Lorentz and *CPT* violation in the Higgs sector

David L. Anderson,* Marc Sher,† and Ismail Turan‡
Particle Theory Group, Department of Physics, College of William and Mary, Williamsburg, Virginia 23187-8795, USA

(Received 7 April 2004; published 1 July 2004)

Colladay and Kostelecký have proposed a framework for studying Lorentz and *CPT* violation in a natural extension of the standard model. Although numerous bounds exist on the Lorentz and *CPT* violating parameters in the gauge boson and fermion sectors, there are no published bounds on the parameters in the Higgs sector. We determine these bounds. The bounds on the *CPT*-even asymmetric coefficients arise from the one-loop contributions to the photon propagator, those from the *CPT*-even symmetric coefficients arise from the equivalent $c_{\mu\nu}$ coefficients in the fermion sector, and those from the *CPT*-odd coefficient arise from bounds on the vacuum expectation value of the Z boson.

DOI: 10.1103/PhysRevD.70.016001 PACS number(s): 11.30.Er, 12.60.−i

I. INTRODUCTION

The scale of the unification of gravity with the other interactions is expected to be near the Planck scale of 10^{19} GeV. This is far out of reach of any future accelerators and thus is not directly experimentally accessible. However, the nonlocality of string theory leads to the possibility that Lorentz and *CPT* symmetry violations might exist at that scale [1], and hence high-precision studies of these symmetries might be able to probe Planck-scale physics.

It is difficult to write the most general Lorentz and *CPT* violating theory—even the meaning of a Lagrangian becomes questionable in such a theory. However, with some reasonable assumptions, one can study Lorentz and *CPT* violation. To develop a framework for studying Lorentz and *CPT* violation in the standard model, Colladay and Kostelecký [2] constructed the standard model extension (SME). This is a theory based on the standard model but which includes additional Lorentz and *CPT* violating terms. These terms satisfy the $SU(3)\times SU(2)\times U(1)$ gauge symmetry of the standard model, and they also satisfy invariance under observer Lorentz transformations [2–4]. This means that any Lorentz indices that the additional term contains must be contracted (i.e., it must be an observer Lorentz scalar), and that rotations and boosts of the observer inertial frame do not affect the physics. This ensures that the physics does not depend on the choice of coordinates. In addition, the Lorentz violation is assumed independent of position and time, and thus energy and momentum are conserved. The Lorentz-violating terms considered in the SME violate invariance under particle Lorentz transformations, i.e. under rotations and boost of a particle within a fixed observer inertial frame. An example of two such terms in the pure electron sector is $\bar{\psi}M\psi$, where $M\equiv a_\mu\gamma^\mu+b_\mu\gamma^\mu\gamma_5$. This term is clearly $SU(3)\times SU(2)\times U(1)$ invariant, and the coefficients are position independent, but a_μ and b_μ are constant vectors and do not transform under a particle Lorentz transformation. It should be noted that this is the "minimal" extension. Non-Minkowski spacetimes [5] will lead to spacetime-dependent coefficients, and some models can lead to nonrenormalizable terms. Such minimal extensions are beyond the scope of this paper.

In the SME, the additional terms in the Higgs sector are given by [2]

$$\mathcal{L}_{CPT\ \text{even}}=\left[\frac{1}{2}(k^S_{\phi\phi}+ik^A_{\phi\phi})_{\mu\nu}(D^\mu\Phi)^\dagger D^\nu\Phi+\text{H.c.}\right]-\frac{1}{2}k^{\mu\nu}_{\phi B}\Phi^\dagger\Phi B_{\mu\nu}-\frac{1}{2}k^{\mu\nu}_{\phi W}\Phi^\dagger W_{\mu\nu}\Phi, \quad (1.1)$$

and

$$\mathcal{L}_{CPT\ \text{odd}}=ik^\mu_\phi\Phi^\dagger D_\mu\Phi+\text{H.c.} \quad (1.2)$$

Here, we have broken the $k_{\phi\phi}$ term up into its real symmetric and imaginary antisymmetric parts. Note that the $k_{\phi B}$ and $k_{\phi W}$ coefficients are real antisymmetric, the *CPT* even coefficients are all dimensionless, and the complex-valued *CPT* odd coefficient has units of mass.

To our knowledge, there are no published limits on the possible values of these coefficients. The purpose of this article is to explore the current bounds on these terms. In Sec. II, we consider the bounds on the *CPT*-even antisymmetric coefficients, $k^A_{\phi\phi}, k_{\phi B}$ and $k_{\phi W}$. In Sec. III, the bounds of the *CPT*-even symmetric coefficients $k^S_{\phi\phi}$ are determined, and the bounds on the *CPT*-odd coefficient k_ϕ are discussed in Sec. IV. Section V contains our conclusions and a summary of the bounds.

II. BOUNDS ON THE *CPT*-EVEN ANTISYMMETRIC COEFFICIENTS

Whenever new particles or new interactions are proposed, there are two approaches to discovery. One can look for direct detection of these particles or interactions (as in searches for supersymmetric particles or for flavor-changing neutral currents). Alternatively, one can look at the loop effects of the new physics on lower energy processes, such as in precision electroweak measurements. In studying the above co-

*Electronic address: dlande@wm.edu
†Electronic address: sher@physics.wm.edu
‡Electronic address: ituran@newton.physics.metu.edu.tr

efficients, direct detection would necessitate producing large numbers of Higgs bosons, and the resulting bounds would be quite weak. However, there are extremely stringent bounds on Lorentz violation at low energies, and thus searching for the effects of these new interactions through loop effects will provide the strongest bounds. The most promising of these effects will be on the photon propagator.

In this section, we will consider the bounds on the *CPT*-even antisymmetric coefficients, $k^A_{\phi\phi}, k_{\phi B}$ and $k_{\phi W}$. These interactions will lead to modified vertices and propagators, and will thus affect the one-loop photon propagator. We first look at the most general *CPT*-even photon propagator, and then relate the $k^A_{\phi\phi}$ coefficients to the Lorentz-violating terms in the photon propagator. Then, the experimental constraints on such terms lead directly to stringent bounds on the $k^A_{\phi\phi}$ coefficients. We then consider the $k_{\phi B}$ and $k_{\phi W}$ coefficients.

