

Semileptonic decays of doubly heavy baryons in the relativistic quark model

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Semileptonic decays of doubly heavy baryons are studied in the framework of the relativistic quark model. The doubly heavy baryons are treated in the quark-diquark approximation. The transition amplitudes of heavy diquarks bb and bc going, respectively, to bc and cc are explicitly expressed through the overlap integrals of the diquark wave functions in the whole accessible kinematic range. The relativistic baryon wave functions of the quark-diquark bound system are used for the calculation of the transition matrix elements, the Isgur-Wise function, and decay rates in the heavy quark limit.

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I. INTRODUCTION

The description of doubly heavy baryon properties has acquired in the last few years the status of an actual physical problem that can be studied experimentally. The appearance of experimental data on B_c mesons [1] and heavy-light baryons [2] stimulate the investigation of heavy quark-bound states and can help in discriminating numerous quark models. Recently, the first experimental indications of the existence of doubly charmed baryons were published by Mattson *et al.* [3]. Although these data need further experimental confirmation and clarification, it manifests that in the near future the mass spectra and decay rates of doubly heavy baryons will be measured. This gives additional grounds for the theoretical investigation of the doubly heavy baryon properties. The success of the heavy quark effective theory (HQET) [4] in predicting properties of the heavy-light $q\bar{Q}$ mesons (B and D) suggests to apply these methods to heavy-light baryons, too. The semileptonic decays of heavy hadrons present also an important tool for determining the parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

The relativistic quark model has been remarkably successful in describing the observed hadronic states and their decay rates [5–8]. This includes the heavy-light mesons and heavy quarkonia. The quark model predicts the existence of doubly heavy baryons containing two heavy quarks (cc, bc, bb) and one light quark (u, d, s). The energies necessary to produce these particles are already reached. The main difficulty remains in their reconstruction, as these states have in general a large number of decay modes, and thus high statistics is required [1].

Doubly heavy baryons occupy a special position among existing baryons because they can be studied in the quark-diquark approximation and the two-particle bound state methods can be applied. The two heavy quarks compose, in this case, a bound diquark system in the antitriplet color

state, which serves as a localized color source. The light quark q is orbiting around this heavy source at a distance much larger ($\sim 1/m_q$) than the source size ($\sim 2/m_Q$). The estimates of the light-quark velocity in these baryons show that its value is $v/c \sim 0.7$ – 0.8 , and the light quark should be treated fully relativistically. Thus the doubly heavy baryons look effectively like a two-body bound system and strongly resemble the heavy-light B and D mesons [9,10]. Then the HQET expansion in the inverse heavy diquark mass can be performed. We used a similar approach for the calculation of the mass spectra of doubly heavy baryons [8]. The ground-state baryons with two heavy quarks can be composed from a compact doubly heavy diquark of spin 0 or 1 and a light quark. According to the Pauli principle the diquarks (bb) or (cc) have the spin 1 whereas diquark (bc) can have both the spin 0 and 1.

There exists already a number of papers devoted to studying both the mass spectra of doubly heavy baryons and their decay rates [2,5,11–22]. Whereas the results for the mass spectra are in agreement, the calculations of exclusive semileptonic decays lead to essentially different values for the decay rates [13,15–17] obtained with the help of the Bethe-Salpeter equation, QCD sum rules, and the relativistic three-quark model.

Here we study semileptonic decay rates of doubly heavy baryons using the relativistic quark model in the quark-diquark approximation. The covariant expressions for the semileptonic decay amplitudes of the baryons with the spin $1/2, 3/2$ are obtained in the limit $m_c, m_b \rightarrow \infty$ and compared with the predictions of HQET. The calculation of semileptonic decays of doubly heavy baryons (bbq) or (bcq) to doubly heavy baryons (bcq) or (ccq) can be divided into two steps (see Fig. 1). The first step is the study of form factors of the weak transition between initial and final doubly heavy diquarks. The second step consists of the inclusion of

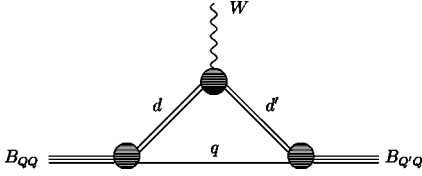


FIG. 1. Weak transition matrix element of the doubly heavy baryon in the quark-diquark approximation.

the light quark in order to compose a baryon with spin 1/2 or 3/2.

The paper is organized as follows. In Sec. II we describe our relativistic quark model and present predictions for the masses of ground-state heavy diquarks and doubly heavy baryons. We apply our model to the investigation of the heavy diquark transition matrix elements in Sec. III. The transition amplitudes of heavy diquarks are explicitly expressed in a covariant form through the overlap integrals of the diquark wave functions. The obtained general expressions reproduce in the appropriate limit the predictions of heavy quark symmetry. Section IV is devoted to the construction of transition matrix elements between doubly heavy baryons in the quark-diquark approximation. The corresponding Isgur-Wise function is determined. In Sec. V semi-leptonic decay rates of doubly heavy baryons are calculated in the nonrelativistic limit for heavy quarks. Our conclusions are given in Sec. VI.

II. RELATIVISTIC QUARK MODEL

In the quasipotential approach and quark-diquark picture of doubly heavy baryons, the interaction of two heavy quarks in a diquark and of the light quark with a heavy diquark in a baryon are described by the diquark wave function (Ψ_d) of the bound quark-quark state and by the baryon wave function (Ψ_B) of the bound quark-diquark state, respectively. These wave functions satisfy the two-particle quasipotential equation [23,24] of the Schrödinger type [25]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R} \right) \Psi_{d,B}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M) \Psi_{d,B}(\mathbf{q}), \quad (1)$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \quad (2)$$

and the center of mass energies of particles on the mass shell E_1, E_2 are given by

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}. \quad (3)$$

Here $M = E_1 + E_2$ is the bound state mass (diquark or baryon), $m_{1,2}$ are the masses of heavy quarks (Q_1 and Q_2), which form the diquark, or of the heavy diquark (d) and light quark (q), which form the doubly heavy baryon (B), and \mathbf{p}

TABLE I. Mass spectrum of ground states of doubly heavy baryons (in GeV) [8]. $\{QQ\}$ denotes the diquark in the axial vector state and $[QQ]$ denotes the diquark in the scalar state.

Baryon	Quark content	J^P	Mass
Ξ_{cc}	$\{cc\}q$	$1/2^+$	3.620
Ξ_{cc}^*	$\{cc\}q$	$3/2^+$	3.727
Ω_{cc}	$\{cc\}s$	$1/2^+$	3.778
Ω_{cc}^*	$\{cc\}s$	$3/2^+$	3.872
Ξ_{bb}	$\{bb\}q$	$1/2^+$	10.202
Ξ_{bb}^*	$\{bb\}q$	$3/2^+$	10.237
Ω_{bb}	$\{bb\}s$	$1/2^+$	10.359
Ω_{bb}^*	$\{bb\}s$	$3/2^+$	10.389
Ξ_{cb}	$\{cb\}q$	$1/2^+$	6.933
Ξ'_{cb}	$[cb]q$	$1/2^+$	6.963
Ξ_{cb}^*	$\{cb\}q$	$3/2^+$	6.980
Ω_{cb}	$\{cb\}s$	$1/2^+$	7.088
Ω'_{cb}	$[cb]s$	$1/2^+$	7.116
Ω_{cb}^*	$\{cb\}s$	$3/2^+$	7.130

is their relative momentum. In the center-of-mass system, the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}. \quad (4)$$

The kernel $V(\mathbf{p},\mathbf{q};M)$ in Eq. (1) is the quasipotential operator of the quark-quark or quark-diquark interaction. It is constructed with the help of the off-mass shell-scattering amplitude, projected onto the positive energy states. Here we closely follow the similar construction of the quark-antiquark interaction in heavy mesons, which were extensively studied in our relativistic quark model [26,27]. For the quark-quark interaction in a diquark we use the relation $V_{QQ} = V_{Q\bar{Q}}/2$ arising under the assumption about the octet structure of the interaction from the difference of the projection onto QQ and $Q\bar{Q}$ color states. The quasipotential of the quark-antiquark interaction is the sum of the usual one-gluon exchange term and the confining part, which is the mixture of long-range vector and scalar linear potentials, where the vector confining potential contains the Pauli terms. The explicit expressions for these quasipotentials are given in Ref. [8]. The quark masses have the following values $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_s = 0.50$ GeV, and $m_{u,d} = 0.33$ GeV.

We calculated, in the framework of the relativistic quark model, the mass spectra of heavy diquarks and doubly heavy baryon masses in the quark-diquark approximation in Ref. [8]. The masses of the ground state axial vector diquarks were found to be $M_{cc}^{AV} = 3.226$ GeV, $M_{bb}^{AV} = 9.778$ GeV, and $M_{bc}^{AV} = 6.526$ GeV, and the mass of the scalar diquark $M_{bc}^S = 6.519$ GeV. The calculated masses of the ground-state doubly heavy baryons are listed in Table I.

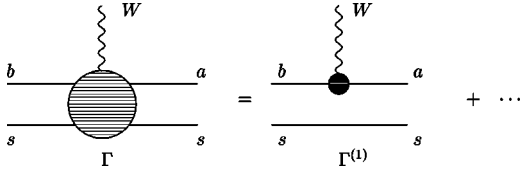


FIG. 2. Leading order contribution $\Gamma^{(1)}$ to the diquark vertex function Γ .

