PHYSICAL REVIEW D
PARTICLES AND FIELDS

THIRD SERIES, VOL. 7, No. 4 15 February 1973

Extraction of Energy and Charge from a Black Hole*

Jacob D. Bekenstein
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540
and Center for Relativity Theory, The University of Texas at Austin, Austin, Texas 78712†
(Received 19 September 1972)

Misner has shown that in the scattering of massless wave fields by a Kerr black hole, certain modes are amplified at the expense of the rotational energy of the hole. We show here that the existence of this effect can be deduced from simple considerations based on Hawking's theorem that the area of a black hole can never decrease; no detailed examination of the wave equations is necessary. This new approach also reveals the existence of an analog of the Misner process in which a charged wave field is amplified upon being scattered by a charged black hole with consequent extraction of charge and Coulomb energy from the hole. Additional byproducts of this approach are a concise demonstration of the existence of a spin-spin force between a Kerr black hole and a nearby spinning particle, examples of the transcendence of the conservation laws for baryon and lepton numbers, and an example of a quantum violation of Hawking's theorem. The origin of this violation is discussed.

I. INTRODUCTION

That energy may be extracted from a rotating (Kerr) black hole was first shown by Penrose.[1] In the typical Penrose process a particle of energy $E_0>0$ splits in the ergosphere[2] of the black hole into a particle with energy $E_1<0$ which is captured by the hole, and a second particle with energy $E_2=E_0+|E_1|$ which escapes to infinity. The process clearly results in the extraction of energy $|E_1|$ from the hole. As later shown by Misner,[3] one may also extract energy from a Kerr black hole by scattering waves upon it. Misner finds that those wave modes for which the frequency ω and the azimuthal quantum number m are related by

$$\omega<m\Omega, \tag{1}$$

where Ω is the rotational frequency of the hole [see (5a) below], are amplified upon scattering with consequent extraction of energy from the hole. Misner's original analysis applied in detail only to the scalar case; however, Teukolsky[4] has recently verified Misner's conjecture that the amplification process occurs also for electromagnetic and gravitational waves when the same condition (1) is satisfied.

The conventional treatment of the Misner amplification process[3,4] depends on a detailed analysis of the relevant wave equations in the Kerr background. We show, in Sec. II of this paper, that the existence of the Misner process when the condition (1) is satisfied may also be deduced from Hawking's theorem[5] that the surface area of a black hole cannot decrease. This new approach has the advantage that it immediately reveals an analog of the Misner process by means of which one can extract charge and electrical energy from a charged black hole (Sec. III). Another bonus is a simple demonstration of the existence of a spin-spin force between a Kerr black hole and a spinning particle (Sec. IV). The new approach also reveals the existence of quantum violations of Hawking's theorem (Sec. V), and it provides illustrations of the transcendence of the laws of baryon-

Copyright © 1973 by The American Physical Society.

and lepton-number conservation in black-hole physics.

First some preliminaries. Consider the expression for the area A of a charged Kerr black hole of mass M, charge Q, and angular momentum $\vec{L}$ (Refs. 5, 6):

$$A = 4\pi(r_+{}^2 + a^2)\,, \tag{2}$$

where

$$\vec{a} = \vec{L}/M\,, \tag{3a}$$

$$r_\pm = M \pm (M^2 - Q^2 - a^2)^{1/2}\,. \tag{3b}$$

Differentiation of (2) gives

$$dA = 4A(r_+ - r_-)^{-1}(dM - \vec{\Omega}\cdot d\vec{L} - \Phi\, dQ)\,, \tag{4}$$

where

$$\vec{\Omega} = \vec{a}(r_+{}^2 + a^2)^{-1}\,, \tag{5a}$$

$$\Phi = Qr_+(r_+{}^2 + a^2)^{-1}\,. \tag{5b}$$

We use vector notation here in order to emphasize that to first order only the increment in that component of the angular momentum which is in the direction of $\vec{L}$ enters into the expression for the increment in area. If the black hole is *slightly* perturbed by an exterior system, it will depart slightly from equilibrium. If the perturbation is then removed, the hole will settle down into a nearby Kerr state. (It is nearly certain by now that a bare black hole in equilibrium must be Kerr.[2]) Thus, the over-all changes in the black-hole parameters will satisfy (4). Now, the area of a black hole which is disturbed in any way can only increase.[5] It follows from (4) that necessarily