Considering *CPT*-even terms only, the photon Lagrangian can be written as [2]

$$\mathcal{L}_{\text{photon}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu}. \tag{2.1}$$

Here k_F has the symmetries of the Riemann tensor plus a double-traceless constraint, giving 19 independent parameters. The equation of motion from this Lagrangian is

$$M^{\alpha\delta}A_\delta = 0, \tag{2.2}$$

where

$$M^{\alpha\delta}(p) \equiv g^{\alpha\delta}p^2 - p^\alpha p^\delta - 2(k_F)^{\alpha\beta\gamma\delta}p_\beta p_\gamma. \tag{2.3}$$

The propagator is clearly gauge invariant (recall that k_F is antisymmetric under exchange of the first or last two indices).

To bound the coefficients, we calculate the vacuum polarization diagrams for the photon propagator, using the full Lagrangian, including Lorentz-violating terms. The result will be of the form of the above propagator, and one can read off the value of k_F. Note that while the $g^{\mu\nu}p^2 - p^\mu p^\nu$ structure is mandated by gauge invariance, the k_F term is separately gauge invariant and may differ order by order in perturbation theory. For simplicity, we look at the divergent parts of the one loop diagrams only.[1] Consideration of higher orders and finite parts will give similar, although not necessarily identical, results.

In general, due to the large number of Lorentz-violating terms, this yields a bound in a multidimensional parameter space. However, if we do not consider the possibility of fine-tuning, then we can consider each of the possible terms independently. One must keep in mind that some of the parameters may be related by a symmetry, but absent such a symmetry, we expect no high-precision cancellations. We begin by considering the antisymmetric part of $k_{\phi\phi}$, and then $k_{\phi B}$ and $k_{\phi W}$.

[1] At extremely high energies, either energy positivity or microcausality may be lost. [6] However if we cut off the theory at a high, but finite, scale, this will not be an issue.

To calculate the additional vacuum polarization diagrams for the photon propagator due to a non-zero $k^A_{\phi\phi}$ term in Eq. (1.1) (assuming all other parameters are zero), we need to find the vertices and propagators which are dependent on $k^A_{\phi\phi}$. For our purpose, vertices involving at least one photon field are necessary. Two of them, for instance, can be quoted here. The $A_\mu W^-_\nu \phi^+ [A_\mu(p)\phi^+\phi^-]$ coupling is given by $-em_W(k^A_{\phi\phi})_{\mu\nu}[-e(k^A_{\phi\phi})_{\mu\nu}p^\nu]$. Here all momenta are taken toward the vertex, and $\phi^\pm$ is the usual charged Goldstone boson. As in the conventional SM, one can choose acceptable gauge-fixing conditions to remove the redundant degrees of freedom from the theory. In the SM, the following conditions in the R_ξ gauge can be chosen [7]: $f_i = \partial_\mu A^\mu_i + (ig\xi/2)(\Phi'^\dagger\tau_i\langle\Phi\rangle_0 - \langle\Phi^\dagger\rangle_0\tau_i\Phi')$, $i=1,2,3$ for the SU(2) case and $f = \partial_\mu B^\mu + (ig'\xi/2)(\Phi'^\dagger\langle\Phi\rangle_0 - \langle\Phi^\dagger\rangle_0\Phi')$ for the U(1) case, where $g(g')$ is the SU(2)[U(1)] coupling constant, τ_i are the Pauli matrices, and Φ' and $\langle\Phi\rangle_0$ are the Higgs doublet and vacuum expectation value, respectively. Then the gauge-fixing term in the Lagrangian is $\mathcal{L}_{gf} = -(\mathbf{f}\cdot\mathbf{f})^2/2\xi - f^2/2\xi$ and this removes the mixing term between $W^\pm$ and $\phi^\mp$. In the SME, we have additional mixing proportional to $k^A_{\phi\phi}$. A simple generalization of the above gauge-fixing conditions, by adding a $i(k^A_{\phi\phi})_{\mu\nu}\partial^\mu A^\nu_i$ term to f_i and a similar $i(k^A_{\phi\phi})_{\mu\nu}\partial^\mu B^\nu$ to the function *f*, would remove such a Lorentz-violating mixing in our case as well. However, such a generalization also leads to an unwanted mixing between the gauge boson Z_μ and the derivative of the Higgs field, $\partial_\nu\phi_1$, which is contracted with $(k^A_{\phi\phi})^{\mu\nu}$, as well as substantially complicating the photon propagator. Instead we use a mixed propagator of the form $m_W(k^A_{\phi\phi})_{\mu\nu}q^\nu$ for $W^\pm_\mu(q)\phi^\mp$ fields [that is, we are treating the mixing term as an interaction, which leads to diagrams like (d), (e), (g), and (h) in Fig. 1]. Here we use the convention that the 4-momentum q of W_μ is incoming to the point where the field turns into a charged Goldstone boson.

Another distinct feature of this model is the presence of a term of the form $im_W(k^A_{\phi\phi})^{\mu\nu}W^+_\mu W^-_\nu$. This term needs to be considered carefully. It obviously represents a new term in the *W* propagator. We will discuss how to deal with this term in the R_ξ gauge, although we use the 't Hooft–Feynman gauge ($\xi=1$) in our vacuum polarization calculations. Since this mixing term can be considered an interaction, one can carry out the Dyson summation. If we pick up the quadratic terms in the *W* boson from the Lagrangian together with $\mathcal{L}_{gf}$, we have $\Delta\mathcal{L}^{(2)}_W = W^-_\mu K^{\mu\nu}(q)W^+_\nu$, where

$$\begin{aligned} iK^{\mu\nu}(q) &\equiv i[-(q^2-m_W^2)g^{\mu\nu} + (1-1/\xi)q^\mu q^\nu \\ &\quad + im_W^2(k^A_{\phi\phi})^{\mu\nu}] \equiv iK^{(0)\mu\nu}(q) - m_W^2(k^A_{\phi\phi})^{\mu\nu}. \end{aligned}$$

