Abnormal number of Nambu-Goldstone bosons in the color-asymmetric dense color superconducting phase of a Nambu–Jona-Lasinio–type model

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We consider an extended Nambu–Jona-Lasinio model including both $(q\bar{q})$ and (qq) interactions with two light-quark flavors in the presence of a single (quark density) chemical potential. In the color superconducting phase of the quark matter the color $SU_c(3)$ symmetry is spontaneously broken down to $SU_c(2)$. If the usual counting of Goldstone bosons would apply, five Nambu-Goldstone (NG) bosons corresponding to the five broken color generators should appear in the mass spectrum. Unlike that expectation, we find only three gapless diquark excitations of quark matter. One of them is an $SU_c(2)$ singlet; the remaining two form an $SU_c(2)$ (anti)doublet and have a quadratic dispersion law in the small momentum limit. These results are in agreement with the Nielsen-Chadha theorem, according to which NG bosons in Lorentz-noninvariant systems, having a quadratic dispersion law, must be counted differently. The origin of the abnormal number of NG bosons is shown to be related to a nonvanishing expectation value of the color charge operator Q_8 reflecting the lack of color neutrality of the ground state. Finally, by requiring color neutrality, two massive diquarks are argued to become massless, resulting in a normal number of five NG bosons with the usual linear dispersion laws.

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I. INTRODUCTION

It is well known that, in accordance with the Goldstone theorem [1,2], N Nambu-Goldstone (NG) bosons appear in Lorentz-invariant systems if an internal continuous symmetry group $\mathcal{G}$ is spontaneously broken down to a subgroup $\mathcal{H}$ (here N is the number of generators in the coset space $\mathcal{G}/\mathcal{H}$); i.e., the number of NG modes is equal to the number of broken generators. However, in Lorentz-noninvariant systems the number of NG bosons can be less than N. In this case, the counting of NG bosons is regulated by the Nielsen-Chadha (NC) theorem [3]: Let n_1 and n_2 be the numbers of gapless excitations that in the limit of long wavelengths have the dispersion laws $E\sim|\vec{p}|$ and $E\sim|\vec{p}|^2$, respectively; then, $N\leqslant n_1+2n_2$. (Here, E is the energy and $\vec{p}$ is the three-momentum of the particle.) In particular, this theorem is valid for relativistically covariant theories as well, since in this case (i) the total number of NG bosons equals N, the number of broken symmetry generators [2]; (ii) evidently, the dispersion law for these N massless excitations looks like $E\sim|\vec{p}|$, thus $N=n_1$.

Recently, in some relativistic models describing the dynamics of the kaon condensate in the color-flavor-locked phase of dense quark matter, an abnormal number of NG bosons has been revealed [4,5]. The same is true for models with a massive relativistic vector fields interaction in the presence of a chemical potential [6]. Since the Lorentz invariance is broken in this case and some of the gapless excitations have a quadratic dispersion law, there are no contradictions with either Goldstone or NC theorems. The superfluid ^{3}He in the A phase [7] and ferromagnets [8,9] are other known examples of condensed-matter systems with an abnormal number of NG bosons.

In the present paper, we demonstrate the abnormal number of NG bosons in the dense color superconducting phase (2SC) of quark matter for a simple version of the Nambu–Jona-Lasinio (NJL) model with two light quarks and a single (quark number) chemical potential. In this phase, which can be realized naturally only at sufficiently large values of the chemical potential (300 MeV$\leqslant\mu<$1 GeV), the initial color $SU_c(3)$ symmetry is spontaneously broken down to the $SU_c(2)$ group. Hence, in accordance with the usual counting of the Goldstone theorem, one might expect five NG bosons, corresponding to the five broken symmetry generators, to

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appear in the NJL model. We shall prove that in the 2SC phase only three gapless excitations that can be identified with NG bosons appear. Two of them have a quadratic dispersion law, thus yielding no contradiction with the NC theorem. For further applications, notice also the following important criterion which is sufficient for the equality between the number of NG bosons and the number of broken generators [5]: If Q_i, $i=1,\ldots,N$, is the full set of broken generators and if $\langle[Q_i,Q_j]\rangle=0$ for any pair (i,j), then the number of NG bosons is equal to the number N of broken generators.

Recall, that there is an alternative approach to study color superconductivity, which is based on weak coupling perturbative QCD [10]. In this case, using the Schwinger-Dyson equation with one-gluon exchange, the color superconducting phase was proved to exist at asymptotically high densities. In the two-flavored QCD investigations of color superconductivity five scalar NG bosons are shown to exist, which are mixing with gluons and required as longitudinal components for five massive gluons (color Meissner effect) [11]. Hence, the results concerning the number of NG bosons of both the QCD and the NJL model approaches seem to be in contradiction.[1] The origin of the above discrepancy is related to the fact that the ground state of the considered simple NJL-type model is not color neutral. Indeed, since the expectation value of the color charge operator Q_8 is nonvanishing, $\langle Q_8\rangle\neq 0$, the criterion [5] is not applicable, and an abnormal number of NG bosons must arise. On the other hand, in quark models with a color-neutral ground state, according to the above criterion, there should arise a normal number of five NG bosons all having normal (linear) dispersion laws. Most interestingly, we find from the analysis of the mass spectrum in our model that the masses of two scalar diquarks are proportional to the ground-state expectation value of the color charge $\langle Q_8\rangle$. Thus, when color neutrality—i.e., $\langle Q_8\rangle=0$—is imposed in NJL-type models as an additional condition (which can be realized by introducing a color-chemical potential μ_8 "by hand" [12]), then two additional massless particles should appear, and the total number of NG bosons would be equal to 5. Recently, it was shown that the ground state of the 2SC phase of QCD is automatically color neutral [13] due to a dynamical generation of the color-chemical potential μ_8 by gluon condensation. So we see that the original discrepancy in the number of NG bosons between the QCD approach and an extended NJL approach would disappear.

The paper is organized as follows. In Sec. II we investigate the considered NJL-type model in the framework of the Nambu-Gor'kov formalism and derive an effective meson-diquark action for a finite chemical potential. Moreover, the gap equations for the quark and diquark condensates are numerically studied. Sections III and IV contain a detailed analysis of the massless and massive excitations in the diquark sectors and explicitly show the influence of the color properties of the ground state on the diquark mass spectrum. Section V contains a summary and discussions. Finally, dispersion laws for gapless diquarks are derived in the Appendix.

II. MODEL AND ITS EFFECTIVE ACTION

It is well known that perturbative methods are not applicable in the low-energy and -density QCD regions. Instead, effective field theories are usually considered. The most popular effective theories are based on Lagrangians with four-fermion interactions, like the NJL-type models. Let us first give several (very approximate) arguments somehow justifying the chosen structure of our QCD-motivated NJL model introduced below. For this aim, consider two-flavor QCD with a nonzero chemical potential and the color group $SU_c(3)$. By integrating in the generating functional of QCD over gluons and further "approximating" the nonperturbative gluon propagator by a δ function, one arrives at an effective local chiral four-quark interaction of the NJL type describing low-energy hadron physics. Finally, by performing a Fierz transformation of the interaction term and taking into account only scalar and pseudoscalar $(\bar{q}q)$- as well as scalar (qq)-interaction channels, one obtains a four-fermionic model given by the Lagrangian[2] (in Minkowski space-time notation)

$$L=\bar{q}[\gamma^\nu i\partial_\nu+\mu\gamma^0-m_0]q+G_1[(\bar{q}q)^2+(\bar{q}i\gamma^5\vec{\tau}q)^2]+G_2[\bar{q}^C\epsilon^\delta\gamma^5\tau^2q][\bar{q}\epsilon^\delta\gamma^5\tau^2q^C]. \quad (1)$$

