All-charm tetraquarks

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We investigate four-body states with only charm quarks. Working in a large but finite oscillator basis, we present a net binding analysis to determine if the resulting states are stable against breakup into a pair of $c\bar{c}$ mesons. We find several close-lying bound states in the two models we examine.

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INTRODUCTION

The spectrum of multiquark states has been investigated for three decades, but very little in the way of quantitative results for the spectrum of all-charm tetraquarks, $c^2\bar{c}^2$, has been published. Iwasaki [1] provides the first treatment that argues bound states of $c^2\bar{c}^2$ exist and estimates their mass. His argument is based on a string model of hadrons to establish the plausibility of $c^2\bar{c}^2$ exotics. The best value for the charm quark mass of 1.5 GeV (in 1975) was used to estimate the mass of the tetraquark bound state in the neighborhood of 6 GeV (or 6.2 GeV if the J/ψ was used).

Some predictions concerning the existence of tetraquark states have already been made. Carlson, Heller, and Tjon [2], for example, argue that a dimeson system, a bound state of two mesons, should be possible in the limit that the two quarks are of infinite mass and the two antiquarks are lighter. They note that their argument fails if one or both of the antiquarks has the same mass as the heavy quarks but do not claim that this prohibits binding in the case where all quarks and antiquarks are of equal mass. In a related vein, Lipkin [3] has proposed a pentaquark state which is of particular interest given the recent experimental detection of these states.

A number of other approaches to multiquark models exist in the literature. Jaffe [4], using an early version of the bag model, made predictions for tetraquark spectroscopy of the type $q^2\bar{q}^2$ with q a light quark (lighter than charm). In a similar vein, Schaffner-Beilich and Vischer [5] give impressive systematic treatments of charmlets (systems with at least one charm quark plus lighter quarks) but do not treat $c^2\bar{c}^2$.

In addition to the bag model, many variants of a wide class of potential models are used to describe multiquark states. Successful models founded upon the work of Isgur and Weinstein [6] tended to confirm the earlier prediction of Jaffe, and identified the $a_0(980)$ and $f_0(980)$ as molecular states of four quarks, for example (see especially Isgur and Godfrey [7]). Models in the spirit of Karl and Ericson [8] employ a pion exchange potential to investigate hadronic molecules, including meson-meson, meson-baryon, and baryon-baryon systems. Stancu [9] investigated the stability of multiquark hadrons and compared the one gluon exchange potential approach with the Goldstone boson exchange models of Glozman [10] and found, for example, that the models differed on the question of the existence of the H-dibaryon. However, all these models include at least one light quark in the bound state and do not consider four bound charmed quarks.

Lattice methods represent another approach to many body states. A massive and comprehensive review of the state of the art of lattice efforts by Bali [11] is worthwhile and instructive reading. To our knowledge, however, lattice methods have not been applied to heavy tetraquark bound states of the type we examine here.

Most recently, for example, the report by BaBar [12] of a narrow state at 2.32 GeV has stimulated a large number of theoretical efforts (see Barnes, Close, and Lipkin [13] and references therein) with tetraquark states that include two flavors of light quarks. Tetraquarks of a single heavy flavor have not received similar attention although it is interesting to note that double charmonium production has been reported at Belle [14] and BaBar [15] at center of mass energy over 10 GeV. One motivation for the present effort is to examine whether similar experiments designed to detect all-charm tetraquarks could be theoretically motivated.

In the remainder of this paper, we present a pair of parametrized Hamiltonians that are used to approximately fit the low-lying charmonium spectrum. These Hamiltonians are then employed to calculate the spectrum of all-charm tetraquarks in a quantum 4-body framework. We then present a net binding analysis to determine if bound states are obtained in our treatment.

THE HAMILTONIAN

The Hamiltonians we employ are nonrelativistic with interactions inspired by the one gluon exchange potential. The main difference is that we treat the coupling strengths of the interactions as free parameters and simply adjust them to achieve an acceptable fit to the low-lying charmonium system. The specific form for our Hamiltonian is

$$H = T_{rel} + \frac{1}{2}\kappa\vec{\lambda}^2 + \alpha\left(H_{cm} - \frac{3}{2}\hbar\Omega\right) + \sum_{i<j}\vec{\lambda}_i\cdot\vec{\lambda}_j\left[\frac{a}{r_{ij}} - \sigma r_{ij} + \beta_0\delta^3(r_{ij})_{S=0} + \beta_1\delta^3(r_{ij})_{S=1} - \omega V_{tens} - \eta V_{so}\right]. \quad (1)$$

The second and third terms of the Hamiltonian serve as projection operators. The first operator, a simple multiple of the SU(3) color Casimir operator, acts on nonsinglet color states so that, with positive κ, it pushes them up relative to the color singlets. The second is a simple harmonic oscillator

TABLE I. Even orbital parity charmonium masses (MeV) for Case 1. The column N=10 presents the fit to the quoted experimental masses. Other columns present masses obtained in truncated spaces.