We know that the inverse of $iK^{(0)\mu\nu}(q)$, say $i\Delta_{(0)\nu\lambda}(q)$ (that is, $K^{(0)\mu\nu}\Delta_{(0)\nu\lambda} = g^\mu_\lambda$), is the usual propagator for the *W* boson. From $K^{\mu\nu}(q)$, one can write the form of the propagator as $\Delta_{\nu\lambda}(q) \equiv \Delta^{(0)}_{\nu\lambda}(q) + B_{\nu\lambda}(k^A_{\phi\phi})$, where all $k^A_{\phi\phi}$ dependence is in the second term. To determine $B_{\nu\lambda}$, we can use the fact that $\Delta_{\nu\lambda}$ is the inverse of $K^{\mu\nu}$. From this equation, one gets $B_{\nu\lambda} = -im_W^2\Delta^{(0)}_{\nu\lambda'}(k^A_{\phi\phi})^{\lambda'\mu}[\Delta^{(0)}_{\mu\lambda} + B_{\mu\lambda}]$. Iterating this equa-

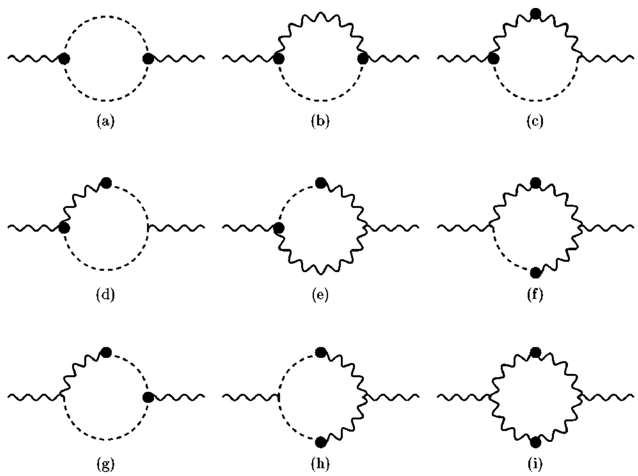

FIG. 1. One-loop contributions to the photon vacuum polarization involving Lorentz-violating interactions to second order. These diagrams are for the $k^A_{\phi\phi}$ case but similar diagrams exist for the other antisymmetric coefficients. Here the wavy (dashed) line circulating in the loop represents the W boson (charged Goldstone boson). Each blob in vertices, W propagator or W-ϕ mixed propagator represents a single Lorentz-violating coefficient insertion. The rest of the diagrams can be obtained by permutations of these 9 diagrams.

tion, one obtains a series. However, we know that $k^A_{\phi\phi}$ parameters are small, so it is sufficient to keep the first few terms. Up to second order, it is straightforward to show that

$$B_{\nu\lambda} = -i m_W^2 \Delta^{(0)}_{\nu\alpha} (k^A_{\phi\phi})^{\alpha\beta} \Delta^{(0)}_{\beta\lambda} - m_W^4 \Delta^{(0)}_{\nu\alpha} (k^A_{\phi\phi})^{\alpha\alpha'} \Delta^{(0)}_{\alpha'\beta'} (k^A_{\phi\phi})^{\beta'\beta} \Delta^{(0)}_{\beta\lambda}.$$

In the 't Hooft–Feynman gauge the propagator has a simple form which can be given as

$$i\Delta_{\nu\lambda}(\xi=1) = i\Delta^{(0)}_{\nu\lambda} + m_W^2 \frac{(k^A_{\phi\phi})_{\nu\lambda}}{(q^2-m_W^2)^2} + i m_W^4 \frac{(k^A_{\phi\phi})_{\nu\alpha}(k^A_{\phi\phi})^{\alpha}{}_{\lambda}}{(q^2-m_W^2)^3}, \tag{2.4}$$

where, for example, the second term is represented as a blob in the W propagator in Fig. 1(c), Fig. 1(f), and Fig. 1(i).

We are now ready to calculate the vacuum polarization diagrams for the photon propagator. It is useful to classify contributions as the ones having first order $k^A_{\phi\phi}$ dependence and the ones with quadratic in $k^A_{\phi\phi}$. The only possible structure in first order is $(k^A_{\phi\phi})_{\mu\nu}$ where $\mu(\nu)$ is the Lorentz index of the incoming(outgoing) photon field. If we add all possible one-loop diagrams, the first order contributions vanish. This is expected from the gauge invariance requirement. It is not difficult to show that getting a gauge invariant transverse structure is only possible with at least two $k^A_{\phi\phi}$ terms. In Fig. 1, we depict the one-loop diagrams which, when permutations are added, give second order Lorentz-violating inclusions. There are two possible structures in second order, which are either $(k^A_{\phi\phi})_{\mu\lambda}(k^A_{\phi\phi})^{\lambda}{}_{\nu}$ or $(k^A_{\phi\phi})_{\mu\lambda}(k^A_{\phi\phi})_{\lambda'\nu} p^{\lambda} p^{\lambda'}$. Here p is the four-momentum of the external photons. Again the first possibility is not gauge invariant and should vanish; thus contributions from the third term in Eq. (2.4) should vanish. We have verified this explicitly. The latter is gauge invariant and gives a non-zero contribution (if we contract with any of two external momenta of photons, p^μ or p^ν, it vanishes due to the antisymmetry property of $k^A_{\phi\phi}$).

Calculating the one-loop diagrams, and comparing with Eq. (2.3), we find that the components of k_F can simply be expressed in terms of $k^A_{\phi\phi}$ as $(k_F)_{\mu\lambda\lambda'\nu} = \frac{1}{3}(k^A_{\phi\phi})_{\mu\lambda}(k^A_{\phi\phi})_{\lambda'\nu}$. We now turn to the experimental bounds on the k_F.

