Tests for Neutral Currents in Neutrino Reactions

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Neutral currents predicted by weak-interaction models of the type discussed by Weinberg may be detected in neutrino reactions. Limits on the ratio R of $\sigma(\nu+N\to\nu+X)$ to $\sigma(\nu+N\to\mu^-+X)$ are obtained independent of any dynamical assumption. For the total cross section for high-energy neutrinos, we find $R\geq 0.18$, provided the Weinberg mixing angle satisfies $\sin^2\theta_W\leq 0.33$. For the production of a single π^0 we find $R'\geq 0.50$ contrasted with the experimental result $R'\leq 0.14$ using only the assumption of (3,3)-resonance dominance. Applications are also given to antineutrino reactions.

GENERAL RESULTS

Weak-interaction models of the type recently discussed by Weinberg[1,2] involve neutral lepton currents which may be coupled to hadrons. There is much interest in testing for such currents in neutrino reactions. In most of the tests that have been discussed[2,3] special dynamical models or approximations have been made. In this note we present tests that do not depend on any such model assumptions. Such tests have also recently been discussed by Pais and Treiman.[4] Our results are improvements on theirs arising from new estimates of the axial-vector contributions leading to (a) a 25% increase of the bound for the total cross sections under zero dynamical assumptions and (b) a derivation of the bound stated in Eq. (23) from a single assumption: the scaling of electroproduction and neutrino-induced production data in the deep-inelastic region.

The effective Lagrangian for strangeness-conserving semileptonic processes involving neutrinos in models of the Weinberg type is given by

$$\mathcal{L}=\frac{G}{\sqrt{2}}[\bar{\mu}\gamma_\alpha(1+\gamma_5)\nu_\mu(J_1^\alpha+iJ_2^\alpha)+\text{H.c.}+\bar{\nu}\gamma_\alpha(1+\gamma_5)\nu(A_3^\alpha+xV_3^\alpha+yJ_s^\alpha)]\,, \quad (1)$$

where $J_i=V_i+A_i$ is one of the isospin components of the usual $V-A$ currents and J_s is an isoscalar current. In the simple Weinberg[5] form

$$x=1-2\sin^2\theta_W \quad\text{and}\quad y=-2\sin^2\theta_W \quad (2)$$

with $J_s=(1/\sqrt{3})V_8$, where θ_W is the mixing angle of the Weinberg model.

We consider in such models the cross-section ratio

$$R=\frac{\frac{1}{2}[\sigma(\nu+p\to\nu+X_1)+\sigma(\nu+n\to\nu+X_2)]}{\frac{1}{2}[\sigma(\nu+p\to\mu^-+X_3)+\sigma(\nu+n\to\mu^-+X_4)]}\equiv\frac{\sigma_0}{\sigma_-}\,. \quad (3)$$

The final states X_i may be chosen as all possible final states in which case we are considering total cross-section ratios. Alternatively, we may consider some limited kinematic range of the standard variables Q^2 and ν. Our considerations also hold if the X_i's are limited to a particular class of final states (such as those involving a single pion) provided *we sum over all possible charge states of the final particles* for each of the reactions. For the denominator in R we write

$$\sigma_-=A+I+V\,, \quad (4)$$

where V comes from the vector current alone, A from the axial-vector current alone, and I is the interference term. It then follows by means of an isotopic-spin rotation, if we set $m_\mu=0$, that

$$\sigma_0=\tfrac{1}{2}(A+xI+x^2V+y^2S)\,, \quad (5)$$

where S is the contribution of the isoscalar current. The averaging over proton and neutron targets, or equivalently the use of an isospin-zero target, is essential to eliminate the interference term between isoscalar and isovector currents. Since $y^2S\geq 0$,

$$R\geq\frac{1}{2}\frac{A+xI+x^2V}{A+I+V}\,. \quad (6a)$$

Furthermore, Schwarz's inequality implies

$$4AV\geq I^2\,. \quad (6b)$$

Combining these two inequalities we obtain

$$R \geq \tfrac{1}{2}\left[1-(1-x)\left(\frac{V}{A+I+V}\right)^{1/2}\right]^2. \tag{7}$$

