Nuclear and nucleon transitions of the H dibaryon

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We consider 3 types of processes pertinent to the phenomenology of an H dibaryon: conversion of two Λ's in a doubly strange hypernucleus to an H, decay of the H to two baryons, and—if the H is light enough—conversion of two nucleons in a nucleus to an H. We compute the spatial wave function overlap using the Isgur-Karl, Miller-Spencer and Bethe-Goldstone wave functions, and treat the weak interactions phenomenologically. The observation of Λ decays from doubly strange hypernuclei puts a constraint on the H wave function which is plausibly satisfied. Imposing this constraint, we obtain model-independent lower limits on the H lifetime; if $m_H < m_N + m_\Lambda$, the H lifetime can be of the order of or longer than the age of the Universe. We discuss limits on a long-lived or stable H, and point out how experiments can improve the constraints.

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I. INTRODUCTION

The most symmetric color-spin representation of six quarks (*uuddss*) is called the H dibaryon. It is flavor singlet with strangeness -2, charge 0, and spin-isospin parity $I\ (J^P) = 0\ (0^+)$. In 1977 Jaffe calculated its mass [1] to be about 2150 MeV in the MIT bag model and thus predicted it would be a strong-interaction-stable bound state, since decay to two Λ particles would not be kinematically allowed. Since then its mass has been estimated in many different models, with results lying in the range 1–2.3 GeV. On the experimental side, there have been many inconclusive or unsuccessful attempts to produce and detect it. See [2] for a review.

The purpose of this paper is to study several processes involving the H which are phenomenologically important if it exists: conversion of two Λ's in a doubly strange hypernucleus to an H, decay of the H to two baryons, and—if the H is light enough—conversion of two nucleons in a nucleus to an H. The amplitudes for these processes depend on the spatial wave function overlap of two baryons and an H. We are particularly interested in the possibility that the H is tightly bound and that it has a mass less than $m_N + m_\Lambda$. In that case, as we shall see, its lifetime can be longer than the age of the Universe.

If the H is tightly bound, it would be expected to be spatially compact. Hadron sizes vary considerably, for a number of reasons. The nucleon is significantly larger than the pion, with charge radius $r_N = 0.87$ fm compared to $r_\pi = 0.67$ fm [3]. Lattice and instanton-liquid studies qualitatively account for this diversity and further predict that the scalar glueball is even more tightly bound: $r_G \approx 0.2$ fm [4,5]. If the analogy suggested in Ref. [6] between H, Λ_{1405} and glueball is correct, it would suggest $r_H \approx r_G \lesssim (1/4) r_N$. The above size relationships make sense: the nucleon's large size is due to the low mass of the pion which forms an extended cloud around it, while the H and glueball do not couple to pions, due to parity and flavor conservation, and thus are small compared to the nucleon. In the absence of an unquenched, high-resolution lattice QCD calculation capable of a reliable determination of the H mass and size, we will consider all values of m_H and take $r_H / r_N \equiv 1/f$ as a parameter, with f in the range 2–6. For a more detailed discussion of the motivation and properties of a stable or long-lived H and a review of experimental constraints on such an H, see Ref. [7].

In this paper we calculate the lifetime for decay of the H to various final states, and we consider two types of experimental constraints on the transition of two baryons to an H in a nucleus, $A_{BB} \to A'_H X$. To estimate the rates for these processes requires calculating the overlap of initial and final quark wave functions. We model that overlap using an Isgur-Karl harmonic oscillator model for the baryons and H, and the Bethe-Goldstone and Miller-Spencer wave functions for the nucleus. The results depend on r_N / r_H and the nuclear hard core radius.

Experiments observing single Λ decays from double Λ hypernuclei $A_{\Lambda\Lambda}$ [8,9] indicate that $\tau(A_{\Lambda\Lambda} \to A'_H X) \gtrsim 10^{-10}$ s. Our calculations show that adequate suppression of $\Gamma(A_{\Lambda\Lambda} \to A'_H X)$ requires $r_H \lesssim 1/2 r_N$ (or less, depending on the short distance nuclear wave function), consistent with expectations. Thus an H with mass $m_H < 2 m_\Lambda$ can still be viable in spite of the observation of double-Λ hypernuclei, as also found in Ref. [10].

We calculate the lifetime of the H, in three qualitatively distinct mass ranges, under the assumption that the conditions to satisfy the constraints from double-Λ hypernuclei are met. The ranges are $m_H < m_N + m_\Lambda$, in which H decay is a doubly weak $\Delta S = 2$ process, $m_N + m_\Lambda < m_H < 2 m_\Lambda$, in which the H can decay by a normal weak interaction, and $m_H > 2 m_\Lambda$, in which the H is strong-interaction unstable. The H lifetime in these ranges is greater than or of order 10^7 yr, ~ 10 s, and $\sim 10^{-14}$ s, respectively.

Finally, if $m_H \lesssim 2 m_N$, nuclei are unstable and $\Delta S = -2$ weak decays convert two nucleons to an H. In this case the stability of nuclei is a more stringent constraint than the double-Λ hypernuclear observations, but our results show that nuclear stability bounds can also be satisfied if the H is sufficiently compact: $r_H \lesssim 1/4 r_N$ depending on mass and nuclear hard core radius. This option is vulnerable to experimental exclusion by Super Kamiokande.

This paper is organized as follows. In Sec. II we describe in greater detail the two types of experimental constraints on the conversion of baryons to an H in a nucleus. In Sec. III we set up the theoretical apparatus to calculate the wave function overlap between H and two baryons. We determine the weak interaction matrix elements phenomenologically in Sec. IV. Lifetimes for various processes are computed in Secs. V B and VI. The results are reviewed and conclusions are summarized in Sec. VII.

II. EXPERIMENTAL CONSTRAINTS

A. Double Λ hyper-nucleus detection

There are five experiments that have reported positive results in the search for single Λ decays from double Λ hypernuclei. We will describe them briefly. The three early emulsion based experiments [11–13] suffer from ambiguities in the particle identification, and therefore we do not consider them further. In the latest emulsion experiment at KEK [9], an event has been observed which is interpreted with good confidence as the sequential decay of ${}^6\mathrm{He}_{\Lambda\Lambda}$ emitted from a Ξ^- hyperon nuclear capture at rest. The binding energy of the double Λ system is obtained in this experiment to be $B_{\Lambda\Lambda}=1.01\pm0.2$ MeV, in significant disagreement with the results of previous emulsion experiments, finding $B_{\Lambda\Lambda}\sim4.5$ MeV.

The BNL experiment [8] used the (K^-,K^+) reaction on a ^{9}Be target to produce $S=-2$ nuclei. That experiment detected pion pairs coming from the same vertex in the Be target. Each pion in a pair indicates one unit of strangeness change from the (presumably) di-Λ system. Observed peaks in the two pion spectrum have been interpreted as corresponding to two kinds of decay events. The pion kinetic energies in those peaks are (114,133) MeV and (104,114) MeV. The first peak can be understood as two independent single Λ decays from $\Lambda\Lambda$ nuclei. The energies of the second peak do not correspond to known single Λ decay energies in hyper-nuclei of interest. The proposed explanation [8] is that they are pions from the decay of the double Λ system, through some specific He resonance. The required resonance has not yet been observed experimentally, but its existence is considered plausible. This experiment does not suffer from low statistics or inherent ambiguities, and one of the measured peaks in the two pion spectrum suggests observation of consecutive weak decays of a double Λ hyper-nucleus. The binding energy of the double Λ system $B_{\Lambda\Lambda}$ could not be determined in this experiment.

The KEK and BNL experiments are generally accepted to demonstrate quite conclusively, in two different techniques, the observation of Λ decays from double Λ hypernuclei. Therefore $\tau_{A_{\Lambda\Lambda}\to A'_H X}$ cannot be much less than $\approx10^{-10}$ s. (To give a more precise limit on $\tau_{A_{\Lambda\Lambda}\to A'_H X}$ requires a detailed analysis by the experimental teams, taking into account the number of hypernuclei produced, the number of observed Λ decays, the acceptance, and so on.) As will be seen below, this constraint is readily satisfied if the H is compact: $r_H\lesssim(1/2)r_N$ or less, depending on the nuclear wave function.