The dimensionless coefficient $(k_F)_{\kappa\lambda\mu\nu}$ has the symmetries of the Riemann tensor and a vanishing double trace, resulting in nineteen independent elements. Following Kostelecký and Mewes [8], we can express these elements in terms of four traceless 3×3 matrices and one coefficient:

$$(\tilde{\kappa}_{e+})^{jk} = \frac{1}{2}(\kappa_{DE}+\kappa_{HB})^{jk},$$

$$(\tilde{\kappa}_{e-})^{jk} = \frac{1}{2}(\kappa_{DE}-\kappa_{HB})^{jk} - \frac{1}{3}\delta^{ij}(\kappa_{DE})^{ll},$$

$$(\tilde{\kappa}_{o+})^{jk} = \frac{1}{2}(\kappa_{DB}+\kappa_{HE})^{jk},$$

$$(\tilde{\kappa}_{o-})^{jk} = \frac{1}{2}(\kappa_{DB}-\kappa_{HE})^{jk},$$

$$\tilde{\kappa}_{tr} = \frac{1}{3}(\kappa_{DE})^{ll}, \tag{2.5}$$

where

$$(\kappa_{DE})^{jk} = -2(k_F)^{0j0k},$$

$$(\kappa_{HB})^{jk} = \frac{1}{2}\epsilon^{jpq}\epsilon^{krs}(k_F)^{pqrs},$$

$$(\kappa_{DB})^{jk} = -(\kappa_{HE})^{kj} = (k_F)^{0jpq}\epsilon^{kpq}. \tag{2.6}$$

There are stringent astrophysical bounds on 10 of the 19 elements, those given by $\tilde{\kappa}_{e+}$ and by $\tilde{\kappa}_{o-}$. These astrophysical bounds have been discussed recently in detail by Kostelecký and Mewes [8]. The observations of radiation propagating in free space over astrophysical distances results in bounds on these elements from velocity and birefringence constraints [3,9–12]. The bound from birefringence constraints is the strongest, and is given by 3×10^{-32}. The bounds on the remaining 9 elements are much weaker (and in fact can be moved into the fermion sector, as will be discussed below).

If one of our coefficients is nonzero, say $(k^A_{\phi\phi})_{01} = -(k^A_{\phi\phi})_{10} \equiv x$, then the only nonzero components of k_F are the $(k_F)_{1010}, (k_F)_{0101}, (k_F)_{1001}$ and $(k_F)_{0110}$ components. This leads to a nonzero $\tilde{\kappa}_{e+}$ matrix, and thus the stringent bounds apply. Extending this one can see that for any single or possible combination of non-zero elements of $(k^A_{\phi\phi})_{\mu\nu}$ it is impossible for both $\tilde{\kappa}_{e+}$ and $\tilde{\kappa}_{o-}$ to be null matrices, and thus the birefringence constraints apply.

One cautionary note should be added. In the above example, the k_F tensor is not double traceless, since $(k_F)^{\mu\nu}_{\mu\nu}$ is proportional to x^2. This means that the kinetic energy for the photon has not been properly normalized. By adding and subtracting a term proportional to the double trace

$$\mathcal{L}=-\frac{1}{4}(1+\varsigma x^2)F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}(k_F)_{\kappa'\lambda'\mu'\nu'}F^{\kappa'\lambda'}F^{\mu'\nu'}+\frac{1}{4}(\varsigma x^2)F_{\mu\nu}F^{\mu\nu}, \tag{2.7}$$

where ς is a constant and the primed indices are summed only over the nonzero elements [in the above example, only over $(k_F)_{1010}, (k_F)_{0101}, (k_F)_{1001}, (k_F)_{0110}$]. A redefinition of the photon field will give a conventional kinetic term, and the remaining terms obey the double traceless condition if one chooses a suitable ς value. This means that, although we started with only a $(k_F)_{0101}$ term (plus permutations), we also have $(k_F)_{0202}, (k_F)_{0303}, (k_F)_{1212}, (k_F)_{1313}$ and $(k_F)_{2323}$ terms (plus permutations). Nonetheless it will still not be possible for the elements of $\tilde{\kappa}_{e+}$ and $\tilde{\kappa}_{o-}$ to become zero, hence these redefinitions do not affect the bounds. From these results, we find an upper bound of 3×10^{-16} for the $k^A_{\phi\phi}$ coefficients, barring, of course, fine-tuned cancellations.

Next, we consider the $k_{\phi B}$ term by setting all other parameters to zero in Eq. (1.1). This term has an interesting new interaction $A_\mu\phi_1\phi_1$, where ϕ_1 is the standard model Higgs boson. There also exists a similar Lorentz-violating vertex with the neutral Goldstone boson, ϕ_2. Therefore, in addition to the charged Goldstone loop, we have diagrams like Fig. 1(a), which are second order in $k_{\phi B}$ with different vertex factors, where now the particles circulating in the loop are the Higgs and the would-be Goldstone bosons. The coupling is $\cos\theta_W(k_{\phi B})_{\mu\nu}p^\nu$, where p is the four-momentum of the photon. Unlike the $k^A_{\phi\phi}$ case, we obviously do not have an additional mixing between the W and charged Goldstone bosons [thus, no diagrams like (d), (e), (g), and (h) in Fig. 1]. But this new term induces a remarkable mixing between the photon and the Higgs scalar, since when the Higgs boson gets a vacuum expectation value, an $A_\mu\partial_\nu\phi$ mixing term appears. This term cannot be removed by gauge fixing, and represents a mixed propagator. In our one-loop calculation of the photon propagator, however, the mixing will not contribute to the divergent part, and is thus not relevant.[2] Therefore, if we look at the structures in the first and the second order in $k_{\phi B}$, there exist $(k_{\phi B})_{\mu\lambda}p^\lambda p_\nu$, $(k_{\phi B})_{\nu\lambda}p^\lambda p_\mu$, and $(k_{\phi B})_{\mu\lambda}(k_{\phi B})_{\lambda'\nu}p^\lambda p^{\lambda'}$. Note that only the scalar loop diagrams with two Lorentz-violating vertices yields the last structure (three scalar loop diagrams with charged Goldstone $\phi^\pm$ and Higgs boson ϕ_1, and would-be neutral Goldstone boson ϕ_2). Gauge invariance makes us expect that the first two non-invariant structures should vanish and this is indeed the case. So, in this framework, the $(k_F)_{\mu\lambda\lambda'\nu}=(5/12e^2)\cos^2\theta_W(k_{\phi B})_{\mu\lambda}(k_{\phi B})_{\lambda'\nu}$ equality holds. Numerically, the bound on the individual $k_{\phi B}$ is stronger than that for $k^A_{\phi\phi}$ by a factor of $(5\cos\theta_W^2/4e^2)^{1/2}\sim3.2$. This gives the upper bound on $k_{\phi B}$ of 0.9×10^{-16}.