$$dM > \vec{\Omega}\cdot d\vec{L} + \Phi\, dQ\,. \tag{6}$$

II. AMPLIFICATION OF WAVES

As a concrete example we consider the scattering of scalar, electromagnetic, or gravitational waves by a (possibly charged) Kerr black hole. In the scalar and gravitational cases the waves may be regarded as propagating in an unperturbed Kerr background to first order. But this is not true in general for the case of electromagnetic waves impinging on a *charged* black hole. In this case the Maxwell equations to first order involve the first-order perturbations of the Kerr metric because the electromagnetic field has a zeroth-order contribution – the black hole's electromagnetic field. (The first-order perturbations of the metric are nonvanishing because the stress-energy tensor has first-order contributions for precisely the same reason.) Therefore, in general, the (first-order) electromagnetic perturbations cannot be regarded as propagating in the unperturbed Kerr background. Since we wish to treat the electromagnetic case on an equal footing with the other cases, we shall suppose (in this case only) that the charge of the black hole is sufficiently small for the above-mentioned difficulty to become entirely negligible.

The above proviso granted, we may regard the various waves as propagating in a stationary axisymmetric (Kerr) background. We may thus write the dependence on the time t and azimuthal angle ϕ of a given scalar wave mode as

$$\psi(r,\theta,\phi,t) = f(\theta, r)e^{im\phi}e^{-i\omega t} + \text{c.c.}\,, \tag{7}$$

with similar expressions for the other fields. The r, θ, ϕ, and t are the Boyer-Lindquist coordinates for the Kerr metric,[2] ω is the (conserved) frequency, and m is the azimuthal quantum number ($m = 0, \pm1, \ldots$). The function f will reduce to ingoing waves at the black hole's surface,[7,3] and it will contain both ingoing and outgoing waves at infinity, which correspond to incident and scattered waves, respectively.

To apply (6) we must know the relation between the energy and angular momentum carried in by the waves. Let us treat the scalar case first. If the stress-energy tensor is $T_{\mu\nu}$, the net radial flux of energy is given by $-T^r{}_t$ while the net radial flux of ϕ angular momentum (the component in the direction of $\vec{L}$) is given by $T^r{}_\phi$. From this, (7), and the standard stress-energy tensor for the scalar field,

$$T_{\mu\nu} = \psi_{,\mu}\psi_{,\nu} - \tfrac{1}{2}g_{\mu\nu}\psi_{,\alpha}\psi_{,}{}^{\alpha}\,, \tag{8}$$

it follows that the ratio of the ϕ angular momentum to the energy carried by the waves across any sphere centered at the black hole is just m/ω. The same ratio holds for electromagnetic and gravitational waves; this is easiest seen by means of the following very general argument Far from the black hole the wave may be regarded as composed of many quanta (scalarons, photons, gravitons, etc.). According to the usual arguments the energy of each quantum is $\hbar\omega$; its ϕ angular momentum is $\hbar m$, where m now labels a mode of definite orbital-plus-spin angular momentum. Thus the ratio of the net ϕ angular momentum to the net energy carried by the wave across a large sphere centered at the black hole must be m/ω (m and ω are conserved through the scattering due to the symmetries).

If the wave, in being scattered by the hole, causes changes dM and $d\vec{L}$ in the hole's parameters, then it follows from the above discussion and from the conservation of energy and angular momentum that $\vec{\Omega}\cdot d\vec{L} = \Omega\, dM m/\omega$. Thus condition (6) becomes

$$dM(1 - m\Omega/\omega) > 0\,. \tag{9}$$

If the mode in question satisfies the Misner condition (1), then it follows from (9) that $dM < 0$, i.e., the mass of the black hole decreases in the process. This means that the scattered wave carries away more energy than the incident wave brought in; the wave is amplified upon scattering with consequent extraction of energy from the hole. (Typically, the amplification factor exceeds unity by a fraction of a percent only.[3])

III. EXTRACTION OF CHARGE

As a second example we consider the scattering of a charged-field wave (examples are charged Klein-Gordon and charged Dirac fields[8]) by a charged Kerr black hole. To first order in the field in question, the relevant wave equation will involve only the unperturbed Kerr metric and electromagnetic potential. Thus we may again concentrate on modes labeled by m and ω. (For a Fermi field m will take on half-integer values.)

