PHYSICAL REVIEW D VOLUME 7, NUMBER 3 1 FEBRUARY 1973

Reaction $\nu+p \rightarrow \nu+\mu^{+}+\mu^{-}+p$ in a Six-Fermion Coupling Theory*

J. Smith and N. Stanko†

The Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790

(Received 7 September 1972)

A theory in which the hypothetical intermediate boson of weak interactions interacts strongly with hadrons becomes a theory with a six-fermion coupling in the limit that the mass of the boson is very large. To facilitate setting an upper limit on such couplings in high-energy neutrino experiments, we give here the theoretical muon spectra for the reaction $\nu + p \rightarrow \nu + \mu^{+} + \mu^{-} + p$ calculated for a typical six-fermion interaction. A comparison is also made with the muon spectra predicted from $V-A$ theory, and a rough upper limit on the coupling constant is made using data from the CERN neutrino experiment.

The failure of high-energy muons to obey the $\sec\theta$ law[1] has aroused interest in mechanisms which produce muons at very high energies. Bjorken, Pakvasa, Simmons, and Tuan[2] have examined theories in which the W boson of weak interactions couples strongly to hadrons. Such theories were considered previously by various authors[3,4] and, due to our lack of knowledge concerning the weak interactions at high energies, they are not ruled out by experiment. In these theories the pairwise interaction of W bosons with hadrons is governed by the strong interactions while the single emission of a W boson is semiweak with respect to the Fermi coupling constant. Obviously it is desirable to set stringent experimental limits on the magnitude of possible strong W-boson–hadron coupling constants.

One reaction which will be investigated at the National Accelerator Laboratory[5] (NAL) is

$$\nu + p \rightarrow \nu + \mu^{+} + \mu^{-} + p . \qquad (1)$$

In conventional theory this reaction involves both the weak and the electromagnetic interactions. Allowing for the possible existence of both charged and neutral (in the manner of Weinberg[6]) intermediate bosons, the relevant diagrams for this reaction are shown in Fig. 1. In $V-A$ theory the proton is required to supply an electromagnetic field and allow conservation of energy and momentum. However, in a theory with a strong W-boson–hadron interaction reaction (1) proceeds via the diagram shown in Fig. 2. Of course we know nothing about the scattering amplitude of the W boson on a proton,[7] so only model-dependent predictions can be made for this reaction. However, Kabir and Kamal[8] have set a lower limit of 10 GeV on the mass of the strongly interacting W boson using the measured muon flux deep underground. Spontaneously broken gauge theories[6] predict very large masses for both charged and neutral W bosons. Obviously neutrino experiments in the range from 1 to 10 GeV cannot possibly produce such bosons as real particles. Fortunately all these theories have a local limit when the mass of the boson is infinitely large and this limit can be checked experimentally. In the $M_W = \infty$ limit the number of diagrams in Fig. 1 reduces from five to the two shown in Fig. 3. Extensive calculations were recently made for these diagrams by Brown, Hobbs, Smith, and Stanko[9] and therefore the energy and angular distributions of the μ^{+} and the μ^{-} mesons in this reaction are known for an arbitrary admixture of vector and axial-vector weak couplings. Previous results obtained by Fujikawa[10] and by Løvseth and Radomski[11] were only given for the special case of $V-A$ coupling. Regarding Fig. 2, which in the limit $M_W = \infty$ reduces to Fig. 4, we know nothing about the corresponding distributions for the muons. We recognize that the interaction is now a direct six-fermion coupling. Hence it is possible to propose models for this coupling, then calculate the muon spectra, and compare them with those predicted by theories with vector and axial-vector couplings.

Actually Ericson and Glashow[3] were among the first to consider reaction (1) in an attempt to set an upper limit on the strength of the six-fermion interaction. They chose the Lagrangian

$$\mathcal{L} = \lambda^{-3} \frac{G}{\sqrt{2}} J_{\mu} J_{\mu}^{\dagger} (\bar{p}p) \qquad (2)$$

with $G \cong 10^{-5}/M_p^{2}$ the Fermi coupling constant. J_μ is the usual lepton current and λ is a coupling parameter with the dimensions of a mass. Their best bound on λ came from an estimate of the total cross section for reaction (1) together with a cutoff on the recoil proton momentum and the assumption that $\sigma < 10^{-39}$ cm^{2}. This very rough estimate yielded $\lambda \geqslant m_{\mu}$, the muon mass. However, they pointed out that more detailed analysis would have to be made to distinguish the six-fermion interaction theory from the four-fermion interaction theo-