In Eq. (1), $\mu\geqslant 0$ is the quark chemical potential, which in isotopically symmetric quark matter is the same for both quark flavors, $q^C=C\bar{q}^t$, $\bar{q}^C=q^tC$ are charge-conjugated spinors, and $C=i\gamma^2\gamma^0$ is the charge conjugation matrix (the symbol t denotes the transposition operation). The quark field $q\equiv q_{i\alpha}$ is a flavor doublet and color triplet as well as a four-component Dirac spinor, where $i=1,2$; $\alpha=1,2,3$. (Latin and Greek indices refer to flavor and color indices, respectively; spinor indices are omitted.) Furthermore, we use the notation $\vec{\tau}\equiv(\tau^1,\tau^2,\tau^3)$ for Pauli matrices in flavor space; $(\epsilon^\delta)^{\alpha\beta}\equiv\epsilon^{\alpha\beta\delta}$ is the totally antisymmetric tensor in color space, respectively. Clearly, Lagrangian (1) is invariant under the chiral $SU(2)_L\times SU(2)_R$ (at $m_0=0$) and color $SU_c(3)$ symmetry groups. The physics of light mesons [14–17], diquarks [18,19], and meson-baryon interactions [20,21] was successfully described in the framework of different NJL models. These effective theories were involved for the investigation of both ordinary [22–24], hot and dilute [16], and color superconducting dense quark matter [25–28]. Usually,

[1]Of course, it is necessary to point out that the QCD weak coupling considerations are valid at asymptotic values of the chemical potential—i.e., at $\mu>1$ GeV—whereas NJL model results are correct for sufficiently lower values $\mu<1$ GeV. Thus, a direct comparison of results seems to be problematic.

[2]The most general Fierz-transformed four-fermion interaction includes additional vector and axial-vector $(\bar{q}q)$ as well as pseudoscalar, vector and axial-vector-like (qq) interactions. However, these terms are omitted here for simplicity.

on the basis of light-meson and baryon phenomenology, the following restrictions on the coupling constants are considered:

$$G_1 > \pi^2/(6\Lambda^2), \quad G_2 < G_1, \tag{2}$$

where Λ is the cutoff parameter in the three-dimensional momentum space, necessary to eliminate the ultraviolet divergences appearing when quantum effects (loops) are taken into account (usually $\Lambda < 1$ GeV). Since Eq. (1) is a low-energy effective theory for QCD, the external parameter μ is restricted from the above: $\mu < \Lambda$. In the region of coupling constants (2) and at small values of μ (the case of low baryon densities), only the operator $\bar{q}q$ has a nonzero vacuum expectation value. Thus, the chiral symmetry of the model is spontaneously broken in this case, while $SU_c(3)$ remains intact. The behavior of quark and meson masses is established rather well in the quark matter of low density [22,23]. In particular, the number of π mesons, which are the three NG bosons in this phase, is equal to the number of broken symmetry generators. Hence, this example is a good demonstration of the fact that Lorentz noninvariance is a necessary but not sufficient condition for the abnormal number of NG bosons to appear in the system. At sufficiently high values of the chemical potential $\mu \sim 300$–350 MeV, the two-flavor color superconductivity phase occurs in model (1) [27]. So a nonzero diquark condensate ($\langle qq \rangle \neq 0$) is formed in this case, and as a consequence, color symmetry is spontaneously broken down to the $SU_c(2)$ subgroup. A naive counting gives us five NG bosons in this case [it is just the number of broken generators of the $SU_c(3)$ group]. However, as will be shown further, there are only three gapless bosonic excitations of the 2SC quark-matter ground state. Two of them satisfy the quadratic dispersion law, in agreement with the NC theorem.

The linearized version of Lagrangian (1) that contains auxiliary bosonic fields has the form

$$\tilde{L} = \bar{q}[\gamma^\nu i\partial_\nu + \mu\gamma^0 - \sigma - m_0 - i\gamma^5 \vec{\tau}\vec{\pi}]q - \frac{\sigma^2 + \vec{\pi}^2}{4G_1} - \frac{\Delta^{*\delta}\Delta^\delta}{4G_2} + \frac{i\Delta^{*\delta}}{2}[\bar{q^C}\epsilon^\delta\gamma^5\tau^2 q] - \frac{i\Delta^\delta}{2}[\bar{q}\epsilon^\delta\gamma^5\tau^2 q^C]. \tag{3}$$

Lagrangians (1) and (3) are equivalent to the equations of motion for bosonic fields, from which it follows that

$$\Delta^\delta \sim \bar{q^C}\epsilon^\delta\gamma^5 q, \quad \sigma \sim \bar{q}q, \quad \vec{\pi} \sim i\bar{q}\gamma^5\vec{\tau}q. \tag{4}$$

However, it is more convenient to start our consideration from Lagrangian (3). Clearly, the σ and $\vec{\pi}$ fields are color singlets. Besides, the (bosonic) diquark field Δ^δ is a color antitriplet and a (isoscalar) singlet under the chiral $SU(2)_L \times SU(2)_R$ group. Note further that σ and Δ^δ are (Lorentz) scalars, but $\vec{\pi}$ are pseudoscalar fields. Hence, if $\langle\sigma\rangle \neq 0$, then the chiral symmetry of the model is spontaneously broken (at $m_0 = 0$), whereas $\langle\Delta^\delta\rangle \neq 0$ indicates the dynamical breaking of color symmetry. In the framework of the Nambu-Gor'kov formalism (for a recent relativistic treatment see, e.g., Ref. [29]) quark fields are represented by a bispinor

$$\Psi = \begin{pmatrix} q \\ q^C \end{pmatrix}.$$

Integrating in the generating functional based on the Lagrangian (3) over the quark fields, one obtains an effective meson-diquark action of the original model (1):

$$\mathcal{S}_{\text{eff}}(\sigma, \vec{\pi}, \Delta^\delta, \Delta^{*\delta}) = -\int d^4x \left[\frac{\sigma^2 + \vec{\pi}^2}{4G_1} + \frac{\Delta^\delta\Delta^{*\delta}}{4G_2}\right] - \frac{i}{2}\mathrm{Tr}_{sfcx}\ln\begin{pmatrix} D^+ & K^- \\ K^+ & D^- \end{pmatrix}. \tag{5}$$

Besides of an evident trace over the two-dimensional Nambu-Gor'kov matrix, the Tr operation in Eq. (5) stands for calculating the trace in spinor (s), flavor (f), and color (c) as well as four-dimensional coordinate (x) spaces, correspondingly. We have used also the notation

$$K^+ = i\Delta^{*\delta}\epsilon^\delta\gamma^5, \quad K^- = -i\Delta^\delta\epsilon^\delta\gamma^5,$$

$$D^\pm = i\gamma^\nu\partial_\nu \pm \mu\gamma^0 - m_0 - \Sigma^\pm, \quad \Sigma^\pm = \sigma \pm i\gamma^5\vec{\pi}\vec{\tau}. \tag{6}$$

Let us introduce notation for the ground-state expectation values of the meson and diquark fields: $\langle\sigma\rangle = \sigma_0$, $\langle\vec{\pi}\rangle = \vec{\pi}_0$, $\langle\Delta^{*\delta}\rangle = \Delta_0^{*\delta}$, $\langle\Delta^\delta\rangle = \Delta_0^\delta$ and

$$(K_0^\pm, D_0^\pm, \Sigma_0^\pm) = (K^\pm, D^\pm, \Sigma^\pm)|_{\sigma=\sigma_0, \vec{\pi}=\vec{\pi}_0, \Delta^\delta=\Delta_0^\delta, \ldots}$$