III. HEAVY DIQUARK TRANSITION FORM FACTORS

The form factors of the subprocess $d(QQ_s) \rightarrow d'(Q'Q_s)e\bar{\nu}$, where one heavy quark Q_s is a spectator, are determined by the weak decay of the active heavy quark $Q \rightarrow Q'e\bar{\nu}$. The local effective Hamiltonian is given by

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{QQ'} [\bar{Q}' \gamma_\mu (1 - \gamma_5) Q] [\bar{l} \gamma_\mu (1 - \gamma_5) \nu_e], \quad (5)$$

where G_F is the Fermi constant and $V_{QQ'}$ is the CKM matrix element. In the relativistic quark model the transition matrix element between two diquark states is determined by the contraction of the wave functions Ψ_d of the initial and final diquarks with the two particle vertex function Γ [24]

$$\begin{aligned} \langle d'(Q) | J_\mu^W | d(P) \rangle \\ = \int \frac{d^3 p d^3 q}{(2\pi)^6} \bar{\Psi}_{d',Q}^{\lambda\sigma}(\mathbf{q}) \Gamma_\mu^{\lambda\sigma,\rho\omega}(\mathbf{p}, \mathbf{q}) \Psi_{d,P}^{\rho\omega}(\mathbf{p}), \\ \lambda, \sigma, \rho, \omega = \pm \frac{1}{2}. \end{aligned} \quad (6)$$

Here we denote mass, energy, and velocity of the initial diquark ($Q_b Q_s$, index b stands for the initial active quark and index s for the spectator) by M_i , $E_i = M_i v^0$ and $v = P/M_i$, and mass, energy, and velocity of the final diquark ($Q_a Q_s$, index a means the final active quark) by M_f , $E_f = M_f v'^0$, and $v' = Q/M_f$.

The leading contribution to the vertex function Γ_μ comes from the diagram in Fig. 2 [24,28,29] (we explicitly show spin indices)

$$\begin{aligned} \Gamma_\mu^{\lambda\sigma,\rho\omega}(\mathbf{p}, \mathbf{q}) = \Gamma_\mu^{(1)} = \bar{u}_a^\lambda(\mathbf{q}_1) \gamma_\mu (1 - \gamma_5) u_b^\rho(\mathbf{p}_1) \\ \times \bar{u}_s^\sigma(\mathbf{q}_2) u_s^\omega(\mathbf{p}_2) (2\pi)^3 \delta(\mathbf{p}_2 - \mathbf{q}_2) \delta^{\sigma\omega}, \end{aligned} \quad (7)$$

where the Dirac spinors are

$$\begin{aligned} u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}} \begin{pmatrix} 1 \\ (\boldsymbol{\sigma}\mathbf{p}) \\ \epsilon(p) + m \end{pmatrix} \chi^\lambda, \\ \epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}, \quad \lambda = \pm \frac{1}{2}. \end{aligned} \quad (8)$$

Relativistic four momenta of the particles in the initial and final states are defined as follows

$$p_{1,2} = \epsilon_{1,2}(p) v \pm \sum_{i=1}^3 n^{(i)}(v) p^i, \quad v = \frac{P}{M_i},$$

$$\mathcal{M}_i = \epsilon_1(p) + \epsilon_2(p),$$

$$q_{1,2} = \epsilon_{1,2}(q) v' \pm \sum_{i=1}^3 n^{(i)}(v') q^i,$$

$$v' = \frac{Q}{M_f}, \quad \mathcal{M}_f = \epsilon_1(q) + \epsilon_2(q) \quad (9)$$

and $n^{(i)}$ are three four vectors defined by

$$n^{(i)}(v) = \left\{ v^i, \delta^{ij} + \frac{1}{1+v^0} v^i v^j \right\}.$$

After making necessary transformations, the expression for Γ should be continued in \mathcal{M}_i and \mathcal{M}_f to the values of initial M_i and final M_f bound state masses. The transformation of the bound state wave functions from the rest frame to the moving one with four-momenta P and Q is given by [24,28]

$$\begin{aligned} \Psi_{d,P}^{\rho\omega}(\mathbf{p}) = D_{b,\rho\alpha}^{1/2}(R_{L_P}^W) D_{s,\omega\beta}^{1/2}(R_{L_P}^W) \Psi_{d,0}^{\alpha\beta}(\mathbf{p}), \\ \bar{\Psi}_{d',Q}^{\lambda\sigma}(\mathbf{q}) = \bar{\Psi}_{d',0}^{\epsilon\tau}(\mathbf{q}) D_{a,\epsilon\lambda}^{\dagger 1/2}(R_{L_Q}^W) D_{s,\tau\sigma}^{\dagger 1/2}(R_{L_Q}^W), \end{aligned} \quad (10)$$

where R^W is the Wigner rotation, L_P is the Lorentz boost from the diquark rest frame to a moving one, and the rotation matrix $D^{1/2}(R)$ is defined by

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D_{b,s}^{1/2}(R_{L_P}^W) = S^{-1}(\mathbf{p}_{1,2}) S(\mathbf{P}) S(\mathbf{p}), \quad (11)$$

where

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p) + m}{2m}} \left(1 + \frac{(\boldsymbol{\alpha}\mathbf{p})}{\epsilon(p) + m} \right)$$

is the usual Lorentz transformation matrix of the four-spinor. Using relations (9) and (10) we can express the matrix element (6) in the form of the trace over spinor indices of both particles. For this aim the following relations are useful

$$\begin{aligned} S_{\alpha\beta}(\Lambda) u_\beta^\lambda(p) = \sum_{\sigma=\pm 1/2} u_\alpha^\sigma(\Lambda p) D_{\sigma\lambda}^{1/2}(R_{\Lambda p}^W), \\ \bar{u}_\beta^\lambda(p) S_{\beta\alpha}^{-1}(\Lambda) = \sum_{\sigma=\pm 1/2} D_{\lambda\sigma}^{\dagger 1/2}(R_{\Lambda p}^W) \bar{u}_\alpha^\sigma(\Lambda p). \end{aligned} \quad (12)$$

Substituting expressions (7) and (10) in Eq. (6) and using relations (11) and (12), we obtain

$$\begin{aligned}
& \langle d'(Q) | J_\mu^W | d(P) \rangle \\
&= \int \frac{d^3 p d^3 q}{(2\pi)^3} \bar{\Psi}_{d',0}^{\varepsilon\tau}(\mathbf{q}) \bar{u}_a^\varepsilon(0) \frac{(\hat{q}_a + m_a)}{\sqrt{2\varepsilon_a(q)[\varepsilon_a(q) + m_a]}} \\
&\quad \times S^{-1}(L_Q) \gamma_\mu (1 - \gamma_5) S(L_P) \frac{(\hat{p}_b + m_b)}{\sqrt{2\varepsilon_b(p)[\varepsilon_b(p) + m_b]}} \\
&\quad \times u_b^\alpha(0) \bar{u}_s^\tau(0) \frac{(\hat{q}_s + m_s)}{\sqrt{2\varepsilon_s(q)[\varepsilon_s(q) + m_s]}} \\
&\quad \times S^{-1}(L_Q) S(L_P) \frac{(\hat{p}_s + m_s)}{\sqrt{2\varepsilon_s(p)[\varepsilon_s(p) + m_s]}} u_s^\beta(0) \\
&\quad \times \delta(\mathbf{p}_2 - \mathbf{q}_2) \Psi_{d,0}^{\alpha\beta}(\mathbf{p}), \tag{13}
\end{aligned}$$

where $p_b = [\varepsilon_b(p), \mathbf{p}]$, $p_s = [\varepsilon_s(p), -\mathbf{p}]$ and $q_a = [\varepsilon_a(q), \mathbf{q}]$, $q_s = [\varepsilon_s(q), -\mathbf{q}]$ (we use the notation $\hat{p} \equiv p^\mu \gamma_\mu$). The spin wave functions for the scalar and axial vector diquark states in the heavy quark rest frame read [15,16]

$$\begin{aligned}
\mathcal{H}^S(0) &= \frac{1 + \gamma^0}{2\sqrt{2}}, \\
\mathcal{H}^{AV}(0) &= \frac{1 + \gamma^0}{2\sqrt{2}} \gamma_5 \hat{\varepsilon}, \tag{14}
\end{aligned}$$

where ε_α is the polarization vector of the axial vector diquark. Using these diquark spin wave functions we get the following expression

$$\begin{aligned}
\langle d'(Q) | J_\mu^W | d(P) \rangle &= \int \frac{d^3 p d^3 q}{(2\pi)^3} \bar{\Psi}_{d',0}(\mathbf{q}) \Psi_{d,0}(\mathbf{p}) \delta(\mathbf{p}_2 - \mathbf{q}_2) \text{Tr} \left\{ S^{T-1}(L_Q) \frac{(\hat{q}_s + m_s)^T}{\sqrt{2\varepsilon_s(q)[\varepsilon_s(q) + m_s]}} \right. \\
&\quad \times \bar{\mathcal{H}}^{S,AV}(0) \frac{(\hat{q}_a + m_a)}{\sqrt{\varepsilon_a(q)[\varepsilon_a(q) + m_a]}} S^{-1}(L_Q) [\gamma_\mu (1 - \gamma_5)] S(L_P) \frac{(\hat{p}_b + m_b)}{\sqrt{2\varepsilon_b(p)[\varepsilon_b(p) + m_b]}} \\
&\quad \left. \times \mathcal{H}^{S,AV}(0) \frac{(\hat{p}_s + m_s)^T}{\sqrt{2\varepsilon_s(p)[\varepsilon_s(p) + m_s]}} S^T(L_P) \right\}, \tag{15}
\end{aligned}$$