B. Stability of nuclei

There are a number of possible reactions by which two nucleons can convert to an H in a nucleus if that is kinematically allowed ($m_H\lesssim2m_N$). The initial nucleons are most likely to be pn or nn in a relative s wave, because in other cases the Coulomb barrier or relative orbital angular momentum suppresses the overlap of the nucleons at short distances that is necessary to produce the H. If $m_H\lesssim2m_N-nm_\pi$,[1] the final state can be $H\pi^+$ or $H\pi^0$ and $n-1$ pions with total charge 0. For $m_H\gtrsim1740$ MeV, the most important reactions are $pn\to He^+\nu_e$ or the radiative doubly weak reaction $nn\to H\gamma$.

The best experiments to place a limit on the stability of nuclei are proton decay experiments. Super Kamiokande (SuperK) can place the most stringent constraint due to its large mass. It is a water Čerenkov detector with a 22.5 kton fiducial mass, corresponding to 8×10^{32} oxygen nuclei. SuperK is sensitive to proton decay events in over 40 proton decay channels [14]. Since the signatures for the transition of two nucleons to the H are substantially different from the monitored transitions, a specific analysis by SuperK is needed to place a limit. We will discuss the order of magnitude of the limits which can be anticipated.

Detection is easiest if the H is light enough to be produced with a π^+ or π^0. The efficiency of SuperK to detect neutral pions, in the energy range of interest (kinetic energy ~0–300 MeV), is around 70%. In the case that a π^+ is emitted, it can charge exchange to a π^0 within the detector, or be directly detected as a non-showering muon-like particle with similar efficiency. More difficult is the most interesting mass range $m_H\gtrsim1740$ MeV, for which the dominant channel $pn\to He^+\nu$ gives an electron with $E\sim(2m_N-m_H)/2\lesssim70$ MeV. The channel $nn\to H\gamma$, whose rate is smaller by a factor of order α, would give a monochromatic photon with energy $(2m_N-m_H)\lesssim100$ MeV.

We can estimate SuperK's probable sensitivity as follows. The ultimate background comes primarily from atmospheric neutrino interactions, $\nu N\to N'(e,\mu)$, $\nu N\to N'(e,\mu)+n\pi$ and $\nu N\to\nu N'+n\pi$, for which the event rate is about 100 per kton yr. Without a strikingly distinct signature, it would be difficult to detect a signal rate significantly smaller than this, which would imply that SuperK might be able to achieve a sensitivity of order $\tau_{A_{NN}\to A'_H X}\gtrsim\mathrm{few}\times10^{29}$ yr. Since the H production signature is not more favorable than the signatures for proton decay, the SuperK limit on $\tau_{A_{NN}\to A'_H X}$ can at best be $0.1\tau_p$, where 0.1 is the ratio of oxygen nuclei to protons in water. A detailed study of the spectrum of the background is needed to make a more precise statement. We can get a lower limit on the SuperK lifetime limit by noting that the SuperK trigger rate is a few hertz [14], putting an immediate limit $\tau_{O\to H+X}\gtrsim\mathrm{few}\times10^{25}$ yr, assuming the decays trigger SuperK.

[1]Throughout, we use this shorthand for the more precise inequality $m_H<m_A-m_{A'}-m_X$ where m_X is the minimum invariant mass of the final decay products.

SuperK limits will apply to specific decay channels, but other experiments potentially establish limits on the rate at which nucleons in a nucleus convert to an H which are independent of the H production reaction. These experiments place weaker constraints on this rate due to their smaller size, but they are of interest because in principle they measure the stability of nuclei directly. Among those cited in Ref. [3], only the experiment by Flerov *et al*. [15] could in principle be sensitive to transitions of two nucleons to the H. It searched for decay products from ^{232}Th, above the Th natural decay mode background of 4.7 MeV α particles, emitted at the rate $\Gamma_\alpha = 0.7\times 10^{-10}$ yr^{-1}. Cuts to remove the severe background of 4.7 MeV α's may or may not remove events with production of an H. Unfortunately Ref. [15] does not discuss these cuts or the experimental sensitivity in detail. An attempt to correspond with the experimental group, to determine whether their results are applicable to the H, was unsuccessful. If applicable, it would establish that the lifetime $\tau_{\mathrm{Th}^{232}\to H+X} > 10^{21}$ yr.

Better channel independent limits on N and NN decays in nuclei have been established recently, as summarized in Ref. [16]. Among them, searches for the radioactive decay of isotopes created as a result of NN decays of a parent nucleus yield the most stringent constraints. This method was first exploited in the DAMA liquid Xe detector [17]. BOREXINO has recently improved these results [16] using their prototype detector, the Counting Test Facility with parent nuclei ^{12}C, ^{13}C, and ^{16}O. The signal in these experiments is the beta and gamma radiation in a specified energy range associated with deexcitation of a daughter nucleus created by decay of outer-shell nucleons in the parent nucleus. They obtain the limits $\tau_{pp} > 5\times 10^{25}$ yr and $\tau_{nn} > 4.9\times 10^{25}$ yr. However, H production requires overlap of the nucleon wave functions at short distances and is therefore suppressed for outer shell nucleons, severely reducing the utility of these limits. Since the SuperK limits will probably be much better, we do not attempt to estimate the degree of suppression at this time.

Another approach could be useful if for some reason the direct SuperK search is foiled. Reference [18] places a limit on the lifetime of a bound neutron, $\tau_n > 4.9\times 10^{26}$ yr, by searching for γ's with energy $E_\gamma = 19$–50 MeV in the Kamiokande detector. The idea is that after the decay of a neutron in oxygen the de-excitation of ^{15}O proceeds by emission of γ's in the given energy range. The background is especially low for γ's of these energies, since atmospheric neutrino events produce γ's above 100 MeV. In our case, some of the photons in the de-excitation process after conversion of pn to H, would be expected to fall in this energy window.

III. OVERLAP OF H AND TWO BARYONS

We wish to calculate the amplitudes for a variety of processes, some of which require one or more weak interactions to change strange quarks into light quarks. By working in pole approximation, we factor the problem into an H-baryon-baryon wave function overlap times a weak interaction matrix element between strange and non-strange baryons, which will be estimated in the next section. For instance, the matrix element for the transition of two nucleons in a nucleus A to an H and nucleus A', $A_{NN}\to A'_H X$, is calculated in the $\Lambda\Lambda$ pole approximation, as the product of matrix elements for two subprocesses: a transition matrix element for formation of the H from the $\Lambda\Lambda$ system in the nucleus, $|\mathcal{M}|_{\{\Lambda\Lambda\}\to HX}$, times the amplitude for a weak doubly-strangeness-changing transition, $|\mathcal{M}|_{NN\to\Lambda\Lambda}$. We ignore mass differences between light and strange quarks and thus the spatial wave functions of all octet baryons are the same. In this section we are concerned with the dynamics of the process and we suppress spin-flavor indices.