The quantities $\sigma_0, \pi_0^k, \Delta_0^\delta, \Delta_0^{*\delta}$ $(k, \delta = 1,2,3)$ correspond to the global minimum of the effective potential V_{eff} and can be found as a solution of the system of equations

$$\frac{\partial V_{\text{eff}}}{\partial\pi^k} = 0, \quad \frac{\partial V_{\text{eff}}}{\partial\sigma} = 0, \quad \frac{\partial V_{\text{eff}}}{\partial\Delta^\delta} = 0, \quad \frac{\partial V_{\text{eff}}}{\partial\Delta^{*\delta}} = 0, \tag{7}$$

usually called gap equations. In Eqs. (7) we used the following definition of the effective potential:

$$\mathcal{S}_{\text{eff}}|_{\sigma, \vec{\pi}, \Delta^\delta, \Delta^{*\delta} = \text{const}} = -V_{\text{eff}}(\sigma, \vec{\pi}, \Delta^\delta, \Delta^{*\delta}) \int d^4x, \tag{8}$$

where, in the spirit of the mean-field approximation, all fields are considered to be independent of x. Without losing any generality of the consideration, one can search for a solution of Eqs. (7) in the form $(\Delta_0^1, \Delta_0^2, \Delta_0^3) \equiv (0,0,\Delta)$. Moreover, we suppose also that $\vec{\pi}_0 = 0$. (The last assumption is maintained by the observation that in the theory of strong interactions P parity is a conserved quantity.) If $\Delta = 0$, the color symmetry of the model remains intact. If $\Delta \neq 0$, then $SU_c(3)$ symmetry is spontaneously broken down to $SU_c(2)$, and the 2SC phase is realized in the model. In this case, the system of gap equations (7) is reduced to

$$\frac{\sigma_0}{2G_1}=4iM\int\frac{d^4q}{(2\pi)^4E}\left\{\frac{E^+}{q_0^2-(E^+)^2}+\frac{E^-}{q_0^2-(E^-)^2}+\frac{2E^+}{D_+(q_0)}+\frac{2E^-}{D_-(q_0)}\right\},\tag{9}$$

$$\frac{\Delta}{4G_2}=4i\Delta\int\frac{d^4q}{(2\pi)^4}\left\{\frac{1}{D_+(q_0)}+\frac{1}{D_-(q_0)}\right\},\tag{10}$$

where $D_\pm(q_0)=q_0^2-(E^\pm)^2-|\Delta|^2$, $E^\pm=E\pm\mu$, $E=\sqrt{\vec{q}^{\,2}+M^2}$, and $M=m_0+\sigma_0$ is the constituent quark mass. In these and other similar expressions, q_0 is a shorthand notation for $q_0+i\varepsilon\,\mathrm{sgn}(q_0)$, where the limit $\varepsilon\to 0_+$ must be taken at the end of all calculations. This prescription correctly implements the role of μ as chemical potential and preserves the causality of the theory (see, e.g., [30]).

Let us now make a field shift $\sigma(x)\to\sigma(x)+\sigma_0$, $\Delta^{*\delta}(x)\to\Delta^{*\delta}(x)+\Delta_0^{*\delta}$, $\Delta^\delta(x)\to\Delta^\delta(x)+\Delta_0^\delta$ and then expand the expression (5) into a power series of the meson and diquark fields. The second-order term $\mathcal{S}_{\rm eff}^{(2)}$ of this expansion is responsible for the mass spectrum of mesons and diquarks. It looks like

$$\mathcal{S}_{\rm eff}^{(2)}(\sigma,\vec{\pi},\Delta^\delta,\Delta^{*\delta})=-\int d^4x\left[\frac{\sigma^2+\vec{\pi}^2}{4G_1}+\frac{\Delta^\delta\Delta^{*\delta}}{4G_2}\right]+\frac{i}{4}\mathrm{Tr}_{sfcx}\left\{S_0\begin{pmatrix}\Sigma^+, & -K^-\\ -K^+, & \Sigma^-\end{pmatrix}\times S_0\begin{pmatrix}\Sigma^+, & -K^-\\ -K^+, & \Sigma^-\end{pmatrix}\right\},\tag{11}$$

where S_0 is the quark propagator, which is an evident 2×2 matrix in the Nambu-Gor'kov space

$$S_0=\begin{pmatrix}D_0^+ & K_0^-\\ K_0^+ & D_0^-\end{pmatrix}^{-1}\equiv\begin{pmatrix}a & b\\ c & d\end{pmatrix},\tag{12}$$

whose matrix elements can be found by means of the projection operator technique [31]:

$$a=\int\frac{d^4q}{(2\pi)^4}e^{-iq(x-y)}\left\{\frac{q_0-E^+}{q_0^2-(E^+)^2+|\Delta|^2\epsilon^3\epsilon^3}\gamma^0\overline{\Lambda}_+ + \frac{q_0+E^-}{q_0^2-(E^-)^2+|\Delta|^2\epsilon^3\epsilon^3}\gamma^0\overline{\Lambda}_-\right\},\tag{13}$$

$$b=-i\Delta\epsilon^3\int\frac{d^4q}{(2\pi)^4}e^{-iq(x-y)}\times\left\{\frac{1}{q_0^2-(E^+)^2+|\Delta|^2\epsilon^3\epsilon^3}\gamma^5\overline{\Lambda}_- + \frac{1}{q_0^2-(E^-)^2+|\Delta|^2\epsilon^3\epsilon^3}\gamma^5\overline{\Lambda}_+\right\},\tag{14}$$

$$c=i\Delta^*\epsilon^3\int\frac{d^4q}{(2\pi)^4}e^{-iq(x-y)}\times\left\{\frac{1}{q_0^2-(E^+)^2+|\Delta|^2\epsilon^3\epsilon^3}\gamma^5\overline{\Lambda}_+ + \frac{1}{q_0^2-(E^-)^2+|\Delta|^2\epsilon^3\epsilon^3}\gamma^5\overline{\Lambda}_-\right\},\tag{15}$$

$$d=\int\frac{d^4q}{(2\pi)^4}e^{-iq(x-y)}\left\{\frac{q_0+E^+}{q_0^2-(E^+)^2+|\Delta|^2\epsilon^3\epsilon^3}\gamma^0\overline{\Lambda}_- + \frac{q_0-E^-}{q_0^2-(E^-)^2+|\Delta|^2\epsilon^3\epsilon^3}\gamma^0\overline{\Lambda}_+\right\},\tag{16}$$

where $\overline{\Lambda}_\pm=\frac{1}{2}[(1\pm\gamma^0(\vec{\gamma}\vec{p}-M)/E]$ are projectors on the solutions of the Dirac equation with positive-negative energy, and $\epsilon^3\epsilon^3=\mathrm{diag}(-1,-1,0)$ is the projector (up to a sign) in the color space. Quantities (13)–(16) are nontrivial operators in the coordinate, spinor, and color spaces, but they are unit operators in flavor space.

A tedious but straightforward analysis of $\mathcal{S}_{\rm eff}^{(2)}$ shows that there arises a mixing between the σ and Δ^3 fields. We have

$$\mathcal{S}_{\rm eff}^{(2)}(\sigma,\vec{\pi},\Delta^\delta,\Delta^{*\delta})=\mathcal{S}_{\pi\pi}^{(2)}(\vec{\pi})+\mathcal{S}_{\sigma 3}^{(2)}(\sigma,\Delta^3,\Delta^{*3})+\mathcal{S}_1^{(2)}(\Delta^1,\Delta^{*1})+\mathcal{S}_2^{(2)}(\Delta^2,\Delta^{*2}),\tag{17}$$

where ($\rho=1,2$)

$$\mathcal{S}_\rho^{(2)}(\Delta^\rho,\Delta^{*\rho})=-\int d^4x\frac{\Delta^\rho\Delta^{*\rho}}{4G_2}+\frac{i}{2}\mathrm{Tr}_{sfcx}\{a\Delta^\rho\epsilon^\rho\gamma^5 d\Delta^{*\rho}\epsilon^\rho\gamma^5\}.\tag{18}$$

In the following we are going to search for gapless bosonic excitations of the 2SC quark-matter ground state of the model only. Since we are mainly interested in the diquark sector, the expression for the π-meson effective action $\mathcal{S}_{\pi\pi}^{(2)}(\vec{\pi})$ is omitted here. Moreover, $\mathcal{S}_{\sigma 3}^{(2)}(\sigma,\Delta^3,\Delta^{*3})$ has a rather cumbersome form, so we also do not show it. [If needed, both these expressions can be obtained immediately from Eq. (11).]