The term V can be deduced from a knowledge of the isovector contribution to the electroproduction cross section

$$\tfrac{1}{2}[\sigma(e+p\to e+X_1)+\sigma(e+n\to e+X_2)]\equiv\sigma_{\rm em},$$

where, as before, a sum over all possible charge states of the final particles in each channel is assumed so that there is no isoscalar-isovector interference. Not knowing the isoscalar contribution, we write

$$V \leq \frac{G^2}{\pi}\frac{Q^4}{4\pi\alpha^2}\sigma_{\rm em}\equiv V_{\rm em}\,.$$

Combining this with Eq. (7), we get

$$R \geq \tfrac{1}{2}\left[1-(1-x)\left(\frac{V_{\rm em}}{\sigma_-}\right)^{1/2}\right]^2, \tag{8a}$$

$$R \geq \tfrac{1}{2}\left[1-2\sin^2\theta_W\left(\frac{V_{\rm em}}{\sigma_-}\right)^{1/2}\right]^2. \tag{8b}$$

The right-hand side is now expressed in terms of experimental quantities,

$$\frac{V_{\rm em}}{\sigma_-}=\frac{G^2}{\pi}\frac{1}{4\pi\alpha^2}\frac{\int Q^4(d\sigma/dQ^2d\nu d\Gamma)_{\rm em}}{\int(d\sigma/dQ^2d\nu d\Gamma)_{\nu\to\mu^-}}, \tag{9}$$

where $d\Gamma$ is a hadronic phase-space factor. It should be obvious that Eq. (8) is completely model-independent and holds for any set of final hadrons and any region of phase space.

Equation (8b) may be contrasted with the limit given by Pais and Treiman[4]:

$$R \geq \tfrac{1}{2}(1-2\sin^2\theta_W)\left(1-2\sin^2\theta_W\frac{V_{\rm em}}{\sigma_-}\right). \tag{10}$$

Our Eq. (8b) gives a limit which is larger by a term

$$\sin^2\theta_W\left[1-\left(\frac{V_{\rm em}}{\sigma_-}\right)^{1/2}\right]^2$$

and is valid for values of $V_{\rm em}/\sigma_-$ less than $(1-x)^{-2}$.

In spite of its transparency, this bound may not be the best one. A bound which may be better, is derived by finding a lower limit [designated $(1/F)$] for the expression

$$\frac{A+V}{I}=\frac{\cos^2(\frac{1}{2}\theta)W_2+2\sin^2(\frac{1}{2}\theta)W_1}{[(E+E')/M]\sin^2(\frac{1}{2}\theta)W_3}$$
$$=\frac{(1-Q^2/4EE')+[(\nu^2+Q^2)/2EE'][(L)+(R)]}{(\nu^2+Q^2)^{1/2}[(E+E')/2EE'][(L)-(R)]}, \tag{11}$$

where the notation is standard in the literature.[6,7] This is minimized for $(R)=0$, $(L)=1$, giving

$$\frac{A+V}{I}\geq\frac{1}{F}=\frac{2EE'+\nu^2+\frac{1}{2}Q^2}{(E+E')(\nu^2+Q^2)^{1/2}}$$
$$=\frac{1-\mathcal{E}}{(1-2\mathcal{E})^{1/2}}, \tag{12}$$

with

$$\mathcal{E}=\frac{2EE'\cos^2(\frac{1}{2}\theta)}{(E+E')^2}.$$

The alternative bound now follows trivially:

$$R\geq\tfrac{1}{2}\left(x+(1-x)\frac{1}{1+F}-(1-x^2)\frac{V_{\rm em}}{\sigma_-}\right). \tag{13}$$

Equation (13) becomes an equality if the isoscalar contributions to $\sigma_{\rm em}$ and σ_0 are zero[8] and if, as a result of hadron dynamics, $\sigma_R=\sigma_S=0$. In applying Eq. (13) over a large range of Q^2 and ν, one must be careful to use the smallest value of F compatible with the data; in general, this involves separately integrating numerator and denominator in Eq. (11).