A. Isgur-Karl model and generalization to the H

The Isgur-Karl (IK) non-relativistic harmonic oscillator quark model [19–21] was designed to reproduce the masses of the observed resonances and it has proved to be successful in calculating baryon decay rates [20]. In the IK model, the quarks in a baryon are described by the Hamiltonian

$$H = \frac{1}{2m}(p_1^2 + p_2^2 + p_3^2) + \frac{1}{2}k\Sigma_{i<j}^3(\vec{r}_i - \vec{r}_j)^2, \qquad (1)$$

where we have neglected constituent quark mass differences. The wave function of baryons can then be written in terms of the relative positions of quarks and the center of mass motion is factored out. The relative wave function in this model is [20,21]

$$\Psi_B(\vec{r}_1,\vec{r}_2,\vec{r}_3) = N_B \exp\left[-\frac{\alpha_B^2}{6}\Sigma_{i<j}^3(\vec{r}_i - \vec{r}_j)^2\right], \qquad (2)$$

where N_B is the normalization factor, $\alpha_B = 1/\sqrt{\langle r_B^2\rangle} = \sqrt{3km}$, and $\langle r_B^2\rangle$ is the baryon mean charge radius squared. Changing variables to

$$\vec{\rho} = \frac{\vec{r_1} - \vec{r_2}}{\sqrt{2}}, \quad \vec{\lambda} = \frac{\vec{r_1} + \vec{r_2} - 2\vec{r_3}}{\sqrt{6}} \qquad (3)$$

reduces the wave function to two independent harmonic oscillators. In the ground state

$$\Psi_B(\vec{\rho},\vec{\lambda}) = \left(\frac{\alpha_B}{\sqrt{\pi}}\right)^3 \exp\left[-\frac{\alpha_B^2}{2}(\rho^2 + \lambda^2)\right]. \qquad (4)$$

One of the deficiencies of the IK model is that the value of the α_B parameter needed to reproduce the mass splittings of lowest lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons, $\alpha_B = 0.406$ GeV, corresponds to a mean charge radius squared for the proton of $\sqrt{\langle r_N^2\rangle} = 1/\alpha_B = 0.49$ fm. This is distinctly smaller than the experimental value of 0.87 fm. Our results depend on the choice of α_B and therefore we also report results using $\alpha_B = 0.221$ GeV which reproduces the observed charge radius at the expense of the mass splittings.

Another concern is the applicability of the non-relativistic IK model in describing quark systems, especially in the case of the tightly bound H. With $r_H/r_N = 1/f$, the quark momenta in the H are $\approx f$ times higher than in the nucleon, which makes the non-relativistic approach more questionable than in the case of nucleons. Nevertheless we adopt the IK model because it offers a tractable way of obtaining a quantitative estimate of the effect of the small size of the H on the transition rate, and there is no other alternative available at this time. For comparison, it would be very interesting to have a Skyrme model calculation of the overlap of an H with two baryons.

We fix the wave function for the H particle starting from the same Hamiltonian (1), but generalized to a six quark system. For the relative motion part this gives

$$\Psi_H = N_H \exp\left[-\frac{\alpha_H^2}{6} \sum_{i<j}^{6} (\vec{r}_i - \vec{r}_j)^2 \right]. \tag{5}$$

The space part of the matrix element of interest, $\langle A'_H | A_{\Lambda\Lambda} \rangle$, is given by the integral

$$\int \prod_{i=1}^{6} d^3\vec{r}_i \Psi_\Lambda^a(1,2,3) \Psi_\Lambda^b(4,5,6) \Psi_H(1,2,3,4,5,6). \tag{6}$$

Therefore it is useful to choose variables for the H wave function as follows, replacing

$$\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4,\vec{r}_5,\vec{r}_6 \rightarrow \vec{\rho}^a,\vec{\lambda}^a,\vec{\rho}^b,\vec{\lambda}^b,\vec{a},\vec{R}_{CM} \tag{7}$$

where $\vec{\rho}^{a(b)}$ and $\vec{\lambda}^{a(b)}$ are defined as in Eq. (3), with a (b) referring to coordinates $1,2,3$ $(4,5,6)$. (Since we are ignoring the flavor-spin part of the wave function, we can consider the six quarks as distinguishable and not worry about Fermi statistics at this stage.) We also define the center-of-mass position and the separation, $\vec{a}$, between initial baryons a and b:

$$\vec{R}_{CM} = \frac{\vec{R}_{CM}^a + \vec{R}_{CM}^b}{2}, \quad \vec{a} = \vec{R}_{CM}^a - \vec{R}_{CM}^b. \tag{8}$$

Using these variables, the H ground state wave function becomes

$$\Psi_H = \left(\frac{3}{2}\right)^{3/4} \left(\frac{\alpha_H}{\sqrt{\pi}}\right)^{15/2} \exp\left[-\frac{\alpha_H^2}{2}\left(\vec{\rho^a}^2 + \vec{\lambda^a}^2 + \vec{\rho^b}^2 + \vec{\lambda^b}^2 \left(+ \frac{3}{2}\vec{a}^2 \right) \right) \right]. \tag{9}$$

As for the 3-quark system, $\alpha_H = 1/\sqrt{\langle r_H^2 \rangle}$.

B. Nuclear wave function

We will use two different wave functions to describe two Λ's or nucleons in a nucleus, in order to study the model dependence of our results and to elucidate the importance of different aspects of the nuclear wave function. A commonly used wave function is the Miller-Spencer (MS) wave function [22]:

$$\psi_{MS} = 1 - \exp^{-c_1 a^2}(1 - c_2 a^2), \tag{10}$$

with the canonical parameter choices $c_1 = 1.1\ \mathrm{fm}^{-2}$ and $c_2 = 0.68\ \mathrm{fm}^{-2}$. It must be emphasized that at the short distances relevant for this calculation, the form and magnitude of the MS wave function are not constrained experimentally and rather are chosen to give a good fit to long-distance physics with a simple functional form. The other wave function we use is a solution of the Bruecker-Bethe-Goldstone (BBG) equation describing the interaction of a pair of fermions in an independent pair approximation; see, e.g., [23]. It is useful because we can explicitly explore the sensitivity of the result to the unknown short-distance nuclear physics by varying the hard-core radius.

The BBG wave function is obtained as follows. The solution of the Schrödinger equation for two fermions in the Fermi sea interacting through a potential $v(\vec{x}_1,\vec{x}_2)$ takes the form

$$\psi(1,2) = \frac{1}{\sqrt{V}} e^{i\vec{P}\vec{R}_{CM}} \psi(\vec{a}) \tag{11}$$

where $\vec{R}_{CM}$ and $\vec{a}$ are defined as in Eq. (8). The first factor contains the center-of-mass motion and the second is the internal wave function of the interacting pair. $\psi(\vec{a})$ is a solution of the Bethe-Goldstone equation [Eq. (36.15) in [23]] which is simply the Schrödinger equation for two interacting fermions in a Fermi gas, where the Pauli principle forbids the appearance of intermediate states that are already occupied by other fermions. Both wave functions are normalized so that the space integral of the modulus squared of the wave function equals one. In the application of this equation to nuclear matter, the interaction of each particle from the pair with all particles in the nucleus through an effective single particle potential is included, in the independent pair approximation known as Bruecker theory [see Eqs. (41.1) and (41.5) in [23]].

We are interested in s-wave solutions to the Bethe-Goldstone equation since they are the ones that penetrate to small relative distances. Following [23], an s-wave solution of the internal wave function is sought in the form

$$\psi(a) \sim \frac{u(a)}{a} \tag{12}$$

which simplifies the Bethe-Goldstone equation to

$$\left(\frac{d^2}{dx^2} + k^2\right) u(a) = v(a)u(a) - \int_0^\infty \chi(a,y)v(y)u(y)dy \tag{13}$$

where $v(a)$ is the single particle potential in the effective-mass approximation, and the kernel $\chi(a,y)$ is given by

$$\chi(a,y) = \frac{1}{\pi}\left[\frac{\sin k_F(a-y)}{a-y} - \frac{\sin k_F(a+y)}{a+y} \right], \tag{14}$$

where k_F is the Fermi wave number. For the interaction potential between two nucleons in a nucleus we choose a hard core potential for the following reasons. The two particle potential in a nucleus is poorly known at short distances. Measurements (the observed deuteron form factors, the sums of longitudinal response of light nuclei, etc.) constrain only two-nucleon potentials and the wave functions they predict at internucleon distances larger than 0.7 fm [24]. The Bethe-Goldstone equation can be solved analytically when a hard-core potential is used. While the hard-core form is surely only approximate, it is useful for our purposes because it enables us to isolate the sensitivity of the results to the short-distance behavior of the wave function. We stress again that more "realistic" wave functions, including the MS wave function, are in fact not experimentally constrained for distances below 0.7 fm. Rather, their form at short distance is chosen for technical convenience or aesthetics.