Before proceeding further, we should fix the parameters of our model. Here, we choose the parametrization procedure given in [15]. We simplify that model by ignoring the corrections to the pion-quark coupling constant that come from the mixing between the pion and axial vector. To fix the model parameters, we require the model to reproduce the Goldberger-Treiman relation $Mg_\pi=F_\pi$, where $F_\pi\approx 93$ MeV is the pion weak-decay constant, and g_π is the pion-quark coupling constant determined in the local NJL model as follows:

$$g_\pi^{-2} = \frac{-iN_c}{(2\pi)^4}\int \frac{d^4q}{(q^2-M^2)^2}. \tag{19}$$

(This integral is divergent, and a regularization is supposed to be implemented here. In our work we use a three-dimensional cutoff to make such integrals meaningful.)

Then, we require the observed pion mass ($\sim$140 MeV) to be reproduced by the local NJL model with our parameters. After this, one more condition is needed to complete the parameter fixing procedure. We consider two possibilities to finalize it: (i) fix the model parameters so that the QCD sum rules estimate for the chiral condensate $\langle \bar{q}q \rangle \approx (-245\ \mathrm{MeV})^3$ is reproduced in our model, (ii) choose the model parameters in a way that allows one to obtain the correct description of the $\rho \to \pi\pi$ decay, as was done in [15]. In the first case (let us refer to this set of parameters as set A), the cutoff Λ is about 618 MeV, and the four-quark interaction constant G_1 is equal to 5.86 GeV^{-2}. The remaining constant G_2 is chosen according to the Fierz transformation as $G_2 = 3G_1/4 = 4.40\ \mathrm{GeV}^{-2}$. The current quark mass m_0 is fixed via the gap equations to the value: 5.7 MeV. The constituent quark mass in the vacuum (at zeroth temperature and chemical potential) amounts to 350 MeV.

In the second case, we obtain a different set of model parameters, which will be referred to, throughout the paper, as set B. Imposing the condition that an extension of this model to the vector channel would give the experimental value for the ρ meson width (the ρ meson decays mostly into a couple of charged pions, and the decay is described by the constant $g_\rho \approx 6.1$.) and keeping in mind that in the local NJL model one gets a natural connection between g_ρ and g_π (discarding pion–axial-vector transitions), $\sqrt{6}g_\rho = g_\pi$, we obtain $\Lambda = 856$ MeV, $G_1 = 2.49\ \mathrm{GeV}^{-2}$, $G_2 = 3G_1/4 = 1.87\ \mathrm{GeV}^{-2}$, and $m_0 = 3.6$ MeV. In the vacuum the constituent quark mass is 233 MeV, which is small, compared to the case, where the chiral condensate is used to fix the model parameters.[3] A similar procedure has been implemented in [28] in the chiral limit. Here, we keep the current quark mass nonzero in order to reproduce the pion mass in the vacuum.

Solving the gap equations at a fixed chemical potential, we obtain the constituent quark mass M and the color gap Δ that satisfy the global minimum of the effective potential. In Figs. 1 and 2, one can see the quark mass M and the color gap Δ versus the chemical potential for parameter sets A and B. For parameter set A, one observes a first-order phase transition from hadron matter to the 2SC phase. The gaps (constituent quark mass and color gap) change abruptly when the chemical potential reaches the vacuum value of M (see Fig. 1). On the contrary, if parameter set B is chosen, the gaps reveal a rather smooth character near the phase transition, which is typical for a phase transition of second order (see Fig. 2).

[3]A larger quark mass can be obtained if one takes into account the mixing between the pion and axial vector. However, we omit it in our paper for simplicity and for the reason that this mixing is negligible in the 2SC phase.

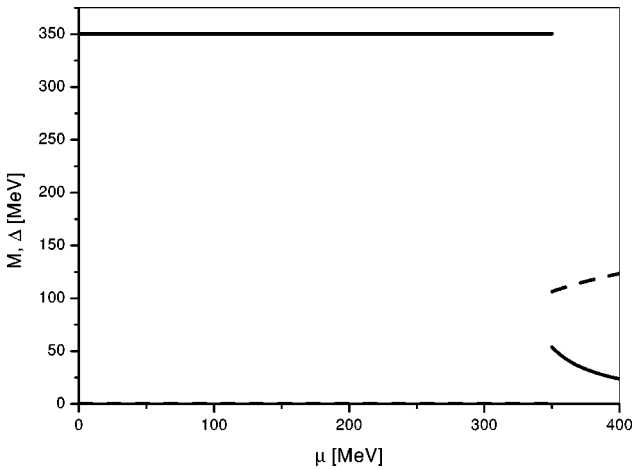

FIG. 1. The constituent quark mass M (solid line) and the color gap Δ (dashed line) as functions of the chemical potential (parameter set A).

III. GAPLESS EXCITATIONS IN THE DIQUARK SECTORS

Since Δ^1 in Eq. (17) is not mixed with other fields, let us, first of all, focus on the diquark Δ^1 sector. If $\Delta^1(x) = [\varphi_1(x) + i\varphi_2(x)]/\sqrt{2}$, $\Delta^{*1}(x) = [\varphi_1(x) - i\varphi_2(x)]/\sqrt{2}$, then the effective action $\mathcal{S}_1^{(2)}(\Delta^1, \Delta^{*1})$ from Eq. (17) can be presented in the form

$$\mathcal{S}_1^{(2)}(\Delta^1, \Delta^{*1}) \equiv \mathcal{S}_1^{(2)}(\varphi_1, \varphi_2) = -\frac{1}{2}\int d^4x d^4y\, \varphi_k(x)\Gamma_{kl}(x-y)\varphi_l(y), \tag{20}$$

where the 2×2 matrix $\Gamma(x-y)$ is the inverse propagator of the φ_1, φ_2 fields [summation over $k,l = 1,2$ is implied in Eq. (20)]. Its matrix elements can be found via a second variation of $\mathcal{S}_1^{(2)}$:

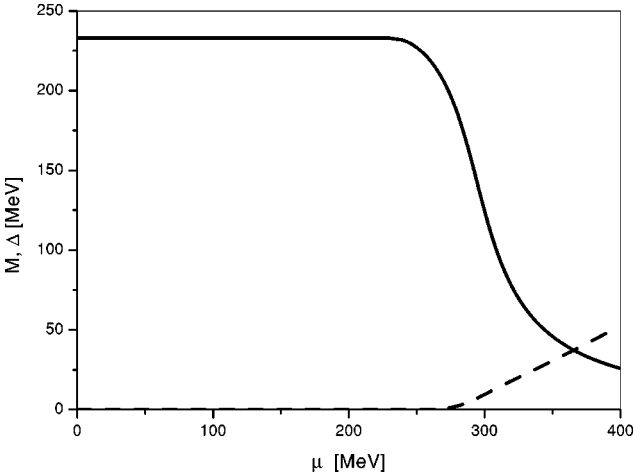

FIG. 2. The constituent quark mass M (solid line) and the color gap Δ (dashed line) as functions of the chemical potential (parameter set B).

$$\Gamma_{kl}(x-y) = -\frac{\delta^2 \mathcal{S}_1^{(2)}}{\delta\varphi_l(y)\,\delta\varphi_k(x)}. \tag{21}$$

In momentum space, the Fourier-transformed components of the Γ matrix have the structure

$$\overline{\Gamma_{11}}(p) = \overline{\Gamma_{22}}(p) = \frac{1}{2}[\overline{\Gamma_{\Delta^*\Delta}}(p) + \overline{\Gamma_{\Delta^*\Delta}}(-p)],$$

$$\overline{\Gamma_{12}}(p) = -\overline{\Gamma_{21}}(p) = \frac{i}{2}[\overline{\Gamma_{\Delta^*\Delta}}(p) - \overline{\Gamma_{\Delta^*\Delta}}(-p)]. \tag{22}$$

The derivation of these relations is given in the Appendix, where the quantity $\overline{\Gamma_{\Delta^*\Delta}}(p)$ is also presented [see Eq. (A5)]. The particle dispersion laws are defined by zeros of $\det\overline{\Gamma}(p)$ in the p_0 plane—i.e., by the equation

$$\begin{aligned}\det\overline{\Gamma}(p) &= \overline{\Gamma_{11}}(p)\overline{\Gamma_{22}}(p) - \overline{\Gamma_{12}}(p)\overline{\Gamma_{21}}(p) \\ &= \overline{\Gamma_{\Delta^*\Delta}}(p)\overline{\Gamma_{\Delta^*\Delta}}(-p) = 0.\end{aligned} \tag{23}$$

