used in the $1/N_c$ expansion several times: for the masses of the **56** [10], for charge radii and quadrupole moments [11–13], for the masses and couplings of the orbitally-excited baryon multiplet **70** [14–18], and even for the $\Delta \to N\gamma$ couplings closely related to the ΔN transition magnetic moments [19].

This paper is organized as follows: In Sec. II we explain why the operator expansion $1/N_c$ truncates at a finite order, and how the complete set of operators may be enumerated. We compute and tabulate all the matrix elements of all these operators in Sec. III. In Sec. IV we compare our approach to previous ones in the literature [with and without perturbative flavor SU(3) breaking], derive new relations, fit to all existing data, and use the results of this fit to predict all unmeasured moments. The casual reader uninterested in calculational details is encouraged to skip directly to Sec. IV. We summarize and conclude in Sec. V.

II. OPERATOR BASIS

Each baryon state belongs to a representation composed of N_c color fundamental representations combined into a color singlet. While it is suggestive to think of each such fundamental representation being associated with a single current quark, such an identification is not necessary; in general, each fundamental representation merely represents an interpolating field whose quantum numbers match those of a single quark in color, spin, and flavor—each of these in the fundamental representation of the corresponding group—and which together exhaust the whole baryon wave function [11]. In general, such a field consists of not only a current quark, but gluons and sea quark-antiquark pair Fock components as well, and indeed may be thought of as a rigorously defined constituent quark. We continue to denote such an interpolating field by the simple label "quark."

An arbitrary baryon bilinear, as appearing in the Hamiltonian for masses, magnetic moments, etc., thus carries the quantum numbers of N_c quarks and N_c antiquarks. Since *fundamental* $\otimes$ *antifundamental* = *adjoint* $\oplus$ *singlet* for all SU groups (in this case, each of spin, flavor, and spin-flavor), each operator that can connect the baryon states may be decomposed into pieces transforming as products of $0,1,2,\ldots$, up to N_c adjoint representations. In terms of spin-flavor SU($2N_F$), where N_F is the number of light flavors, the operators comprising the adjoint are defined:

$$J^i=\sum_\alpha q_\alpha^\dagger\left(\frac{\sigma^i}{2}\otimes\mathbb{1}\right)q_\alpha,$$

$$T^a=\sum_\alpha q_\alpha^\dagger\left(\mathbb{1}\otimes\frac{\lambda^a}{2}\right)q_\alpha,$$

$$G^{ia}=\sum_\alpha q_\alpha^\dagger\left(\frac{\sigma^i}{2}\otimes\frac{\lambda^a}{2}\right)q_\alpha, \tag{2.1}$$

where the index α sums over the N_c quarks, σ^i are the Pauli spin matrices, and λ^a are the Gell-Mann flavor matrices. Thus, each distinct operator may be written as a monomial in J, T, and G of total order n, with $0\leqslant n\leqslant N_c$. Such an operator is termed an *n-body operator*.

A large subset of operators constructed in this way are redundant or give vanishing matrix elements due to group-theoretical constraints. For example, commutators such as $[J^i,J^j]=i\epsilon^{ijk}J^k$ behave exactly as they do for the underlying σ and λ matrices. Furthermore, some combinations of J, T, and G act only on non-symmetric combinations of quarks and hence annihilate the ground-state wave functions, while yet other combinations are spin-flavor Casimirs and hence give the same value for every state of the representation, making them indistinguishable from the identity operator. The *operator reduction rule* for removing all such extra operators was derived for the **56** in Ref. [20], and extended to the **70** in Refs. [15]. For the present case with $N_F\leqslant 3$, the rule states: All operator products in which two flavor indices are contracted using δ^{ab}, d^{abc}, or f^{abc} [28], or two spin indices on G's are contracted using δ^{ij} or ϵ^{ijk}, can be eliminated.

None of the preceding reasoning depends specifically upon the $1/N_c$ expansion. Such $1/N_c$ factors arise from two sources: First, an n-body operator appears in an irreducible diagram in which n quarks are connected by gluons, requiring a minimum of $n-1$ gluons exchanged; the 't Hooft scaling $\alpha_s\propto 1/N_c$ then implies an explicit suppression factor $1/N_c^{n-1}$. Second, the combinatorics of quarks inside the baryon permits the matrix element of J, T, or G to be as large as $O(N_c^1)$ whenever the contributions from the N_c quarks add coherently. However, if the baryons chosen nevertheless have spins, isospins, and strangeness of $O(N_c^0)$—as we choose for the spin-$\frac{1}{2}$ **8** and spin-$\frac{3}{2}$ **10**—then the matrix elements of $J^{1,2,3}$, $T^{1,2,3}$, and $N_s\equiv\frac{1}{3}(\mathbb{1}-2\sqrt{3}T^8)$ are also $O(N_c^0)$.

The replacement of T^8 by N_s in constructing the operator basis presents a trivial example of what, in Ref. [15], is called "operator demotion." Whereas operator reduction rules identify linear combinations of operators that give precisely zero to all orders in $1/N_c$ when acting upon all states in a baryon spin-flavor representation, operator demotion identifies linear combinations of operators whose matrix elements are a higher order in powers of $1/N_c$ than those of the component operators, at least for the observed baryon states.

TABLE I. The 27 linearly independent operators contributing to the magnetic moments of the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ ground-state baryons, organized according to the leading N_c counting of their matrix elements.

$O(N_c^1)$	G^{33}
$O(N_c^0)$	$J^3,\ G^{38},\ \frac{1}{N_c}T^3G^{33},\ \frac{1}{N_c}N_sG^{33},\ \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i3},G^{33}\}$
$O(N_c^{-1})$	$\frac{1}{N_c}T^3J^3,\ \frac{1}{N_c}N_sJ^3,\ \frac{1}{N_c}T^3G^{38},\ \frac{1}{N_c}N_sG^{38},\ \frac{1}{N_c^2}\frac{1}{2}\{J^2,G^{33}\},\ \frac{1}{N_c^2}(T^3)^2G^{33},\ \frac{1}{N_c^2}N_s^2G^{33},$
	$\frac{1}{N_c^2}T^3N_sG^{33},\ \frac{1}{N_c^2}J^iG^{i3}J^3,\ \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i8},G^{33}\},\ \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i3},G^{38}\}$
$O(N_c^{-2})$	$\frac{1}{N_c^2}J^2J^3,\ \frac{1}{N_c^2}N_s^2J^3,\ \frac{1}{N_c^2}(T^3)^2J^3,\ \frac{1}{N_c^2}T^3N_sJ^3,\ \frac{1}{N_c^2}\frac{1}{2}\{J^2,G^{38}\},\ \frac{1}{N_c^2}(T^3)^2G^{38},\ \frac{1}{N_c^2}N_s^2G^{38},$
	$\frac{1}{N_c^2}T^3N_sG^{38},\ \frac{1}{N_c^2}J^iG^{i8}J^3,\ \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i8},G^{38}\}$

Such a result can occur since the J, T, and G matrix elements in general contain both leading and subleading contributions in N_c.

In summary, a complete accounting of the $1/N_c$ expansion thus requires one to take into account the ingredients: (i) a complete set of operators under spin-flavor; (ii) operator reduction rules to remove linearly dependent operators; (iii) a counting of N_c factors arising both from explicit powers of α_s on one hand and coherent contributions due to quark combinatorics on the other; and (iv) operator demotions to identify operators whose matrix elements are linearly dependent at $O(1/N_c^n)$ for some n but are independent at $O(1/N_c^{n+1})$.

A full analysis of all states in the baryon multiplets for N_c large and finite, as discussed above, requires the inclusion of up to N_c-body operators. A parallel analysis carried out for $N_c=3$ states, by the same reasoning, requires only up to 3-body operators. Once the physical $N_c=3$ baryons are identified as states embedded within the $N_c>3$ multiplets, one sees that 4-,5-, . . . ,N_c-body operators do indeed act upon the physical baryons, but give results linearly dependent on those of lower-order operators, and therefore may be discarded.

In the case of magnetic moments for the **56**, we have seen that there are 27 independent parameters with $\Delta J=1, \Delta J^3=0, \Delta Y=0, \Delta I^3=0$, T odd for $N_c=3$. The conditions $\Delta J=1$ and $\Delta J^3=0$ require that each operator has a single unsummed spin index i, which for definiteness we take to be $i=3$. T odd, of course, is the behavior of an angular momentum under time reversal; as it turns out, this is accomplished automatically because all operators containing the structure constants ϵ^{ijk}, f^{abc}, or d^{abc} can be eliminated. The conditions $\Delta Y=0, \Delta I^3=0$ require each unsummed flavor index a to equal 3 or 8. The complete set of 27 such operators, including the demotion $T^8 \rightarrow N_s$ appears in Table I. For those cases in which different orderings of component operators would give different values for matrix elements (such as J^2 and G^{33}), the operators are written in a symmetric form.

In fact, a direct calculation shows that no other operator demotions occur. Consider the first 6 operators in Table I, the complete set up to and including $O(N_c^0)$. If the leading coefficient of each of these operators—at $O(N_c^1)$ for G^{33} and at $O(N_c^0)$ for the other 5—for each of the 27 observables is collected into a 6×27 matrix, then one finds that this matrix has rank 6: No linear combination of the operators has matrix elements that are only $O(N_c^{-1})$. Similarly, no combination of the 17 operators up to $O(N_c^{-1})$ is demoted to $O(N_c^{-2})$.

III. COMPUTING MATRIX ELEMENTS

We compute the matrix elements of the 27 operators listed in Table I using only the Wigner-Eckart theorem (or its variants) and the total spin-flavor symmetry of the **56** baryon states. While the task of computing matrix elements of n-body operators for states containing an arbitrarily large number (N_c) of constituents may naively seem to require a large amount of group-theoretical technology [e.g., SU(6) $9j$ symbols], it turns out that all of the necessary matrix elements can be reduced to simple SU(2) spin and isospin Clebsch-Gordan (CG) coefficients, and nothing worse than an SU(2) $6j$ symbol needs to be computed. All of the necessary tools have been developed in Refs. [12,13], but we present them here for completeness.