Recently, data involving antineutrinos have also been reported.[9] Our general results Eqs. (8) and (13) apply equally well to the ratio

$$\overline{R}=\frac{\frac{1}{2}[\sigma(\overline{\nu}+p\to\overline{\nu}+X_1)+\sigma(\overline{\nu}+n\to\overline{\nu}+X_2)]}{\frac{1}{2}[\sigma(\overline{\nu}+p\to\mu^++X_3)+\sigma(\overline{\nu}+n\to\mu^++X_4)]}\equiv\frac{\overline{\sigma}_0}{\sigma_+} \tag{14}$$

provided σ_- is replaced by σ_+. New results may be obtained by combining ν and $\overline{\nu}$ cross sections. The Eqs. (4) and (5) for σ_- and σ_0 are changed into equations for σ_+ and $\overline{\sigma}_0$, respectively, by changing the sign of I. Defining

$$D=\frac{\sigma_0-\overline{\sigma}_0}{\sigma_--\sigma_+}, \tag{15}$$

we find

$$D=\tfrac{1}{2}x$$
$$=\tfrac{1}{2}(1-2\sin^2\theta_W). \tag{16}$$

Thus limits on D may be used to bound $\sin^2\theta_W$ in the neighborhood of $\frac{1}{2}$. If we use the limit set by electron-antineutrino scattering[10] of $\sin^2\theta_W\leq0.33$, then $D\geq0.17$. Alternatively we can add neutrino and antineutrino cross sections giving

$$\overline{R}'=\frac{\sigma_0+\overline{\sigma}_0}{\sigma_-+\sigma_+}$$
$$=\frac{1}{2}\frac{A+x^2V+y^2S}{A+V}. \tag{17}$$

It then follows simply that

$$\overline{R}'\geq\tfrac{1}{2}\left(1-2(1-x^2)\frac{V_{\rm em}}{\sigma_-+\sigma_+}\right) \tag{18a}$$

or

$$\overline{R}' \geq \tfrac{1}{2}\left(1 - 8\sin^2\theta_W(1-\sin^2\theta_W)\frac{V_{em}}{\sigma_- + \sigma_+}\right). \qquad (18b)$$

Equation (18) is the best possible limit since it becomes an equality if the isoscalar contributions are all zero.

Even if the antineutrino cross sections are not well known, they may still be useful in improving the limits for neutrino events. If it is known experimentally that the ratio

$$\frac{\sigma_-}{\sigma_+} \leq B, \qquad (19)$$

then

$$\frac{I}{A+V} \leq \frac{B-1}{B+1}$$

and from (13)

$$R \geq \tfrac{1}{2}\left(x + (1-x)\,\frac{B+1}{2B} - (1-x^2)\frac{V_{em}}{\sigma_-}\right). \qquad (20)$$

Equation (20) may be applied to $\overline{R}$ if σ_- and σ_+ are interchanged in Eq. (19) and in the last term of Eq. (20).

TOTAL CROSS SECTIONS

To calculate the bounds given by Eqs. (8) all that one needs are the cross sections for electroproduction and neutrino-induced production. For the total cross section of electroproduction, we make use of the scaling property from which

$$V_{em} = \frac{G^2}{\pi}\int \frac{Q^4}{4\pi\alpha^2}\left(\frac{d\sigma}{dQ^2 d\nu}\right)_{em} dQ^2 d\nu$$
$$\leq \frac{G^2}{\pi}\frac{4}{3}\,ME\int F_2(\omega)d\omega\,, \qquad (21)$$

where $F_2(\omega) = \nu W_2$ is the standard scaling function. The above inequality holds for any value of σ_l/σ_T; in case that the ratio is zero, as seems to be borne out by the data for deep-inelastic scattering it becomes an equality. Using Eq. (21), we can substitute for V_{em}/σ_- in Eq. (8) the quantity

$$\frac{\int F_2(\omega)d\omega}{\sigma_-}\,\frac{G^2}{\pi}\,\frac{4}{3}ME = 0.36\,, \qquad (22)$$

where we have[11] used

$$\int F_2(\omega)d\omega = 0.14 \pm 0.02$$

and for the total neutrino cross section[12]

$$\sigma_- = (G^2/\pi)ME(0.52 \pm 0.13)\,.$$

Requiring $\sin^2\theta_W \leq 0.33$, we find from Eq. (10) (Pais and Treiman) $R \geq 0.14$, whereas from our Eq. (8) we obtain $R \geq 0.18$.