where $\Psi_{d,0}(\mathbf{p})$ is the spin-independent part of the diquark wave function and the superscript T denotes transposing. After explicit multiplication of the matrices we get the covariant (see Ref. [24]) expression for the transition matrix element

$$\begin{aligned}
\langle d'(Q) | J_\mu^W | d(P) \rangle &= 2\sqrt{M_i M_f} \int \frac{d^3 p d^3 q}{(2\pi)^3} \text{Tr} \{ \bar{\Psi}_{d'}(Q, q) \\
&\quad \times \gamma_\mu (1 - \gamma_5) \Psi_d(P, p) \} \delta^3(\mathbf{p}_2 - \mathbf{q}_2), \tag{16}
\end{aligned}$$

where the amplitudes Ψ_d for the scalar (S) and axial vector (AV) diquarks (d) are given by

$$\begin{aligned}
\Psi_S(P, p) &= \sqrt{\frac{\varepsilon_b(p) + m_b}{2\varepsilon_b(p)}} \sqrt{\frac{\varepsilon_s(p) + m_s}{2\varepsilon_s(p)}} \left[\frac{\hat{v} + 1}{2\sqrt{2}} \right. \\
&\quad + \frac{\hat{v} - 1}{2\sqrt{2}} \frac{\tilde{p}^2}{[\varepsilon_b(p) + m_b][\varepsilon_s(p) + m_s]} \\
&\quad - \left(\frac{\hat{v} + 1}{2\sqrt{2}} \frac{1}{\varepsilon_s(p) + m_s} \right. \\
&\quad \left. \left. + \frac{\hat{v} - 1}{2\sqrt{2}} \frac{1}{\varepsilon_b(p) + m_b} \right) \hat{p} \right] \gamma_0 \Phi_S(p), \tag{17}
\end{aligned}$$

$$\begin{aligned}
\Psi_{AV}(P, p, \varepsilon) &= \sqrt{\frac{\varepsilon_b(p) + m_b}{2\varepsilon_b(p)}} \sqrt{\frac{\varepsilon_s(p) + m_s}{2\varepsilon_s(p)}} \left[\frac{\hat{v} + 1}{2\sqrt{2}} \hat{\varepsilon} \right. \\
&\quad + \frac{\hat{v} - 1}{2\sqrt{2}} \frac{\tilde{p}^2}{[\varepsilon_b(p) + m_b][\varepsilon_s(p) + m_s]} \hat{\varepsilon} \\
&\quad - \frac{\hat{v} - 1}{2\sqrt{2}} \frac{2(\varepsilon \cdot \tilde{p}) \hat{p}}{[\varepsilon_b(p) + m_b](\varepsilon_s(p) + m_s)} \\
&\quad + \frac{\hat{v} + 1}{2\sqrt{2}} \frac{\hat{\varepsilon} \hat{p}}{\varepsilon_s(p) + m_s} \\
&\quad \left. - \frac{\hat{v} - 1}{2\sqrt{2}} \frac{\hat{p} \hat{\varepsilon}}{\varepsilon_b(p) + m_b} \right] \gamma_0 \gamma^5 \Phi_{AV}(p). \tag{18}
\end{aligned}$$

Here $\Phi_d(p) \equiv \Psi_{d,0}(\mathbf{p}) / \sqrt{2M_d}$ is the diquark wave function in the rest frame normalized to unity. The four-vector

$$\tilde{p} = L_P(0, \mathbf{p}) = \left(\frac{(\mathbf{p}\mathbf{P})}{M}, \mathbf{p} + \frac{(\mathbf{p}\mathbf{P})\mathbf{P}}{M(E+M)} \right) \tag{19}$$

has the following properties

$$\begin{aligned}
\tilde{p}^2 &= -\mathbf{p}^2, \\
(\boldsymbol{\varepsilon} \cdot \tilde{\mathbf{p}}) &= -(\boldsymbol{\varepsilon} \mathbf{p}), \\
(\tilde{\mathbf{p}} \cdot \mathbf{v}) &= 0.
\end{aligned} \tag{20}$$

The presence of $\delta^3(\mathbf{p}_2 - \mathbf{q}_2)$, with momenta \mathbf{p}_2 and \mathbf{q}_2 given by Eq. (9), in the decay matrix element (16) leads to the additional relations

$$\begin{aligned}
\tilde{p}_\mu &= \frac{w\epsilon_s(p) - \epsilon_s(q)}{w^2 - 1} (wv_\mu - v'_\mu) \\
&= \frac{\sqrt{\mathbf{p}^2}}{\sqrt{w^2 - 1}} (wv_\mu - v'_\mu),
\end{aligned} \tag{21}$$

$$\begin{aligned}
\tilde{q}_\mu &= \frac{w\epsilon_s(q) - \epsilon_s(p)}{w^2 - 1} (wv'_\mu - v_\mu) \\
&= \frac{\sqrt{\mathbf{q}^2}}{\sqrt{w^2 - 1}} (wv'_\mu - v_\mu),
\end{aligned} \tag{22}$$

$$\begin{aligned}
\epsilon_s(q) &= w\epsilon_s(p) - \sqrt{w^2 - 1} \sqrt{\mathbf{p}^2}, \\
\mathbf{q}^2 &= [\sqrt{w^2 - 1} \epsilon_s(p) - w\sqrt{\mathbf{p}^2}]^2,
\end{aligned} \tag{23}$$

$$\begin{aligned}
\epsilon_s(p) &= w\epsilon_s(q) - \sqrt{w^2 - 1} \sqrt{\mathbf{q}^2}, \\
\mathbf{p}^2 &= [\sqrt{w^2 - 1} \epsilon_s(q) - w\sqrt{\mathbf{q}^2}]^2,
\end{aligned} \tag{24}$$

which allow to express either \mathbf{q} through \mathbf{p} or \mathbf{p} through \mathbf{q} . The argument of the δ function can then be rewritten as

$$\mathbf{p}_2 - \mathbf{q}_2 = \mathbf{q} - \mathbf{p} - \frac{\epsilon_s(p) + \epsilon_s(q)}{w + 1} (\mathbf{v}' - \mathbf{v}), \tag{25}$$

where $w = (\mathbf{v} \cdot \mathbf{v}')$. Calculating traces in Eq. (16) and using relations (21)–(24) one can see that the spectator quark contribution factors out in all decay matrix elements. Indeed all transition matrix elements have a common factor

$$\begin{aligned}
&\sqrt{\frac{\epsilon_s(p) + m_s}{2\epsilon_s(p)}} \sqrt{\frac{\epsilon_s(q) + m_s}{2\epsilon_s(q)}} \left[1 - \sqrt{\frac{w-1}{w+1}} \left(\frac{\sqrt{\mathbf{p}^2}}{\epsilon_s(p) + m_s} \right. \right. \\
&\quad \left. \left. + \frac{\sqrt{\mathbf{q}^2}}{\epsilon_s(q) + m_s} \right) + \frac{\sqrt{\mathbf{p}^2} \sqrt{\mathbf{q}^2}}{[\epsilon_s(q) + m_s][\epsilon_s(p) + m_s]} \right] \\
&= \sqrt{\frac{2}{w+1}} I_s(p, q).
\end{aligned} \tag{26}$$

If the δ function is used to express \mathbf{q} through \mathbf{p} or \mathbf{p} through \mathbf{q} then $I_s(p, q) = \mathcal{I}_s(p)$ or $I_s(p, q) = \mathcal{I}_s(q)$ with

$$\begin{aligned}
\mathcal{I}_s(p) &= \sqrt{\frac{w\epsilon_s(p) - \sqrt{w^2 - 1} \sqrt{\mathbf{p}^2}}{\epsilon_s(p)}} \theta \left(\sqrt{\epsilon_s(p) - m_s} \right. \\
&\quad \left. - \sqrt{\frac{w-1}{w+1}} \sqrt{\epsilon_s(p) + m_s} \right) \\
&\quad + \frac{m_s}{\sqrt{\epsilon_s(p)[w\epsilon_s(p) - \sqrt{w^2 - 1}]}} \\
&\quad \times \theta \left(\sqrt{\frac{w-1}{w+1}} \sqrt{\epsilon_s(p) + m_s} - \sqrt{\epsilon_s(p) - m_s} \right).
\end{aligned} \tag{27}$$

The weak current matrix elements have the following covariant decomposition: (a) Scalar to scalar diquark transition ($bc \rightarrow bc$)

$$\frac{\langle S_f(v') | J_\mu^V | S_i(v) \rangle}{\sqrt{M_{S_i} M_{S_f}}} = h_+(w)(v + v')_\mu + h_-(w)(v - v')_\mu, \tag{28}$$