We begin by constructing baryon states in the **56**. Since the wave function is completely symmetric under exchange of spin and flavor quantum numbers of any two quarks, it follows that the collection of all N_q quarks of any fixed flavor q must be completely symmetric under spin exchange. The spin J_q carried by them must therefore have its "stretched" value, $N_q/2$.

Next, the u quarks and d quarks combine to give a state with $I^3=\frac{1}{2}(N_u-N_d)=J_u-J_d$. The total isospin I is determined by noting that the exchange symmetry property of the state under u-d flavor exchange must precisely match that of these quarks' spins, in order for the total wave function to be completely symmetric under spin-flavor. It follows that $J_{ud}=I$, where $\mathbf{J}_{ud}\equiv\mathbf{J}_u+\mathbf{J}_d$.

In the final step, one simply combines the state of ud total spin $J_{ud}=I$ and isospin quantum numbers I,I^3 with the symmetrized strange quarks carrying total spin J_s to obtain the complete state with spin eigenvalues J,J^3, where $\mathbf{J}\equiv\mathbf{J}_{ud}+\mathbf{J}_s$:

$$|JJ^3;II^3(J_uJ_dJ_s)\rangle=\sum_{J_{ud}^3,J_s^3}\left(\begin{array}{cc|c} I & J_s & J \\ J_{ud}^3 & J_s^3 & J^3\end{array}\right)\times\sum_{J_u^3,\,J_d^3}\left(\begin{array}{cc|c} J_u & J_d & I \\ J_u^3 & J_d^3 & J_{ud}^3\end{array}\right)\times|J_uJ_u^3\rangle|J_dJ_d^3\rangle|J_sJ_s^3\rangle, \tag{3.1}$$

where the parentheses denote CG coefficients. Now, in order to compute the matrix elements of any particular operator, one need only sandwich it between a bra and ket of the form of Eq. (3.1) and use the Wigner-Eckart theorem.

The basic operators T^3, N_s, J^3, and $\mathbf{J}^2$, acting diagonally on baryon states, are easy to handle even if they are parts of more complicated operators. On the other hand,

$$G^{i8}=\frac{1}{2\sqrt{3}}(J^i-3J_s^i),$$

$$G^{i3}=\frac{1}{2}(J_u^i-J_d^i), \tag{3.2}$$

are in general not diagonal and must be handled more carefully. According to Table I, they appear in the forms G^{38} and G^{33}, and as

$$J^iG^{i8}=\frac{1}{2\sqrt{3}}(\mathbf{J}^2-3\mathbf{J}\cdot\mathbf{J}_s),$$

$$J^iG^{i3}=\frac{1}{2}\mathbf{J}\cdot(\mathbf{J}_u-\mathbf{J}_d), \tag{3.3}$$

which may be simplified by noting that

$$\mathbf{J}\cdot\mathbf{J}_s=-\frac{1}{2}[(\mathbf{J}-\mathbf{J}_s)^2-\mathbf{J}^2-\mathbf{J}_s^2]=\frac{1}{2}(\mathbf{J}^2+\mathbf{J}_s^2-\mathbf{I}^2),$$

$$\mathbf{J}\cdot(\mathbf{J}_u-\mathbf{J}_d)=(\mathbf{J}_u+\mathbf{J}_d+\mathbf{J}_s)\cdot(\mathbf{J}_u-\mathbf{J}_d)=\mathbf{J}_u^2-\mathbf{J}_d^2+\mathbf{J}_s\cdot(\mathbf{J}_u-\mathbf{J}_d). \tag{3.4}$$

It becomes apparent that only a few nontrivial matrix elements need be computed. Denoting the matrix element $\langle\mathbf{J}_\alpha\cdot\mathbf{J}_\beta\rangle$ as $\langle\alpha\beta\rangle^{(0)}$, where α and β are any two quark flavors, the only nontrivial required matrix elements are $\langle J_u^3\rangle$, $\langle J_d^3\rangle$, $\langle J_s^3\rangle$, $\langle us\rangle^{(0)}$, and $\langle ds\rangle^{(0)}$.

Even more simplification is possible, because Eq. (3.1) depends on the exchange of u and d quarks only through the second CG coefficient, and the factor obtained through this exchange is just $(-1)^{J_u+J_d-I}$. Of course, the eigenvalues J_α, which simply count one-half the number of quarks of flavor α in these baryons, remain unchanged from initial to final state. The same is true for $I^3=J_u-J_d$, but the total isospin may change to a value I'. One thus finds for an operator $\mathcal{O}$ that

$$\langle I'I^3|\mathcal{O}(u\leftrightarrow d)|II^3\rangle=(-1)^{I'-I}\langle I'-I^3|\mathcal{O}|I-I^3\rangle. \tag{3.5}$$

Thus the only matrix elements that need be computed are $\langle J_u^3\rangle$, $\langle J_s^3\rangle$, and $\langle us\rangle^{(0)}$. These were computed in Ref. [13] and are reproduced here:

$$\langle J_u^3\rangle=\delta_{J'^3J^3}\delta_{J_u'J_u}\delta_{J_d'J_d}\delta_{J_s'J_s}(-1)^{J-J'+J^3+J_s+I'-I-J_u-J_d}\sqrt{J_u(J_u+1)(2J_u+1)(2I'+1)(2I+1)(2J'+1)(2J+1)}\times\begin{Bmatrix} J_d & J_u & I \\ 1 & I' & J_u\end{Bmatrix}\begin{Bmatrix} J_s & I & J \\ 1 & J' & I'\end{Bmatrix}\begin{pmatrix} 1 & J' & J \\ 0 & J^3 & -J^3\end{pmatrix}, \tag{3.6}$$

$$\langle J_s^3\rangle=\delta_{J'^3J_3}\delta_{I'I}\delta_{J_u'J_u}\delta_{J_d'J_d}\delta_{J_s'J_s}(-1)^{1+J^3+J_s+I}\sqrt{J_s(J_s+1)(2J_s+1)(2J'+1)(2J+1)}\begin{Bmatrix} I & J_s & J \\ 1 & J' & J_s\end{Bmatrix}\begin{pmatrix} 1 & J' & J \\ 0 & J^3 & -J^3\end{pmatrix}, \tag{3.7}$$

$$\langle us\rangle^{(0)}=\delta_{J'J}\delta_{J'^3J^3}\delta_{J_u'J_u}\delta_{J_d'J_d}\delta_{J_s'J_s}(-1)^{1+J+J_s-J_u-J_d}\sqrt{J_u(J_u+1)(2J_u+1)J_s(J_s+1)(2J_s+1)(2I'+1)(2I+1)}\times\begin{Bmatrix} J_d & J_u & I \\ 1 & I' & J_u\end{Bmatrix}\begin{Bmatrix} J & J_s & I \\ 1 & I' & J_s\end{Bmatrix}. \tag{3.8}$$

Note that, in the interest of exhibiting maximal symmetry, the remaining CG coefficients have been written as $3j$ symbols. Despite the fact that a number of their entries are $O(N_c^1)$, all the $3j$ and $6j$ symbols of interest may be computed using analytic forms appearing in the standard text by Edmonds [21].

The matrix elements for all relevant states are presented in Tables II–IX. Tables II and III are lifted directly from Ref.

TABLE II. Matrix elements of the operators $N_{u,d,s}$ [whence $\langle \mathbf{J}_\alpha^2 \rangle = \langle \alpha\alpha \rangle^{(0)} = (N_\alpha/2)(N_\alpha/2+1)$] and the rank-0 tensors $\langle \alpha\beta \rangle^{(0)}$ with $\alpha \neq \beta$. Since spin is unchanged by these operators, the matrix elements vanish for all off-diagonal transitions except $\Sigma^0\Lambda$; in that case, the only nonvanishing entries are $\langle us \rangle^{(0)} = -\langle ds \rangle^{(0)} = -\frac{1}{8}\sqrt{(N_c-1)(N_c+3)}$.

