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Arguments Supporting a Positive Two-Pomeranchukon Discontinuity*

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Abarbanel's derivation of the 2-Pomeranchukon discontinuity is examined at $t=0$, where the physics is especially transparent. By slightly altering the significance of Abarbanel's decomposition of the total cross section, arguments are given to support the crucial and controversial assumption that his "single-fireball" vertices do not contain the 2-Pomeranchukon branch point. It is shown further how Abarbanel's discontinuity formula gives a semiquantitative realization of the Finkelstein-Kajantie requirement of small Pomeranchukon couplings if $\alpha_P(0)$ is close to unity. This demonstration, which shows that the triple-Pomeranchukon coupling $g_P{}^2$ is proportional to $1-\alpha_P(0)$, depends critically on the positive sign of the Abarbanel discontinuity.

I. INTRODUCTION

There has recently occurred the remarkable development that the same general S-matrix formula for the discontinuity across the 2-Pomeranchukon branch cut has been proposed by several different authors,[1-4] but one of these authors argues for a positive discontinuity while all others believe the sign to be negative. The physical argument for a negative discontinuity (the majority position) has arisen from Feynman-graph models where the cut represents an "absorptive" correction to a pole when the latter is regarded as given *ab initio* with arbitrarily assignable strength, the cut being needed to keep the complete amplitude within unitarity bounds. Such an interpretation, however, lacks meaning in S-matrix language, where the strengths of pole, cut, and all other singularities are simultaneously controlled by unitarity.

Feynman-graph models typically represent the amplitude as an infinite superposition of components associated with individual graphs, but without attention to renormalization the inserts in a particular graph cannot be identified with singularities of the full S matrix. Consistent renormalization procedures never having been developed for Reggeon lines, an insert line in existing forms of "Reggeon calculus" does not correspond to an actual J pole of the S matrix. The status in graph models of Regge branch points is equally obscure.

Since the discontinuity formula at issue is expressible entirely through the S matrix, it should be possible to derive the formula without recourse to Feynman graphs, and indeed two attempts of this kind have been made. The first, by Abarbanel, depends on the formulation of a certain integral equation whose kernel has simple analyticity properties near the branch point in question.[3] Abarbanel found a positive discontinuity from his equation, but his arguments to support the crucial property of his kernel have not been entirely convincing, and Abarbanel's result has failed to shake the faith of those who on the basis of Feynman graphs had come to believe in the negative sign. A second attempt at an S-matrix derivation has been made by White,[4] using techniques that in principle seem more straight-forward than those of Abarbanel but that in practice involve intricate technical points where sign errors may occur. Thus White's publication of a negative sign has not settled the issue.

Although the physical importance of the 2-Pomeranchukon branch point (being only one of a welter of Regge singularities) is far from established, a healthy protracted controversy over the sign of the discontinuity should augment the understanding of Regge behavior. The intent of this paper is to fuel the controversy with arguments that support Abarbanel's result.

II. A PHYSICAL INTERPRETATION OF ABARBANEL'S ANALYSIS

Roughly speaking, Abarbanel's analysis depends on classifying high-energy events according to the number of produced "fireballs." At zero momentum transfer ($t=0$) the physics is especially transparent because one may there carry out the discussion through the total cross section which, apart from a simple positive factor, is the s discontinuity of the elastic amplitude. Abarbanel breaks down the total cross section into certain partial cross sections which are recursively related and thence he obtains his integral equation. Because all quantities throughout are real and positive, no technical mistake about algebraic signs can occur. If a mistake is made, it has a more subtle origin.

Abarbanel is not quite precise about his decomposition of the total cross section. We suggest that for each event the produced particles be ordered according to longitudinal rapidity, the event being characterized as "single-fireball," "two-fireball," "three-fireball," etc., according to the number of large rapidity gaps in the ordered chain. Figure 1, for example, depicts a four-fireball event, $AB \to 8$ particles. To make unambiguous our fireball definition we specify a minimum interfireball rapidity gap Δ; given a particular choice of Δ, the total cross section may be uniquely decomposed as

$$\sigma_{AB}^{\text{tot}}(s)=\sigma_{AB}^{0,\Delta}(s)+\sigma_{AB}^{1,\Delta}(s)+\sigma_{AB}^{2,\Delta}(s)+\cdots, \tag{2.1}$$

the superscript indicating the number of rapidity gaps larger than Δ or, equivalently, the number of fireballs minus one.