(b) Scalar to axial vector diquark transition ($bc \rightarrow cc$)

$$\frac{\langle AV(v', \boldsymbol{\varepsilon}') | J_\mu^V | S(v) \rangle}{\sqrt{M_{AV} M_S}} = ih_V(w) \epsilon_{\mu\alpha\beta\gamma} \boldsymbol{\varepsilon}'^{*\alpha} v'^\beta v^\gamma, \tag{29}$$

$$\begin{aligned}
\frac{\langle AV(v', \boldsymbol{\varepsilon}') | J_\mu^A | S(v) \rangle}{\sqrt{M_{AV} M_S}} &= h_{A_1}(w)(w+1) \boldsymbol{\varepsilon}'^*_\mu - h_{A_2}(w) \\
&\quad \times (v \cdot \boldsymbol{\varepsilon}'^*) v_\mu \\
&\quad - h_{A_3}(v \cdot \boldsymbol{\varepsilon}'^*) v'_\mu,
\end{aligned} \tag{30}$$

(c) Axial vector to scalar diquark transition ($bb \rightarrow bc$)

$$\frac{\langle S(v') | J_\mu^V | AV(v, \boldsymbol{\varepsilon}) \rangle}{\sqrt{M_{AV} M_S}} = ih_V(w) \epsilon_{\mu\alpha\beta\gamma} \boldsymbol{\varepsilon}^{\alpha} v'^\beta v^\gamma, \tag{31}$$

$$\begin{aligned}
\frac{\langle S(v') | J_\mu^A | AV(v, \boldsymbol{\varepsilon}) \rangle}{\sqrt{M_{AV} M_S}} &= h_{A_1}(w)(w+1) \boldsymbol{\varepsilon}_\mu - \tilde{h}_{A_2}(w) \\
&\quad \times (v' \cdot \boldsymbol{\varepsilon}) v'_\mu - \tilde{h}_{A_3}(v \cdot \boldsymbol{\varepsilon}') v_\mu,
\end{aligned} \tag{32}$$

(d) Axial vector to axial vector diquark transition ($bb \rightarrow bc, bc \rightarrow cc$)

$$\begin{aligned}
& \frac{\langle AV_f(v', \varepsilon') | J_\mu^V | AV_i(v, \varepsilon) \rangle}{\sqrt{M_{AV_i} M_{AV_f}}} \\
&= -(\varepsilon'^* \cdot \varepsilon) [h_1(w)(v+v')_\mu + h_2(w)(v-v')_\mu] \\
& \quad + h_3(w)(v \cdot \varepsilon'^*) \varepsilon_\mu + h_4(w)(v' \cdot \varepsilon) \varepsilon'_\mu \\
& \quad - (v \cdot \varepsilon'^*)(v' \cdot \varepsilon) [h_5(w)v_\mu + h_6(w)v'_\mu], \quad (33)
\end{aligned}$$

$$\begin{aligned}
& \frac{\langle AV_f(v', \varepsilon') | J_\mu^A | AV_i(v, \varepsilon) \rangle}{\sqrt{M_{AV_i} M_{AV_f}}} \\
&= i \varepsilon_{\mu\alpha\beta\gamma} \{ \varepsilon^\beta \varepsilon'^* \gamma [h_7(w)(v+v')^\alpha + h_8(w)(v-v')^\alpha] \\
& \quad + v'^\beta v^\gamma [h_9(w)(v \cdot \varepsilon'^*) \varepsilon^\alpha + h_{10}(w)(v' \cdot \varepsilon) \varepsilon'^* \alpha] \}. \quad (34)
\end{aligned}$$

The transition form factors are expressed through the overlap integrals of the diquark wave functions and are given in Appendix A. These exact expressions for diquark form factors were obtained without any assumptions about the spectator and active quark masses.

If we consider the spectator quark to be light and then take the limit of an infinitely heavy active quark mass, $m_{a,b} \rightarrow \infty$, we can explicitly obtain heavy quark symmetry relations for the decay matrix elements of heavy-light diquarks that are analogous to those of heavy-light mesons [28,30]

$$\begin{aligned}
h_+(w) &= h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \tilde{h}_{A_3}(w) = h_1(w) \\
&= h_3(w) = h_4(w) = h_7(w) = \xi(w), \\
h_-(w) &= h_{A_2}(w) = \tilde{h}_{A_2}(w) = h_2(w) = h_5(w) = h_6(w) \\
&= h_8(w) = h_9(w) = h_{10}(w) = 0,
\end{aligned}$$

with the Isgur-Wise function

$$\begin{aligned}
\xi(w) &= \sqrt{\frac{2}{w+1}} \int \frac{d^3 p d^3 q}{(2\pi)^3} \bar{\Phi}_F(\mathbf{q}) \Phi_I(\mathbf{p}) I_s(p, q) \\
& \quad \times \delta^3 \left(\mathbf{p} - \mathbf{q} + \frac{\epsilon_s(p) + \epsilon_s(q)}{w+1} (\mathbf{v}' - \mathbf{v}) \right). \quad (35)
\end{aligned}$$

The diquark transition matrix element should be multiplied by a factor 2 if either the initial or final diquark is composed of two identical heavy quarks. For the heavy diquark system we can now apply the v/c expansion. First we perform the integration over q in the form factors (A1)–(A8) and use relations (23). Then, applying the nonrelativistic limit, we get the following expressions for the form factors: (a) Scalar-to-scalar diquark transition

$$\begin{aligned}
h_+(w) &= F(w), \\
h_-(w) &= -(w+1)f(w)F(w), \quad (36)
\end{aligned}$$

(b) Scalar-to-axial vector diquark transition

$$\begin{aligned}
h_V(w) &= [1 + (w+1)f(w)]F(w), \\
h_{A_1}(w) &= h_{A_3}(w) = [1 + (w-1)f(w)]F(w), \\
h_{A_2}(w) &= -2f(w)F(w), \quad (37)
\end{aligned}$$

(c) Axial vector to scalar diquark transition

$$\begin{aligned}
h_V(w) &= \tilde{h}_{A_3}(w) = [1 + (w+1)f(w)]F(w), \\
h_{A_1}(w) &= [1 + (w-1)f(w)]F(w), \\
\tilde{h}_{A_2}(w) &= 0, \quad (38)
\end{aligned}$$

(d) Axial-vector-to-axial-vector diquark transition

$$\begin{aligned}
h_1(w) &= h_7(w) = F(w), \\
h_2(w) &= h_8(w) = -(w+1)f(w)F(w), \\
h_3(w) &= h_4(w) = [1 + (w+1)f(w)]F(w), \\
h_5(w) &= h_9(w) = 2f(w)F(w), \\
h_6(w) &= h_{10}(w) = 0, \quad (39)
\end{aligned}$$

where

$$\begin{aligned}
F(w) &= \sqrt{\frac{1}{w(w+1)}} \left(1 + \frac{m_a}{\sqrt{m_a^2 + (w^2-1)m_s^2}} \right)^{1/2} \\
& \quad \times \int \frac{d^3 p}{(2\pi)^3} \bar{\Phi}_F \left(\mathbf{p} + \frac{2m_s}{w+1} (\mathbf{v}' - \mathbf{v}) \right) \Phi_I(\mathbf{p}) \quad (40)
\end{aligned}$$

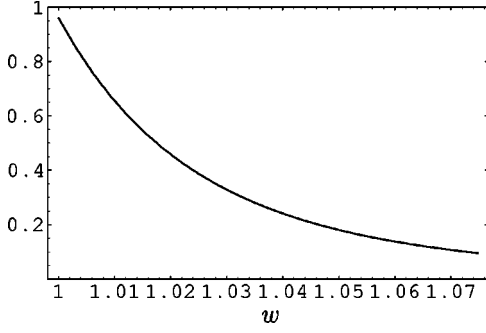
and

$$f(w) = \frac{m_s}{\sqrt{m_a^2 + (w^2-1)m_s^2} + m_a}. \quad (41)$$

The appearance of the terms proportional to the function $f(w)$ is the result of the account of the spectator quark recoil. Their contribution is important and distinguishes our approach from previous considerations [15,18]. We plot the function $F(w)$ for $bb \rightarrow bc$ and $bc \rightarrow cc$ diquark transitions in Figs. 3 and 4.

IV. DOUBLY HEAVY BARYON TRANSITIONS

The second step in studying weak transitions of doubly heavy baryons is the inclusion of the spectator light quark in the consideration. We carry out all further calculations in the limit of an infinitely heavy diquark mass, $M_d \rightarrow \infty$, treating the light quark relativistically. The transition matrix element between doubly heavy baryon states in the quark-diquark approximation (see Figs. 1 and 2) is given by [cf., Eqs. (6) and (7)]

FIG. 3. The function $F(w)$ for the $bb \rightarrow bc$ quark transition.

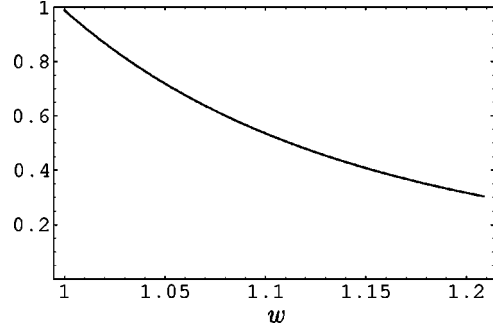
$$\begin{aligned} \langle B'(Q) | J_\mu^W | B(P) \rangle &= 2\sqrt{M_I M_F} \int \frac{d^3 p d^3 q}{(2\pi)^3} \Psi_{B',Q}(\mathbf{q}) \\ &\quad \times \langle d'(Q) | J_\mu^W | d(P) \rangle \\ &\quad \times \Psi_{B,P}(\mathbf{p}) \delta^3(\mathbf{p}_q - \mathbf{q}_q), \end{aligned} \quad (42)$$

where $\Psi_{B,P}(p)$ is the doubly heavy baryon wave function, \mathbf{p} and \mathbf{q} are the relative quark-diquark momenta, and \mathbf{p}_q and \mathbf{q}_q are the light quark momenta expressed in the form similar to (9).

