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Symmetry Breaking and $\eta' \to \eta\pi\pi$ Decay

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A broken-chiral-invariant and broken-dilational-invariant effective Lagrangian with a symmetry-breaking term belonging to the $(3,3^*)\oplus(3^*,3)$ representation of $SU(3)\otimes SU(3)$ is used to obtain the matrix element of the σ term between η and η'. It utilizes both wave-function-renormalization mixing and mass-matrix mixing. The presently available upper limit of the $\eta' \to \eta\pi\pi$ decay rate requires a rather large value for this σ term and, correspondingly, rather large values for the mixing parameters that vanish in the symmetry limit and ought to be small.

The nature of the interaction that breaks chiral $SU(3)\otimes SU(3)$ symmetry is an interesting and, as yet, largely unresolved problem. One method to investigate this problem is to consider processes that involve two pions where the soft-pion limit yields a matrix element of the "σ term" – the double commutator of axial charges with the symmetry-breaking Hamiltonian. This method has been applied to πN scattering, particularly by Cheng and Dashen[1] who find a rather large value of the σ-nucleon matrix element. Such a large value is, however, provided within the framework[2] of broken dilation invariance. Riazuddin and Oneda[3] have pointed out that the σ-$\eta\eta'$ matrix element, which appears in the soft-pion limit of $\eta' \to \eta\pi\pi$ decay, is also probably very large. Recently, Weisz, Riazuddin, and Oneda[4] have calculated the σ-$\eta\eta'$ matrix element by using the method of broken dilation invariance, but they obtain a small value.

We shall review the situation in $\eta' \to \eta\pi\pi$ decay. In particular, we shall show that some of the approximations made by Weisz *et al.*[4] are not supported by an effective Lagrangian that is invariant under dilations and the chiral group except for a symmetry-breaking part that belongs to the $(3, 3^*)\oplus(3^*, 3)$ representation and has a single dimension d. This Lagrangian gives a singlet-octet mixing scheme that utilizes both wave-function-renormalization mixing (i.e., nondiagonal kinetic-energy terms) and mass-matrix mixing. This mixing scheme enables us to obtain a somewhat larger value of the σ term than that found by Weisz *et al.* after their approximations are altered to conform with our model. However, if the decay width turns out to be near its present upper limit, the σ term would be quite large, and large mixing parameters would be required. These parameters vanish in the symmetry limit and ought to be small. Although our calculation is by no means model-independent, a large decay width would cast considerable doubt on the postulated structure of the symmetry-breaking Lagrangian. Thus, although we disagree with some of the details of the work of Weisz *et al.*, our conclusion is substantially the same as theirs. The need for further experimental work to clarify the situation in η' decay is obvious. It is important to have improved measurements on both the decay width and on the slope of the Dalitz plot. We turn now to the details of our work.

We label the four-momentum of the η' by p' and that of the η by p. The pions are labeled by isospin indices a and b, and they carry off four-momenta q_1 and q_2. We use the energy variables

$$\nu = -(q_1 - q_2)\cdot(p+p')/4m_{\eta'} \tag{1}$$

and

$$\nu_B = -q_1\cdot q_2/2m_{\eta'}. \tag{2}$$

The method of current algebra can be applied to $\eta' \to \eta\pi\pi$ decay in precisely the same way that it is used in πN scattering. We use the method advocated by Brown, Pardee, and Peccei.[5] A simple alteration of their formula (6) expresses the η' decay amplitude as

$$T_{ab}(\eta' \to \eta\pi\pi) = \delta_{ab}\left[\sigma_{\eta\eta'} + \left(\frac{\nu_B}{m_{\eta'}}\right)a + \left(\frac{\nu}{m_{\eta'}}\right)^2 b + \left(\frac{m_\pi}{m_{\eta'}}\right)^4 c\right]. \tag{3}$$

Here a, b, and c are dimensionless, nonsingular functions, and $\sigma_{\eta\eta'}$ measures the breaking of chiral symmetry in the sense that

$$F_\pi{}^2\sigma_{\eta\eta'}=\tfrac{1}{3}\sum_{a=1}^{3}\langle\eta|[X_a,[X_a,-\mathcal{L}_{SB}]]|\eta'\rangle\,. \qquad (4)$$

The X_a are the generators of chiral transformations with an isospin normalization, and $F_\pi=92.6$ MeV is the pion decay constant. It is important to observe that at the special (unphysical) point $\nu=\nu_B=0$ the decay amplitude is determined by the σ term save for corrections of order $m_\pi{}^4$. Since the σ term itself is of order $m_\pi{}^2$ it should dominate the amplitude at this point and thus be determined by an extrapolation of the physical decay amplitude into the unphysical region.