Clearly, the solutions of Eq. (23) with positive and negative p_0 correspond to particles and antiparticles, respectively. Let us first put $p=(p_0,0,0,0)$. In this case, $\det\overline{\Gamma}(p_0)$ is an even function of p_0, so solutions of Eq. (23) in the p_0^2 plane might be searched for. They are the squared masses of excited states in the Δ^1 sector of the model. Moreover, all these formulas are then greatly simplified. Indeed, it follows from Eq. (A5) at $\vec{p}^{\,2}=0$ that

$$\overline{\Gamma_{\Delta^*\Delta}}(p_0) = 4ip_0\int \frac{d^4q}{(2\pi)^4}\left\{\frac{1}{(p_0+q_0+E^+)D_+(q_0)} + \frac{1}{(p_0+q_0-E^-)D_-(q_0)}\right\} \equiv -4p_0H(p_0), \tag{24}$$

where it is possible to integrate over q_0, using the following prescription: $(p_0+q_0)\to(p_0+q_0)+i\varepsilon\,\mathrm{sgn}(p_0+q_0)$ and $q_0\to q_0+i\varepsilon\,\mathrm{sgn}(q_0)$ with $\varepsilon\to 0_+$ [see also comments after formula (10)], and

$$H(p_0) = -\frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\left\{\frac{1}{(p_0+E^++E_\Delta^+)E_\Delta^+} + \frac{\theta(E^-)}{(p_0-E^--E_\Delta^-)E_\Delta^-} + \frac{\theta(-E^-)}{(p_0-E^-+E_\Delta^-)E_\Delta^-}\right\}. \tag{25}$$

In Eq. (25) $E_\Delta^\pm = \sqrt{(E^\pm)^2+|\Delta|^2}$. Since the integral on the right-hand side (RHS) of this equation is ultraviolet divergent, we regularize it, as the other divergent integrals, by using a three-dimensional cutoff Λ—i.e., $\vec{q}^{\,2}\le\Lambda^2$. Then, Eq. (23) is transformed to

$$p_0^2 H(p_0)H(-p_0) = 0, \tag{26}$$

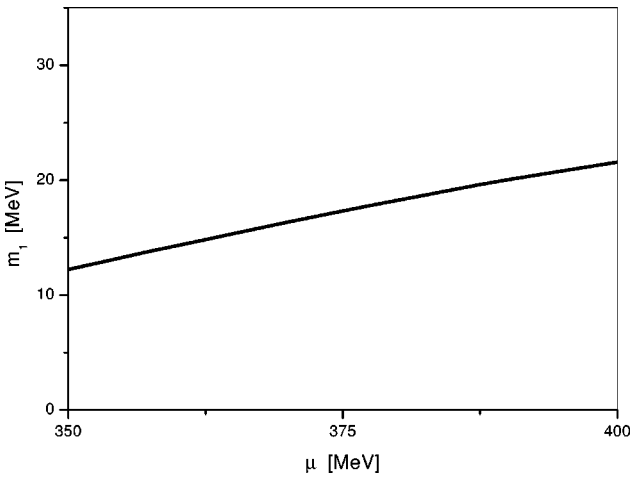

FIG. 3. The diquark mass $m_1 = H(0)/H'(0)$ as a function of the chemical potential (parameter set A).

from which it is evident that there is a solution $p_0^2=0$, corresponding to a gapless excitation of the 2SC ground state. Since the function $H(p_0)$ does not vanish at $p_0=0$ and $\mu\neq 0$, this solution may be identified in the case of finite $\vec{p}^{\,2}$ with a NG boson with the quadratic dispersion law $p_0\sim\vec{p}^{\,2}/H(0)$ (see the Appendix). Now, let us suppose that at some nonzero value $p_0=-m_1$ the function $H(p_0)$ has a zero—i.e., $H(-m_1)=0$. Then, $\det\Gamma(p_0)$ has two zeros $p_0=\pm m_1$—i.e., the point $p_0^2=m_1^2$ is a solution of Eq. (26)—and the second bosonic excitation of this sector has the nonzero mass m_1. In the low-p_0 expansion we have $H(p_0)=H(0)+p_0H'(0)+\cdots$, where

$$H(0) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\left\{\frac{\theta(E^-)}{(E^-+E_\Delta^-)E_\Delta^-} + \frac{\theta(-E^-)}{(E^--E_\Delta^-)E_\Delta^-} - \frac{1}{(E^++E_\Delta^+)E_\Delta^+}\right\},$$

$$H'(0) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\left\{\frac{\theta(E^-)}{E_\Delta^-(E^-+E_\Delta^-)^2} + \frac{\theta(-E^-)}{(E^--E_\Delta^-)^2E_\Delta^-} + \frac{1}{E_\Delta^+(E^++E_\Delta^+)^2}\right\}. \tag{27}$$

Thus, in this approximation $m_1=H(0)/H'(0)$. The m_1 vs μ plots for particular values of the coupling constants G_1, G_2 and the cutoff parameter Λ discussed at the end of Sec. II (or, equivalently, for the values Δ and M from Figs. 1 and 2) are presented in Figs. 3 and 4. In Fig. 3 the values of m_1 are calculated for $\mu=350$–400 MeV when parameter set A is involved. The leftmost point corresponds to the condition of the phase transition ($\mu\approx 350$ MeV). At smaller μ the color gap equals zero and Eqs. (27) cannot be applied, since they have been derived by using Eq. (10) with $\Delta\neq 0$. Similarly, in

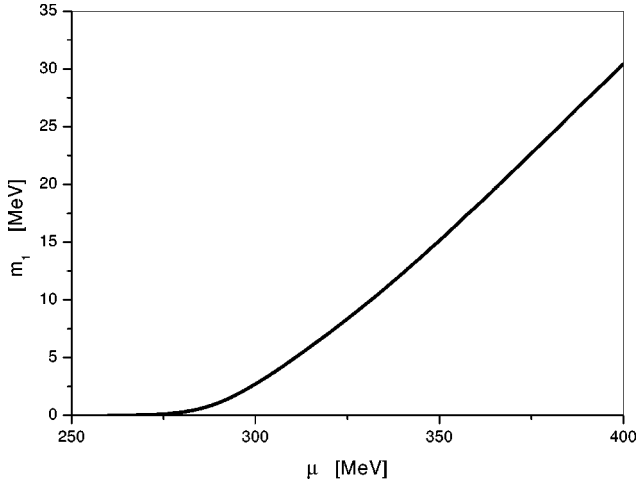

FIG. 4. The diquark mass $m_1 = H(0)/H'(0)$ as a function of the chemical potential (parameter set B).

Fig. 4 the axis μ begins with 250 MeV. At smaller μ (but greater than 233 MeV), the value of m_1 is almost zero. Common to both parameter sets A and B is that the values of m_1 are small and thus justify the series expansion of the function $H(p_0)$ around $p_0 = 0$.[4] Analogous results are obtained for the Δ^2 sector of the model. There, as in the Δ^1 sector, only one massless boson with the quadratic dispersion law as well as a massive one with the mass $m_2 \equiv m_1$ is found. Evidently, the two massless and two massive bosons in the Δ^1, Δ^2 sectors form antidoublets with respect to the unbroken $SU_c(2)$ symmetry.