The baryon ground-state wave function $\Psi_{B,P}(p)$ is a product of the spin-independent part $\Psi_B(p)$ satisfying the related quasipotential equation (1) and the spin part $U_B(v)$

$$\Psi_{B,P}(\mathbf{p}) = \Psi_B(\mathbf{p}) U_B(v). \quad (43)$$

The baryon spin wave function is constructed from the Dirac spinor $u_q(v)$ of the light spectator quark and the diquark wave function. The ground-state spin 1/2 baryons can contain either the scalar or axial vector diquark. The former

FIG. 4. The function $F(w)$ for the $bc \rightarrow cc$ quark transition.

baryon is denoted by $\Xi'_{QQ'}$, and the latter one by $\Xi_{QQ'}$. The ground-state spin 3/2 baryon can be formed only from an axial vector diquark and is denoted by $\Xi_{QQ'}^*$. To obtain the corresponding baryon spin states we use, in the baryon matrix elements, the following replacements

$$\begin{aligned} u_q(v) &\rightarrow U_{\Xi'_{QQ'}}(v), \\ [\varepsilon_\mu(v) u_q(v)]_{\text{spin } 1/2} &\rightarrow \frac{i}{\sqrt{3}} (\gamma_\mu + v_\mu) \gamma_5 U_{\Xi_{QQ'}}(v), \\ [\varepsilon^\mu(v) u_q(v)]_{\text{spin } 3/2} &\rightarrow U_{\Xi_{QQ'}^*}^\mu(v), \end{aligned} \quad (44)$$

where baryon spinor wave functions are normalized by $U_B^* U_B = 1$ ($B = \Xi', \Xi$), and the Rarita-Schwinger wave functions are normalized by $U_{\Xi^*}^\mu U_{\Xi^* \mu} = -1$.

Then the decay amplitudes of doubly heavy baryons in the infinitely heavy diquark limit are given by the following expressions.

$\Xi'_{QQ_s} \rightarrow \Xi'_{Q'Q_s}$ transition

$$\frac{\langle \Xi'_{Q'Q_s}(v') | J_\mu^W | \Xi'_{QQ_s}(v) \rangle}{2\sqrt{M_I M_F}} = [h_+(w)(v+v')_\mu + h_-(w)(v-v')_\mu] \bar{U}_{\Xi'_{Q'Q_s}}(v') U_{\Xi'_{QQ_s}}(v) \eta(w), \quad (45)$$

$\Xi_{QQ_s} \rightarrow \Xi_{Q'Q_s}$ and $\Xi_{QQ_s} \rightarrow \Xi'_{Q'Q_s}$ transitions

$$\begin{aligned} \frac{\langle \Xi_{Q'Q_s}(v') | J_\mu^W | \Xi_{QQ_s}(v) \rangle}{2\sqrt{M_I M_F}} &= \frac{i}{\sqrt{3}} [ih_V(w) \epsilon_{\mu\alpha\beta\gamma} v'^\beta v^\gamma - g_{\mu\alpha} h_{A_1}(w+1) + v_\mu v_\alpha h_{A_2}(w) \\ &\quad + v'_\mu v'_\alpha h_{A_3}(w)] \bar{U}_{\Xi_{Q'Q_s}}(v') \gamma_5 (\gamma^\alpha + v'^\alpha) U_{\Xi_{QQ_s}}(v) \eta(w), \end{aligned} \quad (46)$$

$$\begin{aligned} \frac{\langle \Xi'_{Q'Q_s}(v') | J_\mu^W | \Xi_{QQ_s}(v) \rangle}{2\sqrt{M_I M_F}} &= \frac{i}{\sqrt{3}} [ih_V(w) \epsilon_{\mu\alpha\beta\gamma} v'^\beta v^\gamma - g_{\mu\alpha} h_{A_1}(w+1) \\ &\quad + v'_\mu v'_\alpha \tilde{h}_{A_2}(w) + v_\mu v'_\alpha \tilde{h}_{A_3}(w)] \bar{U}_{\Xi'_{Q'Q_s}}(v') (\gamma^\alpha + v'^\alpha) \gamma_5 U_{\Xi_{QQ_s}}(v) \eta(w), \end{aligned} \quad (47)$$

$\Xi'_{Q_s} \rightarrow \Xi^*_{Q_s}$ and $\Xi^*_{Q_s} \rightarrow \Xi'_{Q_s}$ transitions

$$\frac{\langle \Xi^*_{Q_s}(v') | J_\mu^W | \Xi'_{Q_s}(v) \rangle}{2\sqrt{M_I M_F}} = [i h_V(w) \epsilon_{\mu\alpha\beta\gamma} v'^\beta v^\gamma - g_{\mu\alpha} h_{A_1}(w+1) + v_\mu v_\alpha h_{A_2}(w) + v'_\mu v'_\alpha h_{A_3}(w)] \bar{U}_{\Xi^*_{Q_s}}^\alpha(v') U_{\Xi'_{Q_s}}(v) \eta(w), \quad (48)$$

$$\frac{\langle \Xi'_{Q_s}(v') | J_\mu^W | \Xi^*_{Q_s}(v) \rangle}{2\sqrt{M_I M_F}} = [i h_V(w) \epsilon_{\mu\alpha\beta\gamma} v'^\beta v^\gamma - g_{\mu\alpha} h_{A_1}(w+1) + v'_\mu v'_\alpha \tilde{h}_{A_2}(w) + v_\mu v_\alpha \tilde{h}_{A_3}(w)] \bar{U}_{\Xi'_{Q_s}}(v') U_{\Xi^*_{Q_s}}^\alpha(v) \eta(w), \quad (49)$$

$\Xi_{Q_s} \rightarrow \Xi'_{Q_s}$ transition

$$\begin{aligned} \frac{\langle \Xi'_{Q_s}(v') | J_\mu^W | \Xi_{Q_s}(v) \rangle}{2\sqrt{M_I M_F}} = & -\frac{1}{3} \{ g_{\rho\lambda} [h_1(w)(v+v')_\mu + h_2(w)(v-v')_\mu] - h_3(w) g_{\mu\rho} v_\lambda - h_4(w) g_{\mu\lambda} v'_\rho \\ & + v'_\rho v_\lambda [h_5(w)v_\mu + h_6(w)v'_\mu] + i \epsilon_{\mu\alpha\beta\gamma} (g_\rho^\beta g_\lambda^\gamma [h_7(w)(v+v')^\alpha + h_8(w)(v \\ & - v')^\alpha] + v'^\beta v^\gamma [h_9(w)g_\rho^\alpha v_\lambda + h_{10}(w)g_\lambda^\alpha v'_\rho]) \} \bar{U}_{\Xi'_{Q_s}}(v') \gamma_5 (\gamma^\lambda + v'^\lambda) (\gamma^\rho \\ & + v^\rho) \\ & \times \gamma_5 U_{\Xi_{Q_s}}(v) \eta(w), \end{aligned} \quad (50)$$

$\Xi_{Q_s} \rightarrow \Xi^*_{Q_s}$ and $\Xi^*_{Q_s} \rightarrow \Xi_{Q_s}$ transitions

$$\begin{aligned} \frac{\langle \Xi^*_{Q_s}(v') | J_\mu^W | \Xi_{Q_s}(v) \rangle}{2\sqrt{M_I M_F}} = & -\frac{i}{\sqrt{3}} \{ g_{\rho\lambda} [h_1(w)(v+v')_\mu + h_2(w)(v-v')_\mu] - h_3(w) g_{\mu\rho} v_\lambda - h_4(w) g_{\mu\lambda} v'_\rho \\ & + v'_\rho v_\lambda [h_5(w)v_\mu + h_6(w)v'_\mu] + i \epsilon_{\mu\alpha\beta\gamma} (g_\rho^\beta g_\lambda^\gamma [h_7(w)(v+v')^\alpha + h_8(w)(v \\ & - v')^\alpha] + v'^\beta v^\gamma [h_9(w)g_\rho^\alpha v_\lambda + h_{10}(w)g_\lambda^\alpha v'_\rho]) \} \bar{U}_{\Xi^*_{Q_s}}^\lambda(v') (\gamma^\rho \\ & + v^\rho) \gamma_5 U_{\Xi_{Q_s}}(v) \eta(w), \end{aligned} \quad (51)$$

$$\begin{aligned} \frac{\langle \Xi'_{Q_s}(v') | J_\mu^W | \Xi^*_{Q_s}(v) \rangle}{2\sqrt{M_I M_F}} = & -\frac{i}{\sqrt{3}} \{ g_{\rho\lambda} [h_1(w)(v+v')_\mu + h_2(w)(v-v')_\mu] - h_3(w) g_{\mu\rho} v_\lambda - h_4(w) g_{\mu\lambda} v'_\rho \\ & + v'_\rho v_\lambda [h_5(w)v_\mu + h_6(w)v'_\mu] + i \epsilon_{\mu\alpha\beta\gamma} (g_\rho^\beta g_\lambda^\gamma [h_7(w)(v+v')^\alpha + h_8(w)(v \\ & - v')^\alpha] + v'^\beta v^\gamma [h_9(w)g_\rho^\alpha v_\lambda + h_{10}(w)g_\lambda^\alpha v'_\rho]) \} \bar{U}_{\Xi'_{Q_s}}(v') \gamma_5 (\gamma^\lambda \\ & + v'^\lambda) U_{\Xi^*_{Q_s}}(v) \eta(w), \end{aligned} \quad (52)$$