ID	J	$M_{\rm exp}$	N=0	N=2	N=4	N=6	N=8	N=10
$\eta_c(1S)$	0	2979	3537	3369	3203	3151	3092	3069
J/ψ	1	3097	4302	3466	3343	3198	3155	3097
$\eta_c(2S)$	0	3654		4854	4417	3995	3855	3676
$\psi(2S)$	1	3686		5164	4609	4327	4005	3878

TABLE II. Odd orbital parity charmonium masses (MeV) for Case 1. The column N=9 presents the fit to the quoted experimental masses. Other columns present masses obtained in truncated spaces.

ID	J	$M_{\rm exp}$	N=1	N=3	N=5	N=7	N=9
$\chi_{c0}(1P)$	0	3415	4311	4014	3719	3542	3423
$\chi_{c1}(1P)$	1	3511	4455	4062	3808	3638	3511
$h_c(1P)$	1	3526	4483	4071	3825	3656	3528
$\chi_{c2}(1P)$	2	3556	4498	4076	3834	3666	3537

Hamiltonian that acts on the center of mass (CM) coordinates so that, with positive α, it pushes states of excited CM motion higher in the spectrum relative to the states with pure 0S harmonic oscillator CM motion. For all applications in this paper we will set κ and α equal to 3 GeV and 10 (unitless), respectively. These values are sufficiently large to provide a clean separation between the low-lying states we present here, the ''physical'' states, and states with color nonsinglet character and/or excited CM motion.

The tensor and spin-orbit potentials are defined by the following relationships. The orbital angular momentum referred to in the spin-orbit interaction is the relative orbital angular momentum,

$$V_{tens}=\frac{1}{r^3}[\vec{S}_1\cdot\vec{S}_2-3\vec{S}_1\cdot\hat{r}\vec{S}_2\cdot\hat{r}]$$

$$V_{so}=\frac{\vec{l}\cdot\vec{S}}{r^3}. \tag{2}$$

The other parameter values in the Hamiltonian will be specified with each of the two fits.

We realize that our Hamiltonian admits van der Waals interactions in a multiquark system. Willey [16] has estimated the long-range contribution of these forces to behave as R^{-7} where R is the separation of two color singlet subsystems. An earlier work by Greenberg and Lipkin [17] estimates a different power law dependence that is inconsistent with experimental data. Working in the limited spaces that we employ, we hope that such long-range forces do not affect our conclusions.

THE COMPUTATIONAL FRAMEWORK AND RESULTS

To solve the quantum 2-body and 4-body problems, we select a single particle basis of harmonic oscillator states and construct Slater determinants of good total magnetic projection and good parity. We diagonalize the Hamiltonian in the Slater determinant basis to obtain the mass spectra and wave functions using the Lanczos algorithm [18]. Once the wave functions are obtained, we evaluate the total angular momentum, J, of each eigenstate.

The Lanczos method is an iterative procedure that obtains the eigenvectors and eigenvalues of a symmetric matrix. We choose a trial vector, $|\chi_1\rangle$ (with $b_0=0$) to begin the iteration, which is outlined below. The ''$\pm$'' appearing in the Lanczos algorithm simply reflects the overall phase uncertainty of any quantum state and has no consequences for any observable quantity. By our convention we take the positive root,

$$|\eta_{n+1}\rangle=H|\chi_n\rangle-b_n|\chi_{n-1}\rangle$$

$$a_n=\langle\eta_{n+1}|\chi_n\rangle$$

$$|\eta'_{n+1}\rangle=|\eta_{n+1}\rangle-a_n|\chi_n\rangle$$

$$b_{n+1}=\pm[\langle\eta'_{n+1}|\eta'_{n+1}\rangle]^{1/2}$$

$$|\chi_{n+1}\rangle=\frac{|\eta'_{n+1}\rangle}{b_{n+1}}.$$

This algorithm produces a tri-diagonal matrix that is unitarily equivalent to the original matrix. The Lanczos method is computationally efficient since it minimally requires only two vectors to be stored simultaneously, and it is easily optimized to run in a parallel computational architecture.