The $k_{\phi W}$ term has very similar features to the $k^A_{\phi\phi}$ case except for the photon–Higgs-boson mixing. It additionally allows the Lorentz-violating $A_\mu(p)\phi_1\phi_1$ vertex, which is equal to $-\sin\theta_W k_{\mu\nu}p^\nu$ [leading to diagrams like Fig. 1(a) with ϕ_1 second order in $k_{\phi W}$]. Adapting the same gauge-fixing conditions of $k^A_{\phi\phi}$, one can show that the W propagator with one $k_{\phi W}$ inclusion becomes $2im_W^2(k_{\phi W})_{\mu\nu}/g(q^2-m_W^2)^2$. Computation of diagrams [Figs. 1(a)–1(i) plus their permutations] shows us the $(k_{\phi W})_{\mu\nu}, (k_{\phi W})_{\mu\lambda}p^\lambda p_\nu$, and $(k_{\phi W})_{\lambda\nu}p^\lambda p_\mu$ structures in the first order and $(k_{\phi W})_{\mu\lambda}(k_{\phi W})^\lambda{}_\nu$ and $(k_{\phi W})_{\mu\lambda}(k_{\phi W})_{\lambda'\nu}p^\lambda p^{\lambda'}$ in the second order. The only surviving term is the last one which is gauge invariant. Consequently, as in the $k_{\phi B}$ case, a very similar relation between k_F and $k_{\phi W}$, $(k_F)_{\mu\lambda\lambda'\nu}=-(5/12e^2)\sin^2\theta_W(k_{\phi W})_{\mu\lambda}(k_{\phi W})_{\lambda'\nu}$, yields an upper bound of 1.7×10^{-16}. It is seen that the current bound on all three Lorentz-violating coefficients is of the order of 10^{-16}.

III. COORDINATE AND FIELD REDEFINITIONS AND THE SYMMETRIC COEFFICIENTS

In this section, we consider bounds on the $k^S_{\phi\phi}$ coefficients. In this case, the strongest bounds come from relating, through field redefinitions, these coefficients to other Lorentz violating coefficients in the fermion sector, and then using previously determined bounds on those coefficients.

Once one extends a model by relaxing one or more symmetry properties of the original model, the extended model should involve all possible otherwise invariant structures. However, if the modification is carried out under the assumption that the fields are transformed under this otherwise broken symmetry group in the usual way, not all of the new parameters representing apparent violation of this symmetry may be physical (i.e. the model has some redundant parameters). Therefore an extension should be carefully analyzed to check for redundant parameters. This analysis may yield several Lagrangians which are equivalent to each other by some coordinate and field redefinitions and rescalings [2,13–15]. The same situation applies to the SME case. A simple example is provided by Colladay and Kostelecký [2]. Consider the electron in QED, with the kinetic term $\overline{\psi}\gamma^\mu D_\mu\psi$. Suppose one transforms the electron field as $\psi\to\exp(-ia^\mu x_\mu)\psi$, where a is a constant vector. This is not a gauge transformation, since A_μ is not changed. Plugging into the kinetic term, one finds a term $a_\mu\overline{\psi}\gamma^\mu\psi$. But this is one of the Lorentz-violating terms mentioned in the first section, and thus this term can have no physical effect. Other field redefinitions can eliminate (or, more precisely, make redundant) other possible terms. Recently, the spinor part of the

[2] With the use of this mixing, there is another place where the Lorentz-violating $k_{\phi B}$ term could contribute, namely in the $A_\mu\overline{e}e$ and $\phi_1\overline{e}e$ effective vertices. However, the bounds we obtain below render any such effects negligible.

extended QED has been extensively discussed by Colladay and McDonald [13]. The a_μ term need not be redundant if gravity is included. This has been explored [5] by studying the SME with gravity in the context of Riemann-Cartan spacetimes, and thus new Lorentz-violating coefficients appear in such a framework.

In the Higgs sector, one can also make some of the symmetric coefficients redundant. Here we just consider the U(1) part but the generalization to SU(2)×U(1) is straightforward. A toy model discussed in [8,15] is relevant to our purpose. Consider first a model involving only two Lorentz-violating parameters $k_{\phi\phi}$ and k_F in the scalar and photon sectors, respectively. The Lagrangian is

$$\mathcal{L}=[g_{\mu\nu}+(k_{\phi\phi})_{\mu\nu}](D^\mu\Phi)^\dagger D^\nu\Phi-m^2\Phi^\dagger\Phi-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}-\tfrac{1}{4}(k_F)_{\mu\lambda\lambda'\nu}F^{\mu\lambda}F^{\lambda'\nu},$$

where $D_\mu=\partial_\mu+iqA_\mu$ and $k_{\phi\phi}$ is real and symmetric. First let us assume that only one component of $k_{\phi\phi}$, $(k_{\phi\phi})_{00}\equiv k^2-1$, is nonzero [8,15] and that k_F is taken as zero. By making the coordinate transformations $t\to kt$, $\mathbf{x}\to\mathbf{x}$ and the field redefinitions $A_0\to A_0$, $\mathbf{A}\to k\mathbf{A}$ with rescaling of the electric charge $q\to q/k$, one gets the Lagrangian $\mathcal{L}_{\text{photon}}=(D_\mu\Phi)^\dagger D^\mu\Phi-m^2\Phi^\dagger\Phi+\frac{1}{2}(E^2-k^2B^2)$, where $E(B)$ is the electric (magnetic) field. So we start with a system having a Lorentz violation in the scalar sector ($k_F=0$) and end up with an equivalent Lagrangian involving Lorentz violation in photon sector (some components of k_F are nonzero). Second we can further show that by choosing[3] only $(k_{\phi\phi})_{11}=(k_{\phi\phi})_{22}=(k_{\phi\phi})_{33}=k^2-1$ nonzero it is still possible to get an equivalent Lagrangian as $\mathcal{L}_{\text{photon}}=(D_\mu\Phi)^\dagger D^\mu\Phi-m^2\Phi^\dagger\Phi+\frac{1}{2}(E^2-B^2/k^2)$ under the transformations $t\to t$, $\mathbf{x}\to k\mathbf{x}$ and the redefinitions $A_0\to kA_0$, $\mathbf{A}\to\mathbf{A}$ with the same charge rescaling $q\to q/k$. However, for the other components of $k_{\phi\phi}$, there are no such obvious transformations.

