PHYSICAL REVIEW D VOLUME 7, NUMBER 3 1 FEBRUARY 1973

Comments and Addenda

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How Massive Is the *W* Particle?

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(Received 8 June 1972)

It is suggested that the mass of the hypothetical unit-spin charged particle mediating the weak interactions is ~53 BeV.

Slowly, attention has been drawn to the idea[1] that the weak and electromagnetic interactions are, not merely analogous phenomena, but aspects of a unified dynamical mechanism. Some time ago, I applied this concept to estimate the mass of the hypothetical charged carriers of the weak interaction. That work was reported at a Columbia University colloquium in 1967, but remained unpublished. Recently, I became aware that other authors[2] had subsequently used similar reasoning, but had arrived at a different estimate. This note is devoted to an outline of my quite elementary considerations, and also isolates the hypotheses responsible for the discordant answers.

The leptons will be described in the manner introduced in the 1957 paper cited in Ref. 1. The conserved leptonic charge $L=\pm 1$ distinguishes particles of equal electric charge, $\mu^+(L=+1)$, $e^+(L=-1)$, which is the basis for asserting that different neutrinos accompany μ and e: $\mu^+\bar{\nu}$ $(L=-1)$, $e^+\nu(L=+1)$. For a given leptonic charge, there is an electric-charge triplet; $L=+1: \mu^+, \nu, e^-$. The matrices acting in the electric-charge space are the 3×3 antisymmetrical isotopic-spin matrices t_a, $a=1,2,3$, which are completed by the six symmetrical products $\{t_a, t_b\}$; the electric-charge matrix is identified as t_3. The chiral charge-bearing currents observed in the leptonic processes can be presented as $2j^\mu_{12}$, $2j^\mu_{21}$, in which

$$\begin{aligned} j^\mu &= \tfrac{1}{2}\psi\gamma^0\gamma^\mu T\psi\,, \\ T &= \frac{1+i\gamma_5 t_3}{2}\,2^{1/2}t\,\frac{1+i\gamma_5 t_3}{2} \\ &= 2^{-3/2}(t+i\gamma_5\{t,t_3\})\,, \end{aligned} \tag{1}$$

and

$$t_{12}=t_1+it_2,\quad t_{21}=t_1-it_2\,. \tag{2}$$

Thus, the phenomenological coupling responsible for the decay $\mu^+\to e^+ + \nu_L + \nu_R$ is

$$2^{-1/2}G\,2j^\mu_{12}2j_{\mu 21},\quad Gm_{\rm prot}{}^2=1.0\times 10^{-5}\,. \tag{3}$$

The particular normalization adopted for T_{12}, T_{21} is such that U_2-group commutation relations are obeyed,

$$[T_{12}, T_{21}]=T_{11}-T_{22}=2T_{11}-(T_{11}+T_{22})\,, \tag{4}$$

where

$$T_{11}=t_3 \tag{5}$$

is the electric-charge matrix, and

$$T_{11}+T_{22}=i\gamma_5+\tfrac{3}{2}t_3(1-i\gamma_5 t_3) \tag{6}$$

commutes with the other T matrices. (It reduces to $i\gamma_5$ when multiplying T_{12}, T_{21}.) This is analogous to the U_2 subgroup of strong interactions, where $T_{11}=Q$ and $T_{11}+T_{22}=Y$.

The electromagnetic field A^μ, which couples to electric charge, is now extended to the set of fields A^μ_{ab}, $A^\mu_{11}=A^\mu$, through the hypothesis of U_2-invariant coupling[3]:

$$e\sum_{ab}A^\mu_{ab}j_{\mu ba},\quad e^2/4\pi=\alpha\,. \tag{7}$$

That symmetry is broken by the introduction of a mass, m_W, for the charged particles represented by A^μ_{12}, A^μ_{21}. The resulting quasilocal coupling of the associated currents is then

$$\frac{1}{2}\,\frac{e^2}{m_W{}^2}\sum_{1,2}j^\mu_{ab}j_{\mu ba}=\frac{e^2}{m_W{}^2}j^\mu_{12}j_{\mu 21}\,, \tag{8}$$

and comparison with (3) gives

$$\frac{e^2}{m_W{}^2}=2^{3/2}G\,. \tag{9}$$

This predicts the mass

$$m_W=\left(\frac{4\pi\alpha}{2^{3/2}G}\right)^{1/2}=53\ \text{BeV}\,. \tag{10}$$