State	$\langle N_u \rangle$	$\langle N_d \rangle$	$\langle N_s \rangle$	$\langle ud \rangle^{(0)}$	$\langle us \rangle^{(0)}$	$\langle ds \rangle^{(0)}$
Δ^{++}	$\frac{1}{2}(N_c+3)$	$\frac{1}{2}(N_c-3)$	0	$-\frac{1}{16}(N_c-3)(N_c+7)$	0	0
Δ^{+}	$\frac{1}{2}(N_c+1)$	$\frac{1}{2}(N_c-1)$	0	$-\frac{1}{16}(N_c^2+4N_c-29)$	0	0
Δ^{0}	$\frac{1}{2}(N_c-1)$	$\frac{1}{2}(N_c+1)$	0	$-\frac{1}{16}(N_c^2+4N_c-29)$	0	0
Δ^{-}	$\frac{1}{2}(N_c-3)$	$\frac{1}{2}(N_c+3)$	0	$-\frac{1}{16}(N_c-3)(N_c+7)$	0	0
Σ^{*+}	$\frac{1}{2}(N_c+1)$	$\frac{1}{2}(N_c-3)$	1	$-\frac{1}{16}(N_c-3)(N_c+5)$	$+\frac{1}{16}(N_c+5)$	$-\frac{1}{16}(N_c-3)$
Σ^{*0}	$\frac{1}{2}(N_c-1)$	$\frac{1}{2}(N_c-1)$	1	$-\frac{1}{16}(N_c^2+2N_c-19)$	$+\frac{1}{4}$	$+\frac{1}{4}$
Σ^{*-}	$\frac{1}{2}(N_c-3)$	$\frac{1}{2}(N_c+1)$	1	$-\frac{1}{16}(N_c-3)(N_c+5)$	$-\frac{1}{16}(N_c-3)$	$+\frac{1}{16}(N_c+5)$
Ξ^{*0}	$\frac{1}{2}(N_c-1)$	$\frac{1}{2}(N_c-3)$	2	$-\frac{1}{16}(N_c-3)(N_c+3)$	$+\frac{1}{12}(N_c+3)$	$-\frac{1}{12}(N_c-3)$
Ξ^{*-}	$\frac{1}{2}(N_c-3)$	$\frac{1}{2}(N_c-1)$	2	$-\frac{1}{16}(N_c-3)(N_c+3)$	$-\frac{1}{12}(N_c-3)$	$+\frac{1}{12}(N_c+3)$
Ω^{-}	$\frac{1}{2}(N_c-3)$	$\frac{1}{2}(N_c-3)$	3	$-\frac{1}{16}(N_c-3)(N_c+1)$	0	0
p	$\frac{1}{2}(N_c+1)$	$\frac{1}{2}(N_c-1)$	0	$-\frac{1}{16}(N_c-1)(N_c+5)$	0	0
n	$\frac{1}{2}(N_c-1)$	$\frac{1}{2}(N_c+1)$	0	$-\frac{1}{16}(N_c-1)(N_c+5)$	0	0
Σ^{+}	$\frac{1}{2}(N_c+1)$	$\frac{1}{2}(N_c-3)$	1	$-\frac{1}{16}(N_c-3)(N_c+5)$	$-\frac{1}{8}(N_c+5)$	$+\frac{1}{8}(N_c-3)$
Σ^{0}	$\frac{1}{2}(N_c-1)$	$\frac{1}{2}(N_c-1)$	1	$-\frac{1}{16}(N_c^2+2N_c-19)$	$-\frac{1}{2}$	$-\frac{1}{2}$
Λ	$\frac{1}{2}(N_c-1)$	$\frac{1}{2}(N_c-1)$	1	$-\frac{1}{16}(N_c-1)(N_c+3)$	0	0
Σ^{-}	$\frac{1}{2}(N_c-3)$	$\frac{1}{2}(N_c+1)$	1	$-\frac{1}{16}(N_c-3)(N_c+5)$	$+\frac{1}{8}(N_c-3)$	$-\frac{1}{8}(N_c+5)$
Ξ^{0}	$\frac{1}{2}(N_c-1)$	$\frac{1}{2}(N_c-3)$	2	$-\frac{1}{16}(N_c-3)(N_c+3)$	$-\frac{1}{6}(N_c+3)$	$+\frac{1}{6}(N_c-3)$
Ξ^{-}	$\frac{1}{2}(N_c-3)$	$\frac{1}{2}(N_c-1)$	2	$-\frac{1}{16}(N_c-3)(N_c+3)$	$+\frac{1}{6}(N_c-3)$	$-\frac{1}{6}(N_c+3)$

[13] (except for the repair of typos in the $\Sigma^0\Lambda$ matrix elements in Table III).

IV. RESULTS

If data existed for all of the 27 observables associated with the magnetic moment sector, one would proceed to form a Hamiltonian

$$H = -\boldsymbol{\mu}\cdot\mathbf{B}, \tag{4.1}$$

where the operators $\mathcal{O}_i$ of Table I enter with unknown dimensionless coefficients c_i via

$$\mu_z = \mu_0 \sum_{i=1}^{27} c_i \mathcal{O}_i , \tag{4.2}$$

where μ_0 is the sole scale in the problem, a mean value of magnetic moments in the multiplet, which one expects to be some $O(1)$ multiple of the nuclear magneton μ_N. The $1/N_c$ expansion provides a reliable effective Hamiltonian if the coefficients c_i are not larger than $O(1)$. In fact, a number of them may be smaller than $O(1)$ because certain operators may only contribute once SU(3) flavor symmetry is broken. They may also be smaller if dynamical effects are present that suppress them below the level predicted by naive $1/N_c$ counting. With all 27 observables in hand, one would simply invert the 27×27 matrix whose elements are given in Tables IV–IX and solve for all c_i to test this hypothesis. Essentially this procedure was carried out for the masses of the **56** in Ref. [10].

However, the *Review of Particle Physics* [22] gives unambiguous values for only 10 of the observables: magnetic moments of 7 of the 8 octet baryons (μ_{Σ^0} is unknown), the Ω^-, and the $\Sigma^0\Lambda$ and $\Delta^+ p$ transition moments. The last of these is extracted from the $\Delta\to N\gamma$ helicity amplitudes $A_{1/2}$ and $A_{3/2}$ via the standard formula for the M1 amplitude:

$$\mu_{\Delta^+ p} = -m_p \frac{A_{1/2}+\sqrt{3}A_{3/2}}{\sqrt{4\pi\alpha k}}, \tag{4.3}$$

where $k\simeq 260$ MeV is the photon momentum, from which one finds $\mu_{\Delta^+ p} = 3.51\pm 0.09\ \mu_N$. In addition, we use a recent extraction [23] $\mu_{\Delta^{++}} = 6.14\pm 0.51\ \mu_N$ obtained from an analysis of data that has some model dependence, but that respects both gauge invariance and the finite Δ^{++} width. We therefore include 11 observables in our analysis. There is also a recent experimental determination [24] of $\mu_{\Delta^+} = 2.7^{+1.0}_{-1.3}$ (stat) ± 1.5 (syst) ± 3 (theor) μ_N, but due to the large theoretical uncertainty we do not use this value in our analysis.

With only 11 pieces of information to study a system of 27 observables, one must resort to using the known quantities to fit the coefficients at the lowest orders of the $1/N_c$ expansion, and to use the coefficients so obtained to predict the remaining observables. One may then proceed either by

TABLE III. Matrix elements of the operators J_u^3, J_d^3, and J_s^3 in the state of maximal J^3.

State	$\langle J_u^3\rangle$	$\langle J_d^3\rangle$	$\langle J_s^3\rangle$
Δ^{++}	$+\frac{3}{20}(N_c+7)$	$-\frac{3}{20}(N_c-3)$	0
Δ^{+}	$+\frac{1}{20}(N_c+17)$	$-\frac{1}{20}(N_c-13)$	0
Δ^{0}	$-\frac{1}{20}(N_c-13)$	$+\frac{1}{20}(N_c+17)$	0
Δ^{-}	$-\frac{3}{20}(N_c-3)$	$+\frac{3}{20}(N_c+7)$	0
Σ^{*+}	$+\frac{1}{8}(N_c+5)$	$-\frac{1}{8}(N_c-3)$	$+\frac{1}{2}$
Σ^{*0}	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$
Σ^{*-}	$-\frac{1}{8}(N_c-3)$	$+\frac{1}{8}(N_c+5)$	$+\frac{1}{2}$
Ξ^{*0}	$+\frac{1}{12}(N_c+3)$	$-\frac{1}{12}(N_c-3)$	$+1$
Ξ^{*-}	$-\frac{1}{12}(N_c-3)$	$+\frac{1}{12}(N_c+3)$	$+1$
Ω^{-}	0	0	$+\frac{3}{2}$
p	$+\frac{1}{12}(N_c+5)$	$-\frac{1}{12}(N_c-1)$	0
n	$-\frac{1}{12}(N_c-1)$	$+\frac{1}{12}(N_c+5)$	0
Σ^{+}	$+\frac{1}{12}(N_c+5)$	$-\frac{1}{12}(N_c-3)$	$-\frac{1}{6}$
Σ^{0}	$+\frac{1}{3}$	$+\frac{1}{3}$	$-\frac{1}{6}$
Λ	0	0	$+\frac{1}{2}$
$\Sigma^{0}\Lambda$	$-\frac{1}{12}\sqrt{(N_c-1)(N_c+3)}$	$+\frac{1}{12}\sqrt{(N_c-1)(N_c+3)}$	0
Σ^{-}	$-\frac{1}{12}(N_c-3)$	$+\frac{1}{12}(N_c+5)$	$-\frac{1}{6}$
Ξ^{0}	$-\frac{1}{36}(N_c+3)$	$+\frac{1}{36}(N_c-3)$	$+\frac{2}{3}$
Ξ^{-}	$+\frac{1}{36}(N_c-3)$	$-\frac{1}{36}(N_c+3)$	$+\frac{2}{3}$
$\Delta^{+}p$	$+\frac{1}{6\sqrt{2}}\sqrt{(N_c-1)(N_c+5)}$	$-\frac{1}{6\sqrt{2}}\sqrt{(N_c-1)(N_c+5)}$	0
$\Delta^{0}n$	$+\frac{1}{6\sqrt{2}}\sqrt{(N_c-1)(N_c+5)}$	$-\frac{1}{6\sqrt{2}}\sqrt{(N_c-1)(N_c+5)}$	0
$\Sigma^{*0}\Lambda$	$+\frac{1}{6\sqrt{2}}\sqrt{(N_c-1)(N_c+3)}$	$-\frac{1}{6\sqrt{2}}\sqrt{(N_c-1)(N_c+3)}$	0
$\Sigma^{*0}\Sigma^{0}$	$+\frac{1}{3\sqrt{2}}$	$+\frac{1}{3\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$
$\Sigma^{*+}\Sigma^{+}$	$+\frac{1}{12\sqrt{2}}(N_c+5)$	$-\frac{1}{12\sqrt{2}}(N_c-3)$	$-\frac{\sqrt{2}}{3}$
$\Sigma^{*-}\Sigma^{-}$	$-\frac{1}{12\sqrt{2}}(N_c-3)$	$+\frac{1}{12\sqrt{2}}(N_c+5)$	$-\frac{\sqrt{2}}{3}$
$\Xi^{*0}\Xi^{0}$	$+\frac{1}{9\sqrt{2}}(N_c+3)$	$-\frac{1}{9\sqrt{2}}(N_c-3)$	$-\frac{\sqrt{2}}{3}$
$\Xi^{*-}\Xi^{-}$	$-\frac{1}{9\sqrt{2}}(N_c-3)$	$+\frac{1}{9\sqrt{2}}(N_c+3)$	$-\frac{\sqrt{2}}{3}$

(i) separating the observables into isoscalar and isovector, as well as $I=2$ and 3 isotensor, combinations, or (ii) one may employ the electromagnetic nature of magnetic moments to construct only operators with a flavor dependence in proportion to the quark charges (the ‘‘single-photon ansatz’’). Since both methods have been employed in the literature, we discuss them each in turn.