TABLE III. 2- and 4-body ''free particle'' spectra with computed correction for Case 1.

N=0	N=1	N=2	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10
Lowest 2-body free particle spectra (MeV)										
3940	4580	3568	4023	3406	3761	3315	3606	3256	3503	3215
Lowest 4-body free particle spectra (MeV)										
8840	9480	7979	8488	7538	7966	7261	7632	7070	7397	6929
$\Delta M^{free,N,1280}_{cc\bar{c}\bar{c}}$ (MeV)										
960	960	471	525	402	375	287	303	258	230	208

TABLE IV. Case 1 even parity tetraquarks mass spectra, uncorrected via Eq. (4).

$M^{full,N,1280}_{cc\overline{cc}}$						
J	N=0	N=2	N=4	N=6	N=8	N=10
0	7422	7139	6760	6615	6448	6367
1	7896	7266	6887	6682	6515	6411
2	8252	7450	6995	6762	6576	6459
0	9615	8022	7428	7095	6869	6719

The basis depends on two parameters. One parameter, N, determines the dimension of the basis space. N defines the maximal number of allowed oscillator excitations in a given many body basis state by imposing the condition that $\Sigma_i(2n_i+l_i)\leqslant N$, where the sum is over the single particle oscillator quanta, $2n+l$, present in that state. The second parameter, $\hbar\Omega$, denotes the oscillator energy which we employ as one of our adjustable parameters fit to the charmonium spectrum.

This many-body computational framework has been used in other multifermion applications [19]. Those previous applications also featured a derivation of an effective Hamiltonian that we do not carry out here. Instead, we adapt the established procedures for many-body basis space enumeration, many-body Hamiltonian evaluation, and Lanczos diagonalization for the present effort. We pay special attention to the treatment of the color degree of freedom. That is, we impose the restriction that our physical many-fermion states are global color singlets with the projection method described above.

We first fix our Hamiltonian and basis space parameters by fitting the low-lying charmonium spectra. Two fits are obtained so we may begin to explore the sensitivity of our $c^2\overline{c}^2$ results to variations in acceptable $c\overline{c}$ fits. We select a maximum N, called $N_{\max}=9$ (10) for our negative (positive) orbital parity 2-body states in which we fit our parameters to the lowest few experimentally determined states. We then solve the 4-body problem in a sequence of model spaces by varying N up to $N_{\max}$. The maximum matrix dimension encountered in the 4-body problem was 3 013 782.

For each set of Hamiltonian parameters, designated Case 1 and Case 2 below, the sensitivity of the 2-body charmonium spectrum to spaces of smaller N values indicates that the parameters would require adjustment if $N_{\max}$ is changed.

TABLE V. Case 1 odd parity tetraquark mass spectra, uncorrected via Eq. (4).

$M^{full,N,1280}_{cc\overline{cc}}$					
J	N=1	N=3	N=5	N=7	N=9
0	8247	7752	7320	7073	6876
1	8286	7773	7343	7103	6906
1	8329	7791	7360	7113	6917
2	8355	7800	7369	7123	6926

Method of tetraquark net binding analysis

Since our basis space is comprised of harmonic oscillator states, our spectrum will exhibit an appearance of being bound, but this is not sufficient to conclude that a solution actually corresponds to a physically bound state. Qualitatively, we may view each 4-body state as composed of a pair of 2-body subsystems with a relative kinetic energy between them. Assuming the pairs will break apart, we can identify the minimum possible kinetic energy between the pairs based on the size of the model space, N. Let us refer to this minimum possible kinetic energy between the pairs as the ''kinetic energy penalty,'' the penalty of confining two pairs of free particles in a finite harmonic oscillator basis. By ''free particle'' we mean the solution of our Hamiltonian with 2-body relative interaction terms [Eq. (1) summation term] removed. We quantify this penalty in Eq. (3),

$$\Delta M^{free,N,\hbar\Omega}_{cc\overline{cc}} = M^{free,N,\hbar\Omega}_{cc\overline{cc}} - \min_n [M^{free,n,\hbar\Omega}_{cc\overline{c}} + M^{free,N-n,\hbar\Omega}_{c\overline{c}}]. \quad (3)$$

$M^{free,N,\hbar\Omega}$ denotes the lowest mass of the indicated (subscript) free particle system in the basis space specified by N and $\hbar\Omega$. Note that we subtract the minimum relative kinetic energy of a pair of color singlet 2-body subsystems from the minimum relative kinetic energy of the color singlet 4-body state. This correction tends to zero as $N_{\max}$ increases.