At any finite s there is a maximum n for which $\sigma_{AB}^{n,\Delta}(s)$ is nonvanishing ($n_{\max}$ is of the order $\Delta^{-1}\ln s$), but as $s\to\infty$ the number of terms in the series (2.1) increases without limit. Although $\sigma_{AB}^{n,\Delta}(s)$ does not correspond to any definite set of reactions, each of these partial cross sections must be power bounded, so that the crossed-reaction J projection of the forward amplitude has a decomposition corresponding to Eq. (2.1)[5]:

$$A_{AB}(J)=A_{AB}^{0,\Delta}(J)+A_{AB}^{1,\Delta}(J)+\cdots. \tag{2.2}$$

We note for future purposes that the rightmost J singularity in each $A_{AB}^{n,\Delta}(J)$ is determined by the leading power in the asymptotic expansion of $\sigma_{AB}^{n,\Delta}(s)$. What is the connection between this leading power and the Pomeranchukon?

We shall suppose the Pomeranchukon at $t=0$ to be a simple factorizable Regge pole lying slightly below $J=1$, so that a gap occurs between this pole and other J singularities. Although the contradictory aspects of the 2-Pomeranchukon cut discontinuity may conceivably be related to the failure of such a condition to be realized in the physical S matrix, the Feynman-graph approach should be capable of accommodating an arbitrary pole location, so the controversy is worth pursuing on such a basis. In any event if we fail to assume simple-pole status for the Pomeranchukon, meaning evaporates for the 2-Pomeranchukon discontinuity formula. Brower and Weis have recently given a persuasive argument that *if* the Pomeranchukon is a simple factorizable pole, with finite trajectory slope at $t=0$ and nonvanishing coupling to any channels, then its intercept must lie below $J=1$.[6]

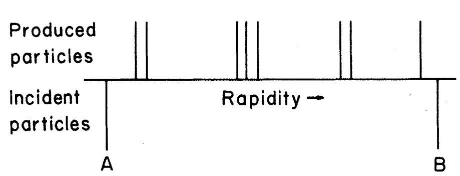

FIG. 1. The rapidity distribution of a four-fireball event.

FIG. 2. Diagrammatic representation of Eq. (2.1).

Supposing a gap in J to occur between the Pomeranchukon pole and other J singularities, we may choose Δ sufficiently large that at each interfireball gap a factorizable Pomeranchukon link becomes a good approximation. The expansion (2.1) may then be represented diagrammatically as in Fig. 2, where the first symbol on the right represents a sum over all types of single fireballs. The reader is cautioned against interpreting Fig. 2 as a Feynman-like expansion. We are representing not the amplitude but the total cross section and each graph depicts the contribution from a distinct class of reactions. The different diagrams of Fig. 2, in other words, correspond to different regions of phase space. To illustrate the meaning of Fig. 2 in a more concrete fashion, consider the second (two-fireball) term in the expansion and define fireball masses s_A and s_B, as well as a squared momentum transfer t_1 in the manner shown in Fig. 3. The factorization property means that $\sigma_{AB}^{1,\Delta}(s)$ at large s has the structure

$$\frac{1}{s^2}\int\int\int ds_A ds_B dt_1 A_{AP}^{0,\Delta}(s_A,t_1) \times\left(\frac{s}{s_A s_B}\right)^{2\alpha_P(t_1)} A_{PB}^{0,\Delta}(s_B,t_1), \tag{2.3}$$

where the factor $A_{iP}^{0,\Delta}(s_i,t)$ may loosely be described as being proportional to the cross section for single-fireball formation when a Pomeranchukon "collides" with a physical particle of type i. The rapidity gap between the "rightmost" particle in the left fireball and the "leftmost" particle in the right fireball is approximately the logarithm of the ratio $s/s_A s_B$ when the interfireball gap is large, so the integration in (2.3) is confined to the region

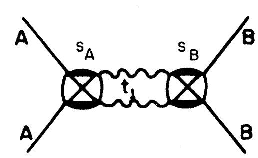

FIG. 3. Two-fireball diagram defining the squared fireball masses, s_A and s_B, and the squared momentum transfer t_1.