A. Analysis in the isoscalar, isovector, isotensor basis

The analysis of Ref. [5] (JM) separates operators, and the corresponding combinations of magnetic moments, into $I=0$ and $I=1$ forms. Since the maximal isospin appearing in the **56** is $\frac{3}{2}$ (for the Δ), isotensor combinations with $I=2$ and $I=3$ are also present:

TABLE IV. Matrix elements of the 1-, 2-, and 3-body operators corresponding to magnetic moments.

State	$\langle J^3\rangle$	$\langle G^{33}\rangle$	$\langle G^{38}\rangle$	$\langle T^3J^3\rangle$	$\langle N_sJ^3\rangle$	$\langle T^3G^{33}\rangle$	$\langle T^3G^{38}\rangle$
Δ^{++}	$\frac{3}{2}$	$\frac{3}{20}(N_c+2)$	$\frac{\sqrt{3}}{4}$	$\frac{9}{4}$	0	$\frac{9}{40}(N_c+2)$	$\frac{3\sqrt{3}}{8}$
Δ^{+}	$\frac{3}{2}$	$\frac{1}{20}(N_c+2)$	$\frac{\sqrt{3}}{4}$	$\frac{3}{4}$	0	$\frac{1}{40}(N_c+2)$	$\frac{\sqrt{3}}{8}$
Δ^{0}	$\frac{3}{2}$	$-\frac{1}{20}(N_c+2)$	$\frac{\sqrt{3}}{4}$	$-\frac{3}{4}$	0	$\frac{1}{40}(N_c+2)$	$-\frac{\sqrt{3}}{8}$
Δ^{-}	$\frac{3}{2}$	$-\frac{3}{20}(N_c+2)$	$\frac{\sqrt{3}}{4}$	$-\frac{9}{4}$	0	$\frac{9}{40}(N_c+2)$	$-\frac{3\sqrt{3}}{8}$
Σ^{*+}	$\frac{3}{2}$	$\frac{1}{8}(N_c+1)$	0	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{8}(N_c+1)$	0
Σ^{*0}	$\frac{3}{2}$	0	0	0	$\frac{3}{2}$	0	0
Σ^{*-}	$\frac{3}{2}$	$-\frac{1}{8}(N_c+1)$	0	$-\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{8}(N_c+1)$	0
Ξ^{*0}	$\frac{3}{2}$	$\frac{1}{12}N_c$	$-\frac{\sqrt{3}}{4}$	$\frac{3}{4}$	3	$\frac{1}{24}N_c$	$-\frac{\sqrt{3}}{8}$
Ξ^{*-}	$\frac{3}{2}$	$-\frac{1}{12}N_c$	$-\frac{\sqrt{3}}{4}$	$-\frac{3}{4}$	3	$\frac{1}{24}N_c$	$\frac{\sqrt{3}}{8}$
Ω^{-}	$\frac{3}{2}$	0	$-\frac{\sqrt{3}}{2}$	0	$\frac{9}{2}$	0	0
p	$\frac{1}{2}$	$\frac{1}{12}(N_c+2)$	$\frac{1}{4\sqrt{3}}$	$\frac{1}{4}$	0	$\frac{1}{24}(N_c+2)$	$\frac{1}{8\sqrt{3}}$
n	$\frac{1}{2}$	$-\frac{1}{12}(N_c+2)$	$\frac{1}{4\sqrt{3}}$	$-\frac{1}{4}$	0	$\frac{1}{24}(N_c+2)$	$-\frac{1}{8\sqrt{3}}$
Σ^{+}	$\frac{1}{2}$	$\frac{1}{12}(N_c+1)$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{12}(N_c+1)$	$\frac{1}{2\sqrt{3}}$
Σ^{0}	$\frac{1}{2}$	0	$\frac{1}{2\sqrt{3}}$	0	$\frac{1}{2}$	0	0
Λ	$\frac{1}{2}$	0	$-\frac{1}{2\sqrt{3}}$	0	$\frac{1}{2}$	0	0
$\Sigma^{0}\Lambda$	0	$-\frac{1}{12}\sqrt{(N_c-1)(N_c+3)}$	0	0	0	0	0
Σ^{-}	$\frac{1}{2}$	$-\frac{1}{12}(N_c+1)$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{12}(N_c+1)$	$-\frac{1}{2\sqrt{3}}$
Ξ^{0}	$\frac{1}{2}$	$-\frac{1}{36}N_c$	$-\frac{\sqrt{3}}{4}$	$\frac{1}{4}$	1	$-\frac{1}{72}N_c$	$-\frac{\sqrt{3}}{8}$
Ξ^{-}	$\frac{1}{2}$	$\frac{1}{36}N_c$	$-\frac{\sqrt{3}}{4}$	$-\frac{1}{4}$	1	$-\frac{1}{72}N_c$	$\frac{\sqrt{3}}{8}$
$\Delta^{+}p$	0	$\frac{1}{6\sqrt{2}}\sqrt{(N_c-1)(N_c+5)}$	0	0	0	$\frac{1}{12\sqrt{2}}\sqrt{(N_c-1)(N_c+5)}$	0
$\Delta^{0}n$	0	$\frac{1}{6\sqrt{2}}\sqrt{(N_c-1)(N_c+5)}$	0	0	0	$-\frac{1}{12\sqrt{2}}\sqrt{(N_c-1)(N_c+5)}$	0
$\Sigma^{*0}\Lambda$	0	$\frac{1}{6\sqrt{2}}\sqrt{(N_c-1)(N_c+3)}$	0	0	0	0	0
$\Sigma^{*0}\Sigma^{0}$	0	0	$\frac{1}{\sqrt{6}}$	0	0	0	0
$\Sigma^{*+}\Sigma^{+}$	0	$\frac{1}{12\sqrt{2}}(N_c+1)$	$\frac{1}{\sqrt{6}}$	0	0	$\frac{1}{12\sqrt{2}}(N_c+1)$	$\frac{1}{\sqrt{6}}$
$\Sigma^{*-}\Sigma^{-}$	0	$-\frac{1}{12\sqrt{2}}(N_c+1)$	$\frac{1}{\sqrt{6}}$	0	0	$\frac{1}{12\sqrt{2}}(N_c+1)$	$-\frac{1}{\sqrt{6}}$
$\Xi^{*0}\Xi^{0}$	0	$\frac{1}{9\sqrt{2}}N_c$	$\frac{1}{\sqrt{6}}$	0	0	$\frac{1}{18\sqrt{2}}N_c$	$\frac{1}{2\sqrt{6}}$
$\Xi^{*-}\Xi^{-}$	0	$-\frac{1}{9\sqrt{2}}N_c$	$\frac{1}{\sqrt{6}}$	0	0	$\frac{1}{18\sqrt{2}}N_c$	$-\frac{1}{2\sqrt{6}}$

TABLE V. First continuation of Table IV.

State	$\langle N_s G^{33}\rangle$	$\langle N_s G^{38}\rangle$	$\langle J^2 J^3\rangle$	$\langle N_s^2 J^3\rangle$	$\langle (T^3)^2 J^3\rangle$
Δ^{++}	0	0	$\frac{45}{8}$	0	$\frac{27}{8}$
Δ^{+}	0	0	$\frac{45}{8}$	0	$\frac{3}{8}$
Δ^{0}	0	0	$\frac{45}{8}$	0	$\frac{3}{8}$
Δ^{-}	0	0	$\frac{45}{8}$	0	$\frac{27}{8}$
Σ^{*+}	$\frac{1}{8}(N_c+1)$	0	$\frac{45}{8}$	$\frac{3}{2}$	$\frac{3}{2}$
Σ^{*0}	0	0	$\frac{45}{8}$	$\frac{3}{2}$	0
Σ^{*-}	$-\frac{1}{8}(N_c+1)$	0	$\frac{45}{8}$	$\frac{3}{2}$	$\frac{3}{2}$
Ξ^{*0}	$\frac{1}{6}N_c$	$-\frac{\sqrt{3}}{2}$	$\frac{45}{8}$	6	$\frac{3}{8}$
Ξ^{*-}	$-\frac{1}{6}N_c$	$-\frac{\sqrt{3}}{2}$	$\frac{45}{8}$	6	$\frac{3}{8}$
Ω^{-}	0	$-\frac{3\sqrt{3}}{2}$	$\frac{45}{8}$	$\frac{27}{2}$	0
p	0	0	$\frac{3}{8}$	0	$\frac{1}{8}$
n	0	0	$\frac{3}{8}$	0	$\frac{1}{8}$
Σ^{+}	$\frac{1}{12}(N_c+1)$	$\frac{1}{2\sqrt{3}}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$
Σ^{0}	0	$\frac{1}{2\sqrt{3}}$	$\frac{3}{8}$	$\frac{1}{2}$	0
Λ	0	$-\frac{1}{2\sqrt{3}}$	$\frac{3}{8}$	$\frac{1}{2}$	0
$\Sigma^{0}\Lambda$	$-\frac{1}{12}\sqrt{(N_c-1)(N_c+3)}$	0	0	0	0
Σ^{-}	$-\frac{1}{12}(N_c+1)$	$\frac{1}{2\sqrt{3}}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$
Ξ^{0}	$-\frac{1}{18}N_c$	$-\frac{\sqrt{3}}{2}$	$\frac{3}{8}$	2	$\frac{1}{8}$
Ξ^{-}	$\frac{1}{18}N_c$	$-\frac{\sqrt{3}}{2}$	$\frac{3}{8}$	2	$\frac{1}{8}$
$\Delta^{+}p$	0	0	0	0	0
$\Delta^{0}n$	0	0	0	0	0
$\Sigma^{*0}\Lambda$	$\frac{1}{6\sqrt{2}}\sqrt{(N_c-1)(N_c+3)}$	0	0	0	0
$\Sigma^{*0}\Sigma^{0}$	0	$\frac{1}{\sqrt{6}}$	0	0	0
$\Sigma^{*+}\Sigma^{+}$	$\frac{1}{12\sqrt{2}}(N_c+1)$	$\frac{1}{\sqrt{6}}$	0	0	0
$\Sigma^{*-}\Sigma^{-}$	$-\frac{1}{12\sqrt{2}}(N_c+1)$	$\frac{1}{\sqrt{6}}$	0	0	0
$\Xi^{*0}\Xi^{0}$	$\frac{\sqrt{2}}{9}N_c$	$\sqrt{\frac{2}{3}}$	0	0	0
$\Xi^{*-}\Xi^{-}$	$-\frac{\sqrt{2}}{9}N_c$	$\sqrt{\frac{2}{3}}$	0	0	0

TABLE VI. Second continuation of Table IV.

