Matter degrees of freedom and string breaking in Abelian projected quenched $SU(2)$ QCD

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In the Abelian projection the Yang-Mills theory contains Abelian gauge fields (diagonal degrees of freedom) and the Abelian matter fields (off-diagonal degrees) described by a complicated action. The matter fields are essential for the breaking of the adjoint string. We obtain numerically the effective action of the Abelian gauge and the Abelian matter fields in quenched $SU(2)$ QCD and show that the Abelian matter fields provide an essential contribution to the total action even in the infrared region. We also observe the breaking of an Abelian analogue of the adjoint string using Abelian operators. We show that the adjoint string tension is dominated by the Abelian and the monopole contributions similarly to the case of the fundamental particles. We conclude that the adjoint string breaking can successfully be described in the Abelian projection formalism.

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I. INTRODUCTION

The mechanism of color confinement in QCD is one of the most important nonperturbative problems in the quantum field theory. One of the most promising approaches to this problem is based on the existence of dual objects, called monopoles, which are condensed in the confinement phase. This approach—known as the dual superconductor hypothesis [1]—is realized with the help of the so-called Abelian projection [2] of $SU(N)$ color degrees of freedom to $U(1)^{N-1}$ degrees of freedom.

The model was shown to be quite successful in explaining the confinement of the fundamental charges such as quarks (see, e.g., reviews in [3]). Abelian and monopole contributions to the interquark potential are dominant in the long-range region of quenched QCD [4,5]. An infrared effective monopole action has been derived in the continuum limit after a block-spin transformation of monopole currents [6,7]. It is a quantum perfect action described by monopole currents. The condensation of monopoles in the confinement phase was observed in various numerical approaches [6,8]. In the language of monopole currents condensation implies the formation of a percolating cluster studied both numerically [9] and analytically [10].

However, this mechanism has a serious problem even in quenched QCD. Although the 't Hooft scenario describes the confinement of quarks correctly, this scenario predicts also the existence of string tension for the adjoint charges (gluons) in the infrared region. On the other hand, gluon charges must be screened at large distances due to the presence of gluons in the QCD vacuum. This screening-confinement problem was extensively discussed in recent publications [11].

The problem of the screening of the adjoint charges in quenched $SU(N)$ QCD has also been discussed in Ref. [12]. The paper provides arguments that the relevant quantity in the confinement mechanism is not the Abelian monopoles but the $Z(N)$ center vortices which can explain the screening problem [13]. In our study we pursue a different approach based on the dual superconductor model.

Consider the screening in a confining Abelian model with charge-2 matter fields (take, for example, the Abelian Higgs model with compact gauge fields). The presence of doubly charged matter fields screens the confining interaction between the external particles with opposite double charges. This happens due to the pair creation from the vacuum at certain separations between the external charges. As a result, the potential between the particles flattens at some distances. It should be stressed that the problem is not only to explain the flattening of the potential but also to show the linear behavior of the potential in the intermediate region. On the other hand, the charge-1 external fields remain unscreened in this model. Namely, the potential is linearly rising at large distances.

The standard model of the dual superconductor in quenched QCD ignores the existence of off-diagonal gluons. However, these gluons have a charge 2 with respect to the Abelian subgroup and they may explain the flattening of the intergluon potential which is usually studied with the help of the adjoint Wilson loop. On the other hand, the introduction of new degrees of freedom—off-diagonal gluons—should not violate the already achieved success of the explanation of the quark confinement in this model. Indeed, quarks have the charge 1 and doubly charged gluons cannot screen them.[1] These and related issues were discussed in Ref. [14] for quenched as well as for full $SU(N)$ QCD.

From the point of view of a realization of the (modified) dual superconductor scenario it seems that we have to keep all charge-2 Abelian Wilson loops in the effective action written by the Abelian link fields to reproduce the screening of charge 2. Indeed, the theory in terms of Abelian link fields

[1]However, we may expect a renormalization of the tension of the string spanned between the quarks due to the presence of double charges.

or Abelian monopole currents alone becomes highly nonlocal if we integrate out all off-diagonal gluon fields after an Abelian projection. Needless to say, such an Abelian effective action is useless. The same problem is more serious in the real full QCD, since a fundamental charge is also screened in this case.

The aim of this paper is to calculate numerically the effective action of quenched QCD within the Abelian projection formalism. Contrary to previous calculations of this kind we include also the doubly charged off-diagonal gluon fields in the effective action and we show that their contribution is essential and thus cannot be neglected. We also calculate correlators of the adjoint Polyakov loops in the Abelian formalism and observe the screening of a properly defined potential between static adjoint sources.

The plan of the paper is the following. In Sec. II we discuss how the screening and confinement problem is solved qualitatively in the framework of Abelian dynamics. Section III is devoted to an investigation of the Abelian action for the Abelian gauge and matter fields obtained by the inverse Monte Carlo method. In Sec. IV we discuss the potential between the adjoint ($Q=2$) charges within the Abelian projection formalism. We show numerically that a properly defined Abelian potential shows screening of the $Q=2$ charges. Moreover, we observe the Abelian and monopole dominance for the adjoint string tension. Our conclusions are presented in the last section.

II. STRING BREAKING IN ABELIAN PROJECTED THEORY

The partition function of the Abelian effective theory of quenched $SU(2)$ QCD in the infrared region may be approximated in the Villain-like form [15]

$$Z_Q[J]=\int_{-\pi}^{\pi}\mathcal{D}\theta\sum_{n\in\mathbb{Z}(c_2)}\times e^{-(1/4\pi^2)[(d\theta+2\pi n),\Delta D(d\theta+2\pi n)]+iQ(\theta,J)}, \tag{1}$$

where D is a differential operator:

$$D\approx\overline{\alpha}+\overline{\beta}\Delta^{-1}+\overline{\gamma}\Delta. \tag{2}$$

This operator contains a local self-interaction term, the Coulomb term described by the inverse Laplacian, Δ^{-1}, and additional interactions between nearest neighbors. The coupling constants $\overline{\alpha}$, $\overline{\beta}$, and $\overline{\gamma}$ were calculated numerically in Ref. [15]. To simplify the notation we use the differential form formalism on the lattice [16].

The partition function (1) can be rewritten as a string model [15]

$$Z_Q[J]\propto\sum_{\substack{\sigma\in\mathbb{Z}(c_2)\\ \delta\sigma+QJ}}e^{-\pi^2(\sigma,(\Delta D)^{-1}\sigma)}, \tag{3}$$

where we have neglected perimeter terms. This model does not contain dynamical matter fields and therefore the string variable σ is always closed on the external current J. Therefore there is no source for string breaking in this model.