Using the hard core potential, the *s*-wave BG wave function is

$$\Psi_{BG}(\vec{a})=\begin{cases} N_{BG}\dfrac{u(a)}{a} & \text{for} \quad a>\dfrac{c}{k_F}, \\ 0 & \text{for} \quad a<\dfrac{c}{k_F}, \end{cases} \tag{15}$$

$$N_{BG}=\frac{1}{\sqrt{\int_{c/k_F}^{R(A)}|u(a)/a|^2 4\pi a^2 da}}, \tag{16}$$

where c/k_F is the hard core radius and $R(A)=1.07A^{1/3}$ is the radius of a nucleus with mass number *A*. Expressions for *u* can be found in [23], Eq. (41.31). The normalization factor N_{BG} is fixed by setting the integral of $|\psi_{BG}|^2$ over the volume of the nucleus equal to one. The function *u* vanishes at the hard core surface by construction and then rapidly approaches the unperturbed value, crossing over that value at the so called "healing distance." At large relative distances and when the size of the normalization volume is large compared to the hard core radius, $u(a)/a$ approaches a plane wave and the normalization factor N_{BG} [Eq. (16)] reduces to the value $1/\sqrt{V_{box}}$, as

$$\psi_{BG}(a)=N_{BG}\frac{u(a)}{a}\rightarrow\frac{1}{\sqrt{V_{box}}}e^{ika}. \tag{17}$$

C. Overlap calculation

The non-relativistic transition matrix element for a transition $\Lambda\Lambda\rightarrow$H inside a nucleus is given by (suppressing spin and flavor)

$$T_{\{\Lambda\Lambda\}\rightarrow \mathrm{H}}=2\pi i\,\delta(E)\int d^3a\,d^3R_{CM} \times\prod_{i=a,b} d^3\rho^i d^3\lambda^i \psi_H^* \psi_\Lambda^a \psi_\Lambda^b \psi_{nuc} e^{i(\vec{k}_H-\vec{k}_{\Lambda\Lambda})\vec{R}_{CM}} \tag{18}$$

where $\delta(E)=\delta(E_H-E_{\Lambda\Lambda})$, $\psi_\Lambda^{a,b}=\psi_\Lambda^{a,b}(\vec{\rho}^{a,b},\vec{\lambda}^{a,b})$, and $\psi_{nuc}=\psi_{nuc}(\vec{a})$ is the relative wave function of the two Λ's in the nucleus. The notation $\{\Lambda\Lambda\}$ is a reminder that the Λ's are in a nucleus. The plane waves of the external particles contain normalization factors $1/\sqrt{V}$ and these volume elements cancel with volume factors associated with the final and initial phase space when calculating decay rates. The integration over the center of mass position of the system gives a 3 dimensional momentum delta function and we can rewrite the transition matrix element as

$$T_{\{\Lambda\Lambda\}\rightarrow \mathrm{H}}=(2\pi)^4 i\,\delta^4(k_f-k_i)\mathcal{M}_{\{\Lambda\Lambda\}\rightarrow \mathrm{H}}, \tag{19}$$

where $|\mathcal{M}|_{\{\Lambda\Lambda\}\rightarrow \mathrm{H}}$ is the integral over the remaining internal coordinates in Eq. (18). In the case of pion or lepton emission, plane waves of the emitted particles should be included in the integrand. For brevity we use here the zero momentum transfer, $\vec{k}=0$, approximation, which we have checked holds with good accuracy; this is not surprising since typical momenta are $\lesssim 0.3$ GeV.

Inserting the IK and BBG wave functions and performing the Gaussian integrals analytically, the overlap of the space wave functions becomes

$$|\mathcal{M}|_{\Lambda\Lambda\rightarrow \mathrm{H}}=\frac{1}{\sqrt{4}}\left(\frac{2f}{1+f^2}\right)^6\left(\frac{3}{2}\right)^{3/4}\left(\frac{\alpha_H}{\sqrt{\pi}}\right)^{3/2} \times N_{BG}\int_{c/k_F}^{R(A)} d^3a\frac{u(a)}{a}e^{-(3/4)\alpha_H^2 a^2} \tag{20}$$

where the factor $1/\sqrt{4}$ comes from the probability that two nucleons are in a relative *s* wave, and *f* is the previously introduced ratio of nucleon to H radius: $\alpha_H=f\alpha_B$. Since N_{BG} has dimensions $V^{-1/2}$ the spatial overlap $\mathcal{M}_{\{\Lambda\Lambda\}\rightarrow \mathrm{H}}$ is a dimensionless quantity, characterized by the ratio *f*, the Isgur-Karl oscillator parameter α_B, and the value of the hard core radius. Figure 1 shows $|\mathcal{M}|^2_{\{\Lambda\Lambda\}\rightarrow \mathrm{H}}$ calculated for oxygen nuclei, versus the hard-core radius, for a range of values of *f*, using the standard value of $\alpha_B=0.406$ GeV for the IK model [21] and also $\alpha_B=0.221$ GeV for comparison.

Figure 1 shows that, with the BBG wave function, the overlap is severely suppressed and that the degree of suppression is very sensitive to the core radius. This confirms that the physics we are investigating depends on the behavior of the nuclear wave function at distances at which it is not directly constrained experimentally. Figure 2 shows a comparison of the overlap using the Miller-Spencer and BBG nuclear wave functions, as a function of the size of the H. One sees that the spatial overlap is strongly suppressed with both wave functions, although quantitatively the degree of suppression differs. We cannot readily study the sensitivity to the functional form of the baryonic wave functions, as there is no well-motivated analytic form we could use to do this calculation other than the IK wave function. However, by comparing the extreme choices of parameter α_B in the IK wave function, also shown in Figs. 1 and 2, we explore the sensitivity of the spatial overlap to the shape of the hadronic wave functions. Fortunately, we will be able to use additional experimental information to constrain the wave function overlap so that our key predictions are insensitive to the overlap uncertainty.

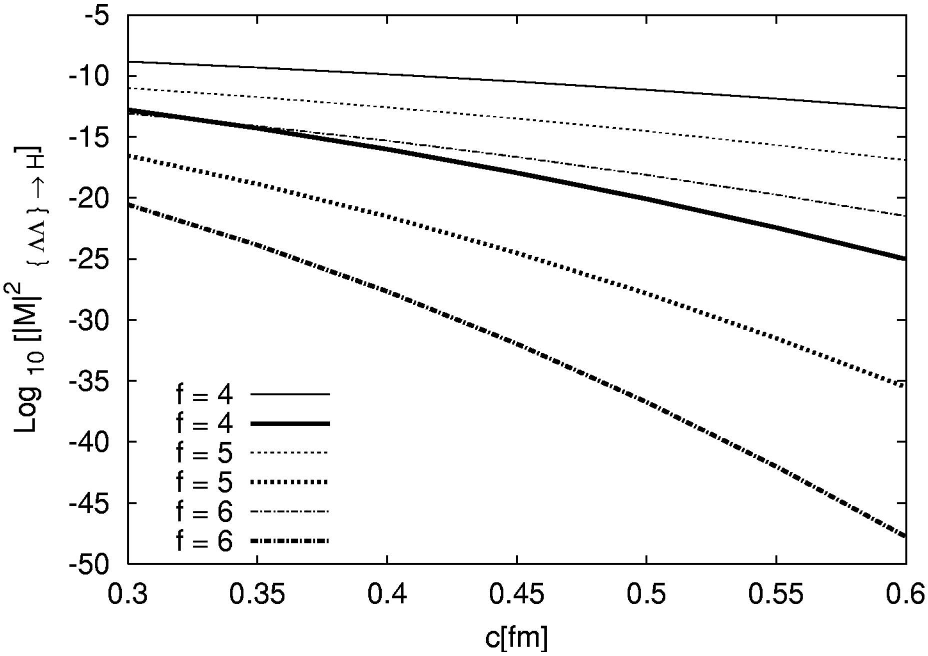

FIG. 1. Log_{10} of $|\mathcal{M}|^2_{\Lambda\Lambda\to\mathrm{H}}$ versus hard core radius in femtometers, for ratio $f=R_N/R_H$ and two values of the Isgur-Karl oscillator parameter: $\alpha_B=0.406$ GeV (thick lines) and $\alpha_B=0.221$ GeV (thin lines).