As before, far from the black hole we may regard the wave as being composed of many quanta (mesons, electrons, etc.).[9] The energy per quantum in the given mode will be $\hbar\omega$; the ϕ angular momentum will be $\hbar m$. If e is the charge per quantum, then the ratios of net ϕ angular momentum to net energy and net charge to net energy carried by the wave across a large sphere centered at the black hole will be m/ω and $e/\hbar\omega$, respectively. We need not be alarmed at the appearance of $\hbar$ in a ratio which is purported to be valid in the classical limit (many quanta). We must remember that the (minimal) coupling of a charged field to the electromagnetic field is accomplished through the replacement[8] $\nabla_\mu \to \nabla_\mu - i(e/\hbar)A_\mu$. This coupling can be carried out even before quantization, so it is clear that $\lambda \equiv e/\hbar$ is a "classical charge parameter."

If the scattering process results in changes dM, $d\vec{L}$, and dQ in the black hole parameters, then it follows from our previous discussion and from the conservation laws for energy, angular momentum, and charge that $\vec{\Omega}\cdot d\vec{L} = \Omega dM m/\omega$ and $dQ = \lambda dM/\omega$. Thus condition (6) becomes

$$dM\left(1 - \frac{m\Omega}{\omega} - \frac{\lambda\Phi}{\omega}\right) > 0\,. \tag{10}$$

If

$$\omega < m\Omega + \lambda\Phi\,, \tag{11}$$

then it follows from (10) that $dM < 0$. This means that the scattered wave carries away more energy than the incident wave brought in. Recalling that the ratio of charge to energy in the field is fixed at λ/ω, we see that a wave mode satisfying (11) is amplified upon scattering by extracting energy and charge from the black hole.

IV. SPIN-SPIN FORCE

We now turn our attention from fields to classical particles. Consider a classical spinning particle with charge e and energy E which starts falling freely along the axis of symmetry towards a Kerr black hole. If the particle's initial spin $\vec{S}$ is parallel to the axis, then by symmetry the particle will continue falling along the axis and its spin will remain parallel to it. Thus the combined system will be precisely axisymmetric, and as a result no angular momentum will be carried off by gravitational and electromagnetic waves, although energy will surely be radiated off. Suppose the particle falls into the black hole. By conservation of charge $dQ = e$, by conservation of angular momentum $\vec{\Omega}\cdot d\vec{L} = \vec{\Omega}\cdot\vec{S}$, and by conservation of energy $dM < E$ (some energy lost by radiation). It follows from (6) that $E > \vec{\Omega}\cdot\vec{S} + \Phi e$. We conclude that if

$$E < \vec{\Omega}\cdot\vec{S} + \Phi e\,, \tag{12}$$

then the particle cannot possibly be captured by the black hole.

We must therefore conclude that whenever $V_0 \equiv \vec{\Omega}\cdot\vec{S} + \Phi e > 0$ there is a potential barrier of height $V \gtrsim V_0$ about the black hole – at least in the direction of the symmetry axis – which prevents a particle with $E < V$ from falling in. Clearly the Φe part of the barrier is just the Coulomb contribution: A charged black hole repels a particle of the same sign of charge. (It is clear that Φ behaves as the electric potential of the hole.) The spin-spin part of the potential barrier, $\vec{\Omega}\cdot\vec{S}$, tends to increase the height of the barrier if $\vec{L}\cdot\vec{S} > 0$, and tends to decrease it if $\vec{L}\cdot\vec{S} < 0$. Thus there exists a spin-spin repulsive force between a Kerr black hole and a particle with spin vector parallel to the hole's, and a spin-spin attractive force if the spins are antiparallel. The existence of such spin-spin forces in general relativity was (first) conjectured by Hawking[10] from considerations of the energy release in collisions of two black holes. One demonstration that such forces do indeed exist between two distant spinning bodies has been given by Wald.[11]