Now let us consider the off-diagonal gluons. The Wilson action of quenched $SU(2)$ QCD is

$$S=\frac{\beta}{2}\sum_{s,\mu,\nu}\mathrm{Tr}\,U_{\mu\nu}(s),$$

$$U_{\mu\nu}(s)=U_\mu(s)U_\nu(s+\hat{\mu})U_\mu^\dagger(s+\hat{\nu})U_\nu^\dagger(s), \tag{4}$$

where $U_\mu(s)$ is the $SU(2)$ gauge field.

It is convenient to parametrize the $SU(2)$ link variable $U_\mu(s)$ as $U_\mu(s)=c_\mu(s)u_\mu(s)$ where

$$c_\mu(s)=\begin{pmatrix}\cos\phi_\mu(s) & i\sin\phi_\mu(s)e^{-i\varphi_\mu(s)}\\ i\sin\phi_\mu(s)e^{i\varphi_\mu(s)} & \cos\phi_\mu(s)\end{pmatrix},$$

$$u_\mu(s)=\begin{pmatrix}e^{i\theta_\mu(s)} & 0\\ 0 & e^{-i\theta_\mu(s)}\end{pmatrix}.$$

Here θ, φ, and ϕ are independent variables defined in the regions $-\pi\leqslant\theta_\mu(s)$, $\varphi_\mu(s)<\pi$, and $0\leqslant\phi_\mu(s)<\pi/2$. The field θ behaves as a $U(1)$ gauge field while the field φ corresponds to the phase of the off-diagonal gluon field because, under an Abelian gauge transformation

$$\Omega^{\mathrm{Abel}}(s)=\mathrm{diag}(e^{i\alpha(s)},e^{-i\alpha(s)}) \tag{5}$$

they behave as follows:

$$\theta_\mu(s)\to\theta_\mu(s)-\partial_\mu\alpha(s)\equiv\theta_\mu(s)+\alpha(s)-\alpha(s+\hat{\mu}),$$

$$\varphi_\mu(s)\to\varphi_\mu(s)+2\alpha(s). \tag{6}$$

The variable $\phi_\mu(s)$ is not affected by the $U(1)$ gauge transformation. After an Abelian projection we can integrate this variable out without harming the $U(1)$ content of the model. In order to get an insight of possible forms of interactions between the Abelian gauge and Abelian matter fields we replace the averages of $\cos\phi_\mu(s)$ and $\sin\phi_\mu(s)$ by their mean values:

$$\cos\phi_\mu(s)\to\langle\cos\phi_\mu(s)\rangle\equiv c,\quad \sin\phi_\mu(s)\to\langle\sin\phi_\mu(s)\rangle\equiv s, \tag{7}$$

where c and s are functions of the $SU(2)$ coupling constant β.

As the Abelian projection, we use the maximal Abelian gauge which is defined by a maximization of the functional,

$$R=\frac{1}{2}\sum_{s,\mu}\mathrm{Tr}[\sigma_3\tilde{U}_\mu(s)\sigma_3\tilde{U}_\mu^\dagger(s)]\equiv\sum_{s,\mu}[2\cos^2\phi_\mu(s)-1], \tag{8}$$

with respect to the $SU(2)$ gauge transformations $U_\mu(s)\to\tilde{U}_\mu(s)=\Omega(s)U_\mu(s)\Omega^\dagger(s+\hat{\mu})$. The functional (8) is invariant under residual $U(1)$ gauge transformations (5). The local condition corresponding to maximization (8) can be written in the continuum limit as the differential equation $(\partial_\mu + igA^3_\mu)(A^1_\mu - iA^2_\mu)=0$.

The maximization of the functional (8) corresponds to the minimization of the ϕ variable. Thus the observation of Refs. [17,18] made for the mean values (7),

$$c \simeq 1, \quad s \ll c, \tag{9}$$

does not come as a surprise. These relations hold in a wide region of the coupling constant β.

Following Ref. [18] we rewrite the action of the model (4) in terms of the variables θ, φ and ϕ with the help of the definitions (5). Applying Eq. (7) to the original action we get

$$\begin{aligned}\frac{1}{2}\mathrm{Tr}\,U_{\mu\nu}(s) &= c^4\cos[\Theta_{\mu\nu}(s)] - c^2s^2\cos[\Theta_{\mu\nu}(s) - H_{\mu\nu}(s) - C_{\mu\nu}(s)]\\ &\quad - c^2s^2\cos[\Theta_{\mu\nu}(s) + H_{\nu\mu}(s) - C_{\mu\nu}(s)]\\ &\quad + c^2s^2\cos[\Theta_{\mu\nu}(s) + H_{\nu\mu}(s)] + c^2s^2\cos[\Theta_{\mu\nu}(s)\\ &\quad - H_{\mu\nu}(s) + H_{\nu\mu}(s) - C_{\mu\nu}(s)] + c^2s^2\cos[\Theta_{\mu\nu}(s)\\ &\quad - H_{\mu\nu}(s)] + c^2s^2\cos[\Theta_{\mu\nu}(s) + C_{\mu\nu}(s)]\\ &\quad + s^4\cos[\Theta_{\mu\nu}(s) - H_{\mu\nu}(s) + H_{\nu\mu}(s) - 2C_{\mu\nu}(s)],\end{aligned} \tag{10}$$

where we have denoted the $U(1)$ gauge invariant variables as follows:

$$\Theta_{\mu\nu}(s) = \theta_\mu(s) + \theta_\nu(s+\hat{\mu}) - \theta_\mu(s+\hat{\nu}) - \theta_\nu(s), \tag{11}$$

$$H_{\mu\nu}(s) = 2\theta_\mu(s) + \varphi_\nu(s) - \varphi_\nu(s+\hat{\mu}), \tag{12}$$

$$C_{\mu\nu}(s) = \varphi_\mu(s) - \varphi_\nu(s). \tag{13}$$

The variable Θ is the $U(1)$ plaquette for the gauge field θ, the variable H describes the interaction of the matter field φ with the gauge field θ, and the variable C corresponds to the self-interaction of the matter field. The validity of the mean-field approximation based on a self-consistent substitution (7) is not known. When we perform the ϕ integration, we generally get an effective action in terms of $\Theta_{\mu\nu}$, $H_{\mu\nu}$, and $C_{\mu\nu}$. Below we use numerical method to find this effective action.