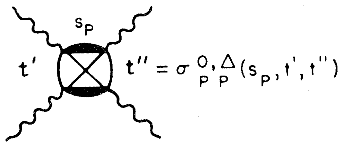

FIG. 4. Four-Pomeranchukon vertex involved in the production of more than two fireballs.

where this ratio is greater than e^{Δ}.

For production of more than two fireballs one encounters additionally the 4-Pomeranchukon vertex shown in Fig. 4, which might be described as the single-fireball production cross section in a Pomeranchukon-Pomeranchukon collision. With such a factor repeated $n-2$ times, formula (2.3) may be generalized so as to construct the physical AB cross section for n-fireball formation. Abarbanel has given a set of variables for the general formula[3]; also suitable are the Toller variables of Ref. 7.

The J projection of (2.3) to obtain $A^{1;\Delta}_{AB}(J)$ involves an integration over s that extends to $s=\infty$. If the factor $A^{0;\Delta}_{iP}(s_i, t)$ is bounded by a sufficiently low power of s_i, the asymptotic s dependence is controlled by the factor $s^{2\alpha_P(t_1)}$ so the leading J singularity of $A^{1;\Delta}_{AB}(J)$ will be a branch point at $J=2\alpha_P(0)-1$, whose discontinuity has the form

$$\int dt_1 A^{0;\Delta}_{AP}(J, t_1) A^{0;\Delta}_{PB}(J, t_1)\delta(J - 2\alpha_P(t_1)+1). \tag{2.4}$$

Similarly, if $\sigma^{0;\Delta}_{PP}(s_P, t', t'')$ is bounded by a sufficiently low power of s_P, the leading singularity of $A^{n;\Delta}_{AB}(J)$ will for any n be a branch point at this same location – with a discontinuity that can be computed. All these discontinuities are positive since they dominate the asymptotic behavior of separately positive pieces of the cross section. We have here essentially the same situation as that analyzed by Finkelstein and Kajantie for finite fireball masses.[8]

It does not immediately follow that the discontinuity of the total amplitude $A_{AB}(J)$ is necessarily positive, because the series (2.2) diverges for $J \leq \alpha_P(0)$. This series, however, may be replaced by an integral equation, whose kernel has the structure

$$K(J; t't'') \propto A^{0;\Delta}_{PP}(J, t', t'')\frac{1}{J-2\alpha_P(t'')+1}, \tag{2.5}$$

and if the J singularities of the factor $A^{0;\Delta}_{PP}(J, t', t'')$ may be ignored one deduces that the discontinuity of the full amplitude has the form

$$\int dt_1 A_{AP}(J_+, t_1) A_{PB}(J_-, t_1)\delta(J - 2\alpha_P(t_1)+1), \tag{2.6}$$

the points J_+ and J_- lying on opposite sides of the cut. The sign of this discontinuity can be shown to be positive in the sense of the preceding discussion, although the magnitude of the full-amplitude discontinuity near the branch point is smaller than that of any of the individual terms in the series (2.2). It should be observed that the final discontinuity formula (2.6) is independent of Δ, although this parameter has been important at every stage of the derivation.

Abarbanel's reasoning depends crucially on the asymptotic behavior of single-fireball cross sections. Is it possible that this behavior could lead to J singularities of $A^{0;\Delta}_{iP}(J)$ and $A^{0;\Delta}_{PP}(J)$ that would alter the result (2.6)? With our definition of a single fireball we find such an eventuality hard to imagine because singularities in J arise from power behavior in the limit as the fireball mass approaches infinity. Now to the extent that transverse momenta are bounded, each fireball cross section for a definite number of produced particles vanishes when the fireball mass exceeds some finite limit, because within the fireball we constrain the magnitude of allowable longitudinal-rapidity gaps. To the extent that the probability for large transverse momenta decreases exponentially we shall have an asymptotic exponential decrease with fireball mass (faster than any power) for each of the partial cross sections. Thus the J projection of each fixed-multiplicity component of a single-fireball cross section will be free from J singularities, apart from those in the left-half J plane due to the projecting group representation function. Singularities in the right-half J plane arise only from a divergence of the infinite series of components.