State	$\langle T^3 N_s J^3\rangle$	$\frac{1}{2}\langle\{J^2,G^{33}\}\rangle$	$\frac{1}{2}\langle\{J^2,G^{38}\}\rangle$	$\langle (T^3)^2 G^{33}\rangle$
Δ^{++}	0	$\frac{9}{16}(N_c+2)$	$\frac{15\sqrt{3}}{16}$	$\frac{27}{80}(N_c+2)$
Δ^{+}	0	$\frac{3}{16}(N_c+2)$	$\frac{15\sqrt{3}}{16}$	$\frac{1}{80}(N_c+2)$
Δ^{0}	0	$-\frac{3}{16}(N_c+2)$	$\frac{15\sqrt{3}}{16}$	$-\frac{1}{80}(N_c+2)$
Δ^{-}	0	$-\frac{9}{16}(N_c+2)$	$\frac{15\sqrt{3}}{16}$	$-\frac{27}{80}(N_c+2)$
Σ^{*+}	$\frac{3}{2}$	$\frac{15}{32}(N_c+1)$	0	$\frac{1}{8}(N_c+1)$
Σ^{*0}	0	0	0	0
Σ^{*-}	$-\frac{3}{2}$	$-\frac{15}{32}(N_c+1)$	0	$-\frac{1}{8}(N_c+1)$
Ξ^{*0}	$\frac{3}{2}$	$\frac{5}{16}N_c$	$-\frac{15\sqrt{3}}{16}$	$\frac{1}{48}N_c$
Ξ^{*-}	$-\frac{3}{2}$	$-\frac{5}{16}N_c$	$-\frac{15\sqrt{3}}{16}$	$-\frac{1}{48}N_c$
Ω^{-}	0	0	$-\frac{15\sqrt{3}}{8}$	0
p	0	$\frac{1}{16}(N_c+2)$	$\frac{\sqrt{3}}{16}$	$\frac{1}{48}(N_c+2)$
n	0	$-\frac{1}{16}(N_c+2)$	$\frac{\sqrt{3}}{16}$	$-\frac{1}{48}(N_c+2)$
Σ^{+}	$\frac{1}{2}$	$\frac{1}{16}(N_c+1)$	$\frac{\sqrt{3}}{8}$	$\frac{1}{12}(N_c+1)$
Σ^{0}	0	0	$\frac{\sqrt{3}}{8}$	0
Λ	0	0	$-\frac{\sqrt{3}}{8}$	0
$\Sigma^{0}\Lambda$	0	$-\frac{1}{16}\sqrt{(N_c-1)(N_c+3)}$	0	0
Σ^{-}	$-\frac{1}{2}$	$-\frac{1}{16}(N_c+1)$	$\frac{\sqrt{3}}{8}$	$-\frac{1}{12}(N_c+1)$
Ξ^{0}	$\frac{1}{2}$	$-\frac{1}{48}N_c$	$-\frac{3\sqrt{3}}{16}$	$-\frac{1}{144}N_c$
Ξ^{-}	$-\frac{1}{2}$	$\frac{1}{48}N_c$	$-\frac{3\sqrt{3}}{16}$	$\frac{1}{144}N_c$
$\Delta^{+}p$	0	$\frac{3}{8\sqrt{2}}\sqrt{(N_c-1)(N_c+5)}$	0	$\frac{1}{24\sqrt{2}}\sqrt{(N_c-1)(N_c+5)}$
$\Delta^{0}n$	0	$\frac{3}{8\sqrt{2}}\sqrt{(N_c-1)(N_c+5)}$	0	$\frac{1}{24\sqrt{2}}\sqrt{(N_c-1)(N_c+5)}$
$\Sigma^{*0}\Lambda$	0	$\frac{3}{8\sqrt{2}}\sqrt{(N_c-1)(N_c+3)}$	0	0
$\Sigma^{*0}\Sigma^{0}$	0	0	$\frac{3}{4}\sqrt{\frac{3}{2}}$	0
$\Sigma^{*+}\Sigma^{+}$	0	$\frac{3}{16\sqrt{2}}(N_c+1)$	$\frac{3}{4}\sqrt{\frac{3}{2}}$	$\frac{1}{12\sqrt{2}}(N_c+1)$
$\Sigma^{*-}\Sigma^{-}$	0	$-\frac{3}{16\sqrt{2}}(N_c+1)$	$\frac{3}{4}\sqrt{\frac{3}{2}}$	$-\frac{1}{12\sqrt{2}}(N_c+1)$
$\Xi^{*0}\Xi^{0}$	0	$\frac{1}{4\sqrt{2}}N_c$	$\frac{3}{4}\sqrt{\frac{3}{2}}$	$\frac{1}{36\sqrt{2}}N_c$
$\Xi^{*-}\Xi^{-}$	0	$-\frac{1}{4\sqrt{2}}N_c$	$\frac{3}{4}\sqrt{\frac{3}{2}}$	$-\frac{1}{36\sqrt{2}}N_c$

TABLE VII. Third continuation of Table IV.

State	$\langle (T^3)^2 G^{38}\rangle$	$\langle N_s^2 G^{33}\rangle$	$\langle N_s^2 G^{38}\rangle$	$\langle T^3 N_s G^{33}\rangle$
Δ^{++}	$\frac{9\sqrt{3}}{16}$	0	0	0
Δ^{+}	$\frac{\sqrt{3}}{16}$	0	0	0
Δ^{0}	$\frac{\sqrt{3}}{16}$	0	0	0
Δ^{-}	$\frac{9\sqrt{3}}{16}$	0	0	0
Σ^{*+}	0	$\frac{1}{8}(N_c+1)$	0	$\frac{1}{8}(N_c+1)$
Σ^{*0}	0	0	0	0
Σ^{*-}	0	$-\frac{1}{8}(N_c+1)$	0	$\frac{1}{8}(N_c+1)$
Ξ^{*0}	$-\frac{\sqrt{3}}{16}$	$\frac{1}{3}N_c$	$-\sqrt{3}$	$\frac{1}{12}N_c$
Ξ^{*-}	$-\frac{\sqrt{3}}{16}$	$-\frac{1}{3}N_c$	$-\sqrt{3}$	$\frac{1}{12}N_c$
Ω^{-}	0	0	$-\frac{9\sqrt{3}}{2}$	0
p	$\frac{1}{16\sqrt{3}}$	0	0	0
n	$\frac{1}{16\sqrt{3}}$	0	0	0
Σ^{+}	$\frac{1}{2\sqrt{3}}$	$\frac{1}{12}(N_c+1)$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{12}(N_c+1)$
Σ^{0}	0	0	$\frac{1}{2\sqrt{3}}$	0
Λ	0	0	$-\frac{1}{2\sqrt{3}}$	0
$\Sigma^{0}\Lambda$	0	$-\frac{1}{12}\sqrt{(N_c-1)(N_c+3)}$	0	0
Σ^{-}	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{12}(N_c+1)$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{12}(N_c+1)$
Ξ^{0}	$-\frac{\sqrt{3}}{16}$	$-\frac{1}{9}N_c$	$-\sqrt{3}$	$-\frac{1}{36}N_c$
Ξ^{-}	$-\frac{\sqrt{3}}{16}$	$\frac{1}{9}N_c$	$-\sqrt{3}$	$-\frac{1}{36}N_c$
$\Delta^{+}p$	0	0	0	0
$\Delta^{0}n$	0	0	0	0
$\Sigma^{*0}\Lambda$	0	$\frac{1}{6\sqrt{2}}\sqrt{(N_c-1)(N_c+3)}$	0	0
$\Sigma^{*0}\Sigma^{0}$	0	0	$\frac{1}{\sqrt{6}}$	0
$\Sigma^{*+}\Sigma^{+}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{12\sqrt{2}}(N_c+1)$	$\frac{1}{\sqrt{6}}$	$\frac{1}{12\sqrt{2}}(N_c+1)$
$\Sigma^{*-}\Sigma^{-}$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{12\sqrt{2}}(N_c+1)$	$\frac{1}{\sqrt{6}}$	$\frac{1}{12\sqrt{2}}(N_c+1)$
$\Xi^{*0}\Xi^{0}$	$\frac{1}{4\sqrt{6}}$	$\frac{2\sqrt{2}}{9}N_c$	$\frac{2\sqrt{2}}{\sqrt{3}}$	$\frac{1}{9\sqrt{2}}N_c$
$\Xi^{*-}\Xi^{-}$	$\frac{1}{4\sqrt{6}}$	$-\frac{2\sqrt{2}}{9}N_c$	$\frac{2\sqrt{2}}{\sqrt{3}}$	$\frac{1}{9\sqrt{2}}N_c$

TABLE VIII. Fourth continuation of Table IV.