The authors cited in Ref. 2 have concentrated on the SU_2 subgroup resembling isotopic spin and, in effect, replaced (7) with

$$e\sum_a A_a^\mu j_{\mu a}\,,\quad a=1,2,3\,, \tag{11}$$

where j_3 is the isovector part of the electric current. (There is also an isoscalar term.) The implied coupling of the charge-bearing currents, produced by the exchange of a particle of mass m_W', is then

$$\frac{1}{2}\,\frac{e^2}{m_W'^2}\sum_{a=1,2} j_a^\mu j_{\mu a}=\frac{1}{2}\,\frac{e^2}{m_W'^2}\,j_{12}^\mu j_{\mu 21}\,. \tag{12}$$

This gives the relation

$$\frac{1}{2}\,\frac{e^2}{m_W'^2}=2^{3/2}G \tag{13}$$

and the mass

$$m_W'=2^{-1/2}m_W=37\ \text{BeV}\,. \tag{14}$$

Since the decision between the rival partial-symmetry groups will not be an immediate one, there may be time to improve these estimates by incorporating coupling-constant symmetry violations induced by the splitting of the mass spectrum. That will also involve m_Z, the mass of the particle represented by the field A_{22}^μ. (In the 1957 paper, the heavy charged particles were symbolized by Z, but custom now dictates otherwise.) We shall only remark that the simplest hypothesis for U_2-symmetry breaking, as expressed by the mass term

$$\begin{gathered}-\sum_{ab} A^\mu_{ab} f_b A_{\mu\, ba}\,,\\ f_1=0,\quad f_2=m_W{}^2\,,\end{gathered} \tag{15}$$

suggests that

$$m_Z=2^{1/2}m_W=75\ \text{BeV}\,, \tag{16}$$

but any value can be accepted.

The theory described by Weinberg uses two coupling constants, g and g'. The situation with $g=g'$ is that of U_2-symmetry, while $g'\gg g$ is required for the SU_2 hypothesis. The $g=g'$ version of Weinberg's theory differs in several respects from the phenomenological U_2-theory presented here. Apart from the omission of muons, presumably to avoid notational complications, there is a specific hypothesis concerning the dynamical origin of the boson and lepton masses (it already appears in the 1957 paper), which is designed to produce a renormalizable operator field theory. Yet, the examples of the compensation between various processes that result in good high-energy behavior[4] do not seem to involve this additional hypothesis, but rather depend on the non-Abelian gauge structure of the theory. From the viewpoint of a phenomenological theory, which has no reference to renormalization, it becomes an open question whether one has need for hypotheses concerning the dynamical origin of experimental masses, at least for the practical questions of the anticipatable future. Source-theory techniques should be useful in exploring this and related problems.

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[1] J. Schwinger, Ann. Phys. (N.Y.) 2, 407 (1957); S. Weinberg, Phys. Rev. Letters 27, 1688 (1971); also see Physics Today 25, (No. 4), 17 (1972).

[2] J. Schechter and Y. Ueda, Phys. Rev. D 2, 736 (1970); T. D. Lee, Phys. Rev. Letters 26, 801 (1971).

[3] Another investigation of the unification of the electromagnetic field with the fields of electrically charged particles has been reported [J. Schwinger, Rev. Mod. Phys. 36, 609 (1964)]. It is noted there that the charged unit-spin particles acquire an additional magnetic moment, as expressed by the gyromagnetic ratio $g=2$.

[4] The matrix notation used here simplifies the consideration of classes of processes. Thus, for lepton collisions that create bosons, $l+l\rightarrow A+A$, specifically, longitudinally polarized heavy bosons (examples are $\nu+\nu\rightarrow W+W$, $e+e\rightarrow W+W$, $e+\nu\rightarrow W+Z$), there are two mechanisms, symbolized by $l+l\rightarrow A+(l)+A$ and $l+l\rightarrow(A)\rightarrow A+A$, where parentheses indicate a virtual particle. The lepton exchange process has a simple high-energy limit, growing with energy, that involves the commutator of the T_{ab} matrices. The boson-exchange process, which is dominated by a magnetic-moment-like coupling, contains an analogous commutator of the boson field matrix A_{ab}. In the limit that all phenomenological particle masses in propagation functions are neglected, there is exact cancellation between the two mechanisms.