Now let us consider the sector with mixing of the complex diquark Δ^3 field and the scalar σ meson. Introducing new real fields $\phi(x) = [\Delta^3(x) + \Delta^{*3}(x)]/\sqrt{2}$, $\psi(x) = i[\Delta^{*3}(x) - \Delta^3(x)]/\sqrt{2}$, we rewrite the effective action $\mathcal{S}^{(2)}_{\sigma 3}$ [here we omit the cumbersome expression for this quantity, however it can be easily reproduced from Eq. (11)] in the form

$$\mathcal{S}^{(2)}_{\sigma 3}(\sigma, \Delta^3, \Delta^{*3}) = -\frac{1}{2}\int d^4x d^4y (\sigma(x), \phi(x), \psi(x))\ \ \Pi(x - y)\ \ (\sigma(y), \phi(y), \psi(y))^t, \tag{28}$$

where $\Pi(x-y)$ is the inverse propagator of the σ meson and the diquark fields ϕ, ψ. Clearly, it is a 3×3 matrix, whose elements can be obtained from Eq. (28) by taking all second variational derivatives of $\mathcal{S}^{(2)}_{\sigma 3}$ over the fields σ, ϕ, and ψ. We have calculated the momentum-space matrix $\overline{\Pi}(p)$ (corresponding explicit formulas and relations are omitted), and then, putting $p = (p_0, 0, 0, 0)$, found that $\det[\overline{\Pi}(p_0)]$ has only one zero at $p_0^2 = 0$. Therefore, only one NG boson, which is an $SU_c(2)$ singlet, exists in this sector (as in the Δ^1, Δ^2 sectors, each zero of $\det[\overline{\Pi}(p_0)]$ in the p_0^2 plane means the mass squared of the ground-state excitation, this time in the mixed σ-Δ^3 sector).[5]

As was mentioned above, in the 2SC phase of the model, where $\Delta \neq 0$, the color $SU_c(3)$ symmetry of the ground state is spontaneously broken down to the $SU_c(2)$ group. Hence, in accordance with the Goldstone theorem, it is generally expected that five NG bosons would appear in the theory. However, we have proved that in the $\Delta^1, \Delta^2, \Delta^3$ sectors of the model only three massless bosons exist, which can be identified with NG bosons. Therefore, there seems to be a deficiency of two NG bosons in the framework of the model under consideration. In spite of this fact, there are, however, no contradictions with the NC theorem, since two of the three found NG bosons have quadratic dispersion laws for $|\vec{p}| \to 0$. In addition, the model contains also light massive diquarks.

Usually, in the framework of the NJL model, the 2SC phase is studied in the region of coupling constants (2). In this case, at $\mu = 0$ we have a chirally noninvariant phase even if $m_0 \to 0$, and the transition to the 2SC phase occurs at some finite value of the chemical potential. Formally, however, one can consider the region

$$\omega = \{(G_1, G_2): G_2 > \pi^2/(4\Lambda^2),\ \ \pi^2(G_2 - G_1) > 2G_1 G_2 \Lambda^2\}, \tag{29}$$

where (even at $\mu = 0$) the 2SC phase is realized [32]. As can easily be seen from Eq. (27), in the case of $\mu = 0$ we have $H(0) = 0$ and $m_1 = m_2 = 0$, and the total number of gapless excitations (NG bosons) in the Δ^1 and Δ^2 sectors of the model equals 4. They form two massless $SU_c(2)$ antidoublets. Apart from this, there is one NG boson, which is an $SU_c(2)$ singlet, in the mixed σ-Δ^3 sector. Naturally, all these excitations have a linear dispersion law, as should be in the relativistically invariant case (see the Appendix). However, at an arbitrary small μ, particles from one of these antidoublets acquire nonzero masses, and the dispersion laws for the two massless particles from the remaining antidoublet are changed essentially from the linear to quadratic laws.

Note finally, that in the above consideration we used the real-imaginary part parametrization for diquark fields:

$$\Delta^1(x) = [\varphi_1(x) + i\varphi_2(x)]/\sqrt{2},$$

$$\Delta^2(x) = (\tilde{\varphi}_1(x) + i\tilde{\varphi}_2(x))/\sqrt{2},$$

$$\Delta^3(x) - \Delta = (\phi(x) + i\psi(x))/\sqrt{2}, \tag{30}$$

where $\Delta^p(x)$ are the unshifted diquark fields from Lagrangian (3) and $\Delta = \langle \Delta^3 \rangle$. However, one can use also an alternative "polar coordinate" parametrization

[4]The numerical investigation shows that the mass value m_1 found in the low-p_0 approximation (see Figs. 3 and 4) differs from the exact solution of the equation $H(m_1) = 0$ by no more than 10%.

[5]A detailed investigation of the meson-diquark spectrum including the σ-Δ^3 mixing is in preparation.

$$\begin{pmatrix}\Delta^1(x)\\ \Delta^2(x)\\ \Delta^3(x)\end{pmatrix}=\exp\left\{-i\sum_a \frac{\lambda_a^t\xi_a(x)}{\sqrt{2}\Delta}\right\}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ 0\\ \sqrt{2}\Delta+\eta(x)\end{pmatrix}$$

$$=\frac{1}{\sqrt{2}}\begin{pmatrix}\xi_5-i\xi_4\\ \xi_7-i\xi_6\\ \sqrt{2}\Delta+\eta+i\frac{2}{\sqrt{3}}\xi_8\end{pmatrix}+o(\xi_i,\eta), \qquad (31)$$

where λ_a are Gell-Mann matrices and the summation over $a=4,\ldots,8$ is implied [the form of an exponential multiplier in Eq. (31) corresponds to the fact that $\Delta^\rho(x)$ is an $SU_c(3)$ antitriplet]. Comparing Eqs. (30) and (31) at small ξ_i,η, we see that

$$\varphi_1=\xi_5,\quad \varphi_2=-\xi_4,\quad \tilde{\varphi}_1=\xi_7,$$

$$\tilde{\varphi}_2=-\xi_6,\quad \phi=\eta,\quad \psi=\frac{2}{\sqrt{3}}\xi_8. \qquad (32)$$

Further, one should insert Eq. (31) into Eq. (5) and expand the resulting expression into a series of meson and ξ_a,η fields. It is easily seen that in the second order of the new variables the effective action is a sum similar to Eq. (17). In particular, it means that the diquark fields ξ_4,ξ_5 are decoupled from other fields, and their effective action looks like Eq. (20) with the replacement (32) used. So some elements of the inverse propagator matrix for ξ_4,ξ_5 fields differ by a sign from $\Gamma_{kl}(x-y)$, Eq. (21), but the determinant of the whole momentum space matrix is not changed. Hence, the mass spectrum, the dispersion laws, etc., in the ξ_4,ξ_5 fields are the same, as in the case with old fields φ_1,φ_2. Similar conclusions are valid for the rest of new variables; i.e., such physical characteristics of the model, such as particle masses, dispersion laws, etc., do not depend on the choice of field parametrizations, Eq. (30) or (31). In particular, the number of abnormal NG bosons, which equals 3 in the model under consideration, is a parametrization-invariant quantity.[6]

IV. COLOR ASYMMETRY AND MASSES OF SCALAR DIQUARKS

As was noted in [13], the color superconducting phase is automatically color neutral in QCD, but this does not hold for the NJL model under consideration. Massive scalar diquarks and, as a consequence, the abnormal number of NG bosons in the model discussed in previous sections testify for this. As mentioned in the Introduction, for the number of NG bosons to equal the number of broken generators of the underlying symmetry group [in our case $SU_c(3)$], the ground-state expectation value for the commutator of any two broken symmetry generators Q_a and Q_b must be zero: $\langle[Q_a,Q_b]\rangle=0$ [5]. This criterion is not fulfilled for the NJL model considered here—namely, the ground-state expectation value of the color charge operator $Q_8=\bar{q}\gamma^0\lambda^8 q$ does not vanish—and, therefore, the usual counting rule for NG bosons is not applicable.