When we compute the spectrum of the full Hamiltonian in the 4-body system, we will then subtract the penalty in Eq. (3) from the results as follows:

$$M^{corrected,N,\hbar\Omega}_{cc\overline{cc}} = M^{full,N,\hbar\Omega}_{cc\overline{cc}} - \Delta M^{free,N,\hbar\Omega}_{cc\overline{cc}}. \quad (4)$$

To be specific, we subtract the same kinetic energy penalty from every tetraquark mass eigenvalue of the ''full'' Hamiltonian to arrive at the ''corrected'' mass of each state regardless of its quantum numbers. The tetraquark spectrum, corrected via Eq. (4), will then be compared to charmonium pairs from the 2-body spectrum to determine if there is net binding of the 4-body state.

Case 1 tetraquark results

We now present the results of our first fit to the lowest 4 states for each parity in the charmonium spectrum, ''Case 1,'' in the $N_{\max}=9$ and 10 model spaces. We also present our tetraquark results with the same Hamiltonian.

Tables I and II show the even and odd orbital parity states of charmonium for the full space of the fit, $N_{\max}$, as well as for spaces $N<N_{\max}$. The results in truncated spaces will be used in the tetraquark threshold analysis below. The experimental masses are taken from the Particle Data Group compilation [20]. The fit of the positive orbital parity states in Table I is only fair. This is due mainly to the fact that the delta functions are crude approximations to the short distance interactions. This aspect of our Hamiltonians limited our ability to simultaneously fit both the 1S and 2S splittings of the positive orbital parity states. We decided to adjust the