$\Xi^*_{Q_s} \rightarrow \Xi^*_{Q_s}$ transition

$$\begin{aligned} \frac{\langle \Xi_{Q_s}^* (v') | J_\mu^W | \Xi_{Q_s}^* (v) \rangle}{2\sqrt{M_I M_F}} = & -\{g_{\rho\lambda}[h_1(w)(v+v')_\mu + h_2(w)(v-v')_\mu] - h_3(w)g_{\mu\rho}v_\lambda - h_4(w)g_{\mu\lambda}v'_\rho \\ & + v'_\rho v_\lambda[h_5(w)v_\mu + h_6(w)v'_\mu] + i\epsilon_{\mu\alpha\beta\gamma}(g_\rho^\beta g_\lambda^\gamma[h_7(w)(v+v')^\alpha + h_8(w)(v-v')^\alpha] \\ & + v'^\beta v^\gamma[h_9(w)g_\rho^\alpha v_\lambda + h_{10}(w)g_\lambda^\alpha v'_\rho])\} \bar{U}_{\Xi_{Q_s}^*}^\lambda(v') U_{\Xi_{Q_s}^*}^\rho(v) \eta(w). \end{aligned} \quad (53)$$

Here $\eta(w)$ is the heavy diquark–light quark Isgur-Wise function, which is determined by the dynamics of the light spectator quark q and is calculated similarly to Eq. (35)

$$\begin{aligned} \eta(w) = & \sqrt{\frac{2}{w+1}} \int \frac{d^3 p d^3 q}{(2\pi)^3} \Psi_B(\mathbf{q}) \Psi_B(\mathbf{p}) I_q(p, q) \\ & \times \delta^3\left(\mathbf{p} - \mathbf{q} + \frac{\epsilon_q(p) + \epsilon_q(q)}{w+1}(\mathbf{v}' - \mathbf{v})\right). \end{aligned} \quad (54)$$

We plot the Isgur-Wise function $\eta(w)$ in Fig. 5. In the non-relativistic limit for heavy quarks, the diquark form factors $h_i(w)$ obey relations (36)–(39). In this limit the baryon transition matrix elements contain the common factor $F(w)\eta(w)$ (cf., [18]).

V. SEMILEPTONIC DECAY RATES OF DOUBLY HEAVY BARYONS

The exclusive differential rate of the doubly heavy baryon semileptonic decay $B \rightarrow B' e \bar{\nu}$ can be written in the form

$$\frac{d\Gamma}{dw} = \frac{G_F^2 |V_{bc}|^2 M_F^3 \sqrt{w^2 - 1}}{48\pi^3} \Omega(w), \quad (55)$$

where $w = (v \cdot v') = (M_I^2 + M_F^2 - k^2)/(2M_I M_F)$, $k = P - Q$ and the function $\Omega(w)$ is the contraction of the hadronic transition matrix elements and the leptonic tensor. For the massless leptons the differential decay rates of the transitions $\Xi_{Q_s}^* \rightarrow \Xi_{Q_s}$ and $\Xi_{Q_s} \rightarrow \Xi_{Q_s}^*$ are as follows:

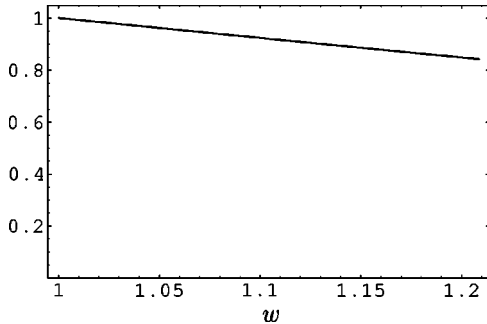


FIG. 5. The Isgur-Wise function $\eta(w)$ of the light quark–heavy diquark bound system.

$$\begin{aligned} \frac{d\Gamma}{dw}(\Xi_{Q_s}' \rightarrow \Xi_{Q_s}) = & \frac{G_F^2 |V_{Q_s}|^2}{72\pi^3} (w^2 - 1)^{1/2} (w + 1)^3 M_F^3 \\ & \times \left\{ 2(M_I^2 + M_F^2 - 2M_I M_F w) \left[h_{A_1}^2(w) \right. \right. \\ & \left. \left. + \frac{w-1}{w+1} h_V^2(w) \right] + \{(M_F - M_I w) h_{A_1}(w) \right. \\ & \left. \left. + (w-1)[M_F h_{A_2}(w) + M_I h_{A_3}(w)]\}^2 \right\} \eta^2(w), \end{aligned} \quad (56)$$

$$\begin{aligned} \frac{d\Gamma}{dw}(\Xi_{Q_s} \rightarrow \Xi_{Q_s}') = & \frac{M_F^3}{M_I^3} \frac{d\Gamma}{dw}(\Xi_{Q_s}' \rightarrow \Xi_{Q_s}, M_I \\ & \leftrightarrow M_F, h_{A_2} \rightarrow \tilde{h}_{A_2}, h_{A_3} \rightarrow \tilde{h}_{A_3}), \end{aligned} \quad (57)$$

The differential decay rates for other transitions are given in the Appendix B. The semileptonic decay rates of doubly heavy baryons are calculated in the nonrelativistic limit for heavy quarks and presented in Tables II and III.

VI. CONCLUSIONS

In this paper we calculated the semileptonic decay rates of doubly heavy baryons in the quark-diquark approximation.

TABLE II. Semileptonic decay rates of doubly heavy baryons Ξ_{bb} and Ξ_{bc} (in $\times 10^{-14}$ GeV).

Decay	Our	Ref. [16]	Ref. [15]	Ref. [17]	Ref. [13]
$\Xi_{bb} \rightarrow \Xi_{bc}'$	1.64	4.28			
$\Xi_{bb} \rightarrow \Xi_{bc}$	3.26	28.5		8.99	
$\Xi_{bb} \rightarrow \Xi_{bc}^*$	1.05	27.2		2.70	
$\Xi_{bb}^* \rightarrow \Xi_{bc}'$	1.63	8.57			
$\Xi_{bb}^* \rightarrow \Xi_{bc}$	0.55	52.0			
$\Xi_{bb}^* \rightarrow \Xi_{bc}^*$	3.83	12.9			
$\Xi_{bc}' \rightarrow \Xi_{cc}$	1.76	7.76			
$\Xi_{bc}' \rightarrow \Xi_{cc}^*$	3.40	28.8			
$\Xi_{bc} \rightarrow \Xi_{cc}$	4.59	8.93	4.0	8.87	0.8
$\Xi_{bc} \rightarrow \Xi_{cc}^*$	1.43	14.1	1.2	2.66	
$\Xi_{bc}^* \rightarrow \Xi_{cc}$	0.75	27.5			
$\Xi_{bc}^* \rightarrow \Xi_{cc}^*$	5.37	17.2			

TABLE III. Semileptonic decay rates of doubly heavy baryons Ω_{bb} and Ω_{bc} (in $\times 10^{-14}$ GeV).

Decay	Γ	Decay	Γ
$\Omega_{bb} \rightarrow \Omega'_{bc}$	1.66	$\Omega'_{bc} \rightarrow \Omega_{cc}$	1.90
$\Omega_{bb} \rightarrow \Omega_{bc}$	3.40	$\Omega'_{bc} \rightarrow \Omega_{cc}^*$	3.66
$\Omega_{bb} \rightarrow \Omega_{bc}^*$	1.10	$\Omega_{bc} \rightarrow \Omega_{cc}$	4.95
$\Omega_{bb}^* \rightarrow \Omega'_{bc}$	1.70	$\Omega_{bc} \rightarrow \Omega_{cc}^*$	1.48
$\Omega_{bb}^* \rightarrow \Omega_{bc}$	0.57	$\Omega_{bc}^* \rightarrow \Omega_{cc}$	0.80
$\Omega_{bb}^* \rightarrow \Omega_{bc}^*$	3.99	$\Omega_{bc}^* \rightarrow \Omega_{cc}^*$	5.76

The weak transition matrix elements between heavy diquark states were calculated with the self-consistent account of the spectator quark recoil. It was shown that recoil effects lead to the additional contributions to the transition matrix elements. Such terms were missed in the previous quark model calculations. If we neglect these recoil contributions, the previously obtained expressions [15,18] for heavy diquark transition matrix elements are reproduced. It was found that these recoil terms, which are proportional to the ratio of the heavy spectator to the final active quark mass [see Eqs. (36)–(41)], give important contributions to transition matrix elements of doubly heavy diquarks even in the nonrelativistic limit. In this limit these weak transition matrix elements are proportional to the function $F(w)$ (40), which is expressed through the overlap integral of the heavy diquark wave functions. The function $F(w)$ falls off rather rapidly, especially for $bb \rightarrow bc$ diquark transition where the spectator quark is the b quark (see Figs. 3 and 4). Such a decrease is the consequence of the large mass of the spectator quark and high recoil momenta ($q_{\max} \approx m_b - m_c \sim 3.33$ GeV) transferred.