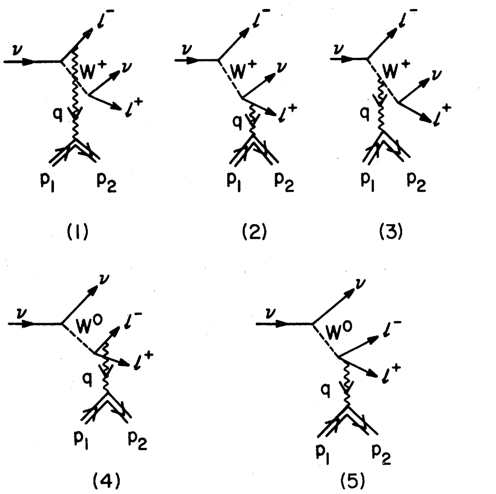

FIG. 1. Feynman diagrams for the reaction $\nu + p \rightarrow \nu + \mu^+ + \mu^- + p$ in a four-fermion theory mediated by charged and neutral vector bosons.

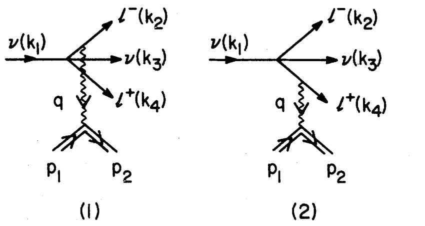

FIG. 3. Feynman diagrams for the reaction $\nu + p \rightarrow \nu + \mu^+ + \mu^- + p$ in a local four-fermion theory.

ry. At that time such analysis would have been highly premature but now that experiments are to be carried out at NAL such information becomes highly relevant. Hence we have used the Lagrangian (2) in our numerical computer programs[9] for reaction (1) to generate both the total cross section and the energy and angular distributions of the muons. The details of the analysis are rather straightforward so we will only present the results here. Since we take λ to be constant one can scale our results to find cross sections and spectra for arbitrary values of λ. For convenience we set $\lambda^{-3} = \lambda' m_\mu^{-3}$, where λ' is a new dimensionless coupling constant which we take for convenience to be of the order of α, the fine-structure constant. We give results below for two cases. The first set of results are calculated for $\lambda' = 6 \times 10^{-4}$ and a bare proton. To see the effect of damping out the proton's recoil spectrum, we then repeated the calculation and arbitrarily included an electromagnetic form factor for the proton. This leads to a small decrease in the cross section ranging from a factor of three for 5-GeV neutrinos to a factor of 5 for 20-GeV neutrinos. We remind the reader that the usual cross section for reaction (1) is very small because the amplitude is first order in the weak interaction and second order in the electromagnetic interaction. Experimentally it is possible to increase the cross section by using a heavy nucleus as target so that we gain an extra factor of Z for the incoherent scatterings we are considering. If the usual four-fermion interaction dominates, then the coherent cross section (with its extra factor of Z^2) is larger than the incoherent cross section for almost all beam energies. Therefore we compare the six-fermion interaction results given here with those of the *coherent* $V-A$ results for an iron nucleus (divided by Z^2) and not with the incoherent $V-A$ results (divided by Z). Obviously total cross-section measurements for reaction (1) will set a limit on the strength of the six-fermion coupling. A more improved limit, however, can be obtained from a knowledge of the energy and angular spectra of the muons in reaction (1). This is due to the fact that the spectra of the muons are completely different in the two theories. The μ^- and μ^+ spectra from reaction (1) are identical in the six-fermion coupling theory with the Lagrangian given by Eq. (2) whereas in $V-A$ theory[9-11] the μ^- is produced with larger average energy and a corresponding smaller average angle than the μ^+.

In Fig. 5 we plot the total cross section for neutrinos incident on a target proton. The curve A gives the results for the six-fermion coupling with no form factor and $\lambda' = 6 \times 10^{-4}$ while B includes the proton's electromagnetic form factor. For comparison we also give the coherent results for iron (divided by Z^2) from standard $V-A$ theory

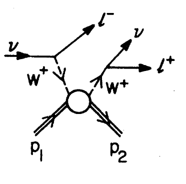

FIG. 2. Feynman diagram for the reaction $\nu + p \rightarrow \nu + \mu^+ + \mu^- + p$ in a theory with strongly interacting W bosons.