Another analysis of the physical effects of the Lorentz-violating coefficients $k^S_{\phi\phi}$ can be found by looking at the effects of field redefinitions over those parameters. These effects in the fermion sector were discussed in detail in the context of extended QED [13]. There it was shown that under the fermion field redefinition $\psi(x)=(1+c_{\mu\nu}x^\mu\partial^\nu)\chi(x)$ it is possible to generate a would-be Lorentz-violating Lagrangian in the free fermion context and $c_{\mu\nu}$ represents the Lorentz violation. Here $c_{\mu\nu}$ is a real symmetric coefficient of the Lorentz violating $c_{\mu\nu}\bar{\psi}\gamma^\mu D^\nu\psi$ term in the fermion sector. However, this transformed Lagrangian can further be expressed in terms of a new coordinate system having a non-diagonal metric, i.e. a skewed coordinate system, and in this way it is possible to restore the form of the original Lagrangian. In this framework, this shows that $c_{\mu\nu}$ is not physical. The redundancy of $c_{\mu\nu}$, however, disappears when the fermion-photon interaction is involved. A very similar analysis for the scalar sector of a toy model, involving a conventional fermion sector with a scalar field ϕ, gives us $\mathcal{L}(\psi,\Phi)=\mathcal{L}^f_0(\psi)+\mathcal{L}^H_0(\varphi)+[1/2\,(k^S_{\phi\phi})_{\mu\nu}(\partial^\mu\varphi^\dagger)\partial^\nu\varphi+\text{H.c.}]$, where the scalar field redefinition $\Phi(x)=[1+\frac{1}{2}(k^S_{\phi\phi})_{\mu\nu}x^\mu\partial^\nu]\varphi(x)$ is assumed. Again expressing the fields in terms of skewed coordinates with a modified metric $\eta_{\mu\nu}=g_{\mu\nu}+(k^S_{\phi\phi})_{\mu\nu}$ the apparent Lorentz-violating $(k^S_{\phi\phi})$ term can be absorbed in the scalar sector but it reappears in the fermion sector as a c term. If we further extend our model by including fermion-photon interactions one can show that there is a mixing among $k^S_{\phi\phi}, c_{\mu\nu}$, and nine unbounded k_F coefficients [17]. Consequently, the observability of $k^S_{\phi\phi}$ is nothing but a matter of convention. The above analysis enables us to move a non-zero $k^S_{\phi\phi}$ term into either a $c_{\mu\nu}$ term or a k_F term. In this article we concentrate on *only* the Lorentz and *CPT* violation in the scalar sector of the SME; hence we assume that the theory has a conventional fermion sector, which means that bounds on $c_{\mu\nu}$ will lead to effective bounds on $k^S_{\phi\phi}$. A full and systematic analysis of all of the field redefinitions and redundancies in the SME would be valuable, but is beyond the scope of this paper. With our normalizations, a bound on $c_{\mu\nu}$ will translate directly into an equivalent bound on $(k^S_{\phi\phi})_{\mu\nu}$.

We thus need the current bounds on the $c_{\mu\nu}$ coefficients. Although numerous bounds appear in the literature, many of them should be taken with a grain of salt. Consider the spatial parts of $c_{\mu\nu}$. The strongest bounds give an upper limit on the diagonal spatial elements of 10^{-27} [18,20,21] and on the off-diagonal elements c_{XZ} and c_{YZ} of 10^{-25} [22,20,21], and c_{XY} of 10^{-27} [18,20,21]. There are several caveats, however. First, these are bounds for $c_{\mu\nu}$ of the neutron. It is conceivable that the mechanism that results in Lorentz violation is proportional to the charge, and these experiments would miss the effect. It is also conceivable that a version of Schiff's theorem (which shows that in the nonrelativistic limit, the electric dipole moment of an atom will vanish, even if it does not vanish for constituents) will cause a screening of the $c_{\mu\nu}$ coefficients of the quarks. The first effect can be eliminated by considering protons or electrons, the second can be eliminated by considering electrons. Another caveat is that the bounds on the diagonal elements are actually bounds on $c_{XX}-c_{YY}$ and $c_{XX}+c_{YY}-2c_{ZZ}$, and thus if the Lorentz violation is isotropic, the bounds will not apply. In this case, the vanishing trace condition will (as in the case of the double-traceless condition on k_F) yield, when the fermion field is properly normalized, a nonzero c_{TT}, and thus the bounds on the diagonal spatial elements will be that of the bound on c_{TT}.

The bound on c_{TT} can be obtained by comparing antiproton cyclotron frequencies with those of a hydrogen ion [23] and a very weak bound of 4×10^{-13} is extracted. An interesting connection between the dispersion relation for fermions and the c_{TT} coefficient has been noted by Bertolami *et al.* [19], and astrophysical experiments to improve the bound are proposed. For the time-space components, there are various studies based on the sensitivities of some planned experiments [20,24–26]; most of the bounds are from the neutrino sector of the SME and the highest proposed sensitivity is around 10^{-25} [24].

[3] This choice was made in Ref. [16], where it was shown that the contribution to Higgs boson decays from this term is negligible.

TABLE I. Estimated upper bounds for the Lorentz and *CPT* violating coefficients in the Higgs sector of the SME.