The location of such singularities will depend on the ratio of successive terms in the series and thus on parameters, such as Δ, other than the Pomeranchukon trajectory.[9] How a branch point could arise at $J=2\alpha_P(0)-1$ is obscure, none of the usual mechanisms being operative. Singularities with other locations, even if they occur to the right of the 2-Pomeranchukon branch point, will not interfere with the 2-Pomeranchukon discontinuity formula.

III. RELATION BETWEEN POLE RESIDUE AND BRANCH POINT DISCONTINUITY

Support for a positive discontinuity also emerges from a study of the relation between the discontinuity and the Pomeranchukon-pole residue when the pole at $t=0$ is very close to $J=1$. Following Abarbanel, we define

$$R(J)=A_{PP}(J, t'=t''=t_0(J)), \tag{3.1}$$

where $t_0(J)$ is defined by

$$2\alpha_P(t_0)-1=J. \tag{3.2}$$

Assuming the trajectory $\alpha_P(t)$ to be analytic in t near the branch point, the discontinuity formula for $R(J)$ (up to the uncertainty in sign) is

$$\operatorname{disc}R(J)=2i\rho(J)R(J_+)R(J_-), \tag{3.3}$$

where

$$\rho(J)=\frac{\beta}{2(d\alpha_P/dt)_{t=t_0(J)}}, \tag{3.4}$$

β being a positive constant that depends on the precise normalization of $R(J)$. Normalizing so that the pole in $R(J)$ at $J=\alpha_P(0)$ has the residue $g_P{}^2(0)$, where $g_P(t)$ is the triple-Pomeranchukon coupling defined in Ref. 10, it turns out that $\beta=\frac{1}{16}$.

Formula (3.3) implies that the function

$$R^{-1}(J)-\frac{1}{\pi}\rho(J)\ln\frac{J-\alpha_c}{\alpha_P-\alpha_c}, \tag{3.5}$$

where $\alpha_c=2\alpha_P-1$ is the branch point position, is free from singularities near $J=\alpha_c$ and in this neighborhood may be expanded in a power series if $R(J)$ has no nearby zeros. To be troublesome to the following argument a zero would have to be located as close to α_c as the pole at α_P. Such a location for a zero is conceivable but would constitute an accident if the pole and branch point are significantly coupled to each other. Since the Finkelstein-Kajantie result[8] suggests an important interaction between branch point and pole we shall ignore possible zeros of $R(J)$ near $J=1$. Since $R^{-1}(J)$ vanishes at $J=\alpha_P$, it is convenient to expand around the pole position:

$$R^{-1}(J)=\frac{1}{\pi}\rho(J)\ln\frac{J-\alpha_c}{\alpha_P-\alpha_c} + b(J-\alpha_P)+O((J-\alpha_P)^2). \tag{3.6}$$

Now,

$$\frac{1}{g_P{}^2}=\left(\frac{d}{dJ}\frac{1}{R(J)}\right)_{J=\alpha_P}, \tag{3.7}$$

so

$$\frac{1}{g_P{}^2}=\frac{1}{\pi}\frac{\rho(\alpha_P)}{\alpha_P-\alpha_c}+b \tag{3.8}$$

if the Abarbanel sign is correct, while the sign of the first term on the right-hand side of (3.8) is reversed if the absorptive sign is correct. With the Abarbanel sign, formula (3.8) smoothly exhibits the Finkelstein-Kajantie mechanism[8] as $\alpha_P \to 1$. In this limit $\alpha_c \to \alpha_P$ from below and $g_P{}^2$ approaches zero from the positive direction.[11] With the absorptive sign for the cut discontinuity, on the other hand, $g_P{}^2$ becomes negative if the difference $\alpha_P-\alpha_c$ is too small.[12]

IV. CONCLUSION

If the Pomeranchukon is not a simple pole with factorizable residue, the entire subject under discussion requires reformulation, but the apparent success of scaling rules for experimentally measured inclusive reactions is understandable in Regge language only with a factorizable Pomeranchukon at $t=0$. On the other hand, the assumption that the Pomeranchukon trajectory is analytic near $t=0$ has little experimental support. If the pole collides with the branch point at $t=0$ and for negative t moves onto an unphysical sheet of the J plane, the Finkelstein-Kajantie line of argument and the closely-related argument of Abarbanel must be reexamined. Both these arguments require factorization of asymptotic amplitudes *near* $t=0$ as well as *at* $t=0$ [as exhibited by the appearance of the Pomeranchukon trajectory slope in formula (3.4)].