One can use Eq. (13) to obtain better bounds by comparing electroproduction and neutrino-induced production processes point by point. Such detailed data are not available and we discuss again the deep-inelastic data. In this case we must, however, assume in addition to the scaling of electroproduction, the scaling of the neutrino-induced production data for which the experimental evidence is still rudimentary. Integrating numerator and denominator in Eq. (11), we find[13] the limit $(1/F) = 2$. This leads to

$$R \geq \tfrac{1}{2}\left(\tfrac{2}{3} + \tfrac{1}{3}x - (1-x^2)\frac{V_{em}}{\sigma_-}\right) \geq 0.23\,. \qquad (23)$$

It is interesting to compare this Eq. (23) with the equation

$$R \geq \tfrac{1}{6}(1 + x + x^2) \geq 0.24 \qquad (24)$$

derived by Pais and Treiman [their Eq. (27)] using the additional dynamical assumption of $V = A$. If $V_{em}/\sigma_- = \frac{1}{3}$, which is approximately correct experimentally, then Eqs. (23 and (24) are identical and the additional assumption is unnecessary. The reason for the identity is that if $V/\sigma_- = \frac{1}{3}$, then our condition $(A+V)/I \geq 2$ leads to $A/(A+I+V) \geq \frac{1}{3}$. The two equations $V/\sigma_- = \frac{1}{3}$ and $A/\sigma_- \geq \frac{1}{3}$ are equivalent, in the sense of inequalities,[14] to $V/\sigma_- = \frac{1}{3}$ and $A = V$.

Equation (23) can also be derived from Eq. (20) using the theoretical result[6] that $\sigma_-/\sigma_+ \leq 3$ for the scaling region. Recent experimental results[9] seem to indicate that the ratio σ_-/σ_+ of the neutrino to the antineutrino cross section is approximately equal to this limiting value of 3, so that it is not possible to use Eq. (20) to improve on Eq. (23). On the other hand, if the experiments are interpreted as giving $\sigma_+/\sigma_- \geq \frac{1}{3}$, then Eq. (23) follows without the scaling assumption. The discussion above provides some insight to this experimental value of σ_-/σ_+. *If it is true that $V/\sigma_- \approx \frac{1}{3}$, then in any model for which $V = A$, it follows that $V = A = I$, and the ratio of 3 to 1 for σ_-/σ_+ follows.*

If we use the approximate experimental results $V_{em}/\sigma_- = \frac{1}{3}$ and $\sigma_-/\sigma_+ = 3$, then in addition to Eq. (24) we obtain for antineutrino experiments

$$\overline{R} \geq \tfrac{1}{2}(1 - x + x^2) \geq 0.39\,, \qquad (25)$$

and combining neutrino and antineutrino experiments,

$$\overline{R}' \geq \tfrac{1}{4}(1 + x^2) \geq 0.28\,. \qquad (26)$$

ISOBAR PRODUCTION

The general results Eqs. (8) and (13) hold when we limit ourselves to the nucleon isobar Δ as the final state. In this case

$$R=\frac{\frac{1}{2}[\sigma(\nu+p\to\nu+\Delta^+)+\sigma(\nu+n\to\nu+\Delta^0)]}{\frac{1}{2}[\sigma(\nu+p\to\mu^-+\Delta^{++})+\sigma(\nu+n\to\mu^-+\Delta^+)]}\equiv\frac{\sigma_0}{\sigma_-}\,. \tag{27}$$

Experimental data have been given by Lee[15] on a related ratio,

$$R'=\frac{\frac{1}{2}[\sigma(\nu+p\to\nu+p+\pi^0)+\sigma(\nu+n\to\nu+n+\pi^0)]}{\sigma(\nu+n\to\mu^-+p+\pi^0)}\equiv\frac{\sigma_0'}{2\sigma_-'}\,. \tag{28}$$