The experimental situation is far from clear. The observed[6] variation of the amplitude in ν_B gives

$$a/\sigma_{\eta\eta'}\approx 59\,. \qquad (5)$$

The precise decay rate is not yet known; only an upper limit[7] $\Gamma_{\eta\pi\pi}\le 1.3$ MeV now exists. If one takes this upper limit, divides out the phase space,[8] takes the square root, and uses a linear extrapolation in ν_B with a slope given by Eq. (5), one gets

$$|\sigma_{\eta\eta'}|\le 8.9\,. \qquad (6)$$

A theoretical estimate of this quantity can be made by using the ideas of broken dilation invariance. First we note that, with the symmetry-breaking Lagrangian

$$-\mathcal{L}_{SB}=u_0+cu_8 \qquad (7)$$

belonging[9] to the $(3,3^*)\oplus(3^*,3)$ representation of the chiral group, Eq. (4) gives

$$F_\pi{}^2\sigma_{\eta\eta'}=\tfrac{1}{3}(c+\sqrt{2})\langle\eta|u_8+\sqrt{2}\,u_0|\eta'\rangle\,. \qquad (8)$$

The constant c can be estimated[9] from the mass splitting of the meson octet to be about

$$c\simeq -1.25\,. \qquad (9)$$

We now assume that, except for a constant, the only part of the Lagrangian that breaks scale invariance[10] is the chiral-symmetry-breaking term u_0+cu_8 and that this term has a single dimension d. This hypothesis yields a constraint on the trace of the stress-energy tensor

$$-T^\mu{}_\mu=(4-d)(u_0+cu_8)+\text{const}\,. \qquad (10)$$

that may be used to eliminate u_0 and put Eq. (8) in the form

$$F_\pi{}^2\sigma_{\eta\eta'}=\tfrac{1}{3}(c+\sqrt{2})(c^{-1}-\sqrt{2})\langle\eta|cu_8|\eta'\rangle - \frac{\sqrt{2}}{3}\,\frac{c+\sqrt{2}}{4-d}\langle\eta|T^\mu{}_\mu|\eta'\rangle\,. \qquad (11)$$

The conservation of the stress tensor requires that its matrix element has the general structure

$$\langle\eta(p)|T^{\mu\nu}|\eta'(p')\rangle=(g^{\mu\nu}k^2-k^\mu k^\nu)B_1(k^2) + [P^\mu P^\nu k^2-(P^\mu k^\nu+k^\mu P^\nu)k\cdot P + g^{\mu\nu}(k\cdot P)^2]B_2(k^2)\,, \qquad (12)$$

where

$$k=p-p'\,,\quad P=p+p'\,. \qquad (13)$$

We shall assume that the form factor $B_2(k^2)$ vanishes, for it is associated with a tensor form involving high (quartic) powers of the momenta. This assumption is akin to that often made in "hard-pion theory" where quantities involving high powers of the momenta in vertex functions are discarded. In terms of an effective Lagrangian, our assumption corresponds to the omission of interactions involving a large number of derivatives. We now have

$$\langle\eta|T^\mu{}_\mu|\eta'\rangle=3k^2B_1(k^2)\,, \qquad (14)$$

which should be a small quantity since the squared momentum transfer k^2 in the $\eta'\to\eta\pi\pi$ decay is small. The omission of this trace gives

$$F_\pi{}^2\sigma_{\eta\eta'}\simeq\tfrac{1}{3}(c+\sqrt{2})(c^{-1}-\sqrt{2})\langle\eta|cu_8|\eta'\rangle\,. \qquad (15)$$

The value of the matrix element $\langle\eta|cu_8|\eta'\rangle$ depends very much on the model employed for mixing the unperturbed octet and singlet states. We shall use a model that contains both wave-function (λ) mixing and mass (θ) mixing and always work only to first order in the mixing parameters with

$$\phi_8\simeq\eta+(\theta-\lambda)\eta'\,,\quad \phi_0\simeq\eta'-(\theta+\lambda)\eta\,. \qquad (16)$$