Let us look at the function $H(p_0)$ at $p_0=0$. One can rewrite the expression (27) in a different form:

$$H(0)=\frac{1}{2}\int\frac{d^3q}{(2\pi)^3}\frac{1}{\Delta^2}\left(-2\theta(\mu-E)+\frac{E^+}{E_\Delta^+}-\frac{E^-}{E_\Delta^-}\right). \qquad (33)$$

This formula can be expressed in terms of quark densities. For this purpose, we need the expression of the thermodynamical potential of the system, which in the mean-field approximation is equal to the value of $V_{\rm eff}$ in its global minimum point, supplied by the gap equations—i.e.,

$$\Omega(\mu)=V_{\rm eff}(\sigma_0,\vec{\pi}_0,\Delta_0^\delta,\Delta_0^{*\delta}), \qquad (34)$$

where $V_{\rm eff}$ was given in Eq. (8). Further, one can divide the thermodynamical potential into several terms

$$\Omega(\mu)=\sum_{c=1}^{3}\Omega_c(\mu)+\frac{(M-m_0)^2}{4G_1}+\frac{|\Delta|^2}{4G_2}, \qquad (35)$$

where $\Omega_c(\mu)$ is the contribution from the color-c quark, defined as follows:

$$\Omega_1(\mu)=\Omega_2(\mu)=-2\int\frac{d^3q}{(2\pi)^3}(E_\Delta^+ + E_\Delta^-),$$

$$\Omega_3(\mu)=-2\int\frac{d^3q}{(2\pi)^3}(|E^+|+|E^-|). \qquad (36)$$

The contributions from color-1 and color-2 quarks, $\Omega_{1(2)}(\mu)$, are equal due to the remaining $SU_c(2)$ color symmetry after the global $SU_c(3)$ symmetry group is spontaneously broken in the color superconducting phase. The third color quark, which does not condensate, gives $\Omega_3(\mu)$. Then, one can calculate the densities n_c of color-1, -2, and -3 quarks separately without introducing additional chemical potentials for each color simply by differentiating $\Omega_c(\mu)$ over μ—i.e.,

$$n_c=-\frac{\partial\Omega_c(\mu)}{\partial\mu},\quad c=1,2,3, \qquad (37)$$

$$n_1=n_2=2\int\frac{d^3q}{(2\pi)^3}\left(\frac{E^+}{E_\Delta^+}-\frac{E^-}{E_\Delta^-}\right),$$

$$n_3=4\int\frac{d^3q}{(2\pi)^3}\theta(\mu-E). \qquad (38)$$

[6]In Refs. [4,5] an abnormal number of NG bosons in the same $SU(2)\times U(1)$ toy model was discovered. But in [4] the parametrization (30) was used for the scalar doublet, whereas in [5] a "polar coordinate" parametrization, similar to Eq. (31), was used. Evidently, their results coincide; i.e., the abnormal number of NG bosons, particle masses, formulas for dispersion laws are the same.

Equation (33) can thus be transformed to

$$H(0)=\frac{1}{8\Delta^2}(n_1+n_2-2n_3)=\frac{\sqrt{3}\langle Q_8\rangle}{8\Delta^2}. \tag{39}$$

From Eq. (39) and the fact that the diquark mass $m_1=H(0)/H'(0)$ is found to be nonzero (see Figs. 3 and 4), one concludes that the 2SC ground state in the NJL model under consideration has finite color charge $\langle Q_8\rangle$. Evidently, this fact is related to the inequality of color quark densities $n_{1,2}>n_3$ for paired (1,2) and unpaired (3) quarks—i.e., to color asymmetry. Moreover, as discussed in the Appendix, the nonzero value of $H(0)$ is responsible for the abnormal dispersion law $p_0\sim\vec{p}^2/H(0)$ for two NG bosons.

To restore the local color neutrality in the NJL model, one can, e.g., include an additional term $\mu_8 Q_8$ and impose the color-neutrality condition

$$\langle Q_8\rangle=-\frac{\partial\Omega(\mu,\mu_8)}{\partial\mu_8}=0 \tag{40}$$

or consider the superconducting matter as composed of colored domains, with the global color charge being equal zero. The latter, however, goes beyond the mean-field approximation exploited here and should be considered as an approximation for the case of large colored domains. Anyway, the true ground state should be checked to give the absolute minimum of the thermodynamical potential.

The inclusion of an additional chemical potential μ_8 "by hand," however, does not contradict QCD. As discussed in [13], a nonvanishing expectation value of the gluon field A_0^8, corresponding to the eighth generator in the $SU_c(3)$ group, gives rise to a nonvanishing value of μ_8, which then cancels certain tadpole contributions and restores the color neutrality. This cannot occur in the NJL model automatically because there are no gluons in it.

V. SUMMARY AND DISCUSSIONS

Recently, it has been shown that in QCD with a nonzero strangeness chemical potential [4,5], as well as in some models for relativistic massive vector field condensation [6], the number of NG bosons can be less than the number of broken symmetry generators, so that the usual counting rule for NG bosons evidently does not hold. In the present paper, we have presented another relativistic model providing us with an abnormal number of NG bosons. This is an NJL-type model with two light-quark flavors, where a single quark-chemical potential is taken into account. In the color superconducting phase of this model, the color $SU_c(3)$ group is spontaneously broken down to $SU_c(2)$; i.e., the number of broken symmetry generators equals 5. Despite this fact, there appeared only three NG bosons in the 2SC phase; two of them form an antidoublet with respect to the unbroken $SU_c(2)$ subgroup and have quadratic dispersion laws. The remaining one is an $SU_c(2)$ singlet, so there are no contradictions with the NC theorem [3]. Moreover, there exists an $SU_c(2)$ antidoublet of light diquarks Δ^1,Δ^2 with masses m_1, the behavior of which at final chemical potential μ is shown in Figs. 3 and 4 for the parameter sets A and B, respectively.

Most interestingly, we found that the diquark masses $m_1=H(0)/H'(0)$ are proportional to the ground-state average of the color charge operator $\langle Q_8\rangle=1/\sqrt{3}(n_1+n_2-2n_3)$. Here, the quark densities n_1, n_2 of paired quarks with color 1,2 are equal due to the remaining $SU_c(2)$ symmetry and larger than the density n_3 of the unpaired color-3 quark. Thus, the ground state of the considered NJL-type model is evidently color asymmetric. It is just the appearance of this nonvanishing expectation value $\langle Q_8\rangle$ which simultaneously leads to a violation of the criterion for the equality of the number of NG bosons and broken symmetry generators [5] and to the appearance of the quadratic dispersion law for the NG diquarks [compare Eqs. (38), (A6)].

In the investigations of color superconductivity in the two-flavored QCD, five scalar NG bosons are argued to exist. They are mixing with gluons and required as longitudinal components for five massive gluons (color Meissner effect) [11]. The fact that we find an abnormal number of three NG bosons does not directly contradict QCD. Namely, in the color superconducting phase of QCD a nonvanishing condensate of the eighth gluon field component A_0^8 appears, which induces a color chemical potential μ_8 and cancels certain tadpole contributions, responsible for the nonvanishing color charge of the ground state [13]. This mechanism just leads to color neutrality. Obviously, there arises the question, how can one reconcile the considered NJL approach with QCD? Insofar as NJL-type models do not contain gluon fields, the required contribution from the condensed eighth gluon field does not follow automatically. Therefore, the tadpole contribution becomes unbalanced by gluon contributions. It can be cancelled "by hand"—e.g., with the help of the additional color chemical potential μ_8 simulating the omitted gluon contribution. In such enlarged NJL models the condition of color neutrality of the ground state—i.e., $\langle Q_8\rangle=0$—can be imposed as an additional physical requirement [12]. According to our results, one gets in this case two additional NG bosons as well as a change of quadratic dispersion laws into normal linear ones. Thus, the abnormal number of three NG bosons might, in principle, become converted into the normal number five found in QCD.

Nevertheless, what to do now with these five NG bosons arising in the color neutral 2SC phase of an enlarged NJL-type model? Clearly, in the framework of the standard NJL model one cannot apply the color Meissner effect, in order to absorb these NG bosons into longitudinal degrees of freedom of massive gluons. A possible way out of this problem could be an extension of the NJL mode via the inclusion of perturbative gluons, fluctuating around the original low-energy gluonic fields (realized, e.g., by instantons or other nonperturbative background fields, which were already integrated out to yield the effective four-quark interactions) [33]. Obviously, the color Meissner effect could then be realized for such perturbative gluons.

In addition, let us remark that the above discussions refer to the case of locally neutral quark matter, where color neutrality should indeed be required. On the other hand, global

color neutrality does not demand local color neutrality: for example, one can assume the ground state as being composed of colored domains. Anyway, to decide which of the considered ground states is in favor, the absolute minimum of the thermodynamic potential should be calculated in each case. (Note that the presence of domains means a violation of translational invariance and is not consistent with the simple mean-field approach.)