Parameters	Sources			Comments
	$\tilde{\kappa}_{e^+},\tilde{\kappa}_{o^-}$	$c_{\mu\nu}$	b_μ (GeV)	
$(k^A_{\phi\phi})_{\mu\nu}$	3×10^{-16}	—	—	—
$(k_{\phi B})_{\mu\nu}$	0.9×10^{-16}	—	—	—
$(k_{\phi W})_{\mu\nu}$	1.7×10^{-16}	—	—	—
$(k^S_{\phi\phi})_{II}$	—	10^{-27}	—	a
$(k^S_{\phi\phi})_{TT}$	—	4×10^{-13}	—	b
$(k^S_{\phi\phi})_{TI}$	—	10^{-25}	—	c
$(k^S_{\phi\phi})_{XZ},(k^S_{\phi\phi})_{YZ}$	—	10^{-25}	—	d
$(k^S_{\phi\phi})_{XY}$	—	10^{-27}	—	d
$(k_\phi)_X,(k_\phi)_Y$	—	—	10^{-31}	e
$(k_\phi)_Z,(k_\phi)_T$	—	—	2.8×10^{-27}	f

[a]Obtained from $c^{\mathrm{neutron}}_{\mu\nu}$ with the assumption that Lorentz violation is not isotropic. If it is isotropic, the bound on $(k^S_{\phi\phi})_{TT}$ applies.
[b]Obtained from the comparison of the anti-proton's frequency with the hydrogen ion's frequency.
[c]Estimated value based on the sensitivity calculations of some planned space experiments.
[d]Obtained from the neutron.
[e]From b^{neutron}_μ with the use of a two-species noble-gas maser. From $b^{\mathrm{electron}}_\mu$, a weaker but cleaner bound of 1.2×10^{-25} can be obtained.
[f]This bound is from the spatial isotropy test of polarized electrons.

IV. BOUNDS ON THE *CPT*-ODD COEFFICIENT

The remaining part of the Higgs sector Lagrangian has one term that violates both Lorentz and *CPT* symmetries, represented by the complex constant coefficient $(k_\phi)^\mu$. One interesting effect of this term is the modification of the conventional electroweak $SU(2)\times U(1)$ symmetry breaking. Minimization of the static potential yields a nonzero expectation value for Z_μ boson field of the form $\langle Z_\mu\rangle_0 =[(\sin 2\theta_W)/q]\mathrm{Re}(k_\phi)_\mu$. Here we have assumed all the other Lorentz-violating coefficients zero. The nonzero expectation value for the Z will, when plugged into the conventional fermion-fermion-Z interaction, yield a $b_\mu\bar{\psi}\gamma^\mu\gamma_5\psi$ term. Alternatively, one can look at the one-loop effects on the photon propagator; however this will yield much weaker bounds. By assuming that k_ϕ is the only Lorentz-violating term in the Higgs sector, one finds that the effective $b_\mu=\frac{1}{4}Re(k_\phi)_\mu$. If we look at the best current bounds on b_μ, from testing of cosmic spatial isotropy for polarized electrons [27], $b^e_{X,Y}\lesssim3.1\times10^{-29}$ GeV and $b^e_Z\lesssim7.1\times10^{-28}$ GeV in the Sun-centered frame. The best bound comes from the neutron with the use of a two-species noble-gas maser [28] and it is of the order of $b^n_{X,Y}\lesssim10^{-32}$ GeV. Note that in order to get this bound there are some assumption about the nuclear configurations, which make the bound uncertain accuracy to within one or two orders of magnitude. The bound on the time component of b_μ is around $b^n_T\lesssim10^{-27}$ GeV [29]. Therefore, the best bounds for the real part of $(k_\phi)_\mu$ are 10^{-31} GeV and 10^{-27} GeV for the X,Y and for the Z,T components, respectively. The imaginary part of k_ϕ is unphysical, since this term in the Lagrangian is a total divergence.

V. CONCLUSION

In this work we have studied the bounds on the Lorentz and/or *CPT* violating coefficients in the Higgs sector of the SME. It is shown that all antisymmetric *CPT*-even Lorentz-violating coefficients give second-order contributions to the photon vacuum polarization at one loop. By comparing with the k_F term and assuming one of them nonzero in each case (without high-precision cancellation), we find $(k^A_{\phi\phi})_{\mu\nu},(k_{\phi B})_{\mu\nu},(k_{\phi W})_{\mu\nu}\lesssim10^{-16}$. For the symmetric part of $k_{\phi\phi}$, after discussing the close connections with the Lorentz-violating coefficients $c_{\mu\nu}$ in the fermion sector by means of coordinate and field redefinitions, we conclude that the bounds could be determined directly from the $c_{\mu\nu}$ term. In a very similar way we obtain the bound on the *CPT* and Lorentz-violating coefficient $(k_\phi)_\mu$ by comparing with the b_μ term in the fermion sector. The existence of a k_ϕ term leads to a nonzero vacuum value for Z_μ which further enables us to relate $(k_\phi)_\mu$ with b_μ and we find an upper bound of 10^{-31} (10^{-27}) GeV for the $X,Y(T,Z)$ components of $(k_\phi)_\mu$. Table I lists all the bounds together with their sources.

Perhaps the most intriguing bounds are for the antisymmetric coefficients. Recent developments in string theory indicate that Lorentz-violating non-commutative geometry might be a low-energy probe of Planck scale physics [14,30], and this geometry will be antisymmetric. It is interesting that our upper bounds on the coefficients are $O(10^{-16})$, which is less than an order of magnitude above the ratio of the electroweak to Planck scale. An improvement in the birefringence bounds of a couple of orders of magnitude (which is feasible [10,31]) could probe this sensitivity. Should a k_F

term actually be discovered, our analysis shows how one can distinguish Higgs sector Lorentz violation from other sectors. Specifically, of the ten observable k_F coefficients, we find nonzero values only for the two independent diagonal elements of $\tilde{\kappa}_{e+}$. Thus, the origin of Lorentz violation might be experimentally accessible. It should be noted that inclusion of gravity might lead to new Lorentz-violating terms, as discussed in Ref. [5].

If the primary effects of an underlying Lorentz and *CPT* violation appear in the Higgs sector, what are the most promising experiments? We have seen that *CPT* violation will be manifested through a vacuum expectation value of the *Z* boson, and the "*b*" coefficient for a fermion will be proportional to the weak axial coupling of that fermion. Testing this would require b_f to be measured for at least two fermions. For antisymmetric *CPT*-even Lorentz violation, there are very specific signatures, discussed in the previous paragraph, and improvement in the birefringence bounds of a couple of orders of magnitude would be valuable. For symmetric *CPT*-even Lorentz violation, there are tight bounds, but with various assumptions and caveats. The relatively weak c_{TT} and c_{TI} bounds, as noted in Ref. [20], could be substantially tightened.