State	$\langle T^3 N_s G^{38}\rangle$	$\langle (J^i G^{i3}) J^3\rangle$	$\langle (J^i G^{i8}) J^3\rangle$	$\frac{1}{2}\langle\{J^i G^{i8}, G^{33}\}\rangle$
Δ^{++}	0	$\frac{9}{16}(N_c+2)$	$\frac{15\sqrt{3}}{16}$	$\frac{3\sqrt{3}}{32}(N_c+2)$
Δ^{+}	0	$\frac{3}{16}(N_c+2)$	$\frac{15\sqrt{3}}{16}$	$\frac{\sqrt{3}}{32}(N_c+2)$
Δ^{0}	0	$-\frac{3}{16}(N_c+2)$	$\frac{15\sqrt{3}}{16}$	$-\frac{\sqrt{3}}{32}(N_c+2)$
Δ^{-}	0	$-\frac{9}{16}(N_c+2)$	$\frac{15\sqrt{3}}{16}$	$-\frac{3\sqrt{3}}{32}(N_c+2)$
Σ^{*+}	0	$\frac{15}{32}(N_c+1)$	0	0
Σ^{*0}	0	0	0	0
Σ^{*-}	0	$-\frac{15}{32}(N_c+1)$	0	0
Ξ^{*0}	$-\frac{\sqrt{3}}{4}$	$\frac{5}{16}N_c$	$-\frac{15\sqrt{3}}{16}$	$-\frac{5}{32\sqrt{3}}N_c$
Ξ^{*-}	$\frac{\sqrt{3}}{4}$	$-\frac{5}{16}N_c$	$-\frac{15\sqrt{3}}{16}$	$\frac{5}{32\sqrt{3}}N_c$
Ω^{-}	0	0	$-\frac{15\sqrt{3}}{8}$	0
p	0	$\frac{1}{16}(N_c+2)$	$\frac{\sqrt{3}}{16}$	$\frac{1}{32\sqrt{3}}(N_c+2)$
n	0	$-\frac{1}{16}(N_c+2)$	$\frac{\sqrt{3}}{16}$	$-\frac{1}{32\sqrt{3}}(N_c+2)$
Σ^{+}	$\frac{1}{2\sqrt{3}}$	$\frac{1}{16}(N_c+1)$	$\frac{\sqrt{3}}{8}$	$\frac{1}{16\sqrt{3}}(N_c+1)$
Σ^{0}	0	0	$\frac{\sqrt{3}}{8}$	0
Λ	0	0	$-\frac{\sqrt{3}}{8}$	0
$\Sigma^{0}\Lambda$	0	$-\frac{1}{16}\sqrt{(N_c-1)(N_c+3)}$	0	0
Σ^{-}	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{16}(N_c+1)$	$\frac{\sqrt{3}}{8}$	$-\frac{1}{16\sqrt{3}}(N_c+1)$
Ξ^{0}	$-\frac{\sqrt{3}}{4}$	$-\frac{1}{48}N_c$	$-\frac{3\sqrt{3}}{16}$	$\frac{1}{32\sqrt{3}}N_c$
Ξ^{-}	$\frac{\sqrt{3}}{4}$	$\frac{1}{48}N_c$	$-\frac{3\sqrt{3}}{16}$	$-\frac{1}{32\sqrt{3}}N_c$
$\Delta^{+}p$	0	0	0	$\frac{1}{16}\sqrt{\frac{3}{2}}\sqrt{(N_c-1)(N_c+5)}$
$\Delta^{0}n$	0	0	0	$\frac{1}{16}\sqrt{\frac{3}{2}}\sqrt{(N_c-1)(N_c+5)}$
$\Sigma^{*0}\Lambda$	0	0	0	$-\frac{1}{16\sqrt{6}}\sqrt{(N_c-1)(N_c+3)}$
$\Sigma^{*0}\Sigma^{0}$	0	0	0	0
$\Sigma^{*+}\Sigma^{+}$	$\frac{1}{\sqrt{6}}$	0	0	$\frac{1}{32\sqrt{6}}(N_c+1)$
$\Sigma^{*-}\Sigma^{-}$	$-\frac{1}{\sqrt{6}}$	0	0	$-\frac{1}{32\sqrt{6}}(N_c+1)$
$\Xi^{*0}\Xi^{0}$	$\frac{1}{\sqrt{6}}$	0	0	$-\frac{1}{6\sqrt{6}}N_c$
$\Xi^{*-}\Xi^{-}$	$-\frac{1}{\sqrt{6}}$	0	0	$\frac{1}{6\sqrt{6}}N_c$

TABLE IX. Fifth continuation of Table IV.

State	$\frac{1}{2}\langle\{J^iG^{i8},G^{38}\}\rangle$	$\frac{1}{2}\langle\{J^iG^{i3},G^{33}\}\rangle$	$\frac{1}{2}\langle\{J^iG^{i3},G^{38}\}\rangle$
Δ^{++}	$\frac{15}{32}$	$\frac{9}{160}(N_c+2)^2$	$\frac{3\sqrt{3}}{32}(N_c+2)$
Δ^{+}	$\frac{15}{32}$	$\frac{1}{160}(N_c+2)^2$	$\frac{\sqrt{3}}{32}(N_c+2)$
Δ^{0}	$\frac{15}{32}$	$\frac{1}{160}(N_c+2)^2$	$-\frac{\sqrt{3}}{32}(N_c+2)$
Δ^{-}	$\frac{15}{32}$	$\frac{9}{160}(N_c+2)^2$	$-\frac{3\sqrt{3}}{32}(N_c+2)$
Σ^{*+}	0	$\frac{5}{128}(N_c+1)^2$	0
Σ^{*0}	0	0	0
Σ^{*-}	0	$\frac{5}{128}(N_c+1)^2$	0
Ξ^{*0}	$\frac{15}{32}$	$\frac{5}{288}N_c^2$	$-\frac{5}{32\sqrt{3}}N_c$
Ξ^{*-}	$\frac{15}{32}$	$\frac{5}{288}N_c^2$	$\frac{5}{32\sqrt{3}}N_c$
Ω^{-}	$\frac{15}{8}$	0	0
p	$\frac{1}{32}$	$\frac{1}{96}(N_c+2)^2$	$\frac{1}{32\sqrt{3}}(N_c+2)$
n	$\frac{1}{32}$	$\frac{1}{96}(N_c+2)^2$	$-\frac{1}{32\sqrt{3}}(N_c+2)$
Σ^{+}	$\frac{1}{8}$	$\frac{1}{96}(N_c+1)^2$	$\frac{1}{16\sqrt{3}}(N_c+1)$
Σ^{0}	$\frac{1}{8}$	$\frac{1}{96}(N_c-1)(N_c+3)$	0
Λ	$\frac{1}{8}$	$\frac{1}{96}(N_c-1)(N_c+3)$	0
$\Sigma^{0}\Lambda$	0	0	0
Σ^{-}	$\frac{1}{8}$	$\frac{1}{96}(N_c+1)^2$	$-\frac{1}{16\sqrt{3}}(N_c+1)$
Ξ^{0}	$\frac{9}{32}$	$\frac{1}{864}N_c^2$	$\frac{1}{32\sqrt{3}}N_c$
Ξ^{-}	$\frac{9}{32}$	$\frac{1}{864}N_c^2$	$-\frac{1}{32\sqrt{3}}N_c$
$\Delta^{+}p$	0	$\frac{1}{48\sqrt{2}}(N_c+2)\sqrt{(N_c-1)(N_c+5)}$	0
$\Delta^{0}n$	0	$-\frac{1}{48\sqrt{2}}(N_c+2)\sqrt{(N_c-1)(N_c+5)}$	0
$\Sigma^{*0}\Lambda$	0	0	$-\frac{1}{16\sqrt{6}}\sqrt{(N_c-1)(N_c+3)}$
$\Sigma^{*0}\Sigma^{0}$	$\frac{1}{8\sqrt{2}}$	0	0
$\Sigma^{*+}\Sigma^{+}$	$\frac{1}{8\sqrt{2}}$	$\frac{7}{384\sqrt{2}}(N_c+1)^2$	$\frac{7}{32\sqrt{6}}(N_c+1)$
$\Sigma^{*-}\Sigma^{-}$	$\frac{1}{8\sqrt{2}}$	$\frac{7}{384\sqrt{2}}(N_c+1)^2$	$-\frac{7}{32\sqrt{6}}(N_c+1)$
$\Xi^{*0}\Xi^{0}$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{108\sqrt{2}}N_c^2$	$\frac{1}{12\sqrt{6}}N_c$
$\Xi^{*-}\Xi^{-}$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{108\sqrt{2}}N_c^2$	$-\frac{1}{12\sqrt{6}}N_c$

$$I=2:\quad (\mu_{\Sigma^+}-2\mu_{\Sigma^0}+\mu_{\Sigma^-}),\ (\mu_{\Delta^{++}}-\mu_{\Delta^+}-\mu_{\Delta^0}+\mu_{\Delta^-}),\ (\mu_{\Sigma^{*+}}-2\mu_{\Sigma^{*0}}+\mu_{\Sigma^{*-}}),$$

$$(\mu_{\Delta^+p}-\mu_{\Delta^0n}),\ (\mu_{\Sigma^{*+}\Sigma^+}-2\mu_{\Sigma^{*0}\Sigma^0}+\mu_{\Sigma^{*-}\Sigma^-}),$$

$$I=3:\quad (\mu_{\Delta^{++}}-3\mu_{\Delta^+}+3\mu_{\Delta^0}-\mu_{\Delta^-}). \tag{4.4}$$

The JM analysis introduces a leading-order operator X^{ia}, which is equivalent to the $O(N_c^0)$ part of G^{ia}/N_c, and a strange quark spin operator

$$J_s^i\equiv\frac{1}{3}(J^i-2\sqrt{3}G^{i8}). \tag{4.5}$$

The JM operator basis then consists of the 6 operators N_cX^{i3} and N_sX^{i3} $(I=1)$, and J^i, J_s^i, N_sJ^i/N_c, and $N_sJ_s^i/N_c$ $(I=0)$. Since no combinations of these operators have $I=2$ or 3, the combinations in Eqs. (4.4) exactly vanish, giving relations I1–I6 (JM Table 2).