A few remarks about the action (10) are now in order. (i) From Eq. (9) one can immediately observe that the leading contribution to the action is provided by the first QED-like term depending on the variables θ only. The coupling between the gauge field θ and the matter field φ is suppressed and the self-interaction of the matter field is suppressed even further. (ii) The action (10) should acquire corrections from the Faddeev-Popov determinant resulting from the fixing of the maximal Abelian gauge. This determinant is an essentially nonlocal functional and the leading local terms were calculated in Ref. [18].

Let us assume for simplicity the following effective action:

$$\begin{aligned} S_{\text{eff.}} &= S^{(1)}(\theta) + S^{(2)}(\theta,\varphi),\\ S^{(2)}(\theta,\varphi) &= -F_1(H) - F_2(H') - F_3(C),\end{aligned} \tag{14}$$

where we put $H=H_{\mu\nu}(s)$, $H'=H_{\nu\mu}(s)$, $C=C_{\mu\nu}(s)$, and F_1, F_2, F_3 are periodic functions. Following Ref. [14] we rewrite the corresponding partition function Z with the external source J as follows:

$$\begin{aligned} Z_Q[J] &= \int_{-\pi}^{\pi} \mathcal{D}\theta\mathcal{D}\varphi e^{-S_{\text{eff.}} + iQ(\theta,J)}\\ &= \int_{-\pi}^{\pi} \mathcal{D}\theta\mathcal{D}\varphi e^{-S^{(1)}(\theta) - S^{(2)}(\theta,\varphi) + iQ(\theta,J)}\\ &= \int_{-\pi}^{\pi} \mathcal{D}\theta e^{-S^{(1)}(\theta) + iQ(\theta,J)}\\ &\quad \times\left[\int_{-\pi}^{\pi}\mathcal{D}\varphi e^{F_1(H)+F_2(H')+F_3(C)}\right].\end{aligned} \tag{15}$$

The part in the square brackets can be expanded in a Fourier series:

$$\begin{aligned}[\cdots] &= \int_{-\pi}^{\pi}\mathcal{D}\varphi \sum_{\substack{n^{(i)}\in\mathbb{Z}(c_2)\\ i=1,2,3}} I_1(n^{(1)})I_2(n^{(2)})I_3(n^{(3)})\\ &\quad \times e^{i(H,n^{(1)}) + i(H',n^{(2)}) + i(C,n^{(3)})},\end{aligned} \tag{16}$$

where $n^{(i)}$, $i=1,2,3$, are integers and the lattice tensors H, H', $n^{(1)}$, $n^{(2)}$ sum only for $\mu>\nu$ because H, H' are not antisymmetric contrary to $(C,n^{(3)})$.

Integrating over φ and summing over $n^{(3)}$ one can rewrite Eq. (16) as

$$[\cdots] = \sum_{\substack{j\in\mathbb{Z}(c_1)\\ \delta j=0}} w(j) e^{2i(\theta,j)}, \tag{17}$$

where $w(j)$ are certain weights for the closed current j which is defined from the variables $n^{(1)}$ and $n^{(2)}$:

$$j_\mu(s) = \sum_{\nu(<\mu)} n^{(1)}_{\mu\nu}(s) + \sum_{\nu(>\mu)} n^{(2)}_{\nu\mu}(s). \tag{18}$$

The general form of Eqs. (17) and (18) follows from the fact that the fields φ are doubly charged and from the gauge invariance of the expression under the exponential function in Eq. (16). We also give a detailed derivation of Eqs. (17) and (18) in Appendix A.

To simplify further considerations let us rewrite the first term in Eq. (14) in the Villain form as in Eq. (1). Then we get, for the partition function (15),

$$Z_Q[J]=\int_{-\pi}^{\pi}\mathcal{D}\theta\sum_{n\in\mathbb{Z}(c_2)}\sum_{\substack{j\in\mathbb{Z}(c_1)\\ \delta j=0}}w(j)\exp\Bigg\{-\frac{1}{4\pi^2}\times((d\theta+2\pi n),\Delta D(d\theta+2\pi n))+i(\theta,2j+QJ)\Bigg\}.$$

Analogously to Eq. (3) we get the following model for the string variables dual to the gauge field θ:

$$Z_Q[J]=\sum_{\substack{j\in\mathbb{Z}(c_1)\\ \delta j=0}}\sum_{\substack{\sigma\in\mathbb{Z}(c_2)\\ \delta\sigma=2j+QJ}}w(j)\exp\{-\pi^2(\sigma,(\Delta D)^{-1}\sigma)\}. \tag{19}$$

The string model (19) is different from the model (3) by the presence of the doubly charged currents representing the contribution of the off-diagonal gluons [the first sum in Eq. (19)]. The second sum in this equation is over the integer-valued string variable which has the dynamical current j as its boundary.

If the external charge has a unit value, $Q=1$, then the dynamical current j cannot screen the external current QJ and therefore the string always spans on the trajectories of the external currents, $\delta\sigma=2j+QJ\neq 0$. However, if the external current is doubly charged, $Q=2$, then there exists the dynamical current $j=-J$ such that $\delta\sigma=0$. This state breaks the string: when the distance between the external charges is large enough the state with $j=-J$ provides a dominant contribution to the partition function.

III. EFFECTIVE ACTION FOR GAUGE AND MATTER FIELDS

In this section we calculate numerically the effective action for the Abelian gauge and the matter fields in quenched $SU(2)$ QCD. We have chosen a trial action in the form

$$S_{\rm eff}(\theta,\varphi)=\alpha_1 S_1(\theta)+\alpha_2 S_2(\theta)+\alpha_3 S_3(\theta)+\beta_1 S_4(\theta,\varphi), \tag{20}$$

where α_i, $i=1,2,3$, and β_1 are the coupling constants to be determined numerically.

The functionals S_i, $i=1,2,3$, describe the action of the gauge field θ:

$$S_1=-\sum_{s,\mu\neq\nu}[\cos\Theta_{\mu\nu}(s)], \tag{21}$$

$$S_2=-\sum_{s,\mu\neq\nu}[\cos 2\Theta_{\mu\nu}(s)], \tag{22}$$

$$S_3=+\sum_{s,\mu\neq\nu}[\sin\Theta_{\mu\nu}(s)\sin\Theta_{\mu\nu}(s+\hat{\mu})], \tag{23}$$

where the plaquette variable Θ is given in Eq. (11). The action S_1 is the leading term in the Abelian action (10) corresponding to quenched $SU(2)$ QCD in the mean-field approximation. The parts $S_{2,3}$ are also included because they may arise naturally from the integration over ϕ.