If we assume that these events are all Δ events, then it follows from the isovector character of the weak currents (the isoscalar does not contribute) that

$$\sigma_0'=\tfrac{2}{3}\sigma_0,\quad \sigma_-'=\tfrac{1}{6}\sigma_-\,,$$
$$R'=2R\,,$$

and from Eq. (8)

$$R'\geq\left[1-(1-x)\left(\frac{V_{em}}{\sigma_-}\right)^{1/2}\right]^2, \tag{29a}$$

$$R'\geq\left[1-2\sin^2\theta_W\left(\frac{V_{em}}{\sigma_-}\right)^{1/2}\right]^2. \tag{29b}$$

In order to determine V_{em} we use the narrow-width approximation and the data of Galster *et al.*[16] As is indicated by their analysis, the nonresonance background i_ _t most 25%. For the corresponding neutrino cross section we use the data of Budagov *et al.*[17] Most of the errors come from the neutrino data, for which new results will soon become available. The value obtained for V_{em}/σ_- is 0.20 ± 0.05, which is consistent with the value of 0.19 used by Lee.[3] The corresponding bound[18] for R' is

$$R\geq0.50\,.$$

This is to be compared with the model-dependent limit of 0.62 from the analysis of Lee. The only assumption we have made is that all the events observed are to be associated with the Δ. The experimental limit[15] given for R' is $R'\leq0.14$. Our result strengthens the conclusions of Lee that this experimental result disagrees with the Weinberg model unless there is a great admixture of $I=\frac{1}{2}$ final states in σ_-'. In the case of Δ production, since there is no isoscalar contribution, it is not necessary to sum over protons and neutrons. Thus we may consider

$$R''=\frac{\sigma(\nu+p\to\nu+\Delta^+)}{\sigma(\nu+p\to\mu^-+\Delta^{++})}\,. \tag{30}$$

We find

$$R''\geq\tfrac{1}{3}\left[1-(1-x)\left(\frac{V_{em}}{\sigma_-}\right)^{1/2}\right]^2=0.17\,. \tag{31}$$

In case both the neutrino and antineutrino cross sections are known, we can obtain an equality relating R'' to experimentally measurable quantities by using Eqs. (11), (19), and (20).

CONCLUSIONS

In this paper we have established lower limits on the ratio R ($\bar{R}$) of the cross sections for ν + nucleon $\to\nu+X$ ($\bar\nu$ + nucleon $\to\bar\nu+X$) to that of ν + nucleon $\to\mu^-+X$ ($\bar\nu$ + nucleon $\to\mu^++X$) as a function of the parameter $x=1-2\sin^2\theta_W$ in Hamiltonians of the form of Eq. (1). The major results are the following:

1. A general result applicable to any ν ($\bar\nu$) process wherein the target can be considered as having isospin-zero is given by Eq. (8). It is important to remember in the definition of R that all charge states must be summed over in the final product X. Combining neutrino and antineutrino cross sections, we obtain the general results, Eqs. (16), (18), and (20).
2. For the same processes a better lower limit may be given by Eqs. (13) and (12), in which the result depends on the kinematic variables Q^2 and ν or E' and θ.
3. For the total cross section of neutrinos, the model-independent result gives $R\geq0.18$. Assuming scaling for the high-energy neutrino cross section, we obtain Eq. (23), which raises this limit to 0.23.
4. Experiments suggest that the vector contribution V to the total ν cross section is approximately $\frac{1}{3}$ and that the ratio of ν to $\bar\nu$ cross section is approximately $\frac{1}{3}$. These two relations together imply that the axial-vector contribution $A=V$, as predicted by most parton models. In this case, the inequalities take the simple form of Eqs. (24)–(26).
5. For the experiment of Lee involving single π^0 production, we find a ratio $R'\geq0.50$ to be compared with an experimental result $R'\geq0.14$. The only assumption is that the final state is pure $I=\frac{3}{2}$.
6. A general result for Δ production using proton targets alone is given by Eq. (31).