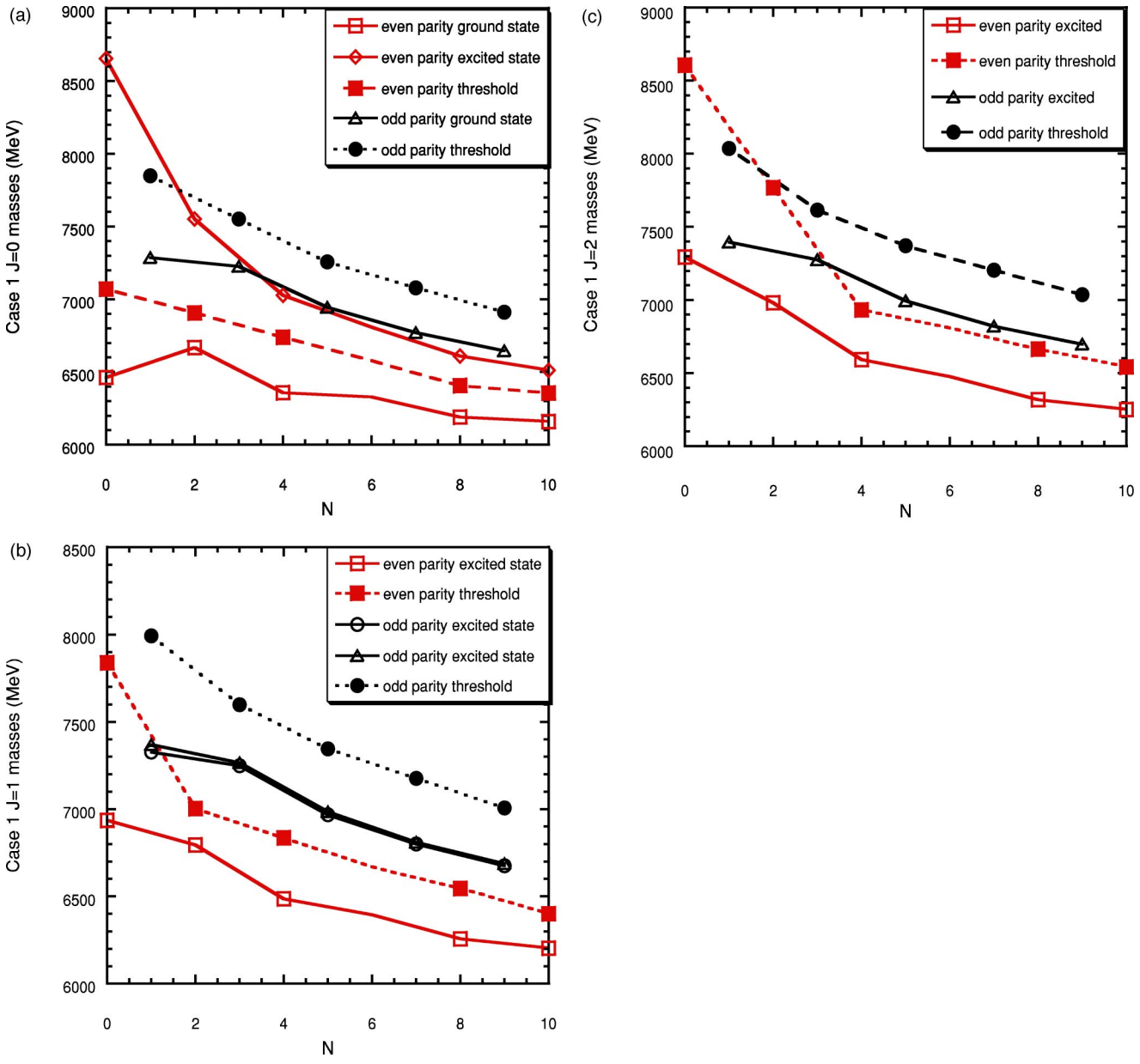

FIG. 1. (a) The J=0 states for case 1. Note that the even parity excited state is unbound. The even parity threshold is 2 η_c, and the odd parity threshold is $\eta_c+\chi_{c0}$. (b) The J=1 states for case 1. The two odd parity states are nearly degenerate. The even parity threshold is η_c+J/ψ, and the odd parity threshold is $\eta_c+\chi_{c1}$. (c) The J=2 states for case 1. Note that the slopes of the thresholds are nearly parallel to the bound state slopes. The even parity threshold is 2 J/ψ, and the odd parity threshold is $\eta_c+\chi_{c2}$.

delta functions to fit the J/ψ and the $\eta_c(2S)$ masses in order to represent states with a range of radial excitations at the 4-body level. The root mean square (RMS) mass difference between the N=10 results and the experimental masses is 106 MeV for all four states, but falls to 54 MeV if one ignores the $\psi(2S)$. The RMS mass difference for the N=9 negative orbital parity results in Table II is 10.4 MeV, an excellent fit. If one uses all 8 states from both parities, the overall RMS mass difference is 75.3 MeV. For comparison, a recent relativistic constituent quark model based upon a covariant constraint dynamics approach [21] provides an RMS mass difference of 19.8 MeV for these same 8 states. For consistency, we will use our calculated $c\overline{c}$ spectra, rather than experimental states, to determine breakup thresholds for our tetraquark states.

Table III presents the lowest "free particle" spectra for $c\overline{c}$ in the first row of masses, the lowest free particle spectra for $c^2\overline{c^2}$ in the second row, and the kinetic energy penalty computed via Eq. (3) in the third row.

The parameters of the Case 1 Hamiltonian are $a=59.4$ MeV fm, $m=1490$ MeV, $\beta_0=-1.8$ MeV fm^3, $\beta_1=0.3$ MeV fm^3, $\omega=\eta=0.24$ MeV fm^3, and we set $\sigma=487.5$ MeV fm^{-1}. The quark mass is 1490 MeV and $\hbar\Omega=1280$ MeV.

TABLE VI. Even parity charmonium masses (MeV) for Case 2. The column N=10 presents the fit to the quoted experimental masses. Other columns present masses obtained in truncated spaces.

ID	J	$M_{\rm exp}$	N=0	N=2	N=4	N=6	N=8	N=10
$\eta_c(1S)$	0	2979	4024	3352	3187	3152	3089	3074
J/ψ	1	3097	4024	3442	3320	3198	3155	3105
$\eta_c(2S)$	0	3654		4637	4301	3884	3783	3602
$\psi(2S)$	1	3686		5053	4545	4268	3943	3822

TABLE VII. Odd parity charmonium masses (MeV) for Case 2. The column N=9 presents the fit to the quoted experimental masses. Other columns present masses obtained in truncated spaces.