IV. WEAK INTERACTION MATRIX ELEMENTS

Transition of a two nucleon system to off-shell $\Lambda\Lambda$ requires two strangeness changing weak reactions. Possible $\Delta S=1$ sub-processes to consider are a weak transition with emission of a pion or lepton pair and an internal weak transition. These are illustrated in Fig. 3 for a three quark system. We estimate the amplitude for each of the sub-processes and calculate the overall matrix element for transition to the $\Lambda\Lambda$ system as a product of the sub-process amplitudes.

The matrix element for weak pion emission is estimated from the $\Lambda\to N\pi$ rate:

$$|\mathcal{M}|^2_{\Lambda\to N\pi}=\frac{1}{(2\pi)^4}\frac{2m_\Lambda}{\Phi_2}\frac{1}{\tau_{\Lambda\to N\pi}}=0.8\times 10^{-12}\ \mathrm{GeV}^2. \tag{21}$$

By crossing symmetry this is equal to the desired $|\mathcal{M}|^2_{N\to\Lambda\pi}$, in the approximation of momentum independence which should be valid for the small momenta in this application. Analogously, for lepton pair emission we have

$$|\mathcal{M}|^2_{\Lambda\to Ne\nu}=\frac{1}{(2\pi)^4}\frac{2m_\Lambda}{\Phi_3}\frac{1}{\tau_{\Lambda\to Ne\nu}}=3\times 10^{-12}. \tag{22}$$

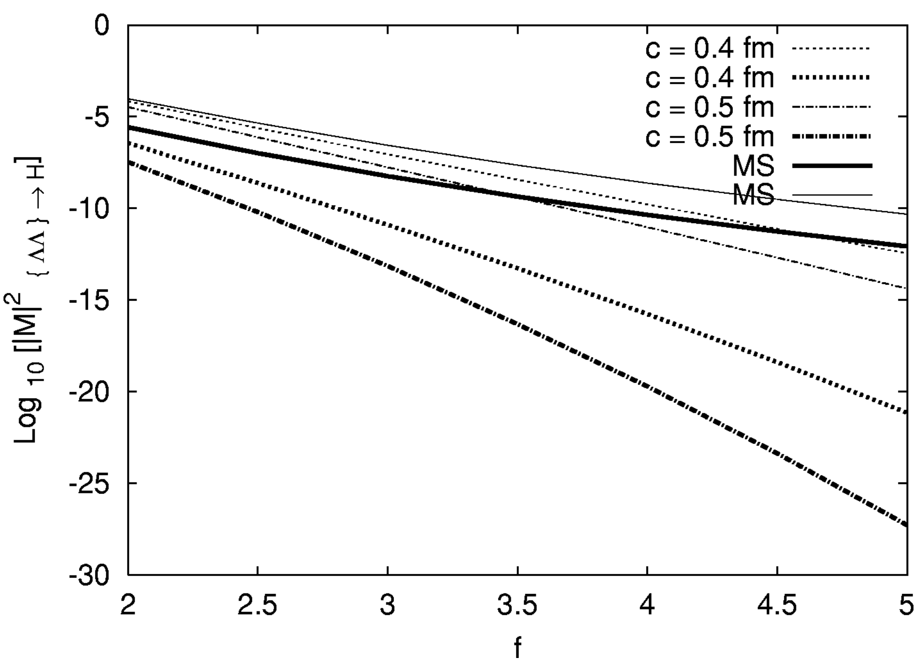

FIG. 2. Log_{10} of $|\mathcal{M}|^2_{\Lambda\Lambda\to\mathrm{H}}$ versus ratio $f=\alpha_H/\alpha_N$, calculated with BBG wave function with core radius 0.4 and 0.5 fm, and with the MS wave function. Thick (thin) lines are for $\alpha_B=0.406$ GeV ($\alpha_B=0.221$ GeV) in the IK wave function.

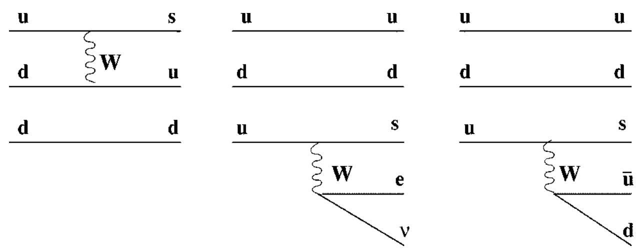

FIG. 3. Some relevant weak transitions for $NN \rightarrow HX$.

The matrix element for internal conversion, $(uds) \rightarrow (udd)$, is proportional to the spatial nucleon wave function when two quarks are at the same point:

$$|\mathcal{M}|_{\Lambda \rightarrow N} \approx \langle \psi_\Lambda | \delta^3(\vec{r}_1 - \vec{r}_2) | \psi_N \rangle \frac{G_F \sin\theta_c \cos\theta_c}{m_q}, \quad (23)$$

where m_q is the quark mass introduced in order to make the 4 point vertex amplitude dimensionless [25]. The expectation value of the delta function can be calculated in the harmonic oscillator model to be

$$\langle \psi_\Lambda | \delta^3(\vec{r}_1 - \vec{r}_2) | \psi_N \rangle = \left(\frac{\alpha_B}{\sqrt{2\pi}} \right)^3 = 0.4 \times 10^{-2} \ \ \mathrm{GeV}^3. \quad (24)$$

The delta function term can also be inferred phenomenologically in the following way, as suggested in [25]. The Fermi spin-spin interaction has a contact character depending on $\vec{\sigma}_1 \vec{\sigma}_2 / m_q^2 \delta(\vec{r}_1 - \vec{r}_2)$, and therefore the delta function matrix element can be determined in terms of electromagnetic or strong hyperfine splitting:

$$(m_{\Sigma^0} - m_{\Sigma^+}) - (m_n - m_p) = \alpha \frac{2\pi}{3 m_q^2} \langle \delta^3(\vec{r}_1 - \vec{r}_2) \rangle, \quad (25)$$

$$m_\Delta - m_N = \alpha_S \frac{8\pi}{3 m_q^2} \langle \delta^3(\vec{r}_1 - \vec{r}_2) \rangle, \quad (26)$$

where m_q is the quark mass, taken to be $m_N/3$. Using the first form to avoid the issue of scale dependence of α_S leads to a value three times larger than predicted by the method used in Eq. (24), namely,

$$\langle \psi_\Lambda | \delta^3(\vec{r}_1 - \vec{r}_2) | \psi_N \rangle = 1.2 \times 10^{-2} \ \ \mathrm{GeV}^3. \quad (27)$$

We average the expectation values (24) and (27) and adopt

$$|\mathcal{M}|^2_{\Lambda \rightarrow N} = 4.4 \times 10^{-15}. \quad (28)$$

In this way we have roughly estimated all the matrix elements for the relevant sub-processes based on weak-interaction phenomenology.