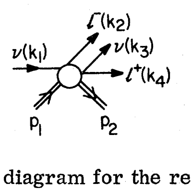

FIG. 4. Feynman diagram for the reaction $\nu + p \rightarrow \nu + \mu^+ + \mu^- + p$ in a six-fermion interaction theory.

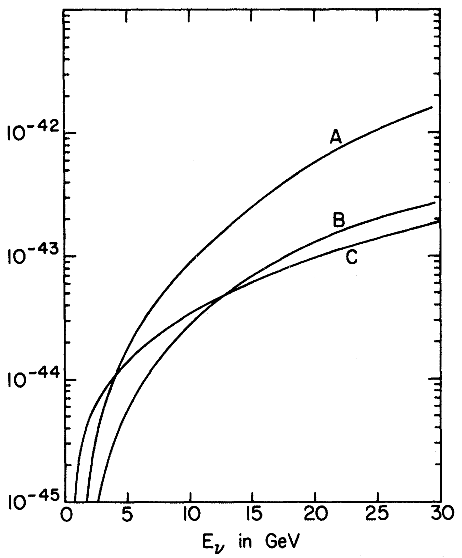

FIG. 5. Total cross sections (per proton) for $\nu + p \rightarrow \nu + \mu^+ + \mu^- + p$. The curves A and B refer to the six-fermion coupling theory without and with a form factor for the proton. C refers to the prediction of standard four-fermion theory for coherent scattering off iron (divided by Z^2).

taken from Ref. 9. Figure 6 shows the energy spectra of the muons in all three cases for an incident neutrino energy of 10 GeV. The spectra for the six-fermion interaction theory have different characteristics from the spectra in the case of $V - A$ theory. Figure 7 shows the corresponding results for the angular distributions. Obviously the muons produced by the six-fermion interaction have much larger angles than those produced via the four-fermion $V - A$ theory. In the event that the four-fermion theory is modified by a neutral current so that it is no longer $V - A$ in the $M_W = \infty$ limit it is still possible to place a bound on λ' using the muon spectra given in Ref. 9. Also in the event that the present theoretical speculations are wrong and a low-mass-charged W boson exists which mediates reaction (1), previous work by the same authors[12] gives the necessary total cross sections and energy and angular distributions of both muons so it will still be possible to set a limit on λ'. Note that we have only chosen $\lambda' = 6 \times 10^{-4}$ so that we can draw all the curves on the same scale. At present experiment does not rule out larger values for λ'.

In fact it is possible to set an approximate upper bound on λ' from the results of Kaftanov *et al.*[13] These authors analyzed the 33 μ-pair candidates from the CERN neutrino experiment and found none compatible with range selection criteria. From the theoretical muon spectra for $V - A$ theory, folded with the neutrino spectrum and range cutoffs, they then estimated the number of events which could be compatible with the background. This analysis led to a bound for G_D, the diagonal $\nu_\mu \mu$ coupling constant, namely, $G_D < 20G$, where

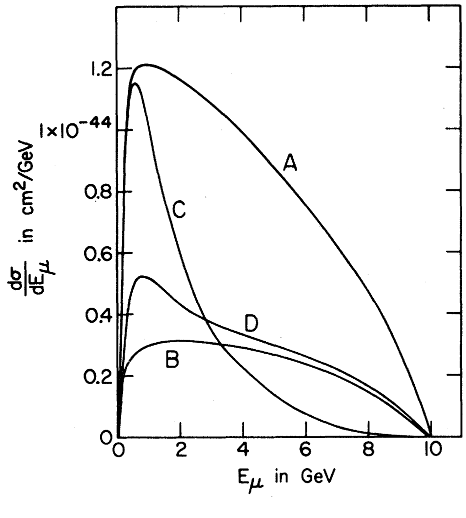

FIG. 6. The energy spectra of the muons in $\nu + p \rightarrow \nu + \mu^+ + \mu^- + p$ with 10-GeV incident neutrinos. The six-fermion theory results are given (A) excluding and (B) including a form factor for the proton. We also show the $V - A$ predictions for the μ^+ (C) and the μ^- (D) obtained from coherent scattering off an iron nucleus (divided by Z^2).