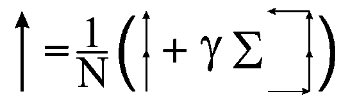

FIG. 1. The visualization of the blockspin transformation, Eq. (25).

As an interaction term between the gauge, θ, and the matter, φ, fields we adopt, for simplicity,

$$S_4=-\sum_{s,\mu\neq\nu}\{\cos[\Theta_{\mu\nu}(s)-H_{\mu\nu}(s)]+\cos[\Theta_{\mu\nu}(s)+H_{\nu\mu}(s)]\}, \tag{24}$$

where the plaquette variable H is given in Eq. (12). We have not included other terms from Eq. (10) into the trial action because it turns out that the minimal form of the action (20) describes the numerical data with a good accuracy.

We have used the standard Monte Carlo procedure to generate the gauge field configurations on the 32^4 lattice. The $SU(2)$ coupling constant was chosen in the range $\beta=2.1$–2.7. In order to express dimensionful quantities in physical units we have followed Ref. [15], providing the values of such quantities in units of the $SU(2)$ string tension. The lattice spacing a at a given value of the gauge coupling β can be the expressed through the (calculated numerically) lattice string tension, σ_{lat}, using the relation $a(\beta)=\sqrt{\sigma_{lat}(\beta)/\sigma_{phys}}$. For illustration purposes we have associated the value of the SU(2) string tension with the phenomenological value of the string tension in the real QCD, $\sigma=(440\ \mathrm{MeV})$. Then the length scale $b=0.45$ fm corresponds approximately to the length $b=1.0\sigma_{phys}^{-1/2}$ in terms of the $SU(2)$ string tension.

We have generated 100 configurations of the gauge field for each value of the coupling constant and then used the simulated annealing method [5] to fix the maximal Abelian gauge. The couplings α_i, $i=1,2,3$, and β_1 were determined by solving the Schwinger-Dyson equations [20]. We describe the details of this method in Appendix B. To make a further improvement of our results towards the continuum limit we used also a block-spin transformation for the $SU(2)$ link variable $U_\mu(s)$: We apply the block-spin transformation to the link variable $U_\mu(s)$:

$$U'_\mu(s')=\frac{1}{N}\Bigg(U_\mu(s)U_\mu(s+\hat{\mu})+\gamma\sum_{\nu(\neq\mu)}U_\nu(s)U_\mu(s+\hat{\nu})\times U_\mu(s+\hat{\mu}+\hat{\nu})U_\nu^\dagger(s+2\hat{\mu})\Bigg), \tag{25}$$

which is visually represented in Fig. 1. Here $N\equiv N(U)$ is the

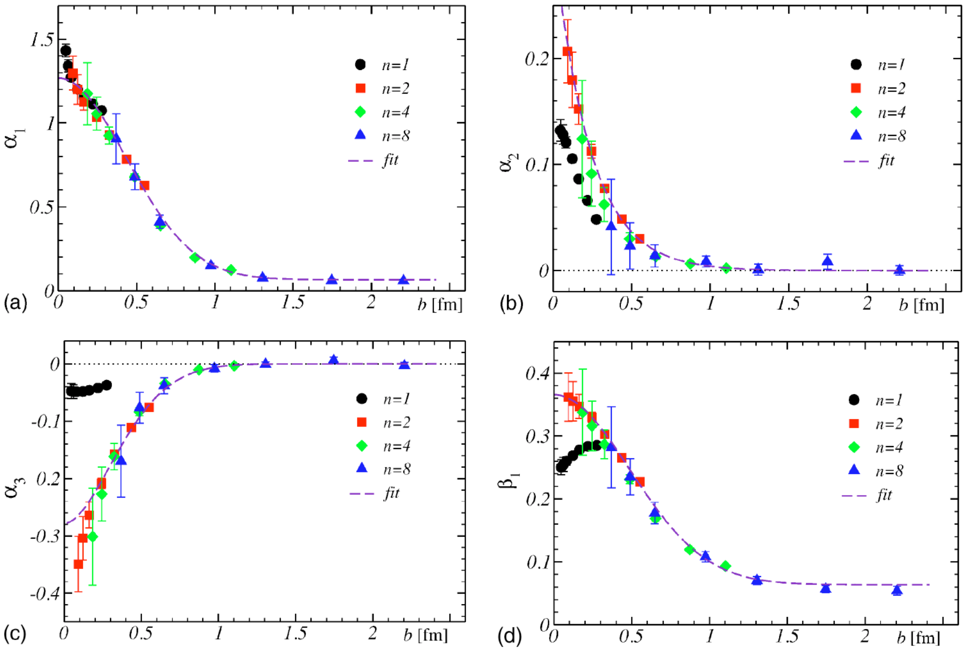

FIG. 2. The parameters α_i, $i=1,2,3$, and β_1 for different blocking steps n vs the scale parameter b. The fits by Eq. (26) are shown by the dashed lines.

normalization factor which is introduced to make the fat link belonging to the $SU(2)$ group. The weight parameter γ was set to $\gamma=0.5$.

The couplings obtained in this way are depicted in Figs. 2(a)–2(d). The coupling α_1 shows a perfect scaling since the coupling constant depends only on the physical length b (and it does not depend on n and a separately). For the couplings α_2, α_3, and β_1 this feature does not work: the original data (no block-spin transformation, $n=1$) is quite different from the cases where the block-spin transformation was done ($n>1$) while the coupling constants with $n>1$ scale almost perfectly. One can make a conclusion that the original data corresponds to very small values of b where the effective action takes more complicated form than Eq. (20).

In order to quantitatively characterize the dependence of the coupling constants on the scale factor b we have fitted the data by a function

$$f(b)=C_0+C_1\exp\{-(b/b_0)^{\nu}\}, \tag{26}$$

where $C_{0,1}$, ν, and b_0 are the fitting parameters. In our fits we have excluded the data without the block-spin transformation, $n=1$, for all coupling constants except for α_1 case. The best fit curves are plotted in Fig. 2 as the dashed lines, and the best fit parameters are shown in Table I.

We have found that in the case of α_1 and β_1 the parameter ν is very close to 2, and therefore in these fits we fixed

TABLE I. The parameters for the exponential fits (26) of the couplings α_i, $i=1,2,3$, and β_1.