ID	J	$M_{\rm exp}$	N=1	N=3	N=5	N=7	N=9
$\chi_{c0}(1P)$	0	3415	4250	3982	3704	3536	3422
$\chi_{c1}(1P)$	1	3511	4384	4028	3788	3625	3503
$h_c(1P)$	1	3526	4417	4039	3808	3647	3523
$\chi_{c2}(1P)$	2	3556	4440	4046	3822	3662	3537

Table IV (V) shows the uncorrected even (odd) parity spectrum for $c^2\overline{c}^2$. The corrected spectra, along with their corresponding breakup thresholds, are plotted in Figs. 1(a)–1(c) vs N.

Care is taken in the net binding calculation to conserve parity and total J. In addition, we allow for the tetraquark system to organize into the two energetically most favored $c\overline{c}$ subsystems subject to the limitation of the total tetraquark oscillator quanta available. As an example of how this is carried out, let us examine the J=0 state at N=10 in Table IV. We find the most stringent threshold. We easily see that breakup into two odd orbital parity $c\overline{c}$ states is unfavorable. Then we take the minimum mass of two $\eta_c(1S)$ particles that, taken together, have the same total oscillator quanta as the N=10 tetraquark state. We add the N=0 and N=10 $\eta_c(1S)$ masses from Table I, the N=2 and N=8 masses, and the N=4 and N=6 masses, and simply choose the minimum of these three combinations, the N=4 and N=6 masses in this case. This minimum is then subtracted from the corrected J=0 state to estimate the net binding.

It is apparent that the three lowest states of even parity in Table IV and all the listed negative parity states in Table V are bound, although the binding generally decreases in magnitude for large N. The states at N=9, and the J=2 state at N=10, would barely be bound without the kinetic energy correction of Eq. (4).

Our main focus is on tetraquark binding relative to its most stringent theoretical breakup threshold. Hence, in our own analysis, we employ $c\overline{c}$ results that arise in model spaces with $N<N_{\rm max}$. However, there is considerable sensitivity of these $c\overline{c}$ masses to the model space dimension. Hence, the corrected tetraquark masses at $N_{\rm max}=9$ (10) in the odd (even) parity should be used for predicting thresholds rather than taken as predictions for the masses of the all-charm tetraquark bound states.

The main import of our results lies in the appreciable binding energies found in the low-lying tetraquark states.

Naturally, we would prefer to carry out this procedure in even larger model spaces (i.e., larger $N_{\rm max}$) with suitably improved Hamiltonians since the underlying logic for the minimum threshold is designed for asymptotically separated and converged $c\overline{c}$ subsystems. In an expanded treatment with Hamiltonians that produce better descriptions of the experimental $c\overline{c}$ states, we would hope to obtain smooth behavior of both the tetraquark and $c\overline{c}$ spectra with increasing $N_{\rm max}$. This would yield greater confidence in the predictions, particularly for the tetraquark masses.

Case 2 tetraquark results

Given the number of adjustable parameters in the Hamiltonian, multiple RMS mass difference minima could be obtained from different fits. In order to explore the sensitivity of our results to allowable variations in our Hamiltonian and basis space parameters, we carried out a second fit to the lowest 8 states of the $c\overline{c}$ spectra. The Case 2 parameters for the Hamiltonian are $\beta_0=-1.2$ MeV fm^3, $\beta_1=0.45$ MeV fm^3, $\omega=0.225$ MeV fm^3, $\eta=0.30$ MeV fm^3, and we have $\hbar\Omega=1200$ MeV. Note that the Coulomb-like interaction coupling and the confining strength are unchanged.

We present the results of a fit to the low-lying even and odd orbital parity spectrum of charmonium in Tables VI and VII. The RMS mass difference for even orbital parity at $N_{\rm max}=10$ is 87.0 (62.7) MeV with (without) the $\psi(2S)$. The RMS mass difference for the odd orbital parity at $N_{\rm max}=9$ is 11 MeV. The RMS mass difference for all 8 states is 62.0 MeV. Case 2 provides a better overall fit to the charmonium spectrum, but if one omits the $\psi(2S)$, Case 1 yields a slightly better fit. The interpretation of Tables VI–X follows the same path as Tables I–V for Case 1.

The tetraquark binding is significantly less than Case 1. Looking at the important relative S-state interaction components, we observe that Case 2 has less attractive delta func-

TABLE VIII. 2- and 4-body free particle spectra with computed correction.