In comparison with our Table I, the choice of JM operators reflects the inclusion of all (2) with $I=1$ at $O(N_c^1)$ and $O(N_c^0)$, and all (4) with $I=0$ at $O(N_c^0)$ and $O(N_c^{-1})$. Since there are (as one may readily count) 10 $I=0$ and 11 $I=1$ magnetic moment combinations in the **56**, it follows that JM predict 6 isoscalar relations (JM Table 3 S1–S6) that receive only $O(N_c^{-2})$ corrections, and 9 isovector relations (JM Table 3 V1–V7, $\mathrm{V8}_1$, and $\mathrm{V9}_1$) that receive only $O(N_c^{-1})$ corrections. As expected, we confirm these predictions in our basis.

The JM analysis makes no use of the electromagnetic behavior of magnetic moments, nor of perturbative SU(3) flavor breaking; its analysis can be said to hold in the presence of arbitrarily large SU(3) breaking. Thus, operators are organized solely by the $1/N_c$ power suppression of their matrix elements. Since 17 operators occur up to and including $O(N_c^{-1})$ while only 11 moment parameters have been measured, it is not yet possible to improve upon the numerical analysis of JM using their same scheme. One must therefore impose a physically natural flavor structure on the magnetic moment operators, a topic to which we next turn.

B. Analysis using the single-photon ansatz

Like all electromagnetic multipole moments, magnetic moments are defined through a particular coupling of a bilinear to an on-shell photon. The lowest-order flavor structure of the coupling to the photon should therefore be such that each quark couples proportionally to its electric charge. In particular, in the limit in which all other sources of SU(3) breaking are suppressed, only operators with one unsummed flavor index a may appear, and then only in the linear combination $(a=3)+(a=8)/\sqrt{3}\equiv(a=Q)$. Specifically, these are the forms

$$Q=T^Q\equiv T^3+\frac{1}{\sqrt{3}}T^8,$$

$$G^{iQ}\equiv G^{i3}+\frac{1}{\sqrt{3}}G^{i8}. \tag{4.6}$$

Implicit in this definition of the quark charge matrix Q is that the quarks assume their usual $N_c=3$ values $q_u=+\frac{2}{3}$, $q_d=q_s=-\frac{1}{3}$. In terms of SU(3) flavor hypercharges, $Y_u=Y_d=\frac{1}{3}$, $Y_s=-\frac{2}{3}$. An alternate choice, $q_u=(N_c+1)/(2N_c)$, $q_d=q_s=(-N_c+1)/(2N_c)$ $(Y_u=Y_d=1/N_c,\ Y_s=-1+1/N_c)$, has the convenient property that all hadrons then have the same electric charges and hypercharges for arbitrary N_c as they do for $N_c=3$. Moreover, with this choice the chiral anomalies of the standard model (with N_c colors) automatically cancel. However, one is also faced with the mysterious prospect of electromagnetic charges dependent upon the number N_c of QCD charges. More significantly, the quantization condition of the Wess-Zumino term permits only baryon SU(3) representations containing states with hypercharge $Y=N_c/3$ [25]; if such states have $O(N_c^0)$ strange quarks, then the N_c-dependent choice $(Y_u=Y_d=1/N_c)$ is disallowed. For the remainder of this paper, we assume the usual N_c-independent quark charges.

The only operators occurring in the single-photon ansatz with no other SU(3) breaking (cf. Table I) are

$$\mathcal{O}_1=G^{3Q},\ \ \mathcal{O}_2=\frac{1}{N_c}QJ^3,\ \ \tilde{\mathcal{O}}_3=\frac{1}{N_c^2}\frac{1}{2}\{J^2,G^{3Q}\},$$

$$\mathcal{O}_4=\frac{1}{N_c^2}J^iG^{iQ}J^3. \tag{4.7}$$

For any value of N_c it turns out that the combination $\mathcal{O}_3\equiv(\tilde{\mathcal{O}}_3-\mathcal{O}_4)$ vanishes for all diagonal moments and survives only for transitions. Since the only transition moment measured at present is μ_{Δ^+p}, using $\mathcal{O}_3$ rather than $\tilde{\mathcal{O}}_3$ provides a more incisive test of the expansion when fitting to current data.

In addition, one may perturbatively break SU(3) symmetry by including effects due to a finite strange quark mass or spin. We incorporate such effects by including a parameter ε that indicates each instance of breaking of the SU(3) symmetry. At first blush, one may suppose that it is proportional to m_s, $\varepsilon\simeq0.3\approx1/N_c$, but as we see below such a rigid identification is not necessary. The list of additional operators with matrix elements up to $O(\varepsilon^1N_c^0)$ reads

$$\varepsilon\mathcal{O}_5=\varepsilon q_sJ_s^3,\ \ \varepsilon\mathcal{O}_6=\frac{\varepsilon}{N_c}N_sG^{3Q},\ \ \varepsilon\mathcal{O}_7=\frac{\varepsilon}{N_c}QJ_s^3. \tag{4.8}$$

These forms are obtained by the simple expedient of inserting sources of SU(3) breaking along the strangeness direction into the operators of Eq. (4.7), and retaining only those with matrix elements up to $O(\varepsilon^1N_c^0)$. The possible substitutions are $J^i\to\varepsilon J_s^i$, $Q/N_c\to\varepsilon q_sN_s/N_c$, $G^{iQ}\to\varepsilon q_sJ_s^i$, or

TABLE X. Best fit values for the coefficients in the expansion Eq. (4.12).

$d_1 = +0.987 \pm 0.038$	$d_2 = +0.076 \pm 0.092$	$d_3 = +1.385 \pm 0.258$	$d_4 = +0.059 \pm 0.255$
$d_5 = -0.348 \pm 0.114$	$d_6 = -0.140 \pm 0.108$	$d_7 = +0.126 \pm 0.108$	

multiplication of an operator by $\varepsilon N_s/N_c$; each of the last three of these replacements also costs a power of $1/N_c$. In terms of this basis, the analysis of Ref. [6] (LMRW) uses the operators $\mathcal{O}_1$ [LMRW Eq. (17)], $\mathcal{O}_2$ [LMRW Eq. (21)], the combination

$$m_s^{1/2}\left[-\left(1+\frac{3}{N_c}\right)\mathcal{O}_5-\mathcal{O}_6+\mathcal{O}_7\right], \tag{4.9}$$

from chiral loop diagrams [LMRW Eq. (25)], and the operators $m_s\mathcal{O}_5$, $m_s\mathcal{O}_6$, $m_s\mathcal{O}_7$ from counterterms to the loop calculation [LMRW Eq. (29)], 5 independent operators in all. Note the characteristic nonanalytic m_s behavior in Eq. (4.9), which suggests that the appropriate SU(3) expansion parameter ε might properly scale as $\simeq N_c^{-1/2}$ rather than $\simeq N_c^{-1}$; we address this issue below.

At the next order, $O(\varepsilon^1 N_c^{-1})$, one finds the 6 operators

$$\varepsilon\mathcal{O}_8=\varepsilon q_s\frac{N_s}{N_c}J^3,\ \varepsilon\mathcal{O}_9=\varepsilon\frac{N_s}{N_c^2}QJ^3,\ \varepsilon\mathcal{O}_{10}=\frac{\varepsilon}{N_c^2}\frac{1}{2}\{\mathbf{J}\cdot\mathbf{J}_s,G^{3Q}\},$$

$$\varepsilon\mathcal{O}_{11}=\frac{\varepsilon}{N_c}J_s^jG^{jQ}J^3,\ \ \varepsilon\mathcal{O}_{12}=\frac{\varepsilon}{N_c^2}\frac{1}{2}\{J^jG^{jQ},J_s^3\}. \tag{4.10}$$

Beyond this collection, the next operators have matrix elements of $O(\varepsilon^2 N_c^{-1})$ and $O(\varepsilon^1 N_c^{-2})$, but significantly there are none of $O(\varepsilon^2 N_c^0)$ or $O(\varepsilon^0 N_c^{-2})$. As a consequence, whether one takes the SU(3)-breaking parameter $\varepsilon\simeq N_c^{-1/2}$ or $\simeq N_c^{-1}$, the series expansion truncates consistently after the inclusion of either the set $\mathcal{O}_1,\dots,\varepsilon\mathcal{O}_7$ (complete to combined orders $\varepsilon^1 N_c^0$ and $\varepsilon^0 N_c^{-1}$) or the larger set $\mathcal{O}_1,\dots,\varepsilon\mathcal{O}_{12}$ [complete to $O(\varepsilon^1 N_c^{-1})$].

In this notation, the operators used in Ref. [7] (DDJM) consist of [DDJM Eq. (2.9)]

$$\mathcal{O}_1,\mathcal{O}_2,\mathcal{O}_3,\varepsilon\mathcal{O}_5,\varepsilon\mathcal{O}_6,\varepsilon\mathcal{O}_7,\varepsilon\mathcal{O}_8, \tag{4.11}$$

and the operator $\varepsilon^2 q_s N_s J_s^3/N_c$ with a coefficient fixed relative to those of $\mathcal{O}_6$ and $\mathcal{O}_7$, 7 independent operators in all, but a somewhat different set than $\mathcal{O}_1,\dots,\varepsilon\mathcal{O}_7$. Note that DDJM does not assign particular powers of ε; DDJM also recognizes the presence of $O(m_s^{1/2})$ loop corrections, so that statements regarding the meaning of ε still apply.

C. A global fit

As we have seen, assigning a particular numerical value to the SU(3)-breaking parameter ε can be problematic, owing to the existence of contributions nonanalytic in m_s. Fortunately, we have also seen that regardless of whether one takes $\varepsilon\simeq N_c^{-1/2}$ or N_c^{-1}, the expansion truncates consistently after the 7 operators in Eqs. (4.7),(4.8) or after the 12 operators including Eqs. (4.10). Powers of ε may simply be left in the coefficients, in which case the size of SU(3) breaking for each operator may be judged directly from a fit to data.