We calculated the doubly heavy baryon transition matrix elements in the heavy diquark limit. The expressions for transition amplitudes and decay rates were obtained for the most general parametrization of the diquark transition matrix elements. The Isgur-Wise function $\eta(w)$ (54) for the light quark–heavy diquark bound system was determined. This function is very similar to the Isgur-Wise function of the

heavy-light meson in our model [28] as it is required by the heavy quark symmetry. In the heavy quark limit, the baryon transition matrix elements contain the common factor, which is the product of the diquark form factor $F(w)$ and the Isgur-Wise function $\eta(w)$.

Our results for the semileptonic decay rates of doubly heavy baryons Ξ_{bb} and Ξ_{bc} are compared with previous predictions in Table II. It is seen from this table that results of different approaches differ substantially. Most of previous papers [13,15,17] give their predictions only for selected decay modes. Their values agree with our in the order of magnitude. Our predictions are smaller than the QCD sum rule results [17] by a factor of ~ 2 . This can be a result of our treatment of the heavy spectator quark recoil in the heavy diquark. On the other hand the authors of Ref. [16], where the Bethe-Salpeter equation is used, give more decay channels. Their results are substantially higher than ours, for some decays the difference reaches almost two orders of magnitude, which seems quite strange [e.g., for the sum of the semileptonic decays $\Xi_{bb} \rightarrow \Xi_{bc}^{(*)}$ Ref. [16] predicts $\sim 6 \times 10^{-13}$ GeV, which almost saturates the estimate of the total decay rate $\Gamma_{\Xi_{bb}}^{\text{total}} \sim (8.3 \pm 0.3) \times 10^{-13}$ GeV [2] and thus is unlikely].

All considerations in the present paper were done in the heavy quark limit. The calculations of the semileptonic decay rates of heavy mesons indicate that the $1/m_Q$ corrections to the heavy quark limit (especially $1/m_c$) are important [4,28–30]. Thus they can give sizeable contributions also to the semileptonic decay rates of doubly heavy baryons. Their account will result in the more complicated structure of the baryon matrix elements. We plan to study these corrections in the future.

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APPENDIX A: FORM FACTORS OF THE DIQUARK TRANSITIONS

$$\begin{aligned}
 h_+(w) = h_1(w) = h_7(w) = & \sqrt{\frac{2}{w+1}} \int \frac{d^3 p d^3 q}{(2\pi)^3} \bar{\Phi}_{d'}(\mathbf{q}) \sqrt{\frac{\epsilon_a(q) + m_a}{2\epsilon_a(q)}} \sqrt{\frac{\epsilon_b(p) + m_b}{2\epsilon_b(p)}} \left(1 - \frac{\sqrt{\mathbf{p}^2} \sqrt{\mathbf{q}^2}}{[\epsilon_a(q) + m_a][\epsilon_b(p) + m_b]} \right) \\
 & \times \Phi_d(\mathbf{p}) I_s(p, q) \delta^3 \left(\mathbf{p} - \mathbf{q} + \frac{\epsilon_s(p) + \epsilon_s(q)}{w+1} (\mathbf{v}' - \mathbf{v}) \right), \tag{A1}
 \end{aligned}$$

$$\begin{aligned}
 h_-(w) = h_2(w) = h_8(w) = & \sqrt{\frac{2}{w+1}} \int \frac{d^3 p d^3 q}{(2\pi)^3} \bar{\Phi}_{d'}(\mathbf{q}) \sqrt{\frac{\epsilon_a(q) + m_a}{2\epsilon_a(q)}} \sqrt{\frac{\epsilon_b(p) + m_b}{2\epsilon_b(p)}} \sqrt{\frac{w+1}{w-1}} \left(\frac{\sqrt{\mathbf{q}^2}}{\epsilon_a(q) + m_a} \right. \\
 & \left. - \frac{\sqrt{\mathbf{p}^2}}{\epsilon_b(p) + m_b} \right) \Phi_d(\mathbf{p}) I_s(p, q) \delta^3 \left(\mathbf{p} - \mathbf{q} + \frac{\epsilon_s(p) + \epsilon_s(q)}{w+1} (\mathbf{v}' - \mathbf{v}) \right), \tag{A2}
 \end{aligned}$$

$$h_V(w) = h_3(w) = h_4(w) = \sqrt{\frac{2}{w+1}} \int \frac{d^3 p d^3 q}{(2\pi)^3} \bar{\Phi}_{d'}(\mathbf{q}) \sqrt{\frac{\epsilon_a(q) + m_a}{2\epsilon_a(q)}} \sqrt{\frac{\epsilon_b(p) + m_b}{2\epsilon_b(p)}} \left[1 + \sqrt{\frac{w+1}{w-1}} \left(\frac{\sqrt{\mathbf{q}^2}}{\epsilon_a(q) + m_a} + \frac{\sqrt{\mathbf{p}^2}}{\epsilon_b(p) + m_b} \right) + \frac{\sqrt{\mathbf{p}^2} \sqrt{\mathbf{q}^2}}{[\epsilon_a(q) + m_a][\epsilon_b(p) + m_b]} \right] \Phi_d(\mathbf{p}) I_s(p, q) \delta^3 \left(\mathbf{p} - \mathbf{q} + \frac{\epsilon_s(p) + \epsilon_s(q)}{w+1} (\mathbf{v}' - \mathbf{v}) \right), \quad (\text{A3})$$

$$h_{A_1}(w) = \sqrt{\frac{2}{w+1}} \int \frac{d^3 p d^3 q}{(2\pi)^3} \bar{\Phi}_{d'}(\mathbf{q}) \sqrt{\frac{\epsilon_a(q) + m_a}{2\epsilon_a(q)}} \sqrt{\frac{\epsilon_b(p) + m_b}{2\epsilon_b(p)}} \left[1 + \sqrt{\frac{w-1}{w+1}} \left(\frac{\sqrt{\mathbf{q}^2}}{\epsilon_a(q) + m_a} + \frac{\sqrt{\mathbf{p}^2}}{\epsilon_b(p) + m_b} \right) + \frac{\sqrt{\mathbf{p}^2} \sqrt{\mathbf{q}^2}}{[\epsilon_a(q) + m_a][\epsilon_b(p) + m_b]} \right] \Phi_d(\mathbf{p}) I_s(p, q) \delta^3 \left(\mathbf{p} - \mathbf{q} + \frac{\epsilon_s(p) + \epsilon_s(q)}{w+1} (\mathbf{v}' - \mathbf{v}) \right), \quad (\text{A4})$$

$$h_{A_2}(w) = h_5(w) = h_9(w) = \sqrt{\frac{2}{w+1}} \int \frac{d^3 p d^3 q}{(2\pi)^3} \bar{\Phi}_{d'}(\mathbf{q}) \sqrt{\frac{\epsilon_a(q) + m_a}{2\epsilon_a(q)}} \sqrt{\frac{\epsilon_b(p) + m_b}{2\epsilon_b(p)}} \frac{2\sqrt{\mathbf{q}^2}}{\sqrt{w^2-1}[\epsilon_a(q) + m_a]} \times \left(1 + \sqrt{\frac{w+1}{w-1}} \frac{\sqrt{\mathbf{p}^2}}{\epsilon_b(p) + m_b} \right) \Phi_d(\mathbf{p}) I_s(p, q) \delta^3 \left(\mathbf{p} - \mathbf{q} + \frac{\epsilon_s(p) + \epsilon_s(q)}{w+1} (\mathbf{v}' - \mathbf{v}) \right), \quad (\text{A5})$$

$$\tilde{h}_{A_2}(w) = h_6(w) = h_{10}(w) = \sqrt{\frac{2}{w+1}} \int \frac{d^3 p d^3 q}{(2\pi)^3} \bar{\Phi}_{d'}(\mathbf{q}) \sqrt{\frac{\epsilon_a(q) + m_a}{2\epsilon_a(q)}} \sqrt{\frac{\epsilon_b(p) + m_b}{2\epsilon_b(p)}} \frac{2\sqrt{\mathbf{p}^2}}{\sqrt{w^2-1}[\epsilon_b(p) + m_b]} \times \left(1 + \sqrt{\frac{w+1}{w-1}} \frac{\sqrt{\mathbf{q}^2}}{\epsilon_a(q) + m_a} \right) \Phi_d(\mathbf{p}) I_s(p, q) \delta^3 \left(\mathbf{p} - \mathbf{q} + \frac{\epsilon_s(p) + \epsilon_s(q)}{w+1} (\mathbf{v}' - \mathbf{v}) \right), \quad (\text{A6})$$