N=0	N=1	N=2	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10
Lowest 2-body free particle spectra (MeV)										
3900	4500	3551	3977	3400	3732	3314	3587	3259	3491	3220
Lowest 4-body free particle spectra (MeV)										
8700	9300	7893	8370	7479	7880	7220	7567	7041	7347	6908
$\Delta M_{cc\overline{c}\overline{c}}^{free,N,1200}$ (MeV)										
900	900	442	493	377	352	269	284	241	215	194

TABLE IX. Case 2 even parity tetraquarks mass spectra, uncorrected via Eq. (4).

	$M_{cc\bar{c}\bar{c}}^{full,N,1200}$					
J	N=0	N=2	N=4	N=6	N=8	N=10
0	7519	7196	6854	6700	6555	6477
1	7870	7320	6974	6774	6624	6528
2	8146	7458	7068	6845	6680	6573
0	8941	7922	7315	7026	6829	6695

TABLE X. Case 2 odd parity tetraquark mass spectra, uncorrected via Eq. (4).

	$M_{cc\bar{c}\bar{c}}^{full,N,1200}$				
J	N=1	N=3	N=5	N=7	N=9
0	8310	7806	7400	7154	6969
1	8363	7830	7429	7190	7004
1	8388	7843	7440	7197	7013
2	8420	7856	7454	7214	7033