Finally, notice that in the framework of the considered NJL-type of model there arises a mixing between the σ meson and Δ^3 diquark in the 2SC phase. The behavior of σ-, Δ^3- as well as π-meson masses versus μ in the 2SC phase is now under consideration.

ACKNOWLEDGMENTS

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APPENDIX: THE Γ-MATRIX ELEMENTS AND DISPERSION LAWS

Using the effective action (18), let us introduce auxiliary quantities

$$\Gamma_{\Delta^*\Delta}(z) = -\frac{\delta^2 \mathcal{S}_1^{(2)}}{\delta\Delta^1(y)\,\delta\Delta^{*1}(x)} = \frac{\delta(z)}{4G_2} - i\,\mathrm{Tr}_{sc}\{a(z)\epsilon^1\gamma^5 d(-z)\epsilon^1\gamma^5\},$$

$$\Gamma_{\Delta\Delta^*}(z) = -\frac{\delta^2 \mathcal{S}_1^{(2)}}{\delta\Delta^{*1}(y)\,\delta\Delta^{1}(x)} = \frac{\delta(z)}{4G_2} - i\,\mathrm{Tr}_{sc}\{d(z)\epsilon^1\gamma^5 a(-z)\epsilon^1\gamma^5\}, \quad \text{(A1)}$$

where $\delta(z)$ is the Dirac delta function, $z=x-y$, and $a(z)$ and $d(z)$ are operators (13) and (16), respectively. Since $\delta^2\mathcal{S}_1^{(2)}/(\delta\Delta^1\delta\Delta^1) = \delta^2\mathcal{S}_1^{(2)}/(\delta\Delta^{*1}\delta\Delta^{*1}) = 0$ and

$$\frac{\delta}{\delta\varphi_1} = \frac{1}{\sqrt{2}}\left(\frac{\delta}{\delta\Delta} + \frac{\delta}{\delta\Delta^*}\right), \quad \frac{\delta}{\delta\varphi_2} = \frac{i}{\sqrt{2}}\left(\frac{\delta}{\delta\Delta} - \frac{\delta}{\delta\Delta^*}\right),$$

the matrix elements of the inverse propagator matrix (21) have the form

$$\Gamma_{11}(z) = \Gamma_{22}(z) = \frac{1}{2}[\Gamma_{\Delta^*\Delta}(z) + \Gamma_{\Delta\Delta^*}(z)],$$

$$\Gamma_{12}(z) = -\Gamma_{21}(z) = \frac{i}{2}[\Gamma_{\Delta^*\Delta}(z) - \Gamma_{\Delta\Delta^*}(z)]. \quad \text{(A2)}$$

For an arbitrary function $F(z)$, it is possible to define the Fourier-transformed one, $\overline{F}(p)$, by the relation

$$\overline{F}(p) = \int d^4 z F(z) e^{ipz}, \quad \text{i.e.} \quad F(z) = \int \frac{d^4 p}{(2\pi)^4}\overline{F}(p) e^{-ipz}. \quad \text{(A3)}$$

Now, using Eq. (A3), it is possible to get, from Eq. (A1),

$$\overline{\Gamma_{\Delta^*\Delta}}(p) = \frac{1}{4G_2} - i\,\mathrm{Tr}_{sc}\int \frac{d^4 q}{(2\pi)^4}\{\overline{a}(q+p)\epsilon^1\gamma^5\overline{d}(q)\epsilon^1\gamma^5\},$$

$$\overline{\Gamma_{\Delta\Delta^*}}(p) = \frac{1}{4G_2} - i\,\mathrm{Tr}_{sc}\int \frac{d^4 q}{(2\pi)^4}\{\overline{d}(q+p)\epsilon^1\gamma^5\overline{a}(q)\epsilon^1\gamma^5\}, \quad \text{(A4)}$$

where the Fourier-transformed expressions $\overline{a}(q)$, $\overline{d}(q)$ can be easily derived from Eqs. (13), (16). It follows from Eqs. (A4) that $\overline{\Gamma_{\Delta^*\Delta}}(-p) = \overline{\Gamma_{\Delta\Delta^*}}(p)$. So, taking into account Eqs. (A2), one can easily obtain the relations (22).

After tedious calculations we arrive at the expression

$$\begin{aligned}\overline{\Gamma_{\Delta^*\Delta}}(p) = {} & \frac{4ip_0}{(2\pi)^4}\int \frac{d^4 q}{[(p_0+q_0+\mu)^2 - E_{p+q}^2]} \\ & \times\left[\frac{p_0+q_0+\mu+E_q}{D_-(q_0)} + \frac{p_0+q_0+\mu-E_q}{D_+(q_0)}\right] \\ & - \frac{4i}{(2\pi)^4}\int \frac{d^4 q}{[(p_0+q_0+\mu)^2 - E_{p+q}^2]} \\ & \times\left[\frac{E_q \vec{p}^{\,2} + (q_0+\mu+E_q)(\vec{p}\cdot\vec{q})}{E_q D_-(q_0)}\right. \\ & \left. + \frac{E_q \vec{p}^{\,2} - (q_0+\mu-E_q)(\vec{p}\cdot\vec{q})}{E_q D_+(q_0)}\right]. \end{aligned} \quad \text{(A5)}$$

Here, the notation that was introduced after Eq. (10) is used. Moreover, $E_q \equiv E = \sqrt{\vec{q}^{\,2} + M^2}$. Note that expression (A5) is obtained under the assumption that $\Delta \neq 0$; i.e., it is valid only in the 2SC phase. In this case, the gap equation (10) was used to eliminate the coupling constant G_2 from relation (A4) in favor of other model parameters.

Quasiparticle dispersion laws are defined by Eq. (23) or, equivalently, by the two equations $\overline{\Gamma_{\Delta^*\Delta}}(\pm p) = 0$. One can easily see that at $\vec{p}^{\,2} = 0$ the first integral in RHS of Eq. (A5) coincides with Eq. (24) having an evident zero at $p_0 = 0$. This is just the dispersion law of a gapless particle (NG boson) at $\vec{p}^{\,2} = 0$. Let us find the dispersion law for the NG boson at small nonzero values of $\vec{p}^{\,2}$—i.e., try to solve the equation $\overline{\Gamma_{\Delta^*\Delta}}(p) = 0$ at $p \to 0$. Expanding expression (A5) into a Taylor series of $(p_0, \vec{p})$ at the point $(0,\vec{0})$ and taking into account only leading terms, we have, at $\mu \neq 0$, the following equation connecting the energy and momentum of the massless particle (in Sec. III, it was proved that such a particle, NG boson, exists in the Δ^1 sector of the model):

$$\overline{\Gamma_{\Delta^*\Delta}}(p) \equiv 4ip_0 H(0) - \frac{4i\vec{p}^{\,2}}{(2\pi)^4}\int \frac{d^4q}{D_-(q_0)[(q_0+\mu)^2-E_q^2]} \times\left\{1-\frac{2\vec{q}^{\,2}}{3E_q(q_0+\mu-E_q)}\right\} - \frac{4i\vec{p}^{\,2}}{(2\pi)^4}\int \frac{d^4q}{D_+(q_0)[(q_0+\mu)^2-E_q^2]} \times\left\{1+\frac{2\vec{q}^{\,2}}{3E_q(q_0+\mu+E_q)}\right\}+\cdots = 0, \quad \text{(A6)}$$

where the quantity $H(0)$ (which is nonzero at $\mu \neq 0$) is given in Eq. (27). Obviously, the quadratic dispersion law, $p_0 \sim \vec{p}^{\,2}$, for a massless particle follows from Eq. (A6). However, in case of relativistic invariance of the system [i.e., at $\mu=0$ and for coupling constants from the region ω, Eq. (29)] or for a color-neutral ground state, when $H(0)=0$, the dispersion law for a NG boson changes. Indeed, in this case the term $4ip_0H(0)$ from Eq. (A6) should be replaced by $4ip_0^2H'(0)$ [$H'(0)$ is presented in Eq. (27)], and one arrives at a linear dispersion law for NG bosons.

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