$$h_{A_3}(w) = \sqrt{\frac{2}{w+1}} \int \frac{d^3 p d^3 q}{(2\pi)^3} \bar{\Phi}_{d'}(\mathbf{q}) \sqrt{\frac{\epsilon_a(q) + m_a}{2\epsilon_a(q)}} \sqrt{\frac{\epsilon_b(p) + m_b}{2\epsilon_b(p)}} \left[1 + \sqrt{\frac{w+1}{w-1}} \left(\frac{w-1}{w+1} \frac{\sqrt{\mathbf{q}^2}}{\epsilon_a(q) + m_a} + \frac{\sqrt{\mathbf{p}^2}}{\epsilon_b(p) + m_b} \right) + \frac{w+1}{w-1} \frac{\sqrt{\mathbf{p}^2} \sqrt{\mathbf{q}^2}}{[\epsilon_a(q) + m_a][\epsilon_b(p) + m_b]} \right] \Phi_d(\mathbf{p}) I_s(p, q) \delta^3 \left(\mathbf{p} - \mathbf{q} + \frac{\epsilon_s(p) + \epsilon_s(q)}{w+1} (\mathbf{v}' - \mathbf{v}) \right), \quad (\text{A7})$$

$$\tilde{h}_{A_3}(w) = \sqrt{\frac{2}{w+1}} \int \frac{d^3 p d^3 q}{(2\pi)^3} \bar{\Phi}_{d'}(\mathbf{q}) \sqrt{\frac{\epsilon_a(q) + m_a}{2\epsilon_a(q)}} \sqrt{\frac{\epsilon_b(p) + m_b}{2\epsilon_b(p)}} \left[1 + \sqrt{\frac{w+1}{w-1}} \left(\frac{\sqrt{\mathbf{q}^2}}{\epsilon_a(q) + m_a} + \frac{w-1}{w+1} \frac{\sqrt{\mathbf{p}^2}}{\epsilon_b(p) + m_b} \right) + \frac{w+1}{w-1} \frac{\sqrt{\mathbf{p}^2} \sqrt{\mathbf{q}^2}}{[\epsilon_a(q) + m_a][\epsilon_b(p) + m_b]} \right] \Phi_d(\mathbf{p}) I_s(p, q) \delta^3 \left(\mathbf{p} - \mathbf{q} + \frac{\epsilon_s(p) + \epsilon_s(q)}{w+1} (\mathbf{v}' - \mathbf{v}) \right). \quad (\text{A8})$$

APPENDIX B: DIFFERENTIAL DECAY RATES OF DOUBLY HEAVY BARYONS

$$\frac{d\Gamma}{dw} (\Xi'_{QQ_s} \rightarrow \Xi'_{Q'Q_s}) = \frac{G_F^2 |V_{QQ'}|^2}{24\pi^3} (w^2-1)^{3/2} (w+1) M_F^3 (M_I + M_F)^2 \left[h_+(w) - \frac{M_I - M_F}{M_I + M_F} h_-(w) \right]^2 \eta^2(w), \quad (\text{B1})$$

$$\begin{aligned} \frac{d\Gamma}{dw} (\Xi'_{QQ_s} \rightarrow \Xi^*_{Q'Q_s}) &= \frac{G_F^2 |V_{QQ'}|^2}{36\pi^3} (w^2-1)^{1/2} (w+1)^3 M_F^3 \left\{ (M_I^2 + M_F^2 - 2M_I M_F w) \left[3h_{A_1}^2(w) + 2\frac{w-1}{w+1} h_V^2(w) \right] \right. \\ &\quad + (w-1) \{ M_I^2 (w+1) h_{A_1}^2(w) + (w-1) [M_F h_{A_2}(w) + M_I h_{A_3}(w)]^2 \\ &\quad \left. + 2(M_F - M_I w) h_{A_1}(w) [M_F h_{A_2}(w) + M_I h_{A_3}(w)] \right\} \eta^2(w), \quad (\text{B2}) \end{aligned}$$

$$\frac{d\Gamma}{dw} (\Xi^*_{QQ_s} \rightarrow \Xi'_{Q'Q_s}) = \frac{1}{2} \frac{M_F^3}{M_I^3} \frac{d\Gamma}{dw} (\Xi'_{QQ_s} \rightarrow \Xi^*_{Q'Q_s}, M_I \leftrightarrow M_F, h_{A_2} \rightarrow \tilde{h}_{A_2}, h_{A_3} \rightarrow \tilde{h}_{A_3}), \quad (\text{B3})$$

$$\begin{aligned}
\frac{d\Gamma}{dw}(\Xi_{Q\bar{Q}_s} \rightarrow \Xi_{Q'\bar{Q}_s}) &= \frac{G_F^2 |V_{Q\bar{Q}'}|^2}{216\pi^3} (w^2-1)^{1/2} (w+1)^3 M_F^3 \left\{ 2(M_I^2 + M_F^2 - 2M_I M_F w) \left((2-w)\{4h_7^2(w) \right. \right. \\
&\quad \left. \left. - (w-1)[h_9(w) + h_{10}(w)]^2\right\} + \frac{w-1}{w+1} [h_3(w) + h_4(w)]^2 + (w-1)[2h_7(w) + h_9(w) + h_{10}(w)]^2 \right) \\
&\quad + 4 \left[(M_I - M_F)h_7(w) - \frac{w-1}{w+1} (M_I + M_F)h_8(w) \right]^2 + \frac{w-1}{w+1} \{(w+2)[(M_I + M_F)h_1(w) \\
&\quad - (M_I - M_F)h_2(w)] + (M_I - M_F w)h_3(w) + (M_F - M_I w)h_4(w) + (w^2-1)[M_F h_5(w) \\
&\quad + M_I h_6(w)]\}^2 \Big\} \eta^2(w), \tag{B4}
\end{aligned}$$

$$\begin{aligned}
\frac{d\Gamma}{dw}(\Xi_{Q\bar{Q}_s} \rightarrow \Xi_{Q'\bar{Q}_s}^*) &= \frac{G_F^2 |V_{Q\bar{Q}'}|^2}{108\pi^3} (w^2-1)^{1/2} (w+1)^3 M_F^3 \left\{ (M_I^2 + M_F^2 - 2M_I M_F w) \left(h_7^2(w) + [h_7(w) + (w-1) \right. \right. \\
&\quad \times (h_9(w) + h_{10}(w))]^2 + 3 \left(\frac{w-1}{w+1} \right)^2 \{h_8^2(w) + [h_8(w) + (w+1)(h_9(w) - h_{10}(w))]^2\} + 2 \frac{w-1}{w+1} ([h_3(w) \\
&\quad - h_4(w)]^2 + h_3(w)h_4(w) - (w^2-1)\{[h_9(w) - h_{10}(w)]^2 + h_9(w)h_{10}(w)\}) \Big) + \left[(M_I - M_F)h_7(w) \right. \\
&\quad \left. - \frac{w-1}{w+1} (M_I + M_F)h_8(w) \right]^2 + \frac{w-1}{w+1} \{(w-1)[(M_I + M_F)h_1(w) - (M_I - M_F)h_2(w)] + (M_I \\
&\quad - M_F w)h_3(w) + (M_F - M_I w)h_4(w) + (w^2-1)[M_F h_5(w) + M_I h_6(w)]\}^2 \Big\} \eta^2(w), \tag{B5}
\end{aligned}$$

$$\frac{d\Gamma}{dw}(\Xi_{Q\bar{Q}_s}^* \rightarrow \Xi_{Q'\bar{Q}_s}) = \frac{1}{2} \frac{d\Gamma}{dw}(\Xi_{Q\bar{Q}_s} \rightarrow \Xi_{Q'\bar{Q}_s}^*), \tag{B6}$$

$$\begin{aligned}
\frac{d\Gamma}{dw}(\Xi_{Q\bar{Q}_s}^* \rightarrow \Xi_{Q'\bar{Q}_s}^*) &= \frac{G_F^2 |V_{Q\bar{Q}'}|^2}{216\pi^3} (w^2-1)^{1/2} (w+1)^3 M_F^3 \left\{ (M_I^2 + M_F^2 - 2M_I M_F w) \left[5\{h_7^2(w) + [h_7(w) + (w-1) \right. \right. \\
&\quad \times (h_9(w) + h_{10}(w))]^2\} + 3 \left(\frac{w-1}{w+1} \right)^2 \{h_8^2(w) + [h_8(w) + (w+1)(h_9(w) - h_{10}(w))]^2\} + 2 \frac{w-1}{w+1} (2[h_3(w) \\
&\quad - h_4(w)]^2 + 5h_3(w)h_4(w) - (w^2-1)\{2[h_9(w) - h_{10}(w)]^2 + 5h_9(w)h_{10}(w)\}) \Big] + 5 \left[(M_I - M_F)h_7(w) \right. \\
&\quad \left. - \frac{w-1}{w+1} (M_I + M_F)h_8(w) \right]^2 + \frac{w-1}{w+1} \{(2w+1)[(M_I + M_F)h_1(w) - (M_I - M_F)h_2(w)] + (M_I \\
&\quad - M_F w)h_3(w) + (M_F - M_I w)h_4(w) + (w^2-1)[M_F h_5(w) + M_I h_6(w)]\}^2 + \{(M_I - M_F w)h_3(w) + (M_F \\
&\quad - M_I w)h_4(w) + (w^2-1)[M_F h_5(w) + M_I h_6(w)]\}^2 - 2(w+2)(w-1)((M_I + M_F)h_1(w) - (M_I \\
&\quad - M_F)h_2(w))^2 \Big\} \eta^2(w). \tag{B7}
\end{aligned}$$

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