V. NUCLEAR DECAY RATES

A. Lifetime of doubly strange nuclei

The decay rate of a doubly strange nucleus is

$$\Gamma_{A_{\Lambda\Lambda} \rightarrow A'_H \pi} \approx K^2 (2\pi)^4 \frac{m_q^2}{2(2 m_{\Lambda\Lambda})} \Phi_2 |\mathcal{M}|^2_{\Lambda\Lambda \rightarrow H}, \quad (29)$$

where Φ_2 is the two body final phase space factor, defined as in [3], and $m_{\Lambda\Lambda}$ is the invariant mass of the Λ's, $\approx 2 m_\Lambda$. The factor K contains the transition element in spin flavor space. It can be estimated by counting the total number of flavor-spin states a $uuddss$ system can occupy, and taking K^2 to be the fraction of those states which have the correct quantum numbers to form the H. That gives $K^2 \sim 1/1440$, and therefore we write $K^2 = (1440 \kappa_{1440})^{-1}$. Combining these factors we obtain the estimate for the formation time of an H in a doubly strange hypernucleus:

$$\tau_{form} \equiv \tau_{A_{\Lambda\Lambda} \rightarrow A'_H \pi} \approx \frac{3(7) \kappa_{1440} \times 10^{-18} \ \mathrm{s}}{|\mathcal{M}|^2_{\Lambda\Lambda \rightarrow H}}, \quad (30)$$

where the phase space factor was evaluated for $m_H = 1.8(2)$ GeV.

Figure 2 shows $|\mathcal{M}|^2_{\{\Lambda\Lambda\} \rightarrow \mathrm{H}}$ in the range of f and hard-core radius where its value is in the neighborhood of the experimental limits, for the standard choice $\alpha_B = 0.406$ GeV and comparison value $\alpha_B = 0.221$ GeV. In order to suppress $\Gamma(A_{\Lambda\Lambda} \rightarrow A'_H X)$ sufficiently that some Λ's in a double-Λ hypernucleus will decay prior to formation of an H, we require $|\mathcal{M}|^2_{\Lambda\Lambda \rightarrow H} \lesssim 10^{-8}$. If the nucleon hard core potential is used, this is satisfied even for relatively large H, e.g., $r_H \lesssim r_N/2.3$ ($r_N/2.1$) for a hard-core radius 0.4 (0.5) fm and can also be satisfied with the MS wave function as can be seen in Fig. 2. Thus the observation of single Λ decay products from double-Λ hypernuclei cannot be taken to exclude the existence of an H with mass below $2 m_\Lambda$ unless it can be demonstrated that the wave function overlap is large enough.

B. Nuclear conversion to an H

If the H is actually stable ($m_H < 2 m_p + 2 m_e$) two nucleons in a nucleus may convert to an H and cause nuclei to disintegrate. $NN \rightarrow HX$ requires two weak reactions. Thus the rate for the process $A_{NN} \rightarrow A'_H \pi\pi$ is approximately

$$\Gamma_{A_{NN} \rightarrow A'_H \pi\pi} \approx K^2 \frac{(2\pi)^4}{2(2 m_N)} \Phi_3 \left(\frac{|\mathcal{M}|^2_{N \rightarrow \Lambda\pi} |\mathcal{M}|_{\Lambda\Lambda \rightarrow H}}{(2 m_\Lambda - m_H)^2} \right)^2, \quad (31)$$

where the denominator is introduced to correct the dimensions in a way suggested by the $\Lambda\Lambda$ pole approximation. Since other dimensional parameters relevant to this process, e.g., $m_q = m_N/3$ or Λ_{QCD}, are comparable to $2 m_\Lambda - m_H$ and we are aiming only for an order-of-magnitude estimate, any of them could equally well be used. The lifetime for nuclear disintegration with two pion emission is thus

TABLE I. The final particles (in addition to A' and H) and momenta for nucleon-nucleon transitions to H in nuclei. For the 3-body final states marked with an asterisk, the momentum given is for the configuration with H produced at rest.

Mass m_H (GeV)	Final state	Final momenta p (MeV)	Partial lifetime $\times K^2 \vert\mathcal{M}\vert^2_{\Lambda\Lambda\to \mathrm{H}}$ (yr)
1.5	π	318	2×10^{-3}
1.5	$\pi\pi$	170*	0.03
1.8	$e\nu$	48*	70
1.8	γ	96	2×10^{3}

$$\tau_{A_{NN}\to A'_H\pi\pi} \approx \frac{40\kappa_{1440}}{|\mathcal{M}|^2_{\Lambda\Lambda\to \mathrm{H}}} \text{ yr}, \tag{32}$$

taking $m_H = 1.5$ GeV in the phase space factor. For the process with one pion emission and an internal conversion, our rate estimate is

$$\Gamma_{A_{NN}\to A'_H\pi} \approx K^2 \frac{(2\pi)^4}{2(2m_N)} \Phi_2(|\mathcal{M}|_{N\to\Lambda\pi}|\mathcal{M}|_{N\to\Lambda}|\mathcal{M}|_{\Lambda\Lambda\to \mathrm{H}})^2 \tag{33}$$

leading to a lifetime, for $m_H = 1.5$ GeV, of

$$\tau_{A_{NN}\to A'_H\pi} \approx \frac{3\kappa_{1440}}{|\mathcal{M}|^2_{\Lambda\Lambda\to \mathrm{H}}} \text{ yr}. \tag{34}$$

If $m_H \gtrsim 1740$ MeV, pion emission is kinematically forbidden and the relevant final states are $e^+\nu$ or γ; we now calculate these rates. For the transition $A_{NN}\to A'_H e\nu$, the rate is

$$\Gamma_{A_{NN}\to A'_H e\nu} \approx K^2 \frac{(2\pi)^4}{2(2m_N)} \Phi_3(|\mathcal{M}|_{N\to\Lambda e\nu}|\mathcal{M}|_{N\to\Lambda}|\mathcal{M}|_{\Lambda\Lambda\to \mathrm{H}})^2. \tag{35}$$

In this case, the nuclear lifetime is

$$\tau_{A_{NN}\to A'_H e\nu} \approx \frac{\kappa_{1440}}{|\mathcal{M}|^2_{\Lambda\Lambda\to \mathrm{H}}} \times 10^5 \text{ yr}, \tag{36}$$

taking $m_H = 1.8$ GeV. For $A_{NN}\to A'_H\gamma$, the rate is approximately

$$\Gamma_{A_{NN}\to A'_H\gamma} \approx K^2 (2\pi)^4 \frac{\alpha_{EM} m_q^2}{2(2m_N)} \Phi_2(|\mathcal{M}|^2_{N\to\Lambda}|\mathcal{M}|_{\Lambda\Lambda\to \mathrm{H}})^2, \tag{37}$$

leading to the lifetime estimate

$$\tau_{A_{NN}\to A'_H\gamma} \approx \frac{2\kappa_{1440}}{|\mathcal{M}|^2_{\Lambda\Lambda\to \mathrm{H}}} \times 10^6 \text{ yr}, \tag{38}$$

for $m_H = 1.8$ GeV.

One sees from Fig. 1 that a lifetime bound of $\gtrsim$few $\times 10^{29}$ yr is not a very stringent constraint on this scenario if m_H is large enough that pion final states are not allowed. For example, with $\kappa_{1440} = 1$ the right-hand side (rhs) of Eq. (36) is $\gtrsim$few$\times 10^{29}$ yr, for standard α_B, a hard core radius of 0.45 fm, and $r_H \approx 1/5 r_N$—in the middle of the range expected based on the glueball analogy. If m_H is light enough to permit pion production, experimental constraints are much more powerful. We therefore conclude that $m_H \lesssim 1740$ MeV is disfavored and is likely to be excluded, depending on how strong limits SuperK can give. Table I summarizes predictions for various final states and m_H values.

VI. LIFETIME OF AN UNSTABLE H

If $2m_N \lesssim m_H < m_N + m_\Lambda$, the H is not stable but it proves to be very long lived if its wave function is compact enough to satisfy the constraints from doubly strange hypernuclei discussed in Secs. II A and V A. The limits on nuclear stability discussed in the previous section do not apply here because nuclear disintegration to an H is not kinematically allowed.