If the White method for calculating the 2-Pomeranchukon discontinuity conclusively yields a result different from that of Abarbanel, it is reasonable to infer a $t=0$ singularity of the Pomeranchukon trajectory as the source of contradiction.

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[5]To avoid the signature complication we may suppose that particle A is self-conjugate, so that left and right

(s and u) discontinuities are equal.

[6]R. Brower and J. Weis, Phys. Letters 41B, 631 (1972).

[7]G. F. Chew and C. DeTar, Phys. Rev. 180, 1577 (1969).

[8]J. Finkelstein and K. Kajantie, Nuovo Cimento 56, 659 (1968).

[9]Note that Δ-dependent singularities must disappear from the full amplitude when the integral equation is solved. J. Koplik [Phys. Rev. D 7, 558 (1973)] has applied the Finkelstein-Kajantie technique to prove that the leading J singularity of $A_{PP}^{0\Delta}(J)$ lies to the left of $\alpha_P(0)$.

[10]H. D. I. Abarbanel *et al.*, Phys. Rev. Letters 26, 937 (1971).

[11]In fact the entire amplitude vanishes in this limit.

[12]It can be shown that any Pomeranchukon-communicating amplitude $A_{ij}(J)$ is related to $R(J)$ by

$$A_{ij}(J) = P_{ij}^{(1)}(J)R(J) + P_{ij}^{(2)}(J),$$

where $P_{ij}^{(1)}(J)$ and $P_{ij}^{(2)}(J)$ are analytic near the branch point. Thus the relative sign of pole residue and cut discontinuity as well as the magnitude ratio is universal when these two singularities are close to one another.

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Asymptotic Behavior of an Analytically Solvable Nova Model*

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We formulate an analytically solvable version of the nova model and obtain the s dependence of single- and two-particle inclusive distributions. The nature of the singularity in the two-particle distribution at $x_1 = x_2 = 0$ and numerical predictions for National Accelerator Laboratory energies are discussed.

The nova model of multiparticle production has been utilized to obtain strikingly good fits to single-particle and two-particle inclusive distributions at present accelerator energies, within the framework of a simple diffractive excitation picture.[1] To obtain the explicit s dependence of the nova model so we can make specific predictions about the behavior of one- and two-particle distributions at National Accelerator Laboratory (NAL) energies, we construct an analytically solvable version of the nova model. Having exhibited analytical forms for the distributions, we can determine the rate at which these distributions approach their limiting forms, and in particular obtain a form for the singularity[2] in the two-particle distribution at $x_1 = x_2 = 0$ which makes its nature more apparent than would be the case if only numerical results were presented. Our specific model should not be taken too seriously, but it does exhibit clearly the expected properties of diffractive models.

To establish our notation and assumptions, we briefly review the nova model: Particles a and s (the spectator) scatter diffractively, and a is quasielastically excited into a fireball or nova which decays isotropically in the nova rest frame with average decay multiplicity linearly proportional to the nova mass. We shall concentrate on the single-particle distribution for pions, since they constitute most of the produced particles and since we can then neglect the nova's transverse momentum[1]; the distribution can be written as

$$\frac{d\sigma_i}{d^3k} = \sum_a \int_{M_a}^{\sqrt{s} - M_s} \rho_a(M) N_i(M) \frac{d^3D}{d^3q}\frac{d^3q}{d^3k}\, dM\,, \tag{1}$$

where $\vec{k}$ is the c.m. momentum of the observed pion, and $\vec{q}$ its momentum in the nova rest frame; M_a and M_s are the masses of a and s;

$$\rho_a(M) = C_{a,s}\frac{\exp[-\beta_a^2/(M - M_a)^2]}{(M - M_a)^2} \tag{2}$$

is the cross section for exciting a nova of mass M;

$$N_i(M) = \frac{M - M_a}{E} \tag{3}$$

is the decay multiplicity of pions from a nova of mass M, and

$$\frac{d^3D}{d^3q} = A\exp\left(-\frac{q_L^2 + q_T^2}{Q^2}\right) \tag{4}$$

is the decay distribution in the nova rest frame, normalized to unity. The Jacobian d^3q/d^3k is just q^0/k^0, and a Lorentz transformation yields