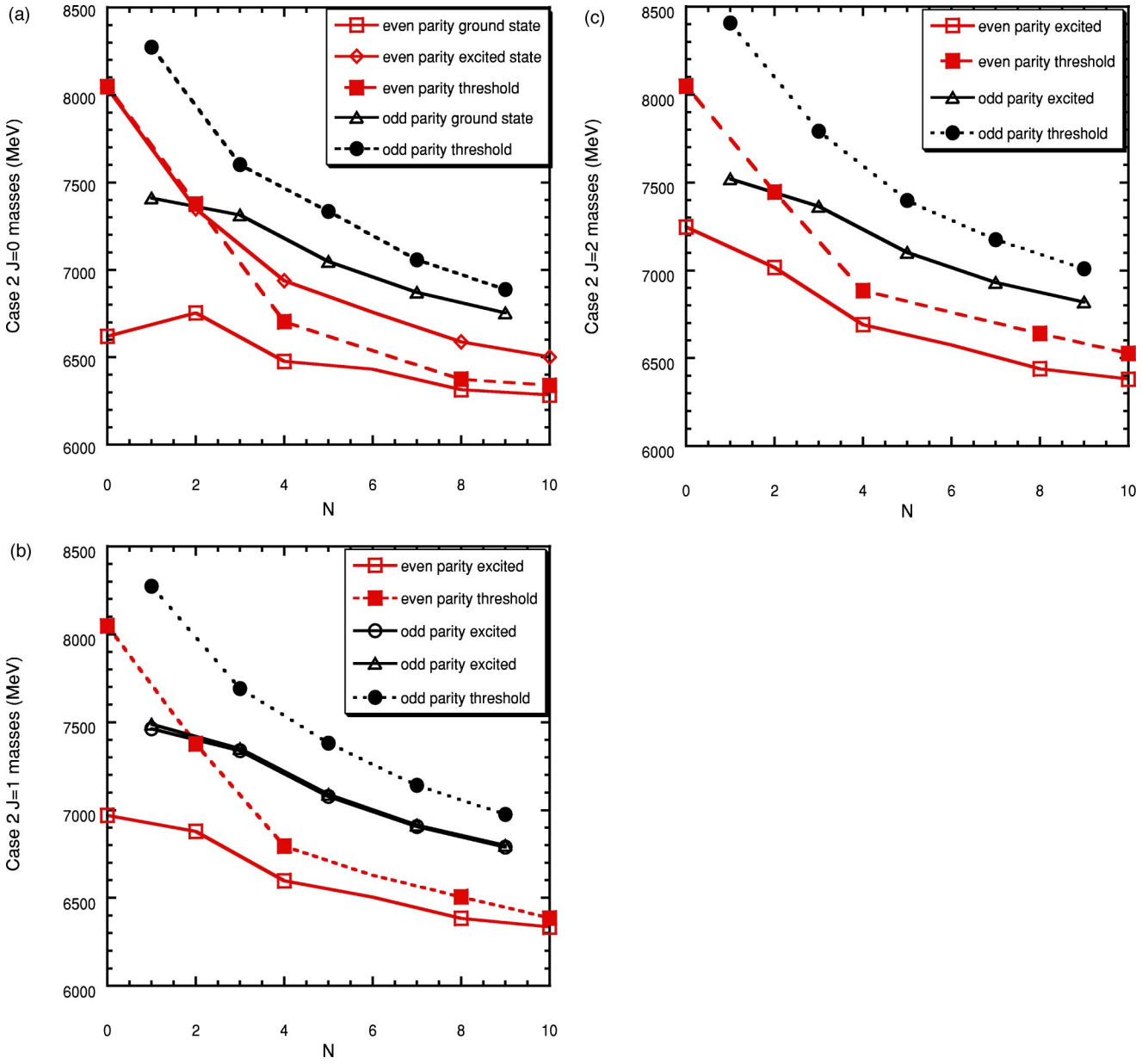

FIG. 2. (a) The J=0 states for case 2. The even parity threshold is 2 η_c, and the odd parity threshold is $\eta_c+\chi_{co}$. These states have less net binding than case 1. (b) The J=1 states for case 2. The even parity threshold is η_c+J/ψ, and the odd parity threshold is $\eta_c+\chi_{c1}$. Note that the odd parity states are nearly degenerate as in case 1 and that the binding is generally less. (c) The J=2 states for case 2. The even parity threshold is 2 J/ψ, and the odd parity threshold is $\eta_c+\chi_{c2}$.

tion strengths and a less repulsive tensor term. The decreased tetraquark binding may well be correlated with the S-state properties of our effective interaction. A more extensive exploration of this sensitivity is needed to establish this correlation.

The ''kinetic energy deficit'' in Table VIII for large N is very similar to Case 1, which is expected since the deficit depends only on the basis space parameters and the quark mass. The uncorrected masses in Tables IX and X for the tetraquark are about 100 MeV higher than Case 1 results, which leads to the reduced binding since the decay thresholds are virtually unchanged. We note that, unlike Case 1, none of the states would be bound at $N_{\max}=9$ or 10 if the kinetic energy correction was omitted. The computed masses, corrected via Eq. (4), are shown in Figs. 2(a)–2(c), along with their corresponding thresholds.

SUMMARY AND CONCLUSIONS

We have used a parametrized Hamiltonian to compute the spectrum of all-charm tetraquark states. After fitting the lowest $c\bar{c}$ states with two different parameter sets and performing a net binding analysis, we obtain bound tetraquark states with both sets of parameters. For example, the lowest tetraquark state with $J=0$ in the positive parity spectrum has a mass below the threshold of two $\eta_c(1S)$ masses computed in our framework.

A simple extrapolation of the binding energies might lead one to suspect some states would become unbound with increasing basis space size, $N_{\max}$. However, the trends in the binding energies are not smooth functions of increasing N and make extrapolation a risky proposition. Furthermore, simple extrapolation is not warranted since the Hamiltonian parameters were fit within the $N_{\max}=9$ (10) basis spaces to the negative (positive) orbital parity experimental meson masses.

Experimental investigations could identify the states we predict at e^+e^- facilities. Indeed, Iwasaki [22] originally proposed measuring the recoil-mass spectrum in coincidence with a J/ψ around CM energy of 6 GeV to look for all-charm tetraquark resonances. In an interesting and provocative paper there has been a theoretical calculation by Ioffe and Kharzeev [23] showing that certain nonperturbative processes in e^+e^- annihilation can explain the enhanced production of $J/\psi+c\bar{c}$ at CM energies greater than 10 GeV that was recently reported at Belle and BaBar. This leads us to speculate that searching for tetraquarks may be fruitful at CM energies higher than the direct production resonance around 6 GeV such that tetraquark formation could capitalize on additional processes similar to those examined in [23].

The lifetimes and decay products of all-charm tetraquark states have not yet been computed. It is reasonable to expect that the lifetime would be of the same order as the J/ψ or η_c depending on which threshold is the closest as listed in the tables. In addition, if the all-charm tetraquark has a two-body correlation similar to J/ψ in its substructure, the favored decay would be $e^+e^-+c\bar{c}$. If there is a two-body correlation similar to η_c in the substructure, then the decay would be predominantly hadronic and more difficult to detect and the expected lifetime would be much shorter.

In a further extension of this work, we intend to employ a Hamiltonian with broader phenomenological success [24] that includes relativistic kinematics effects as well as a treatment of negative frequency states.

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