ACKNOWLEDGMENTS

We are extremely grateful to Alan Kostelecký for many useful discussions, much advice, and encouragement. We also thank Chris Carone for useful discussions. The work of D.A. and M.S. was supported by the National Science Foundation under grant PHY-0243400, and the work of I.T. was supported by the Scientific and Technical Research Council of Turkey (TÜBITAK) in the framework of the NATO-B1 program.

[1] V.A. Kostelecký and R. Potting, Nucl. Phys. **B359**, 545 (1991); Phys. Lett. B **381**, 89 (1996); V.A. Kostelecký and S. Samuel, Phys. Rev. D **40**, 1886 (1989); **39**, 683 (1989); Phys. Rev. Lett. **66**, 1811 (1991); **63**, 224 (1989); V.A. Kostelecký and R. Potting, Phys. Rev. D **63**, 046007 (2001); V.A. Kostelecký, M. Perry, and R. Potting, Phys. Rev. Lett. **84**, 4541 (2000).

[2] D. Colladay and V.A. Kostelecký, Phys. Rev. D **58**, 116002 (1998).

[3] D. Colladay and V.A. Kostelecký, Phys. Rev. D **55**, 6760 (1997).

[4] R. Lehnert, Phys. Rev. D **68**, 085003 (2003); hep-ph/0401124.

[5] V.A. Kostelecky, Phys. Rev. D **69**, 105009 (2004).

[6] V.A. Kostelecky and R. Lehnert, Phys. Rev. D **63**, 065008 (2001); V.A. Kostelecky, C.D. Lane, and A.G.M. Pickering, *ibid.* **65**, 056006 (2002).

[7] T.-P. Cheng and L.-F. Li, *Gauge Theory of Elementary Particle Physics* (Oxford University Press, New York, 1992).

[8] V.A. Kostelecký and M. Mewes, Phys. Rev. D **66**, 056005 (2002).

[9] J.H. Hough *et al.*, Mon. Not. R. Astron. Soc. **224**, 1013 (1987); C. Brindle *et al.*, *ibid.* **244**, 577 (1990); A. Cimatti *et al.*, Astrophys. J. **465**, 145 (1996); A. Dey *et al.*, *ibid.* **465**, 157 (1996); A. Cimatti *et al.*, *ibid.* **476**, 677 (1997); M.S. Brotherton *et al.*, Astrophys. J. Lett. **487**, L113 (1997); Astrophys. J. **501**, 110 (1998); **546**, 134 (2001); M. Kishimoto *et al.*, *ibid.* **547**, 667 (2001); J. Vernet *et al.*, Astron. Astrophys. **366**, 7 (2001).

[10] V.A. Kostelecký and M. Mewes, Phys. Rev. Lett. **87**, 251304 (2001).

[11] S.M. Carroll, G.B. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).

[12] P. Wolf, M.E. Tobar, S. Bize, A. Clairon, A.N. Luiten, and G. Santarelli, gr-qc/0401017.

[13] D. Colladay and P. McDonald, J. Math. Phys. **43**, 3554 (2002).

[14] D. Colladay, in *Short distance behavior of fundamental interactions*, edited by B. N. Kursunoglu *et al.*, AIP Conf. Proc. No. 672 (AIP, Melville, NY, 2003), p. 65.

[15] H. Muller, S. Herrmann, A. Saenz, A. Peters, and C. Lammerzahl, Phys. Rev. D **68**, 116006 (2003).

[16] E.O. Iltan, Mod. Phys. Lett. A **19**, 327 (2004).

[17] V. A. Kostelecký (private communication).

[18] S.K. Lamoreaux *et al.*, Phys. Rev. Lett. **57**, 3125 (1986); Phys. Rev. A **39**, 1082 (1989); T.E. Chupp *et al.*, Phys. Rev. Lett. **63**, 1541 (1989).

[19] O. Bertolami and C.S. Carvalho, Phys. Rev. D **61**, 103002 (2000); O. Bertolami, Gen. Relativ. Gravit. **34**, 707 (2002); O. Bertolami, hep-ph/0301191.

[20] R. Bluhm, V.A. Kostelecky, C. Lane, and N. Russell, Phys. Rev. D **68**, 125008 (2003).

[21] V.A. Kostelecky and C.D. Lane, Phys. Rev. D **60**, 116010 (1999).

[22] J.D. Prestage *et al.*, Phys. Rev. Lett. **54**, 2387 (1985).

[23] G. Gabrielse, A. Khabbaz, D. S. Hall, C. Heimann, H. Kalinowsky, and W. Jhe, presented at Meeting on *CPT* and Lorentz Symmetry (CPT 98), Bloomington, Indiana, 1998.

[24] V.A. Kostelecký and M. Mewes, Phys. Rev. D **69**, 016005 (2004).

[25] V.A. Kostelecký and M. Mewes, hep-ph/0308300.

[26] A. Datta, R. Gandhi, P. Mehta, and S.U. Sankar, hep-ph/0312027.

[27] L.-S. Hou, W.-T. Ni, and Y.-C.M. Li, Phys. Rev. Lett. **90**, 201101 (2003).

[28] D. Bear, R.E. Stoner, R.L. Walsworth, V.A. Kostelecký, and C.D. Lane, Phys. Rev. Lett. **85**, 5038 (2000); **89**, 209902(E) (2002).

[29] F. Cane *et al.*, physics/0309070.

[30] S.M. Carroll, J.A. Harvey, V.A. Kostelecký, C.D. Lane, and T. Okamoto, Phys. Rev. Lett. **87**, 141601 (2001); C.E. Carlson, C.D. Carone, and R.F. Lebed, Phys. Lett. B **549**, 337 (2002); C.E. Carlson, C.D. Carone, and N. Zobin, Phys. Rev. D **66**, 075001 (2002); C.E. Carlson, C.D. Carone, and R.F. Lebed, Phys. Lett. B **518**, 201 (2001); Z. Guralnik, R. Jackiw, S.Y. Pi, and A.P. Polychronakos, *ibid.* **517**, 450 (2001); A. Anisimov, T. Banks, M. Dine, and M. Graesser, Phys. Rev. D **65**, 085032 (2002).

[31] E. Costa *et al.*, Nature (London) **411**, 662 (2001).