It should be noted that the wave-function mixing is not an orthogonal transformation. The effective Lagrangian that will be presented below gives rise to this pattern of mixing with only one constraint on the two parameters, namely,

$$|\theta+\lambda|\simeq\left(\frac{m_8{}^2-m_\eta{}^2}{m_{\eta'}{}^2-m_\eta{}^2}\right)^{1/2} \simeq\left(\frac{\frac{4}{3}m_K{}^2-\frac{1}{3}m_\pi{}^2-m_\eta{}^2}{m_{\eta'}{}^2-m_\eta{}^2}\right)^{1/2} \simeq 0.18\,. \qquad (17)$$

It should be emphasized that this is only a very crude estimate, because the mass splitting $m_8{}^2-m_\eta{}^2$ is of second order in the symmetry breaking. The formula (17) attributes the entire second-order effect to mixing and neglects second-order corrections to the Gell-Mann–Okubo mass relation.[11] At any rate, our effective Lagrangian gives

$$\langle \eta | cu_8 | \eta' \rangle \simeq \tfrac{1}{2}(2m_{\eta'}^2 - m_\eta^2 - m_\pi^2)(\theta - \lambda), \tag{18}$$

and thus

$$\sigma_{\eta\eta'} \simeq \frac{1}{3F_\pi^2}(c+\sqrt{2})(\sqrt{2}-c^{-1}) \times\tfrac{1}{2}(2m_{\eta'}^2 - m_\eta^2 - m_\pi^2)(\theta-\lambda) \simeq 11(\theta-\lambda). \tag{19}$$

Our result (19) will accommodate the experimental limit (6) with $|\lambda-\theta|\simeq 0.8$. If the rate turns out to be close to its present upper limit, this would require an uncomfortably large difference $\lambda-\theta$ for parameters that vanish in the symmetry limit and ought to be small. Our result (19) should correspond to that of Weisz *et al.*[4] if we set $\lambda=0$ to conform with their mixing scheme. It does not, except for the special case when $d=2$. This disparity arises, we believe, because some of their approximations are invalid. In particular, their Eq. (32) is not correct, as shown by an examination of the tree graphs to the effective Lagrangian which we now present.

We write

$$\mathcal{L} = \mathcal{L}_0 - u_0 - cu_8, \tag{20}$$

with $\mathcal{L}_0$ invariant under dilation and under the transformations of the chiral SU(3)⊗SU(3) group while $u_0 + cu_8$ breaks these symmetries. We shall use an octet matrix

$$\Phi = \frac{1}{F_\pi}\sum_{a=1}^{8}\lambda_a\phi_a, \tag{21}$$

where λ_a are eight of the nine (3×3)-dimensional matrices that provide the defining representation of U(3), normalized according to

$$\mathrm{tr}\lambda_a\lambda_b = 2\delta_{ab}, \quad a,b=0,\ldots,8. \tag{22}$$

We can write $\mathcal{L}_0$ in terms of this octet matrix and an additional singlet field ϕ_0:

$$\mathcal{L}_0 = \tfrac{1}{2}e^{2b\chi}[\tfrac{1}{2}F_\pi^2\,\mathrm{tr}(\partial_\mu e^{-i\Phi})(\partial^\mu e^{i\Phi}) + (\partial_\mu\phi_0)^2 + (\partial_\mu\chi)^2] - \tfrac{1}{2}e^{4b\chi}(m_0^2\phi_0^2 + \sqrt{3}\,\epsilon d). \tag{23}$$

It is well known[12] that Lagrangians of this form are invariant under the chiral group. The presence of the scalar field χ makes $\mathcal{L}_0$ dilationally invariant as well, as has been shown by Ellis and others.[10] The symmetry-breaking parameter ϵ appears here so as to forbid the coupling of the dilaton field χ to the vacuum in the complete Lagrangian. We take the symmetry-breaking field to have the form

$$u_a = \epsilon e^{d b\chi}\,\mathrm{tr}\lambda_a\{\cos\Phi + \sin\Phi[\alpha\phi_0 + \beta e^{-4b\chi}\partial_\mu(e^{2b\chi}\partial^\mu\phi_0)]\}. \tag{24}$$

This structure belongs to the $(3,3^*)\oplus(3^*,3)$ representation of the chiral group.[12] The terms identified by the factors α and β account for the ϕ_0-ϕ_8 mixing with the β term giving a wave-function-renormalization mixing. In this term we have exploited the freedom of adding a total divergence to the Lagrangian, an addition that does not alter physical amplitudes. Thus, we take the derivatives to act on the singlet fields χ and ϕ_0 and not on the octet field Φ. This is done to ensure that the equal-time commutator $[A_a^0, \partial_\mu A_b^\mu]$ contains no derivative of the δ function. In order to explain the precise structure of the derivatives, we need to review briefly some formal aspects of dilation invariance.