Coupling	C_0	C_1	b_0 [fm]	ν
α_1	0.066(10)	1.20(2)	0.61(1)	2
α_2	0	0.32(2)	0.231(7)	1
α_3	0	−0.28(3)	0.46(3)	1.8(2)
β_1	0.064(5)	0.30(1)	0.69(2)	2

this parameter, $\nu=2$. Similarly, we have also fixed $\nu=1$ for α_2 and $C_0=0$ for $\alpha_{2,3}$. Note that the fit cannot describe the coupling α_1 accurately at small scales, $b\leqslant 0.2$ fm. A similar deviation can be found for the coupling α_3. We expect that at small scales the Abelian action becomes much more complicated than the trial action (20), (21), (22), (23), (24) which we used to solve the Schwinger-Dyson equations. A small similar effect is observed for the effective monopole action obtained by inverse Monte Carlo methods [15].

The functional S_1, Eq. (21), makes the leading contribution to the action since the corresponding coupling α_1 is the largest. The actions S_2 and S_3, in addition to the expected action S_1, play an essential role at small scales since the corresponding couplings α_2 and α_3 are nonvanishing. The action S_4, which describes the interaction of the matter fields with the gauge fields, has a nonvanishing coupling both at small and large scales similarly to S_1. Moreover, according to Table I the couplings α_1 and β_1, corresponding to these parts of the total action, have relatively large lengths b_0 compared to the coupling constants α_2 and α_3. Thus, at large scales, $b\sqrt{\sigma}\gg 1$, the effective Abelian action for the $SU(2)$ gauge theory can be approximated as a sum of the QED-like action for the gauge field, $S_1(\theta)$, and the interaction term $S_4(\theta,\varphi)$.

We interpret the results obtained in this section as the manifestation of the Abelian dominance (nonvanishing dominant coupling α_1) and the importance of the off-diagonal (matter) degrees of freedom (nonvanishing coupling β_1). The matter fields are essential for the breaking of the adjoint string. From the point of view of further analytical study the results of this section are qualitative because in order to make a quantitative analytical predictions at a finite value of the scale b we need much more terms in the trial action (20) than we have imposed. Indeed, in Ref. [15] the monopole contribution to the string tension has been calculated using the effective monopole action. The monopole action was obtained numerically and it turns out that in order to get a correct analytical result for the string tension one should take into account not only the most local terms in the effective monopole action but also a series of the nonlocal terms. The situation with the effective action for the Abelian fields (20) should be similar to the case of the monopole action since these actions are related to each other [15]. Nevertheless, the adjoint string breaking can *quantitatively* be discussed within the numerical approach on the basis of maximal Abelian gauge fixing. This topic is discussed in the next section.

IV. $Q=2$ POTENTIAL FROM POLYAKOV LOOPS

The easiest way to observe numerically the string breaking effect is to consider the theory at finite temperature and define the potential with the help of the Polyakov loop correlators [14,19]:

$$\langle P(\vec{x})P^{\dagger}(\vec{y})\rangle = e^{-V(\vec{x}-\vec{y})/T}. \tag{27}$$

Here T is temperature.

The adjoint Polyakov loop P_1 is defined as follows:

$$P_1=\frac{1}{3}\mathrm{Tr}\left(\prod_{i\in\mathcal{C}} D_1[U_i]\right)=\frac{1}{3}(4p_0^2-1), \tag{28}$$

where the color vector $p=p_0+i\vec{p}\cdot\vec{\sigma}$ defines the fundamental Polyakov loop, $P_{1/2}=1/2\,\mathrm{Tr}\,p$, $p=\Pi_{i\in\mathcal{C}}U_i$, and $\mathcal{C}$ is the straight line parallel to the temperature direction. The adjoint Polyakov loop (28) contains the charged term, $Q=2$, and neutral term, $Q=0$:

$$P_{Q=2}=\frac{2}{3}(p_0^2-p_3^2),\quad P_{Q=0}=\frac{1}{3}(2p_0^2+2p_3^2-1). \tag{29}$$

The Abelian dominance in the most general sense means that a non-Abelian observable can be calculated with good accuracy with the help of the corresponding Abelian operator in a suitable Abelian projection. The Abelian dominance was first established for the tension of the chromoelectric string spanned between the fundamental sources [4]. In this case the non-Abelian Wilson (or Polyakov) loop was replaced by its Abelian counterpart.

However, in the case of the adjoint potential we immediately encounter a problem [21]: in the Abelian projection the $Q=2$ charged component of the Wilson loop shows the area law while the neutral $Q=0$ component is constant. Therefore, strictly speaking, a straightforward Abelian projection of the adjoint operators leads to vanishing Abelian string tension. The simplest way to overcome this difficulty is to introduce the obvious prescription for the adjoint operators proposed originally in Ref. [22]. Namely, one should disregard the $Q=0$ component of the Wilson loop operator and consider the $Q=2$ Abelian component of the Wilson loop as the Abelian analogue of the full (non-Abelian) loop. In Ref. [22] some numerical arguments in favor of the validity of this prescription were given. Below we follow this recipe and show that the string breaking effect can indeed be seen in the $Q=2$ Abelian and monopole components of the potential. Moreover, we have observed the Abelian and monopole dominance for the adjoint string tension.

After the Abelian projection the $Q=2$ component becomes

$$P^{\mathrm{ab}}_{Q=2}=\cos 2\vartheta_{\mathcal{C}},\quad \vartheta_{\mathcal{C}}=\sum_{i\in\mathcal{C}}\theta_i, \tag{30}$$

where $\vartheta_{\mathcal{C}}$ enters the $Q=1$ Abelian Polyakov loop, $P^{\mathrm{ab}}_{1/2}=\cos\vartheta_{\mathcal{C}}$.

We calculate numerically the static potential between the adjoint particles using the Polyakov loop correlators (27). We use four types of the Polyakov loops: non-Abelian, Abelian, monopole, and photon Polyakov loops:

$$P_{Q=2}=p_0^2-p_3^2,\quad P^{\mathrm{ab}}_{Q=2}=\cos 2\vartheta_{\mathcal{C}},$$
$$P^{\mathrm{mon}}_{Q=2}=\cos 2\vartheta_{\mathcal{C}\,\mathrm{mon}},\quad P^{\mathrm{ph}}_{Q=2}=\cos 2\vartheta_{\mathcal{C}\,\mathrm{ph}}, \tag{31}$$

respectively.