V. QUANTUM VIOLATIONS OF HAWKING'S THEOREM

Comparing (12) with (11) multiplied through by $\hbar$ we see that the condition for wave amplification is precisely the same as that for it to be impossible for the quanta corresponding to the mode in question, when treated classically, to be captured by the black hole. This was first pointed out by Misner.[3] The above observation raises an interesting question. Every particle in nature has a wave aspect. This means that there is some *small but nonzero* probability for a particle to tunnel through the potential barrier we spoke of and fall into the black hole even if it satisfies condition (12) and is thus classically forbidden to fall in. By reversing our derivation of (12) we see that the black hole's area will necessarily *decrease* in such a process $(dM < \vec{\Omega} \cdot d\vec{L} + \Phi\, dQ)$. Thus we see that if the particle satisfies (12), there is a *small* probability for a quantum violation of Hawking's theorem to occur (quantum because it is the wave aspect of the particle which is responsible).

It appears likely that it is the breakdown of the (weak) positive-energy condition assumed by Hawking[5] which is responsible for the violation of the theorem. This condition certainly holds in the classical limit (many quanta), but it might presumably break down under certain conditions in the quantum limit. As we have shown, the condition does appear to break down in cases when a black hole interacts with a single quantum. It should be stressed that a quantum violation of Hawking's theorem has only a small probability of occurring in any given trial. Moreover, such violations need not be cumulative, for suppose that many particles satisfying condition (12) are sent towards a Kerr black hole one after the other. Occasionally a particle will tunnel in through the barrier and cause a decrease in black-hole area, but in most cases the particle will merely be scattered. In being scattered it will always radiate into the hole some gravitational and/or electromagnetic waves which will cause an increase in area. It is natural to expect that the sum total of such increases in area will more than compensate for the rare occasional decrease, so that the area will increase in the long run.

VI. TRANSCENDENCE OF THE CONSERVATION LAWS

The wave aspect of particles also allows one to construct dramatic examples of the transcendence of the laws of conservation of baryon and lepton numbers.[12-14] Consider the scattering by a Kerr black hole of a group of many identical baryons, the quantum numbers of each of which satisfy condition (11) or (12). We regard the group of baryons as a wave.[9] We know from our previous discussion that the scattered wave will be more intense than the incident one, which means in particular that the baryon number of the scattered wave will be larger than that of the incident one. Where did the extra baryons come from? Certainly not from inside the black hole.

Since we believe in the local conservation of baryon number, we must suppose that the extra baryons are created together with the appropriate number of antibaryons during the scattering process. The mechanism for pair creation is not far to seek. We see from (12) that the potential barrier the baryons are impinging on is higher than their energy, and thus higher than their rest mass. Such a strong potential barrier will presumably mix the positive- and negative-energy levels of the baryon spectrum and thus give rise to pair creation. (This is just an example of the well-known Klein paradox.[15]) The antibaryons will have a charge and an m of opposite signs from those of the baryons, and will thus be attracted and captured by the black hole. On the other hand, the freshly created baryons will be repelled and scattered together with the incident baryons.

The above process transcends the law of baryon-number conservation in that an exterior observer sees an increase in the baryon number of the visible universe, but he cannot verify that the baryon number of the black hole has decreased by the same amount. This is because the baryon number of a black hole is not measurable from the exterior.[12,13] To that observer, then, the law of conservation of baryon number is transcended. Everything that has been said about baryon number here is also applicable to lepton number. Again the lepton number of a black hole is not measurable from the exterior.[13,14]

ACKNOWLEDGMENTS

I benefited from conversations with Professor Leonard Parker and Dr. Bahram Mashhoon; I also wish to thank Professor C. W. Misner and Dr. Saul Teukolsky for communicating some of their results to me in advance of publication.

*Supported in part by the National Science Foundation under Grant No. GP 30799X to Princeton University and Grant No. GP 32039X to the University of Texas at Austin.

†Present address.

[1]R. Penrose, Riv. Nuovo Cimento 1, 252 (1969); R. Penrose and R. M. Floyd, Nature Phys. Sci. 229, 177 (1971).