The only result of dilation invariance that is really needed is the trace condition on the stress tensor, Eq. (10). The stress tensor can be defined as the response of the action to variations in the metric tensor $g_{\mu\nu}(x)$ when the Lagrangian has been extended to be invariant against general coordinate transformations,

$$T^{\mu\nu}(x) = 2\frac{\delta}{\delta g_{\mu\nu}(x)}\int(dx')\sqrt{-g}\,\mathcal{L}. \tag{25}$$

Such an extension is trivial, all that one need do is to make replacements of the sort

$$\partial^\mu\phi\partial_\mu\phi \to g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi. \tag{26}$$

We shall, however, not make this minimal substitution for the $(\partial_\mu e^{2b\chi})^2$ contribution to the kinetic energy. We shall instead replace it by $R(e^{2b\chi}g_{\mu\nu})$, where R is the fully contracted Riemann tensor with the metric tensor replaced by $e^{2b\chi}g_{\mu\nu}$. We do this because $\sqrt{-g}\,\mathcal{L}_0$ is now invariant under a space-time-dependent scale transformation

$$g_{\mu\nu}(x) \to e^{2\xi(x)}g_{\mu\nu}(x), \tag{27a}$$

$$\chi(x) \to \chi(x) - b^{-1}\xi(x), \tag{27b}$$

with the other fields remaining unchanged. The corresponding infinitesimal variation of the complete Lagrangian is

$$\delta\mathcal{L} = -\delta\xi(4-d)(u_0 + cu_8). \tag{28}$$

This result is obtained because the derivatives in Eq. (24) are placed in a manner that enables a covariant extension $\partial_\mu(e^{2b\chi}\sqrt{-g}\,g^{\mu\nu}\partial_\nu\phi_0)$ that is scale-invariant. If we now consider the variation of the action, the field variation (27b) does not contribute by virtue of the field equations, and we learn that

$$\delta\int(dx)\sqrt{-g}\,\mathfrak{L}=\int(dx)\sqrt{-g}\,T^{\mu\nu}\tfrac{1}{2}\delta g_{\mu\nu}$$
$$=\int(dx)\sqrt{-g}\,T^{\mu\nu}g_{\mu\nu}\,\delta\xi$$
$$=-(4-d)\int(dx)\sqrt{-g}\,(u_0+cu_8)\delta\xi\,. \qquad (29)$$

Since this holds for arbitrary $\delta\xi(x)$, its flat-space limit gives the trace condition (10).

It is a straightforward matter to compute the σ term in the tree approximation using the Lagrangian (20). Such a calculation verifies the result (18) that we previously quoted and, in addition, gives a value for the matrix element of the trace of the stress tensor

$$\langle\eta|-T^{\mu}{}_{\mu}|\eta'\rangle=(4-d)\frac{k^2}{\mu^2+k^2}m_{\eta'}{}^2(\lambda-\theta)\,. \qquad (30)$$

Here

$$\mu^2=2\sqrt{3}\,\epsilon b^2 d(4-d) \qquad (31)$$

is the mass squared of the particle associated with the dilaton field χ. For reasonable values of this mass, $\mu\sim 1$ GeV, this trace contribution to the σ term (11) is indeed negligible.

The effective Lagrangian can also be used to calculate the nonsingular remainder functions which appear in Eq. (3). A short calculation shows that it gives $b=c=0$ and

$$a=\frac{\sqrt{2}}{3d}\,\frac{4m_{\eta'}{}^4}{F_\pi{}^2 m_\eta{}^2}(\sqrt{2}-c)\,\frac{\mu^2}{\mu^2+k^2}(\lambda-\theta)\,. \qquad (32)$$

We shall take $d=3$ as suggested by the quark model and by the broken dilation invariance calculations[2] of the σ term in πN scattering. We shall also, somewhat arbitrarily,[13] take $\mu=1$ GeV. These parameters give a slope

$$a/\sigma_{\eta\eta'}\simeq 49\,. \qquad (33)$$

This result is independent of the factor $(\lambda-\theta)$ which cancels in the ratio. The number in Eq. (33) is close to the experimental number 59 given in Eq. (5). This agreement is probably fortuitous. It is easy to add additional terms to the effective Lagrangian which preserve the symmetries that we have imposed but which alter the value of this ratio. Nonetheless, it is interesting that the imposition of dilation invariance leads to a derivative coupling of the dilaton field with the meson fields. And this derivative coupling leads to the result (32), a result that produces a large slope.