The functions $\vartheta_{\mathcal{C}\,\mathrm{mon}}$ and $\vartheta_{\mathcal{C}\,\mathrm{ph}}$ represent the contributions to the Polyakov loop coming from the monopole currents and the photon fields, respectively [4,5]:

$$\vartheta\mathcal{C}^{\mathrm{mon}}=-\sum_t\sum_{\vec{x}',t'}D(\vec{x}-\vec{x}',t-t')\partial'_\mu\overline{\Theta}_{\mu4}(\vec{x}',t'), \tag{32}$$

$$\vartheta\mathcal{C}^{\mathrm{ph}}=-2\pi\sum_t\sum_{\vec{x}',t'}D(\vec{x}-\vec{x}',t-t')\partial'_\mu n_{\mu4}(\vec{x}',t'), \tag{33}$$

where the variables $\overline{\Theta}\in(-\pi,\pi)$ and $n\in\mathbb{Z}$ are extracted from the Abelian plaquette variable, $\Theta_{\mu\nu}(s)\equiv\theta_\mu(s)+\theta_\nu(s+\hat{\mu})-\theta_\mu(s+\hat{\nu})-\theta_\nu(s)=\overline{\Theta}_{\mu\nu}(s)+2\pi n_{\mu\nu}(s)$. Here $D(s)$ is the inverse Laplacian, $\partial'_\mu\partial_\mu D(s)=-\delta_{0,s}$.

We numerically measured the potential between the static adjoint sources on the $16^3\times4$ lattice at $\beta=2.2$ (confinement phase) using 2000 configurations. The Abelian, monopole, and the photon components of the potential were measured in the maximal Abelian gauge. In order to reduce the statistical errors in our calculations of the potentials we have applied the hypercubic blocking [23] procedure to ensembles of the non-Abelian, Abelian, and photon gauge fields. We have not applied the blocking to the monopole contribution of the potential because in this particular case the blocking makes the data noisier. The hypercubic blocking method is briefly described in Appendix C.

We present the numerical results in Fig. 3. One can clearly see that all potentials become flat in the infrared region, clearly indicating the presence of string breaking. The non-Abelian potential as well as the Abelian and the monopole contributions contain linear pieces at small enough distances while the photon contribution to the potential does not contain a linear part. These observations are in qualitative agreement with the Abelian (monopole) dominance hypothesis [4].

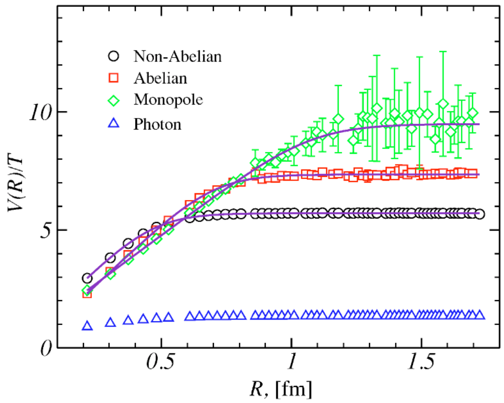

FIG. 3. The potential between adjoint static sources and the Abelian, the monopole, and the photon contributions to it. The fits by the function (34) are shown by the solid lines.

To make a quantitative characterization of the potentials we fit our data by a function

$$\exp\left\{-\frac{V^{\rm fit}(R)}{T}\right\}=\exp\left\{-\frac{V_0+2m}{T}\right\}+\exp\left\{-\frac{V_0+V_{\rm str}(R)}{T}\right\}, \quad (34)$$

where we have chosen the string potential in the simplest form, $V_{\rm str}(R)=\sigma_{Q=2}R$. The fitting parameters are the adjoint string tension $\sigma_{Q=2}$, the mass parameter m, and the self-energy V_0. The first term in Eq. (34) corresponds to the broken string state and the parameter m, is the mass of a state of "external heavy adjoint source"—"light gluon." The second term is the unbroken string state. Here we neglect other states including the string excitations.

We perform fits in the range starting from two lattice spacings, $r_{\rm min}=2a$. The reason for this restriction is twofold: (i) the hypercubic blocking modifies the potential at small distances; (ii) in our fitting function (34) the perturbative Coulomb interaction (which is essential at small distances) is not included.[2]

The best fit functions are shown in Fig. 3 by the dashed lines and the best fit parameters are presented in Table II. One can clearly see the existence of the Abelian dominance for the string tension: $\sigma^{\rm ab}_{Q=2}\approx 0.94\sigma_{Q=2}$, where $\sigma_{Q=2}$ is the string tension extracted from the non-Abelian Polyakov loop correlator. The monopole dominance can also be observed: $\sigma^{\rm mon}_{Q=2}\approx 0.83\sigma^{\rm ab}_{Q=2}\approx 0.78\sigma_{Q=2}$. The monopole dominance is less manifest than the Abelian dominance in agreement with precise observations at $\beta=2.5115$ in the case of fundamental external sources [5].

In Ref. [5] the potential between the static $Q=2$ Abelian sources has been measured in the zero-temperature case. Despite string breaking not being observed in this case, the ratio between $Q=2$ and $Q=1$ Abelian string has been measured: $\sigma_{Q=2}/\sigma_{Q=1}=2.23(5)$. Taking into account that the ratio between $Q=1$ Abelian and $SU(2)$ string tensions is [5], $\sigma_{Q=2}/\sigma=0.92(4)$, we get the prediction of Ref. [5] for the ratio $\sigma_{Q=2}/\sigma=2.42(12)$. We observe a very good agreement with our result, $\sigma_{Q=2}/\sigma=2.33(3)$, given in Table II.

According to our numerical results the Abelian and monopole contributions to the masses of the heavy-light adjoint particles, m, do not coincide with the corresponding mass measured with the help of the non-Abelian Polyakov loops. On the other hand, we do not expect either Abelian or monopole dominance to hold in this case since these types of dominance are usually valid for infrared (nonlocal) quantities in accordance with the ideas of Ref. [1]. Because of the local nature of the mass m, the Abelian and monopole dominance may not work in this case.

The absence of the Abelian dominance for the mass parameter m implies the absence of Abelian dominance for the string breaking distance. Indeed, the simplest definition of the string breaking distance $R_{\rm sb}$ corresponds to a value of R at which both terms in Eq. (34) are equal. For the linear string potential $V_{\rm str}=\sigma_{Q=2}R$, this distance is defined as $R_{\rm sb}=2m/\sigma_{Q=2}$. In other words, the string breaking distance is the distance where the energy of the string, $\sigma_{Q=2}R_{\rm sb}$, is equivalent to the energy of the two heavy-light states, $2m$. Since the Abelian dominance works only for the string tension $\sigma_{Q=2}$, the string breaking distance $R_{\rm sb}$ should not be an Abelian- and monopole-dominated quantity.