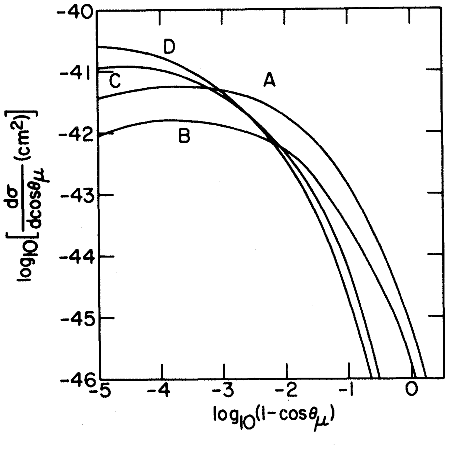

FIG. 7. The angular spectra of the muons in $\nu + p \rightarrow \nu + \mu^+ + \mu^- + p$ with 10-GeV incident neutrinos. The six-fermion theory results are given (A) excluding and (B) including a form factor for the proton. We also show the $V - A$ predictions for the μ^+ (C) and the μ^- (D), obtained for coherent scattering off an iron nucleus (divided by Z^2).

G is the Fermi coupling constant. It is therefore possible to repeat the analysis from our spectra to set a limit on λ'. We have not carried out the actual computation because we do not know the neutrino spectrum and cutoffs used in that particular experiment. However, we will get a reasonable estimate by accepting the value $\lambda' = 6\times10^{-4}$ and scaling it by a factor of 20. Therefore at present the CERN experiment implies values for $\lambda' < 1.2\times10^{-2}$, and we expect the NAL experiment to reduce this number considerably.

Note added in proof. After this paper was submitted, the Utah group (Bergeson, Carlson, Keuffel, and Morrison) withdrew their evidence for a failure of the $\sec\theta$ law. Obviously speculations about theories with strongly interacting W bosons will now subside. Nevertheless, it is still possible to set bounds on six-fermion coupling constants using present (and future) accelerator data so the results given in this paper are still of interest.

We would like to thank Brookhaven National Laboratory for use of its computing facilities.

---

*Work supported in part by the National Science Foundation under Grant No. GP-32998X.

†Present address: CERN, Geneva, Switzerland.

[1] H. Bergeson, J. Keuffel, M. Larson, E. Martin, and G. Mason, Phys. Rev. Letters 19, 1487 (1967); H. Bergeson, G. L. Bolingbroke, G. Carlson, D. E. Groom, J. W. Keuffel, J. L. Morrison, and J. L. Osborne, *ibid.* 27, 160 (1971).

[2] J. D. Bjorken, S. Pakvasa, W. Simmons, and S. F. Tuan, Phys. Rev. 184, 1345 (1969). See also related papers by: P. V. Ramana Murthy, Phys. Letters 28B, 38 (1968); J. B. Kogut, Phys. Rev. 186, 1540 (1969); C. H. Woo, Phys. Rev. Letters 21, 1419 (1968).

[3] T. Ericson and S. L. Glashow, Phys. Rev. 133, B130 (1964).

[4] G. Feinberg, Phys. Rev. 134, B1295 (1964); S. V. Pepper, C. Ryan, S. Okubo, and R. E. Marshak, *ibid.* 137, B1259 (1965); C. G. Callan, Jr., Phys. Rev. Letters 20, 809 (1968).

[5] An experiment is planned by E. W. Beier, D. Cline, A. K. Mann, J. Pilcher, D. D. Reeder, and C. Rubbia (private communication from A. K. Mann).

[6] S. Weinberg, Phys. Rev. Letters 27, 1688 (1971); 19, 1264 (1967).

[7] Under certain assumptions about this amplitude Appelquist and Goldman have estimated a lower bound on the first-order weak contribution to the forward amplitude for $\nu + p \rightarrow \nu + p$. See T. Appelquist and T. Goldman, Phys. Rev. D 6, 2048 (1972).

[8] P. K. Kabir and A. N. Kamal, Lett. Nuovo Cimento 1, 1 (1971).

[9] R. W. Brown, R. H. Hobbs, J. Smith, and N. Stanko, Phys. Rev. D 6, 3273 (1972).

[10] K. Fujikawa, Ann. Phys. (N.Y.) 68, 102 (1971); Ann. Phys. (N.Y.) (to be published).

[11] J. Løvseth and M. Radomski, Phys. Rev. D 3, 3686 (1971).

[12] R. W. Brown and J. Smith, Phys. Rev. D 3, 207 (1971); R. W. Brown, R. H. Hobbs, and J. Smith, *ibid.* 4, 794 (1971).

[13] V. S. Kaftanov, V. D. Khovansky, and M. A. Kubantsev, Phys. Letters 40B, 215 (1972).