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[11]We should remark that $\theta+\lambda$ is the significant mixing parameter in determining the $\gamma\gamma$ decay width of the η' from those of the π^0 and η. Together with the known branching ratios, this width gives an estimate for the $\eta\pi\pi$ width. In view of the considerable uncertainty in the value of $\theta+\lambda$, as we remarked in the text, such an estimate is not very reliable.

[12]See, for example, S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. 41, 531 (1969).

[13]We do not identify μ with the mass of the broad S-wave

π-π resonance that apparently lies somewhere in the region of 750 MeV. We do not make this identification because a bootstrap calculation [L. S. Brown and R. L. Goble, Phys. Rev. D 4, 723 (1971)] indicates that the mass of this S-wave resonance is determined by the pion decay constant F_π and does not vanish in the symmetry limit as does the dilaton mass μ.

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Some Evidence Against the Existence of Sehgal's Low-Mass Fermion*

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Sehgal has proposed that the anomalously low rate observed for the decay $K_L^0 \to 2\mu$ can be explained if there exists a neutral fermion λ with mass $\ll \frac{1}{2}M_K$. We discuss the relevance of a recent experiment at SLAC by Rothenberg *et al.* to Sehgal's proposal. We estimate that if Sehgal's hypothesis were correct, with a tantalum production target the SLAC group should have observed between 10 and 10^5 lepton pairs produced by λ interactions in their spark chambers, while in fact only 2 two-prong events were observed. This poses considerable difficulty for Sehgal's hypothesis. However, because of the uncertainty in our estimates, we do not believe it can be completely excluded.

Sehgal[1] has suggested that the anomalously low rate observed for the decay $K_L^0 \to \mu^+ + \mu^-$ (Ref. 2) can be explained if there exists a neutral fermion λ which has a mass that is small compared to $\frac{1}{2}M_K$ and which couples to the known particles through an interaction of the form

$$\mathfrak{L} = C(\bar{\lambda}\gamma_5\lambda)(l_5 + h_5), \tag{1}$$

where l_5 is a pseudoscalar neutral lepton current, $l_5 = \bar{\mu}\gamma_5\mu + \bar{e}\gamma_5 e$, and h_5 is a pseudoscalar neutral hadron current. We wish to point out that a recent experiment at SLAC by Rothenberg *et al.*[3] provides strong evidence against the existence of such a particle. Rothenberg *et al.* set stringent upper bounds on the cross section for the production of neutrinolike particles by electron beams and their subsequent detection in a massive spark-chamber array. We shall show that the results of this experiment put a severe restriction on the magnitude of the coupling constant C in Eq. (1).

With the interaction described by Eq. (1), the diagram shown in Fig. 1 gives an additional contribution to the amplitude for the decay $K_L^0 \to 2\mu$. With a suitable choice of the coupling constant C this contribution can be made to almost cancel the absorptive contribution of the two-photon intermediate state, and thus explain the anomalously low value observed for the decay rate. This leads Sehgal to the condition

$$CM_K^2/4\pi \gtrsim \tfrac{1}{10}\alpha,$$

where α is the fine-structure constant. Furthermore, in order that the contribution of $\lambda\bar{\lambda}$ pairs to the anomalous magnetic moment of the muon be acceptably small, it is necessary that

$$CM_K^2/4\pi \lesssim \alpha.$$

Thus Sehgal concludes that

$$\tfrac{1}{10}\alpha \lesssim CM_K^2/4\pi \lesssim \alpha, \tag{2}$$

if his hypothesis is to be consistent with existing knowledge.

The experiment of Rothenberg *et al.* was similar to most neutrino experiments conducted at other accelerators except that an intense electron beam from the SLAC machine was used as a source. The detector was an array of spark chambers with aluminum plates that weighed a total of 20 tons. These were situated at 0° with respect to a target in the electron beam and were shielded by approximately 11 000 g/cm^2 of rock, which was sufficient to stop all charged particles. Details of the experiment are given in Ref. 3. In the course of the experiment ~100 neutrino events were observed with several different production configurations.