[2]C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[3]C. W. Misner (unpublished).
[4]S. A. Teukolsky, private communication; see also W. H. Press and S. A. Teukolsky, Nature 238, 211 (1972).
[5]S. W. Hawking, Phys. Rev. Letters 26, 1344 (1971); contribution in *Black Holes,* edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1973).
[6]D. Christodoulou, Ph.D. thesis, Princeton University, 1971 (unpublished).
[7]R. A. Matzner, J. Math. Phys. 9, 163 (1968).
[8]S. S. Schweber, H. A. Bethe, and F. de Hoffmann, *Mesons and Fields* (Row, Peterson, Evanston, Illinois, 1955), Vol. I, p. 117.
[9]For Fermi fields there can only be one quantum per state. In this case we regard the wave as being a superposition of many modes, all with the same ω and m.
[10]S. W. Hawking, Commun. Math. Phys. 25, 152 (1972).
[11]R. M. Wald, Phys. Rev. D 6, 406 (1972).
[12]J. D. Bekenstein, Phys. Rev. Letters 28, 452 (1972); Phys. Rev. D 5, 1239, 2403 (1972); C. Teitelboim, Lett. Nuovo Cimento 3, 326 (1972).
[13]C. Teitelboim, Phys. Rev. D 5, 2941 (1972).
[14]J. B. Hartle, Phys. Rev. D 3, 2938 (1971); C. Teitelboim, Lett. Nuovo Cimento 3, 397 (1972).
[15]J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, Reading, Mass., 1967), p. 120.

PHYSICAL REVIEW D VOLUME 7, NUMBER 4 15 FEBRUARY 1973

Pion Condensation in Superdense Nuclear Matter

R. F. Sawyer* and D. J. Scalapino†
Department of Physics, University of California, Santa Barbara, California 93106
(Received 7 August 1972)

Previous work, based on the interaction of only one mode of the pion field with nucleons, indicated the possibility that the ground state of nuclear matter at densities above some onset density would consist of a coherent mixture of protons, neutrons, and pions, the latter condensed in a plane-wave state of momentum ≈170 MeV/c. The model is now made more realistic by inclusion of ordinary nuclear forces, emission and absorption of noncondensed pion modes, S-wave pion-nucleon interactions, and pion-pion interactions. A pion condensation is again predicted, with onset between $\rho = 0.5$ baryons/F^3 and $\rho = 1$ baryon/F^3. Some possible consequences for neutron stars are discussed.

I. INTRODUCTION

Superdense nuclear matter, or infinite matter at densities higher than those of nuclei, is of interest in connection with neutron-star theory, supernova theory, and the theory of the early stages of the big bang. Of particular interest is the equation of state for cold superdense matter (where the equation of state is concerned the temperature must be in the MeV region in order to be a significant factor). To find the equation of state it is necessary to find the energy density as a function of the baryon density in the ground state of an electrically neutral system with a certain number of baryons confined in a volume V.[1] The equation of state then can be used to determine neutron-star parameters, most notably the mass-radius relationship and the maximum allowable mass.[2]

In the neutron-star regime the baryon densities will range from nuclear densities, 0.15 baryons/F^3, to perhaps 6 baryons/F^3. In almost all theoretical treatments the matter has been assumed to consist entirely of fermions, that is, baryons and leptons.[3] In the lower ranges of density, $0.15 < \rho < 1$ F^{-3}, it turns out, in these theories, that the composition is largely neutrons, with a small fraction of protons neutralized by electrons. It is at least conceivable that the successful theory of nuclear matter, based on phenomenological nucleon-nucleon potentials, can be extended reliably into this density regime, and this has been the aim of considerable recent work.[4-6]

Above a density of about 1 baryon/F^3, other baryons, p, Λ, Σ^-, Σ^0, etc., are thought to enter in appreciable fractions.[4,5] In this regime it is clear that theoretical calculations become shakier. The forces between most of the baryon pairs are unknown experimentally. Important contributions of many-body forces become a likelihood as densities are increased. There is also always the question of whether the list of constituents is complete.

In the present work we investigate the possibility that pions are a significant constituent of the ground state of superdense matter, even in the lower density region, $\rho < 1$ F^{-3}, in which neutrons are presently thought to be the dominant constituent. It will turn out that π^- particles are the only serious contender (they are a natural possibility