All numerical limits above are subject to considerable uncertainty because of the inaccuracies in the experimental numbers inserted into the equations including the limit $\sin^2\theta_W\leq0.33$. Thus they must be considered illustrative rather than definitive.

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[5]In Eq. (1) the Cabibbo angle was set equal to zero. In the model of S. L. Glashow, J. Iliopoulos, and L. Maiani [Phys. Rev. D 2, 1285 (1970)] the isoscalar current has both a vector and an axial-vector part. All our results also hold for such models.

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[8]Small improvements to Eq. (13) can be made providing lower bounds for the isoscalar contribution. For instance, Schwarz's inequality again implies

$$s \geq \frac{|I_{vs}|^2}{4v} \geq \frac{|I_{vs}|^2}{4(v+s)} = \lambda\,(\sigma^p_{\text{em}} + \sigma^n_{\text{em}}),$$

where I_{vs} is the isovector-isoscalar interference contribution to σ_{em}, v is the isovector contribution, and λ can be expressed in terms of proton and neutron cross sections,

$$\lambda = \frac{|\sigma^p_{\text{em}} - \sigma^n_{\text{em}}|^2}{4(\sigma^p_{\text{em}} + \sigma^n_{\text{em}})^2}\,.$$

The bound given in Eq. (13) can be improved by the addition of the term

$$\tfrac{1}{2}\lambda(1-x^2)\frac{V_{\text{em}}}{\sigma_-}\,.$$

For deep-inelastic scattering or total cross sections at high energies, the averaging over protons and neutrons may not be essential, because if we assume that the current scatters incoherently from (Z) protons and $(A-Z)$ neutrons within a stable nucleus, then

$$R^N = \frac{Z\sigma(\nu + p \to \nu + X_1) + (A-Z)\sigma(\nu + n \to \nu + X_2)}{Z\sigma(\nu + p \to \mu^- + X_3) + (A-Z)\sigma(\nu + n \to \mu^- + X_4)} \geq \frac{Z}{A-Z}R\,.$$

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[13]This result can be seen directly from Eq. (3.13) of Ref. 6 where σ_{tot} is proportional to $[\frac{1}{2} + \frac{1}{6}(\langle L\rangle + \langle R\rangle) + \frac{1}{3}(\langle L\rangle - \langle R\rangle)]$. Clearly, $(V+A) = \frac{1}{2} + \frac{1}{6}(\langle L\rangle + \langle R\rangle)$ and $I = \frac{1}{3}(\langle L\rangle - \langle R\rangle)$. The lower limit of $(V+A)/I$ is obtained by setting $\langle R\rangle = 0$, $\langle L\rangle = 1$.

[14]For the simple Weinberg model the additional dynamical assumption of $V=A$ leads to an equality obtained in Ref. 4, Eq. (28).

[15]W. Lee, Phys. Letters 40B, 423 (1972).

[16]S. Galster *et al.*, Phys. Rev. D 5, 519 (1972).

[17]I. Budagov *et al.*, Phys. Letters 29B, 524 (1969).

[18]This bound is not the most conservative one. Ambiguities arise from (a) the extrapolation of the form factor of the (3,3) resonance $G^*_M(Q^2)$ for $Q^2 \gtrsim 2.5\ (\text{GeV}/c)^2$, and (b) from the admixture of an $I=\frac{1}{2}$ component. If we assume that σ_{em} for π production contains a 25% incoherent background of $I=\frac{1}{2}$ and that σ_- also contains 25% background of $I=\frac{1}{2}$, then

$$\frac{V^{(3/2)}_{\text{em}}}{\sigma^{(3/2)}_-} = \frac{4}{3}\,\frac{V^{(3/2)}_{\text{em}}}{\sigma_-} = \frac{4}{3}\,(0.20)$$

and

$$R' \geq 0.70\left[1 - 2\sin^2\theta_W\left(\frac{V^{(3/2)}_{\text{em}}}{\sigma^{(3/2)}_-}\right)^{1/2}\right]^2 = 0.30\,.$$