A. Wave function overlap

To calculate the decay rate of the H we start from the transition matrix element (18). In contrast to the calculation of nuclear conversion rates, the outgoing nucleons are asymptotically plane waves. Nonetheless, at short distances their repulsive interaction suppresses the relative wave function at short distances much as in a nucleus. It is instructive to compute the transition amplitude using two different approximations. First, we treat the nucleons as plane waves so the spatial amplitude is

$$T_{\mathrm{H}\to\Lambda\Lambda} = 2\pi i\,\delta(E_{\Lambda\Lambda} - E_H) \int \prod_{i=a,b} d^3\rho^i d^3\lambda^i d^3 a d^3 \times R_{CM} \psi_H \psi_\Lambda^{*a} \psi_\Lambda^{*b} e^{i(\vec{k}_N^a + \vec{k}_N^b - \vec{k}_H)\vec{R}_{CM}}. \tag{39}$$

The integration over $\vec{R}_{CM}$ gives the usual 4D δ function. Using the Isgur-Karl wave function and performing the remaining integrations leading to $|\mathcal{M}|_{\mathrm{H}\to\Lambda\Lambda}$, as in Eq. (19), the amplitude is

TABLE II. $|\mathcal{M}|^2_{\mathrm{H}\to\Lambda\Lambda}$ in GeV$^{-3/2}$ for different values of f (rows) and nuclear wave function (columns), using the standard value $\alpha_{B1}=0.406$ GeV and the comparison value $\alpha_{B2}=0.221$ GeV in the IK wave function of the quarks.

	BBG, 0.4 fm		BBG, 0.5 fm		MS	
	α_{B1}	α_{B2}	α_{B1}	α_{B2}	α_{B1}	α_{B2}
4	6×10^{-14}	6×10^{-8}	7×10^{-18}	4×10^{-9}	1×10^{-8}	8×10^{-7}
3	5×10^{-9}	3×10^{-5}	3×10^{-11}	7×10^{-6}	2×10^{-6}	9×10^{-5}
2	1×10^{-4}	0.0	1×10^{-5}	0.01	9×10^{-4}	0.03

$$|\mathcal{M}|_{\mathrm{H}\to\Lambda\Lambda}=\left(\frac{2f}{1+f^2}\right)^6\left(\frac{3}{2}\right)^{3/4}\left(\frac{\alpha_H}{\sqrt{\pi}}\right)^{3/2}\times\int_0^\infty d^3a\, e^{-(3/4)\alpha_H^2a^2-i[(\vec{k}_N^a-\vec{k}_N^b)/2]\vec{a}}$$
$$=\left(\frac{8}{3\pi}\right)^{3/4}\left(\frac{2f}{1+f^2}\right)^6\alpha_H^{-3/2}e^{-(\vec{k}_N^a-\vec{k}_N^b)^2/12\alpha_H^2}. \tag{40}$$

The amplitude depends on the size of the H through the factor $f=r_N/r_H$. Note that the normalization N_{BG} in the analogous result (20) which comes from the Bethe-Goldstone wave function of Λ's in a nucleus has been replaced in this calculation by the plane wave normalization factor $1/\sqrt{V}$ which cancels with the volume factors in the phase space when calculating transition rates.

Transition rates calculated using Eq. (40) provide an upper limit on the true rates, because the calculation neglects the repulsion of two nucleons at small distances. To estimate the effect of the repulsion between nucleons we again use the Bethe-Goldstone solution with the hard core potential. It has the desired properties of vanishing inside the hard core radius and rapidly approaching the plane wave solution away from the hard core. As noted in Sec. III B, $N_{BG}\to1/\sqrt{V}$, for $a\to\infty$. Therefore, we can write the transition amplitude as in Eq. (20), with the normalization factor $1/\sqrt{V}$ canceled with the phase space volume element:

$$|\mathcal{M}|_{\mathrm{H}\to\Lambda\Lambda}=\left(\frac{2f}{1+f^2}\right)^6\left(\frac{3}{2}\right)^{3/4}\left(\frac{\alpha_H}{\sqrt{\pi}}\right)^{3/2}\times\int_0^\infty d^3a\frac{u(a)}{a}e^{-(3/4)\alpha_H^2a^2}. \tag{41}$$

This should give a more realistic estimate of decay rates. Table II shows the overlap values for a variety of choices of r_H, hard-core radii, and α_B. Also included are the results with the MS wave function.

B. Empirical limit on wave function overlap

As discussed in Sec. V A, the H can be lighter than 2 Λ's without conflicting with hypernuclear experiments if it is sufficiently compact, as suggested by some models. The constraint imposed by the hypernuclear experiments can be translated into an empirical upper limit on the wave function overlap between an H and two baryons. Using Eq. (30) for the formation time τ_{form} of an H in a double-Λ oxygen-16 hypernucleus we have

$$|\mathcal{M}|^2_{\Lambda\Lambda\to\mathrm{H}}=7\times10^{-8}\frac{\kappa_{1440}}{f_{form}}\left(\frac{\tau_{form}}{10^{-10}\ \mathrm{s}}\right)^{-1}, \tag{42}$$

where $f_{form}=\Phi_2(m_H)/\Phi_2(m_H=2\ \mathrm{GeV})$ is the departure of the phase space factor for hypernuclear H formation appearing in Eq. (29), from its value for $m_H=2$ GeV. By crossing symmetry the overlap amplitudes $|\mathcal{M}|_{\mathrm{H}\to\Lambda\Lambda}$ and $|\mathcal{M}|_{\Lambda\Lambda\to\mathrm{H}}$ differ only because the Λ's in the former are asymptotically plane waves while for the latter they are confined to a nucleus; comparing Eqs. (41) and (20) we obtain

$$|\mathcal{M}|^2_{\mathrm{H}\to\Lambda\Lambda}=\frac{4}{N_{BG}^2}|\mathcal{M}|^2_{\Lambda\Lambda\to\mathrm{H}}. \tag{43}$$

For oxygen-16, $N_{BG}^2/4\approx(1/5\times10^4)$ GeV^3. Using Eqs. (42) and (43) will give us an upper limit on the overlap for the lifetime calculations of the next section.

C. Decay rates and lifetimes

Starting from $|\mathcal{M}|_{\mathrm{H}\to\Lambda\Lambda}$ we can calculate the rates for H decay in various channels, as we did for nuclear conversion in the previous section. The rate of $H\to nn$ decay is

$$\Gamma_{H\to nn}\approx K^2\frac{(2\pi)^4m_q^5}{2m_H}\Phi_2(m_H)(|\mathcal{M}|^2_{N\to\Lambda}|\mathcal{M}|_{\mathrm{H}\to\Lambda\Lambda})^2, \tag{44}$$

where Φ_2 is the phase space factor defined for $H\to nn$ normalized as in [3]. Using Eqs. (43) and (42), the lifetime for $H\to nn$ is

$$\tau_{H\to NN}\approx9(4)\times10^7\mu_0\ \ \mathrm{yr}, \tag{45}$$

for $m_H=1.9(2)$ GeV, where $\mu_0\gtrsim1$ is defined to be $(\tau_{form}f_{form})/(10^{-10}\ \mathrm{s})\times(5\times10^4N_{BG}^2)/4$. The H is therefore cosmologically stable, with a lifetime longer than the age of the Universe, if $|\mathcal{M}|^2_{\Lambda\Lambda\to\mathrm{H}}$ is 10^{2-3} times smaller than needed to satisfy double hypernuclear constraints. As can be seen in Fig. 2, this corresponds to $r_H\lesssim(1/3)r_N$ in the IK model discussed above. Note that κ_{1440} and the sensitivity to the wave function overlap have been eliminated by using τ_{form}.