V. CONCLUSIONS

We have calculated the effective action for the Abelian gauge and the Abelian charged matter fields in the maximal Abelian projection of quenched $SU(2)$ QCD. We have shown that in the infrared limit the contribution of the matter field to the action is nonvanishing. Thus we have shown at the qualitative level that the matter fields, carrying Abelian charge $Q=2$, must lead to adjoint string breaking. To check this effect on the quantitative level we have studied the potential between adjoint static sources as well as the Abelian and monopole contributions to this potential. We have observed that string breaking (flattening of the adjoint potential) manifests itself in Abelian and monopole contributions similarly to the non-Abelian case. Moreover, we show that the adjoint string tension is dominated by the Abelian and monopole contributions analogously to the case of fundamental particles. Thus we conclude that adjoint string breaking can qualitatively be described in the Abelian projection formalism. The key role in adjoint string breaking in the

[2] Nevertheless, we have checked the effect of the Coulomb interaction shifting the string potential as $V_{\rm str}(R)\rightarrow V_{\rm str}(R)-\alpha/R$, where α is an additional fitting parameter. We have observed that the best fit values of the parameters $\sigma_{Q=2}$ and m had a shift of about 1%–2% which is of the order of the statistical errors for these parameters.

TABLE II. The parameters for the fits of the potential by the function (34). Here $\sigma\equiv\sigma_{1/2}(T=0)$.

Type	$\sigma_{Q=2}/\sigma$	$m/\sqrt{\sigma}$
Non-Abelian	2.49(3)	1.28(1)
Abelian	2.33(3)	1.84(1)
Monopole	1.94(1)	2.27(2)

Abelian picture is played by the off-diagonal gluons which become doubly charged Abelian vector fields in the Abelian projection.

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APPENDIX A: DERIVATION OF EQ. (17)

In this appendix we present a detailed derivation of Eq. (17):

$$\int_{-\pi}^{\pi}\mathcal{D}\varphi e^{F_1(H)+F_2(H')+F_3(C)}=\sum_{\substack{j\in\mathbb{Z}(c_1)\\ \delta j=0}} w(j)e^{2i(\theta,j)}. \tag{A1}$$

The Fourier transformation applied to each of the terms on the left-hand side (LHS) of this equation gives

$$\int_{-\pi}^{\pi}\mathcal{D}\varphi e^{F_1(H)+F_2(H')+F_3(C)} =\int_{-\pi}^{\pi}\mathcal{D}\varphi\prod_s \sum_{\substack{n^{(i)}_{\mu\nu}\in\mathbb{Z}\\ i=1,2,3}}\left[\prod_{i=1}^{3} I_i(n^{(i)}_{\mu\nu}(s))\right]e^{i\Phi(\varphi,n^{(i)})}, \tag{A2}$$

where I_i are the Fourier components of e^{F_i} and Φ is the phase:

$$\Phi(\varphi,n^{(i)})=\sum_s\left[\sum_{\mu>\nu}[H_{\mu\nu}(s)n^{(1)}_{\mu\nu}(s)+H_{\nu\mu}(s)n^{(2)}_{\mu\nu}(s)] +\sum_{\mu\neq\nu}C_{\mu\nu}(s)n^{(3)}_{\mu\nu}(s)\right]. \tag{A3}$$

Using the definitions (11)–(13) we get

$$\Phi(\varphi,n^{(i)})=\sum_s\sum_{\mu>\nu}\{2\theta_\mu(s)n^{(1)}_{\mu\nu}(s)+2\theta_\nu(s)n^{(2)}_{\mu\nu}(s) +\varphi_\nu(s)[n^{(1)}_{\mu\nu}(s)-n^{(1)}_{\mu\nu}(s-\hat{\mu})-2n^{(3)}_{\mu\nu}(s)] +\varphi_\mu(s)[n^{(2)}_{\mu\nu}(s)-n^{(2)}_{\mu\nu}(s-\hat{\nu})+2n^{(3)}_{\mu\nu}(s)]\}. \tag{A4}$$

Integration over the field φ gives two constraints

$$n^{(1)}_{\mu\nu}(s)-n^{(1)}_{\mu\nu}(s-\hat{\mu})-2n^{(3)}_{\mu\nu}(s)=0,$$
$$n^{(2)}_{\mu\nu}(s)-n^{(2)}_{\mu\nu}(s-\hat{\nu})+2n^{(3)}_{\mu\nu}(s)=0, \tag{A5}$$

which lead to

$$n^{(1)}_{\mu\nu}(s)-n^{(1)}_{\mu\nu}(s-\hat{\mu})+n^{(2)}_{\mu\nu}(s)-n^{(2)}_{\mu\nu}(s-\hat{\nu})=0. \tag{A6}$$

Equation (A4) gives the natural definition for the current of the matter fields:

$$j_\mu(s)=\sum_{\nu(<\mu)}n^{(1)}_{\mu\nu}(s)+\sum_{\nu(>\mu)}n^{(2)}_{\nu\mu}(s). \tag{A7}$$

Note that as a result of the constraint (A6), the current (A7) is closed:

$$\delta j\equiv\sum_\mu \partial'_\mu j_\mu(s)=\sum_{\mu\neq\nu}[n^{(1)}_{\mu\nu}(s)-n^{(1)}_{\mu\nu}(s-\hat{\mu})+n^{(2)}_{\mu\nu}(s) -n^{(2)}_{\mu\nu}(s-\hat{\nu})]=0.$$

Combining Eqs. (A2), (A4), and (A7), we get the RHS of Eq. (A1) with

$$\omega(j)=\sum_{\substack{n^{(i)}_{\mu\nu}\in\mathbb{Z}\\ i=1,2,3}}\prod_s\left[\prod_{i=1}^{3} I_i(n^{(i)}_{\mu\nu}(s))\right]\delta\Big(j_\mu(s)-\sum_{\nu(<\mu)}n^{(1)}_{\mu\nu}(s) +\sum_{\nu(>\mu)}n^{(2)}_{\nu\mu}(s)\Big)\delta(n^{(1)}_{\mu\nu}(s)-n^{(1)}_{\mu\nu}(s-\hat{\mu}) -2n^{(3)}_{\mu\nu}(s))\,\delta(n^{(2)}_{\mu\nu}(s)-n^{(2)}_{\mu\nu}(s-\hat{\nu})+2n^{(3)}_{\mu\nu}(s)). \tag{A8}$$

APPENDIX B: SCHWINGER-DYSON EQUATIONS

Consider a model of the gauge field θ. The expectation value of an arbitrary operator $O(\theta)$ measured at the ensemble $\{\theta_i\}$ of the gauge fields θ is

$$\langle O(\theta)\rangle=\int\mathcal{D}\theta O(\theta)e^{-S(\theta)}=\prod_i\int_{-\pi}^{\pi}d\theta_i O(\{\theta_i\})e^{-S(\{\theta_i\})}. \tag{B1}$$