Unfortunately, with only 11 measured magnetic moment parameters, only a fit to the first 7 operators is possible at present. We therefore perform a least-squares fit to the coefficients in the expansion

$$\mu_z=\mu_0\sum_{i=1}^{7} d_i\mathcal{O}_i\,. \tag{4.12}$$

We choose the scale μ_0 by reasoning that the best known value among the moments is μ_p, and that there is only one operator, $\mathcal{O}_1$, at leading order (N_c^1), whose value for the proton is $(N_c+3)/12=1/2$ for $N_c=3$. Therefore, a natural choice that makes the sole leading-order coefficient d_1 of order unity is to set $\mu_0=2\mu_p$ [in alternate choices one may average over several measured magnetic moments, but this merely renormalizes all d_i by the same $O(1)$ multiple]. Since the expansion is truncated by ignoring effects of $O(\varepsilon^1 N_c^{-1})$, one must include in addition to the statistical uncertainty of each magnetic moment a "theoretical uncertainty" of order $\mu_0\varepsilon/N_c$. In fact, the χ^2/DOF obtained from a theoretical uncertainty of $\mu_p/N_c^2\simeq 0.3\ \mu_N$, for example, is only about 0.13, suggesting that the naive theoretical uncertainty is a gross overestimate. We find empirically that choosing it to have a value about $\mu_p/N_c^3\simeq 0.1\ \mu_N$ gives a χ^2/DOF$=1.05$, meaning that the fit is as good as one might hope for *even if all* $O(\varepsilon^1 N_c^{-1})$ *effects are suppressed, and uncertainties are effectively only* $O(\varepsilon^2 N_c^{-1})$. This result far supersedes that expected from a naive $1/N_c$ expansion.

The fit values for the coefficients are given in Table X. One immediately notes that no coefficients are larger than $O(1)$; had any of them been so, one would conclude that the $1/N_c$ expansion fails. But in fact, d_1 and d_3 are of $O(1)$, while d_2 and d_4 are actually consistent with zero. The SU(3)-breaking coefficients $d_{5,6,7}$, do indeed have central values about 1/3 or less, but only d_5 is statistically different from zero. Such a pattern of suppression beyond naive $1/N_c$ counting has in fact been seen before, in the orbitally excited baryons [15–17]. Moreover, a hint of this effect is visible in the results of DDJM Table VI [although their operator basis, and especially their treatment of SU(3) breaking, is rather different]. We do not understand the source of this suppression beyond that expected from $1/N_c$ counting, and find it to be the most intriguing feature of our analysis.

One may also use the fit values for d_i to predict all of the remaining 16 magnetic moments to within the theoretical uncertainty; the results are presented in Table XI. Note that the recent μ_{Δ^+} measurement easily agrees with our predic-

TABLE XI. Best fit values for the 16 unknown magnetic moments in units of μ_N.

$\mu_{\Delta^+} = +3.04 \pm 0.13$	$\mu_{\Delta^0} = +0.00 \pm 0.10$	$\mu_{\Delta^-} = -3.04 \pm 0.13$	$\mu_{\Sigma^{*+}} = +3.35 \pm 0.13$
$\mu_{\Sigma^{*0}} = +0.32 \pm 0.11$	$\mu_{\Sigma^{*-}} = -2.70 \pm 0.13$	$\mu_{\Xi^{*0}} = +0.64 \pm 0.11$	$\mu_{\Xi^{*-}} = -2.36 \pm 0.14$
$\mu_{\Sigma^0} = +0.77 \pm 0.10$	$\mu_{\Delta^0 n} = +3.51 \pm 0.11$	$\mu_{\Sigma^{*0}\Lambda} = +2.93 \pm 0.11$	$\mu_{\Sigma^{*0}\Sigma^0} = +1.39 \pm 0.11$
$\mu_{\Sigma^{*+}\Sigma^+} = +2.97 \pm 0.11$	$\mu_{\Sigma^{*-}\Sigma^-} = -0.19 \pm 0.11$	$\mu_{\Xi^{*0}\Xi^0} = +2.96 \pm 0.12$	$\mu_{\Xi^{*-}\Xi^-} = -0.19 \pm 0.11$

tion. An important point that may not be obvious from this compilation is that, with only 7 operators included, there remain 20 relations among the magnetic moments. Among these are vanishing of the $I=2$ and 3 combinations in Eq. (4.4): The leading-order operators in Eq. (4.7) contain only $I=0$ and 1 pieces, while inserting SU(3) breaking along the strangeness direction induces only $I=0$ corrections. The combinations in Eqs. (4.4) receive contributions only from tiny isospin-breaking effects due to either $(m_u - m_d)$ or loop diagrams containing an additional photon $[O(\alpha/4\pi)]$. Furthermore, μ_{Δ^0} vanishes for all 7 operators when $N_c=3$, and receives contributions only from SU(3) breaking not solely in the strangeness direction (since Δ^0 contains no s quarks), which induces an additional $m_{u,d}/m_s$ suppression factor. A similar statement holds for all nonstrange observables: None of them receive a contribution from any operators beyond $\mathcal{O}_1, \ldots, \mathcal{O}_4$; indeed, the famous SU(6) relation $\mu_n = -2\mu_p/3$ for $N_c=3$ receives a contribution only from the anomalously suppressed operator $\mathcal{O}_2$.

Predictions of the diagonal moments appearing in LMRW Table I agree well with the results of Table XI, but ours have smaller uncertainties due to the larger number of operators and the treatment of subleading effects as described above.

Since the next order of the expansion contains 12 operators while 11 observables are well known, it is tempting to suppose that just one more moment measured—say, an improvement on μ_{Δ^+}—will permit such a fit. However, the $I=3$ and $I=2$ relations among the Δ's combined with the result $\mu_{\Delta^0}=0$, as satisfied by all operators in our list [and any others breaking SU(3) solely in the strangeness direction], predict that the Δ magnetic moments are exactly proportional to electric charge for $N_c=3$: $\mu_{\Delta^{++}} = 2\mu_{\Delta^+} = -2\mu_{\Delta^-}$. Moreover, there is precisely one relation satisfied by the first 12 operators among the measured moments:

$$\mu_n - \frac{1}{4}(\mu_{\Sigma^+} + \mu_{\Sigma^-}) - \frac{3}{2}\mu_\Lambda - \sqrt{3}\mu_{\Sigma^0\Lambda} + \mu_{\Xi^0} = O(\varepsilon^2 N_c^{-1}), \tag{4.13}$$

which has a numerical value of $0.22 \pm 0.14\ \mu_N$, the uncertainty being completely dominated by that of $\mu_{\Sigma^0\Lambda}$. In fact, Eq. (4.13) was originally derived in heavy baryon chiral perturbation theory [26], where it was found to have no $O(m_s^{1/2})$, $O(m_s)$, or $O(m_s \ln m_s)$ corrections—i.e., no $O(\varepsilon^1)$ corrections in the current formalism. Converting Eq. (4.13) into a scale-invariant result by dividing by the average of the same expression with all negative values turned to positive ones [giving an $O(N_c^1)$ combination], one obtains 0.057 ± 0.036, in good agreement with expected magnitude $\varepsilon^2 N_c^{-2}$. A better measurement of $\mu_{\Sigma^0\Lambda}$, certainly within current experimental means, would decisively test the expansion at this order.

It follows that a measurement of at least two moments with nonzero strangeness is required to perform a fit to the 12 operators at $O(\varepsilon^1 N_c^{-1})$, and to determine whether the effects at this order are truly suppressed, as the χ^2/DOF suggests. The $\Sigma^*\Lambda$, $\Sigma^*\Sigma$, and $\Xi^*\Xi$ transitions are natural candidates, since the associated radiative decays are presumably being recorded (although not yet studied) at Jefferson Lab, as well as other facilities. To date, the decay $\Sigma^{*0} \to \Lambda\gamma$ has been seen in precisely one event [27]; the opportunity to improve on this meager set clearly exists.

Finally, we note in passing that once the unmeasured moments are included, a number of relations with only $O(\varepsilon^2 N_c^{-1})$ corrections, in addition to Eqs. (4.4), (4.13), and $\mu_{\Delta^0}=0$, remain (7, to be precise). A particularly elegant example is $\mu_{\Sigma^{*-}\Sigma^-} = \mu_{\Xi^{*-}\Xi^-}$.

V. CONCLUSIONS

We have developed a basis of operators representing every possible observable pattern of magnetic moments for the ground-state spin-1/2 and spin-3/2 baryon multiplets, and organized them according to the counting of $1/N_c$ factors. We have furthermore computed the group-theoretical parts of all of these operators, thus producing a complete effective Hamiltonian for magnetic moments. Our analysis of this operator expansion examined the consequences both in the case of arbitrarily large SU(3) breaking and perturbative SU(3) breaking (in powers of a parameter ε) beyond that produced by coupling quarks to photons in proportion to their electric charges. In both cases we have compared to previous results and showed how this work extends earlier analyses.

In particular, we have found in the case of nonperturbative SU(3) breaking that the measurement of several additional magnetic moments is necessary to improve numerically upon previous analysis [i.e., from relative order N_c^{-2} to N_c^{-3}]. However, in the more physically meaningful case of perturbative SU(3) breaking, the series may be truncated consistently after 7 operators (including up to orders $\varepsilon^1 N_c^0$ and $\varepsilon^0 N_c^{-1}$) or after 12 operators [up to $O(\varepsilon^1 N_c^{-1})$]. Since 11 observables are currently well measured, we presented results of a fit to 7 operators, and found not only that several of the effective Hamiltonian coefficients are smaller than expected, but also that a good fit can be obtained if the terms neglected are actually $1/N_c$ smaller than naively expected.

A number of relations among the magnetic moments survive the expansion to 12 operators. After enumerating a number of them [e.g., Eq. (4.13)], we suggested that the most probative tests of the $1/N_c$ expansion: an improved measurement of the $\Sigma^0\Lambda$ transition and observation of $\Sigma^*\Lambda$, $\Sigma^*\Sigma$, and $\Xi^*\Xi$ transitions, should lie with current experimental means.

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