If m_N+m_Λ (2.05 GeV)$<m_H<2m_\Lambda$ (2.23 GeV), H decay requires only a single weak interaction so the rate in Eq. (44) must be divided by $|\mathcal{M}|^2_{N\to\Lambda}$ given in Eq. (28). Thus we have

$$\tau_{H\to N\Lambda}\approx 10\mu_0 \ \text{s}. \tag{46}$$

Finally, if $m_H>2m_\Lambda$ (2.23 GeV), there is no weak interaction suppression and

$$\tau_{\mathrm{H}\to\Lambda\Lambda}\approx 4\times 10^{-14}\mu_0 \ \text{s}. \tag{47}$$

Equations (45)–(47) with $\mu_0=1$ give the lower bound on the H lifetime, depending on its mass. This result for the H lifetime differs sharply from the classic calculation of Donoghue, Golowich, and Holstein [26], because we rely on experiment to put an upper limit on the wave function overlap $|\mathcal{M}|^2_{\mathrm{H}\to\Lambda\Lambda}$. Our treatment of the color-flavor-spin and weak interaction parts of the matrix elements is approximate, but it should roughly agree with the more detailed calculation of Ref. [26], so the difference in lifetime predictions indicates that the spatial overlap is far larger in their bag model than using the IK and Bethe-Goldstone or Miller-Spencer wave functions with reasonable parameters consistent with the hypernuclear experiments. The bag model is not a particularly good description of sizes of hadrons, and in the treatment of [26] the H size appears to be fixed implicitly to some value which may not be physically realistic. Furthermore, it is hard to tell whether their bag model analysis gives a good accounting of the known hard core repulsion between nucleons. As our calculation of previous sections shows, these are crucial parameters in determining the overlap. The calculation of the weak interaction and color-flavor-spin matrix elements in Ref. [26] could be combined with our phenomenological approach to the spatial wave function overlap to provide a more accurate yet general analysis. We note that due to the small size of the H, the *p*-wave contribution should be negligible.

If the H is long lived enough to conceivably be the dark matter, i.e., $m_H\lesssim 2.05$ GeV, one would like to use experiment to investigate the possibility that the dark matter could consist of H and/or $\overline{\mathrm{H}}$'s. The Sudbury Neutrino Observatory (SNO) can probably place good limits on the rate of $H\to nn$ in that detector. The next most important channel $H\to nn\gamma$ should be easy to detect in SuperK for H mass such that the photon energy is in the low-background range $\approx$20–100 MeV [27], or in Kamland for lower photon energies.[2] The rate is

$$\Gamma_{H\to nn\gamma}\approx K^2\alpha_{EM}\frac{(2\pi)^4 m_q^3}{2m_H}\Phi_3(m_H)(|\mathcal{M}|^2_{N\to\Lambda}|\mathcal{M}|_{\mathrm{H}\to\Lambda\Lambda})^2 \tag{48}$$

leading to

$$\tau_{H\to NN\gamma}\approx 4\times 10^{14}(6\times 10^{12})\mu_0 \ \text{yr}, \tag{49}$$

for $m_H=1.9$ (2) GeV. The lifetime for $H\to npe\nu$ is similar in magnitude but is more sensitive to m_H due to the 4-body phase space:

$$\Gamma_{H\to pne\nu}\approx K^2\frac{(2\pi)^4 m_q}{2m_H}\Phi_4(m_H) \times(|\mathcal{M}|_{N\to\Lambda}|\mathcal{M}|_{N\to\Lambda e\nu}|\mathcal{M}|_{\mathrm{H}\to\Lambda\Lambda})^2. \tag{50}$$

For $m_H=1.9$ (2) GeV

$$\tau_{H\to pne\nu}\approx 10^{15} \ (5\times 10^{11})\mu_0 \ \text{yr}. \tag{51}$$

Estimates of the local number density of H's at various depths in the Earth, assuming the dark matter consists of H's and $\overline{\mathrm{H}}$'s, will be discussed in Ref. [28].

VII. SUMMARY

We have considered the constraints placed on the H dibaryon by the stability of nuclei and hypernuclei with respect to conversion to an H, and we have calculated the lifetime of the H if it is heavier than two nucleons. First we performed calculations using specific models for the relevant wave functions. In the model calculations we used the Isgur-Karl wave functions for quarks in baryons and the H, and the Miller-Spencer and Bruecker-Bethe-Goldstone wave functions for nucleons in a nucleus, to obtain a rough estimate of the H-baryon-baryon wave function overlap. By varying the IK oscillator strength parameter and the hard-core radius in the BBG wave function over extreme ranges, we find that the wave function overlap is very sensitive to the size and shape of the hadronic and nuclear wave functions. With the BBG (MS) wave function, the hypernuclear formation time of an H is comparable to or larger than the decay time for the Λ's and thus the H is not excluded, if $r_H\lesssim 1/2$ $(1/3)r_N$.[3] We conclude that the observation of Λ decays in double-Λ hypernuclei cannot be used to exclude $m_H<2m_\Lambda$, given our present lack of understanding of the hadronic and nuclear wave functions.

In the second part of our work we abstracted empirical relations which give us relatively model-independent predictions for the H lifetime. By crossing symmetry, the overlap of the wave functions of an H and two baryons can be constrained using experimental limits on the formation time of an H in a hypernucleus. Using the empirically constrained wave function overlap and phenomenologically determined weak interaction matrix elements, we can estimate the lifetime of the H with relatively little model uncertainty. We find the following.

If $m_N+m_\Lambda\lesssim m_H\lesssim 2m_\Lambda$, the H lifetime is $\gtrsim 10$ s.

If $2m_N\lesssim m_H\lesssim m_N+m_\Lambda$, the H lifetime is $\gtrsim 10^8$ yr. For $r_H\lesssim(1/3)r_N$ as suggested by some models, the H lifetime is

[2]G.R.F. thanks T. Kajita for informative discussions on these issues.

[3]The overlap between an H and two nucleons should be strongly suppressed also in the Skyrme model, in view of the totally different nature of the nucleon and H solitons [29,30]. However, a method for computing the overlap has not been developed so we are unable to explore this here.

comparable to or greater than the age of the Universe.

If $m_H > 2m_\Lambda$, the hypernuclear constraint is not applicable but the H would still be expected to be long lived, in spite of decaying through the strong interactions. For example, with the BBG wave function and $r_H \lesssim (1/2) r_N$, $\tau_H \gtrsim 4\times10^{-14}$ s.

Our results have implications for several experimental programs:

(1) The observation of Λ decays from double Λ hypernuclei excludes that τ_{form}, the formation time of the H in a double Λ hypernucleus, is much less than τ_Λ. However if τ_{form} is of order τ_Λ, some double Λ hypernuclei would produce an H. One might hope these H's could be observed by reconstructing them through their decay products, e.g., $H\rightarrow\Sigma^- p$. Unfortunately, our calculation shows that $\tau_H \gtrsim 10$ s for the relevant range of m_H, so any H's produced would diffuse out of the apparatus before decaying.[4]

(2) Some calculations have found $m_H < 2(m_p + m_e)$, in which case the H is absolutely stable and nucleons in nuclei may convert to an H. We showed that SuperK can place important constraints on the conjecture of an absolutely stable H, or conceivably discover evidence of its existence, through observation of the pion(s), positron, or photon produced when two nucleons in an oxygen nucleus convert to an H. We estimate that SuperK could achieve a lifetime limit $\tau \gtrsim \text{few}\times10^{29}$ yr. This is the lifetime range estimated with the BBG wave function for $m_H \gtrsim 1740$ MeV and $r_H \approx (1/5) r_N$. An H smaller than this seems unlikely, so $m_H \lesssim 1740$ MeV is probably already ruled out.

(3) If $m_H \lesssim 2.05$ GeV *and* $r_H \lesssim (1/3) r_N$ the H lifetime is comparable to the age of the Universe. It is possible that H and anti-H were produced in sufficient abundance in the early Universe to account for dark matter and the baryon asymmetry, as will be discussed elsewhere [28]. We have shown that SuperK and SNO can place limits on signatures of H decay and anti-H annihilation in this scenario and calculated the rates of relevant reactions.

[4]G.R.F. thanks K. Imai for bringing the idea for this experiment to her attention.

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