Shifting one of the link fields θ_{i_0} at the link i_0 by an infinitesimal value ϵ we get

$$\prod_i\int_{-\pi}^{\pi}d\theta_i O(\{\theta_i\})e^{-S(\{\theta_i\})} =\prod_{i\neq i_0}\int_{-\pi}^{\pi}d\theta_i\int_{-\pi}^{\pi}d\theta_{i_0}O(\theta_{i_0},\{\theta_i\}_{i\neq i_0})e^{-S(\theta_{i_0},\{\theta_i\}_{i\neq i_0})} \to\prod_{i\neq i_0}\int_{-\pi}^{\pi}d\theta_i\int_{-\pi+\epsilon}^{\pi+\epsilon}d\theta_{i_0}O(\theta_{i_0}+\epsilon,\{\theta_i\}_{i\neq i_0}) \times e^{-S(\theta_{i_0}+\epsilon,\{\theta_i\}_{i\neq i_0})}$$

$$=\prod_{i\neq i_0}\int_{-\pi}^{\pi}d\theta_i\int_{-\pi}^{\pi}d\theta_{i_0}\Bigg\{O(\theta_{i_0},\{\theta_i\}_{i\neq i_0})\times e^{-S(\theta_{i_0},\{\theta_i\}_{i\neq i_0})}+\epsilon\frac{\partial}{\partial\theta_{i_0}}[O(\theta_{i_0},\{\theta_i\}_{i\neq i_0})\times e^{-S(\theta_{i_0},\{\theta_i\}_{i\neq i_0})}]+\mathcal{O}(\epsilon^2)\Bigg\}. \tag{B2}$$

The requirement that this shift not change the partition function gives the Schwinger-Dyson equation

$$\prod_{i\neq i_0}\int_{-\pi}^{\pi}d\theta_i\int_{-\pi}^{\pi}d\theta_{i_0}\frac{\partial}{\partial\theta_{i_0}}[O(\theta_{i_0},\{\theta_i\}_{i\neq i_0})e^{-S(\theta_{i_0},\{\theta_i\}_{i\neq i_0})}]=\int\mathcal{D}\theta\frac{\partial}{\partial\theta_{i_0}}[O(\theta)e^{-S(\theta)}]=0, \tag{B3}$$

which can also be rewritten in the form

$$\left\langle\frac{\partial O(\theta)}{\partial\theta_{i_0}}\right\rangle-\left\langle O(\theta)\frac{\partial S(\theta)}{\partial\theta_{i_0}}\right\rangle=0. \tag{B4}$$

To determine the parameters of the trial action (20)–(24) we solve Eq. (B4) with the following set of operators:

$$O_I=\frac{\partial S_I}{\partial\theta_\mu(s)}\quad(I=1,2,3,4),\quad O_5=\frac{\partial S_4}{\partial\varphi_\mu(s)}. \tag{B5}$$

The expectation values of these operators give a set of five Schwinger-Dyson equations:

$$\left\langle\frac{\partial^2 S_I}{\partial\theta_\mu(s)^2}\right\rangle=\sum_{J=1}^{3}\alpha_J\left\langle\frac{\partial S_I}{\partial\theta_\mu(s)}\frac{\partial S_J}{\partial\theta_\mu(s)}\right\rangle+\beta_1\left\langle\frac{\partial S_4}{\partial\theta_\mu(s)}\frac{\partial S_J}{\partial\theta_\mu(s)}\right\rangle\quad(I=1,2,3,4), \tag{B6}$$

$$\left\langle\frac{\partial^2 S_4}{\partial\varphi_\mu(s)^2}\right\rangle=\beta_1\left\langle\left(\frac{\partial S_4}{\partial\varphi_\mu(s)}\right)^2\right\rangle. \tag{B7}$$

Since we have five equations (B6), (B7) to determine four independent couplings α_i, $i=1,2,3$, and β_1, the system of equations (B6), (B7) is overdefined. Thus we find the couplings with the help of Eq. (B6) and then use Eq. (B7) as a consistency check. We find that for the original fields the LHS of Eq. (B7) is approximately 10% larger than the RHS. However, after applying the block-spin transformation the discrepancy becomes much smaller (it becomes of the order of the statistical errors), and the solution of Eqs. (B6), (B7) becomes self-consistent.

APPENDIX C: HYPERCUBIC BLOCKING

The hypercubic blocking (hyp) procedure is a version of the smearing method which allows us to reduce the noises of the lattice gauge fields [23]. As a result the statistical errors of ensemble averages of various operators are reduced. hyp is replacing gauge link fields, $U_\mu(s)$, by "fat links" $V_\mu(s)$, according to the following scheme:

$$V_\mu(s)=\frac{1}{k_1}\Bigg[(1-\alpha_1)U_\mu(s)+\frac{\alpha_1}{6}\sum_{\nu\neq\pm\mu}\tilde{V}_{\nu;\mu}(s)\times\tilde{V}_{\mu;\nu}(s+\hat{\nu})\tilde{V}^\dagger_{\nu;\mu}(s+\hat{\mu})\Bigg],$$

$$\tilde{V}_{\mu;\nu}(s)=\frac{1}{k_2}\Bigg[(1-\alpha_2)U_\mu(s)+\frac{\alpha_2}{4}\sum_{\rho\neq\pm\mu,\pm\nu}\bar{V}_{\rho;\mu\nu}(s)\bar{V}_{\mu;\nu\rho}(s+\hat{\rho})\times\bar{V}^\dagger_{\rho;\mu\nu}(s+\hat{\mu})\Bigg],$$

$$\bar{V}_{\mu;\nu\rho}(s)=\frac{1}{k_3}\Bigg[(1-\alpha_3)U_\mu(s)+\frac{\alpha_3}{2}\times\sum_{\sigma\neq\pm\mu,\pm\nu,\pm\rho}U_\sigma(s)U_\mu(s+\hat{\sigma})U^\dagger_\sigma(s+\hat{\mu})\Bigg], \tag{C1}$$

where k_i, $i=1,2,3$, are chosen in such a way that the matrices (C1) belong to the $SU(2)$ group. We choose the parameters of the hyp, α_1, α_2, $\alpha_3\in[0,1]$, following Ref. [23]: $\alpha_1=0.75$, $\alpha_2=0.60$, and $\alpha_3=0.30$. At these values the smoothing of the gauge field configurations is most efficient.

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